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# New findings on structure and production of ${ }^{10} \mathrm{He}$ from ${ }^{11} \mathrm{Li}$ with the $\left(d,{ }^{3} \mathrm{He}\right)$ reaction 

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#### Abstract

We present the first missing mass spectrum of the unbound nucleus ${ }^{10} \mathrm{He}$, measured at RIKEN using the ${ }^{11} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right)$ reaction at $50 \mathrm{~A} \mathrm{MeV} .{ }^{10} \mathrm{He}$ is believed to be a three-body ${ }^{8} \mathrm{He}+n+n$ resonance beyond the limit of nuclear binding. Our observation of a new decay branch, ${ }^{6} \mathrm{He}+4 n$, and of a puzzling reduction of the ${ }^{11} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right){ }^{10} \mathrm{He}$ cross section challenges this view. Moreover, our experiment shows a new trend in the evolution of the proton spectroscopic strength in Li isotopes deduced from the comparison of the $\left(d,{ }^{3} \mathrm{He}\right)$ cross sections on ${ }^{7,8,9,11} \mathrm{Li}$ with theoretical predictions. We discuss new questions about physics beyond the limits of nuclear existence raised by these findings.


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High beam intensities at modern radioactive ion beam facilities together with powerful detection systems allow light nuclei beyond the limits of nuclear binding to be studied, in particular, those with two neutrons above the last particle-stable isotopes, challenging nowadays experimental and theoretical nuclear physics. Five such isotopes are known today: ${ }^{5} \mathrm{H}[1],{ }^{10} \mathrm{He}[2,3],{ }^{13} \mathrm{Li}[4,5],{ }^{16} \mathrm{Be}[6]$, and ${ }^{26} \mathrm{O}$ [7].
${ }^{10} \mathrm{He}$, with the largest neutron-to-proton $(N / Z)$ ratio is perhaps the most important among them. Within the basic shell model (SM), it corresponds to a doubly closed-shell nucleus ( $Z=2, N=8$ ) but its magic character is expected to be lost since the $N=8$ gap quickly erodes when approaching the neutron drip line [8]. The treatment of ${ }^{10} \mathrm{He}$ in the alternative framework of three-body models is a priori natural with the relatively well-bound ${ }^{8} \mathrm{He}$ as the core interacting with two neutrons. However, the characteristics of core-neutron resonances of the ${ }^{8} \mathrm{He}+n$ subsystem, the basic ingredient of such a model, are still not well established and are controversial despite many theoretical and experimental studies [9].
${ }^{10} \mathrm{He}$ is generally observed as a broad resonance which makes the determination of its energy $E$ and width $\Gamma$ very difficult so that their actual values are still subject to controversy. Experiments in which ${ }^{10} \mathrm{He}$ has been populated by proton removal from ${ }^{11} \mathrm{Li}$ [noted as ${ }^{11} \mathrm{Li}(-1 p)$ hereafter], agree that the energy of the ground unbound state, ${ }^{10} \mathrm{He}$
g.s., with respect to the ${ }^{8} \mathrm{He}+n+n$ threshold is $E \sim 1.2-$ $1.6 \mathrm{MeV}[2,10,11]$, while the recent missing-mass spectrum from the ${ }^{8} \mathrm{He}(t, p)^{10} \mathrm{He}$ transfer experiment [12] suggests a higher value, $E \sim 2.1 \mathrm{MeV}$. Among the few theoretical calculations solving the unbound three-body problem for ${ }^{10} \mathrm{He}$, only the recent Faddeev-type calculation which includes ${ }^{8} \mathrm{He}$ core excitations [13] provides a relative agreement with ${ }^{11} \mathrm{Li}(-1 p)$ results, predicting a g.s. energy of ${ }^{10} \mathrm{He}$ at $E=0.8$ MeV and a width of $\Gamma=0.67 \mathrm{MeV}$.

In the present work, we have investigated ${ }^{10} \mathrm{He}$ by using the ${ }^{11} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right){ }^{10} \mathrm{He}$ proton pickup reaction, studied by missing mass spectroscopy. In order to go beyond spectroscopic aspects, we have measured for the first time differential cross sections, giving access to the $\left\langle{ }^{11} \mathrm{Li} \mid{ }^{10} \mathrm{He}\right\rangle$ wave-function overlap. Branching ratios of the decaying channels of ${ }^{10} \mathrm{He}$ were also measured, which provides detailed information on the wave function of decaying states. Information extracted from our measurement reveals that (i) the three-body ${ }^{8} \mathrm{He}+2 n$ decay mode in ${ }^{10} \mathrm{He}$ is weaker than the five-body ${ }^{6} \mathrm{He}+4 n$ decay mode thus giving evidence of the importance of manybody dynamics in ${ }^{10} \mathrm{He}$, and (ii) the cross section of the ${ }^{10} \mathrm{He}$ ground-state production is reduced compared to theoretical predictions in a proportion never seen before in transfer and/or knockout experiments [14-16], while the transfer to excited states of ${ }^{10} \mathrm{He}$ is enhanced. These experimental facts shed new


FIG. 1. (Color online) ${ }^{8} \mathrm{He}$ (a) and ${ }^{10} \mathrm{He}$ (b) spectra obtained from measurement of ${ }^{3} \mathrm{He}$ in coincidence with ${ }^{8} \mathrm{He}$ (solid histograms) and ${ }^{6} \mathrm{He}$ (dashed histogram). ${ }^{6} \mathrm{He}+4 n$ threshold and excitation energy efficiency (a.u.) of our setup are shown by solid and dotted lines, respectively. (b) The solid curve represents the result from the fit of the ${ }^{8} \mathrm{He}$ coincident data for ${ }^{10} \mathrm{He}$ for $K=0$ and $K=2$ (indistinguishable). R1 and R2 are respectively the g.s. region and the second peak region used to obtain branching ratio and cross sections.
light on the ${ }^{10} \mathrm{He}$ and ${ }^{11} \mathrm{Li}$ structure while raising new questions about its structure and about the structure beyond the neutron drip line in general.

An ${ }^{18} \mathrm{O}$ beam at 100 A MeV was fragmented on a 10 mm Be target to produce ${ }^{11} \mathrm{Li}$ and ${ }^{9} \mathrm{Li}$ beams at 50 A MeV on the RIPS line of the Radioactive Isotope Beam Factory (RIBF), operated by the RIKEN Nishina Center and the Center for Nuclear Study of the University of Tokyo. The mean intensity of ${ }^{11} \mathrm{Li}$ and ${ }^{9} \mathrm{Li}$ are $1.7 \times 10^{4} \mathrm{pps}$ and $1.0 \times 10^{5} \mathrm{pps}$, respectively. Taking into account dead time and running time, this is equivalent to $6.44 \times 10^{9}$ and $4.07 \times 10^{9}$ incident beam particles for ${ }^{11} \mathrm{Li}$ and ${ }^{9} \mathrm{Li}$ respectively. The data from the ${ }^{9} \mathrm{Li}$ beam provide a reference to validate the analysis procedure, which is crucial for studying unbound nuclei such as ${ }^{10} \mathrm{He}$. A deuterated polypropylene target $\left(\mathrm{CD}_{2}\right)$ of $1.9 \mathrm{mg} / \mathrm{cm}^{2}$ thickness was used to measure simultaneously the $(d, d)$ and $\left(d,{ }^{3} \mathrm{He}\right)$ reaction channels. The background coming from the carbon contained in the $\mathrm{CD}_{2}$ target was evaluated using a $1 \mathrm{mg} / \mathrm{cm}^{2}$ thick ${ }^{\text {nat }} \mathrm{C}$ target and was found to be negligible for data in coincidence with anything but ${ }^{4} \mathrm{He}$. The ${ }^{3} \mathrm{He}$ recoil particles were detected at forward angles by four MUST2 [17] telescopes placed at 18 cm from the target in a wall configuration covering the $9^{\circ}-53^{\circ}$ angular region. A single telescope located around $90^{\circ}$, 15 cm from the target, was additionally installed to detect the elastically scattered deuterons. To achieve an $E-\Delta E$ identification of the recoil particles, the four forward telescopes were equipped with $20 \mu \mathrm{~m}$ thick single sided silicon detectors (SSSDs) of $5 \times 5 \mathrm{~cm}^{2}$ active area, placed 65 mm from the MUST2 telescopes. Four parallel plate avalanche counters (PPACs) were placed upstream of the target, improving the angular resolution of the recoil particles to $0.4^{\circ}$. The beam-like ejectiles were detected by a two-stage plastic detector to enable particle identification.

This setup allows the missing mass spectra to be extracted from scattering angles and kinetic energies of the light particles with a typical resolution of 1 MeV (FWHM) for the studied reactions. As an illustration, the ${ }^{8} \mathrm{He}$ spectrum from ${ }^{9} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right)$ is shown in Fig. 1(a). It shows a peak at $-0.2(2) \mathrm{MeV}$ with

TABLE I. Energies $E$ and widths $\Gamma$ of states populated by $\left(d,{ }^{3} \mathrm{He}\right)$ reaction, all in MeV , gated by ${ }^{8} \mathrm{He}$ residues.

| Nucleus | $E$ | $\Gamma(E)$ | $K$ | $\chi^{2} / \mathrm{NDF}$ |
| :--- | ---: | ---: | :---: | :---: |
| ${ }^{8} \mathrm{He}$ | $-0.2(2)$ |  |  | 0.5 |
| ${ }^{10} \mathrm{He}$ | $1.4(3)$ | $1.4(2)$ | 0 | 0.22 |
| ${ }^{10} \mathrm{He}$ | $6.3(7)$ | $3.2(2)$ | 0 | 0.22 |
| ${ }^{10} \mathrm{He}$ | $1.4(3)$ | $1.5(2)$ | 2 | 0.22 |
| ${ }^{10} \mathrm{He}$ | $6.3(7)$ | $3.2(2)$ | 2 | 0.22 |

a resolution of 1.3 MeV which agrees with the 1.16 MeV obtained from the Monte Carlo simulation. As expected, no ${ }^{8} \mathrm{He}$ excited states are observed in coincidence with ${ }^{8} \mathrm{He}$ as they all are unbound and decay through the ${ }^{6} \mathrm{He}+2 n$ channel.

The ${ }^{10} \mathrm{He}$ missing mass spectra for the ${ }^{11} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right)$ reaction gated on the ${ }^{6,8} \mathrm{He}$ residues are displayed in Fig. 1(b). The spectrum gated on the ${ }^{8} \mathrm{He}$ residues reveals two resonant-like structures which cannot be explained by the phase-space contribution, obtained in, e.g., Monte Carlo simulations. This spectrum was fitted by a two-resonance function. Their energy $E$ and width $\Gamma$ were extracted by fitting the spectrum by the convolution of a Gaussian function with an energy-dependent width $\sigma_{\mathrm{MC}}(E)$, given by Monte Carlo simulation, and the Breit-Wigner distribution with energy-dependent $\Gamma$ of the following form from [18]:

$$
\Gamma(E)=\frac{2 \gamma_{0}}{\left(\pi M \rho_{\mathrm{ch}}^{2}\right)\left[J_{K+2}^{2}\left(\chi \rho_{\mathrm{ch}}\right)+N_{K+2}^{2}\left(\chi \rho_{\mathrm{ch}}\right)\right]}
$$

where $\chi=\sqrt{2 M E}$ and $\rho_{\text {ch }}$ is the penetrability of the decay channel, fixed at $40 \mathrm{fm}, M$ the nucleon mass, and $K$ the hypermomentum of the neutron pair. $J$ and $N$ are the regular and irregular Bessel functions, respectively. The reduced width $\gamma_{0}$ was left as a free parameter to the fit while the case of $K=0$ and $K=2$, respectively corresponding to $\nu\left[s_{1 / 2}\right]$ and $\nu\left[p_{1 / 2}-p_{3 / 2}\right]$ configurations, were tested; results are presented in Table I.

The first peak is attributed to ${ }^{10} \mathrm{He}$ (g.s.). Its energy, $1.4(3) \mathrm{MeV}$, is compatible with all previous ${ }^{11} \mathrm{Li}(-1 p)$ measurements [2,10,11], and with ${ }^{14} \mathrm{Be}(-2 p 2 n)$ [19], but disagrees with that from ${ }^{8} \mathrm{He}(t, p){ }^{10} \mathrm{He}$ reaction [12]. The deduced width of the g.s. peak for both $K=0$ and $K=2$ is similar to all previous measurements except in the case of the double-charge exchange study. [3]. Our result is in relative agreement with recent three-body calculations that include the ${ }^{8} \mathrm{He}$ core excitations and predict the g.s. of ${ }^{10} \mathrm{He}$ at $E=0.8 \mathrm{MeV}$ with $\Gamma=0.67 \mathrm{MeV}$ [13]. It does not support existence of a three-body virtual state with the $\nu\left[s_{1 / 2}^{2}\right]$ structure close to the ${ }^{8} \mathrm{He}+2 n$ threshold, predicted in [20], nor the high-energy ${ }^{10} \mathrm{He}$ ground state predicted by the no-core shell model [21]. The second peak in the ${ }^{10} \mathrm{He}$ spectrum is located at $E=6.3(7)$ MeV with a width $\Gamma=3.2(2) \mathrm{MeV}$. Its observation has not been reported in earlier works. The structure of the excitation spectrum gated on ${ }^{6} \mathrm{He}$ residues, shown in Fig. 1(b), cannot be resolved due to the lack of statistics.

For the first time the ${ }^{6} \mathrm{He}+4 n$ decay channel of ${ }^{10} \mathrm{He}$ has been observed and the decay branching ratio to this channel measured. The branching ratio to ${ }^{8} \mathrm{He}+2 n$ were found to be
$64(18) \%$ for the ground state region $\mathrm{R} 1(-1$ to 3.5 MeV$)$ and $46(8) \%$ for the second peak region R2 ( $4-10 \mathrm{MeV}$ ). This direct observation alone proves that ${ }^{10} \mathrm{He}$ cannot be described as a simple three-body system, as suggested in [22] through the failure of the rigid core three-body system description. The $4 n$ channel can be populated by $2 n$ sequential decay via known excited states ${ }^{8} \mathrm{He}\left(2_{1}^{+}\right)$and ${ }^{8} \mathrm{He}\left(1_{1}^{-}\right)$at 3.1 MeV and 4.4 MeV , respectively, suggesting sizable ${ }^{8} \mathrm{He}$ excitation in the wave function of the ${ }^{10} \mathrm{He}$ ground state. The spectroscopic factors (SFs) for $\left\langle{ }^{10} \mathrm{He} \mid{ }^{8} \mathrm{He}\right\rangle$, obtained using $2 \hbar \omega$ shell model (SM) calculations with the WBP interaction [23], ${ }^{1}$ show that ${ }^{10} \mathrm{He}\left(2_{1}^{+}\right)$should decay mostly to ${ }^{8} \mathrm{He}$ (g.s.), while ${ }^{10} \mathrm{He}\left(1_{1}^{-}\right)$and ${ }^{10} \mathrm{He}\left(0_{2}^{+}\right)$should have significant ${ }^{8} \mathrm{He}\left(2_{1}^{+}\right)+2 n$ and ${ }^{8} \mathrm{He}\left(1_{1}^{-}\right)+2 n$ decay branches. Therefore, the second peak is not attributed to only ${ }^{10} \mathrm{He}\left(2_{1}^{+}\right)$but rather a mixture of several states with different spins and parities. The same SM predicts that the SF for $\left\langle\left.{ }^{10} \mathrm{He}(\mathrm{g} . \mathrm{s})\right|^{8} .\mathrm{He}\left(2_{1}^{+}\right)\right\rangle$is four times larger than the SF for $\left\langle{ }^{10} \mathrm{He}\right.$ (g.s.) $\left.\right|^{8} \mathrm{He}($ g.s. $\left.)\right\rangle$. Then, the high energy tail of the (g.s.) resonance can undergo the $4 n$ decay, which is indeed observed around the ${ }^{6} \mathrm{He}+4 n$ threshold. Thus, the important role of the ${ }^{6} \mathrm{He}+4 n$ decay channel is experimentally evidenced for both the ground state and excited states of ${ }^{10} \mathrm{He}$.

Concerning the ${ }^{4} \mathrm{He}+6 n$ decay branch, unambiguous extraction of the corresponding spectrum failed due to a large number of ${ }^{4} \mathrm{He}$ coming from the carbon-induced reactions background unavoidable with the $\mathrm{CD}_{2}$ target.

In the present study, differential cross sections for the population of resonances in ${ }^{10} \mathrm{He}$ through the ${ }^{11} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right)$ reaction have been extracted for the first time. The angular distribution associated with the ground-state resonance was obtained by reconstructing the $\theta_{\text {c.m. }}$ event by event for all events in coincidence with ${ }^{8} \mathrm{He}$ or ${ }^{6} \mathrm{He}$ in the ground-state region R1 ( -1 to 3.5 MeV ) and separately for the first excited-state region R2 $(4-10 \mathrm{MeV})$. As we shall see, results provide further indication that many-body dynamics is at work in ${ }^{10} \mathrm{He}$ and also in ${ }^{11} \mathrm{Li}$ itself. In the following, measured cross sections $\sigma_{\text {expt }}$ are compared to theoretical predictions $\sigma_{\text {th }}$ of the standard transfer reaction model, the distortedwave Born approximation (DWBA). A large suppression is inferred in the ground-state population for both ${ }^{9} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right)$ and ${ }^{11} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right)$ reactions. Finite-range DWBA calculations were performed using the DWUCK5 code [25]. Parameters for the $d+{ }^{9,11} \mathrm{Li}$ optical potentials were adjusted to fit our measured elastic scattering cross sections. The analysis of the elastic scattering data is reported in [26]. For the exit channel, the ${ }^{3} \mathrm{He}$ optical potentials were taken from Ref. [27]. Both the deuteron and ${ }^{3} \mathrm{He}$ distorted waves were corrected for nonlocality. The $\left\langle\left. d\right|^{3} \mathrm{He}\right\rangle$ overlap was taken from the latest $a b$ initio calculations [28]. In a first step, the overlap functions $\left\langle{ }^{9} \mathrm{Li} \mid{ }^{8} \mathrm{He}\right\rangle$ and $\left\langle{ }^{11} \mathrm{Li} \mid{ }^{10} \mathrm{He}\right\rangle$ were represented by single-particle (s.p.) wave functions obtained in a standard potential model (SPM) with the Woods-Saxon potential of reduced radius $r_{0}=1.25 \mathrm{fm}$ and diffuseness $a=0.65 \mathrm{fm}$. The cross sections

[^0]

FIG. 2. (Color online) Experimental ( $d,{ }^{3} \mathrm{He}$ ) cross sections for populating (a) ${ }^{8} \mathrm{He}(\mathrm{g} . \mathrm{s}$.$) , (b) { }^{10} \mathrm{He}$ (g.s.), and (d) the second peak in ${ }^{10} \mathrm{He}$ in comparison to DWBA predictions with SPM+SM, STA(+GMF when applicable), and VMC form factors. (c) Evolution of ratio $R_{s}=\sigma_{\text {expt }} / \sigma_{\mathrm{th}}$ for different Li targets.
$\sigma_{\text {th }}$ include shell model SFs, 0.93 for ${ }^{9} \mathrm{Li}$ and 0.90 for ${ }^{11} \mathrm{Li}$, obtained in the $0 \hbar \omega$ and $2 \hbar \omega$ model spaces respectively.

Figures 2(a) and 2(b) compare DWBA cross sections with experimental data. While calculations reproduce well the shape of $\left(d,{ }^{3} \mathrm{He}\right)$ angular distributions, their absolute value is strongly overestimated. This overestimation persists even when more sophisticated models of the nuclear overlaps are used in the DWBA. For ${ }^{9} \mathrm{Li}$, the ratio $R_{s}=0.38(5)$, shown in Table II, is smaller than all reduction factors deduced from ( $\left.{ }^{3} \mathrm{He}, d\right)$ and ( $d, p$ ) reactions for other nuclei [16] but consistent with the systematics of $R_{s}$ observed in nucleon knockout reactions in [14]. Using the ab initio variational Monte Carlo (VMC) $\left\langle{ }^{9} \mathrm{Li} \mid{ }^{8} \mathrm{He}\right\rangle$ overlap ${ }^{2}$ from [29] leads to a similar reduction factor, $R_{s}=0.36(5)$ despite a smaller SF , 0.57 as compared to the SM SF of 0.93 . This happens because the VMC overlap is very similar to the SPM wave function (multiplied by the SM SF) in the most contributing asymptotic region, while having a larger radius and smaller norm. Using $\left\langle{ }^{9} \mathrm{Li} \mid{ }^{8} \mathrm{He}\right\rangle$ overlap from the source term approach (STA) [30], $R_{s}$ increases to $0.58(9)$ but this value is still significantly different from unity.
$R_{s}$ obtained from this work are compared to those from ${ }^{7,8} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right)$ [31] and ${ }^{7} \mathrm{Li}\left(e, e^{\prime} p\right){ }^{6} \mathrm{He}$ [32] in Fig. 2(c) and compiled in Table II. For VMC, which gives an excellent description of ${ }^{7} \mathrm{Li}\left(e, e^{\prime} p\right)$ data in both the shape and magnitude,

[^1]TABLE II. SFs from different theoretical models and the corresponding ratios $R_{s}=\sigma_{\text {expt }} / \sigma_{\text {th }}$ obtained for different reactions leading to ground states of ${ }^{6-10} \mathrm{He}$.

| Source | Product | SPM |  | STA |  | VMC |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | SM SF | $R_{s}$ | SF | $R_{s}$ | SF | $R_{s}$ |
| ${ }^{7} \mathrm{Li}$ | ${ }^{6} \mathrm{He}$ |  |  | 0.28 |  | $0.42^{\text {a }}$ | 1.02(4) ${ }^{\text {a }}$ |
| ${ }^{7} \mathrm{Li}$ | ${ }^{6} \mathrm{He}$ |  |  | 0.28 |  | $0.42{ }^{\text {b }}$ | 0.95(6) ${ }^{\text {b }}$ |
| ${ }^{8} \mathrm{Li}$ | ${ }^{7} \mathrm{He}$ |  |  | 0.33 |  | $0.58{ }^{\text {b }}$ | $0.62(7)^{\text {b }}$ |
| ${ }^{9} \mathrm{Li}$ | ${ }^{8} \mathrm{He}$ | 0.93 | 0.38(5) | 0.39 | 0.58(9) | 0.57 | 0.36(5) |
| ${ }^{11} \mathrm{Li}$ | ${ }^{10} \mathrm{He}$ | 0.90 | 0.13(2) | 0.16 | 0.49(9) ${ }^{\text {c }}$ |  |  |
| ${ }^{\text {a }}\left(e, e^{\prime} p\right)$ [32]. |  |  |  |  |  |  |  |
| ${ }^{\mathrm{b}}\left(d,{ }^{3} \mathrm{He}\right)$ [31]. |  |  |  |  |  |  |  |
| ${ }^{\text {c }}$ Calculated using GMF. |  |  |  |  |  |  |  |

$R_{s}$ decreases with increasing number of valence neutrons. It is unlikely that these $R_{s}$ could be increased by tuning optical potentials since typical uncertainties due to their choice do not exceed $20-30 \%$. Also, for the population of the ${ }^{6,8} \mathrm{He}$ ground states, the DWBA formalism is reasonably well adapted and the observed decrease of $R_{s}$ can hardly be explained by a failure of the reaction model. Finally, we note that no significant dependence of missing strength in singleparticle transfer cross section on nucleon separation energy asymmetry were found in recent works ([15] and references therein).

More surprising is the $R_{s}$ for ${ }^{11} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right){ }^{10} \mathrm{He}$ for which the standard DWBA gives the smallest value, $R_{s}=0.13(2)$, ever seen before. We note that this result is in line with the clear trend observed from stability to the drip line [Fig. 2(c)]. The question of the applicability of the DWBA to the $\left(d,{ }^{3} \mathrm{He}\right)$ reactions when residual nuclei are unbound three-body systems can be raised. One can note that the transfer of a proton from a neutron-rich nucleus is indeed the transfer of a well bound nucleon, which allows us to define the form factor of the reaction in a standard way within the DWBA framework. The remaining potential issue is the one of the optical potential to be used in the outgoing channel, where the residual nucleus is unbound; however, the choice of the exit channel potential weakly affects the predicted cross section. A more evolved theory should at least account for five-body dynamics. Reaction models of this kind do not exist, and replacing them by any coupled-reaction-channel calculations would require dealing with interactions between two unbound systems in intermediate channels, for which no theory is available either. Besides, we stress that in, e.g., the case of ${ }^{8} \mathrm{He}(p, d){ }^{7} \mathrm{He}$ at 15.7 A MeV [33] where the residue is also unbound, a spectroscopic factor compatible with a full occupation of the $p_{3 / 2}$ shell was deduced. We also note that for ${ }^{8} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right)^{7} \mathrm{He}$ [31] the cross-section reduction found is well in the systematic established by ${ }^{7} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right)^{6} \mathrm{He}$ and ${ }^{9} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right){ }^{8} \mathrm{He}$. This leads us to consider the origin of the small $R_{s}$ for ${ }^{10} \mathrm{He}$ as mainly due to overestimation of the nuclear overlap itself.

We have calculated the $\left\langle{ }^{11} \mathrm{Li} \mid{ }^{10} \mathrm{He}\right\rangle$ overlap function in the STA, which accounts for nucleon-nucleon correlations in a phenomenological way $[34,35]$. We assumed here that
the last two neutrons ${ }^{11} \mathrm{Li}$ and ${ }^{10} \mathrm{He}$ are $40 \%$ in the $\left[1 s_{1 / 2}\right.$ ] state and $60 \%$ in the $\left[0 p_{1 / 2}\right]^{2}$ state and center of-mass effects were neglected. With this overlap, $\sigma_{\mathrm{th}}$ is still much larger than $\sigma_{\text {expt }}\left[R_{s}=0.10(2)\right]$ suggesting that some physics is still missing. In particular the difference between the binding energies of the valence neutrons in ${ }^{11} \mathrm{Li}$ and ${ }^{10} \mathrm{He}$ may have a significant effect on the $\left\langle{ }^{11} \mathrm{Li} \mid{ }^{10} \mathrm{He}\right\rangle$ overlap. In this case the geometrical mismatch factor (GMF), deduced from the overlap between single-particle wave functions of different geometry, can be used to correct the SM+SFs, as originally suggested in [36]. It was pointed out in Ref. [37] that the GMF is small for medium- and strongly-bound nuclei but near the drip lines it can mimic the SF reduction obtained in exact calculations [38] and simulate the cusps arising in theoretical SFs due to coupling to continuum near thresholds [39]. We have estimated the GMF assuming that the valence neutron in ${ }^{11} \mathrm{Li}$ has half the $2 n$-separation energy and that the s.p. wave function of the valence nucleon in ${ }^{10} \mathrm{He}$ is described by one continuum bin that contains a resonance at the energy equal to half the experimental energy in ${ }^{10} \mathrm{He}$. Correcting the STA overlap by GMFs, we get much smaller $\sigma_{\text {th }}$ than that obtained with the SPM overlap [see Fig. 2(b)] but still larger than $\sigma_{\text {expt }}$, with $R_{s}=0.49(9)$. Despite the introduction of GMF, calculated assuming no core excitation in ${ }^{10} \mathrm{He}, R_{s}$ is still small, thus suggesting that, as evidenced by our above-mentioned branching ratios, these core excitations may play an essential role in the $\left\langle{ }^{11} \mathrm{Li} \mid{ }^{10} \mathrm{He}\right\rangle$ overlap.

In striking contrast with the ground-state case, ${ }^{11} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right){ }^{10} \mathrm{He}$ * cross sections [corresponding to the R2 region in Fig. 1(b)] are found to be enhanced. The calculated cross sections $\sigma_{\text {th }}$, taking into account results of the $2 \hbar \omega \mathrm{SM}$ calculations for the SFs for $2_{1}^{+}, 1_{1}^{-}$, and $0_{2}^{+}$in ${ }^{10} \mathrm{He}$, are much smaller than $\sigma_{\text {expt }}$, as shown in Fig. 2(d), obtained by adding the contribution of the ${ }^{8} \mathrm{He}+2 n$ and ${ }^{6} \mathrm{He}+4 n$ decay channel populating the higher resonances. The experimental cross sections should be considered as lower limits since the contribution from the ${ }^{4} \mathrm{He}+6 n$ channel is missing. The combined observation of weak population of the ${ }^{10} \mathrm{He}$ ground state and strong population of excited states in the ${ }^{11} \mathrm{Li}\left(d,{ }^{3} \mathrm{He}\right)$ reaction that we assign to the weakness or strength of the corresponding nuclear overlaps points consistently toward the important contribution of ${ }^{10} \mathrm{He}$ core excitations in the ground-state wave function of ${ }^{11} \mathrm{Li}$.

Thus, our measurement reveals for the first time the importance of core excitations in ${ }^{10} \mathrm{He}$. Our results also confirm their presence in the ground state of ${ }^{11} \mathrm{Li}$, underlining the crucial role of many-body dynamics for these light neutron-rich nuclei located at and beyond the drip line. This discovery raises a range of new questions starting with a possible distortion of experimental spectra due to the interplay between few-body and many-body dynamics and ending by the role of reaction models in possible enhancement of the structure evolution effects beyond the edge of stability. Answering these questions appears vital for correct understanding of the physics beyond the drip line, especially for unbound nuclei with loosely bound cores such as ${ }^{13} \mathrm{Li}$ and ${ }^{16} \mathrm{Be}$. Finally, our study also points out the failure of cross-section calculations making use of $a b$ initio $\langle\mathrm{Li} \mid \mathrm{He}\rangle$ overlaps for neutron-rich nuclei, which raises the question of their validity far from stability.

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[^0]:    ${ }^{1}$ Calculation performed using the NUSHELL code written by Brown and Rae [24].

[^1]:    ${ }^{2}$ Due to the large statistical errors of the VMC overlap and its wrong behavior in the asymptotic region we used the result of a fit by a wave function obtained from a Woods-Saxon potential model at $r<5 \mathrm{fm}$. A good fit is obtained for $r_{0}=1.45 \mathrm{fm}$ and $a=0.91 \mathrm{fm}$.

