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New Homotopy Perturbation Method to Solve Non-Linear Problems

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Abstract

In this article, we introduce a new homotopy perturbation method (NHPM) for solving non-linear problems, such that it can be converted a non-linear differential equations to some simple linear differential. We will solve linear differential equation by using analytic method that it is better than the variational iteration method and to find parameter α , we use projection method, which is easier and decrease computations in comparison with similar works. Also in some of the references perturbation method are depend on small parameter but in our proposed method it is not depend on small parameter, finally we will solve some example for illustrating validity and applicability of the proposed method.

Keywords: Non-linear, Differential Equations, Homotopy, Perturbation, Galerkin Method.

1. Introduction

In $[8]$, perturbation method depend on small parameter and choose unsuitable small parameter can be lead to wrong solution. Homotopy is an important part of topology $[7]$ and it can convert any non-linear problem in to a finite linear problems and it doesn't depend on small parameter (see $[2,3,5]$).

For introduce homotopy perturbation method corresponding the above mentioned references, we consider to non-linear problem with boundary condition the following form:

$$\begin{aligned} &A(u)-f(t)=0, && t\in\Omega \\ &B\left(u,\frac{\partial u}{\partial n}\right)=0, && n\in\Gamma \end{aligned} \tag{1}$$

Where A is a general differential operator, B is a boundary operator, $f(t)$ is a known

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analytic function and Γ is the boundary of the domain Ω. The operator A can be divided into two operators L and N, where L and N are linear and non-linear operators sequentially. So, we can write equation (1) as the following form:

$$L(u)+N(u)-f(t)=0. \tag{2}$$

Homotopy perturbation $H(v,p)$ may be written as follows:

$$\begin{aligned} &H:\Omega\times[0,1]\to R,\\ &H(v,p)=L(v)-L(u_0)+pL(u_0)+p[N(v)-f(t)]=0, \end{aligned} \tag{3}$$

Where $p\in[0,1], t\in\Omega, v$ is an approximation of u, u_0 is an initial approximation of u, also P is an embedding parameter.

Now we introduce a new NHPM:

$$H(v,p)=N(v)+(p-1)N(u_0)=0, \tag{4}$$

In other words we assume only non-linear operator and $p\in[0,1]$.

2. Application of (NHPM) and comparison with (HPM)

For showing ability and validity the proposed method we compare it with homotopy perturbation method to solve non-linear problem that used in [2, 5, 8],

$$\begin{cases} u''+\omega^2 u+4q^2u^2u''+4q^2uu'^2=0, t\in\Omega\\ u(0)=A, \quad u'(0)=0 \end{cases} \tag{5}$$

Where ω and q are known constants, also linear operator and non-linear operator chose as,

$$\begin{cases} L(u)=u''+\omega^2u,\\ N(u)=u^2u''+uu'^2, \end{cases} \tag{6}$$

Where $v_0(t)=a\cos\alpha\omega t$ and v_1 problem has been given as follows:

$$\begin{cases} v_1''+\omega^2v_1=(\alpha^2+2q^2A^2\alpha^2-1)\omega^2A\cos\alpha\omega t+2q^2\alpha^2\omega^2A^3\cos 3\alpha\omega t,\\ v_1(0)=0, v_1'(0)=0. \end{cases} \tag{7}$$

In [2], problem (7) was solved by using variational iteration method and for obtaining α they chose $\alpha^2+2q^2A^2\alpha^2-1=0$, and α is given by,

$$\alpha=\frac{1}{\sqrt{1+2q^2A^2}}. \tag{8}$$

But in the proposed method for finding α we use Galerkin method and to solve v_1 problem we employ analytic method because variational iteration method is an approximate method and analytic technique is given exact solution. Also in the Galerkin method we obtain parameter α exactly, in generally NHPM convert non-linear problem to some easier linear problems in compare to HPM.

In [1], by using Galerkin method for $n = 2$, is equal to (8), but in this article we use NHPM according to (4) and we have,

$$\begin{aligned}&(1+2q^2A^2+2q^2A^2\cos 2\alpha\omega t)v_1'' -(4q^2A^2\alpha\omega\sin 2\alpha\omega t)v_1' \\ &+\omega^2(1-2q^2A^2\alpha^2-6q^2\alpha^2A^2\cos 2\alpha\omega t)v_1 \\ &=(\alpha^2-1+2q^2\alpha^2A^2)\omega^2 A\cos\alpha\omega t+(2q^2\alpha^2\omega^2A^3)\cos 3\alpha\omega t\end{aligned} \tag{9}$$

In the case of $n = 3$ in Galerkin method we obtain α the following form:

$$\alpha = \frac{1}{\sqrt{5-9q^2A^2-\sqrt{16+56q^2A^2+53q^4A^4}}}. \tag{10}$$

3. Discussion of parameter α

Problem (5) has a periodic solution with exact period T and $T_0 = \frac{2\pi}{\omega}$ that ω is constant and also

$$\frac{T}{T_0} = \left(\frac{2}{\pi}\right)\int_0^{\frac{\pi}{2}} \sqrt{1+4q^2A^2\cos^2\Phi}\quad d\Phi \tag{11}$$

In [8], first and second order perturbation method was used and $\frac{T}{T_0}$ was given as follows:

$$first\ order: \frac{T_1}{T_0} = 1+q^2A^2, \tag{12}$$

$$second\ order: \frac{T_2}{T_0} = 1+q^2A^2-q^4A^4, \tag{13}$$

Relations of (12-13) are valid only for $(qA)^2 \le 0.1$ (see [8]), but (10) is valid for any values $(qA)^2$. Also we have

$$\lim_{(qA)^2\to\infty} \frac{T_{exact}}{T_{(He)}} = \lim_{(qA)^2\to\infty} \frac{\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\sqrt{1+4q^2A^2\cos^2\Phi}\quad d\Phi}{\sqrt{1+2q^2A^2}} \approx 0.90,$$

and

$$\lim_{(qA)^2 \to \infty} \frac{T_{exact}}{T_{(NHPM)}} = \frac{\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sqrt{1+4q^2A^2\cos^2\Phi}\ \ d\Phi}{\sqrt{5-9q^2A^2-\sqrt{16+56q^2A^2+53q^4A^4}}} \approx 0.97,$$

Also, for any value of $(qA)^2$, relative error is,

$$0 \le \left|\frac{T_{exact} - T_{He}}{T_{He}}\right| \le \%10$$

but for proposed method according (10) is,

$$0 \le \left|\frac{T_{exact} - T_{(NHPM)}}{T_{(NHPM)}}\right| \le \%3$$

4. Conclusion

In this paper we introduce a new homotopy method with Galerkin method for obtaining α parameter such that it has a high accuracy in comparison with similar work such that HPM in $[2,3,8,1]$.

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