New Image Transforms Using Hybrid Wavelets and Directional Filter Banks: Analysis and Design

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Abstract—We propose a new family of perfect reconstruction, non-redundant, and multiresolution geometrical image transforms using the wavelet transform in conjunction with modified versions of directional filter banks (DFB). In the proposed versions of DFB, we use either horizontal or vertical directional decomposition. Taking advantage of the wavelet transform that has efficient nonlinear approximation property, we add the important feature of directionality by applying the modified and regular DFB to the subbands of a few finest wavelet levels. This way we can eliminate a major portion of the artifacts usually introduced when DFB are used. The proposed Hybrid Wavelets and DFB (HWD) transform family provides visual and PSNR improvements over the wavelet and contourlet transforms.

Keywords-geometrical image transforms; directional filter banks; wavelet transforms

I. INTRODUCTION

Wavelets have been successfully applied to many image processing tasks such as low bit-rate compression and denoising [8]. However, they lack the important feature of directionality and hence, they are not efficient in retaining textures and fine details in these applications [3][4]. There have been several efforts towards developing geometrical image transforms. Directional wavelet transforms [1], steerable pyramids [13], complex wavelets [6], curvelets [2] and contourlets [3] are a few examples where all of them are redundant.

For image coding, however, a non-redundant transform is required. Recently, some non-redundant geometrical image transforms based on the DFB have been introduced. The octave-band directional filter banks [5] represent a new family of the DFB that achieves radial decomposition as well. The CRISP-contourlet [7] is another transform, which is developed based on the contourlet structure, but nonseparable filter banks are only utilized. Non-uniform DFB [10], a modified version of the CRISP-contourlets, is the other non-redundant directional transform, which also provides multiresolution.

Neither of the above non-redundant directional schemes has been used in a practical image processing application. In [4] we introduced *Wavelet-Based Contourlet Transform* (WBCT), where we applied the DFB to all the detail subbands of wavelets in a similar way that one constructs contourlets. The main difference is that we used wavelets instead of the Laplacian pyramids employed in contourlets. Therefore, the WBCT is non-redundant and can be adapted for some efficient wavelet-based image coding methods [4].

The main disadvantage of the WBCT (and other contourlet-based transforms) is the occurrence of artifacts that are caused by setting some transform coefficients to zero for nonlinear approximation and also due to quantizing the coefficients for coding. In this paper, we introduce *Hybrid Wavelets and Directional filter banks* (HWD) as a remedy for this problem. Here again we employ wavelets as the subband multiresolution decomposition. Then we apply the DFB and *modified* versions of the DFB to *some* of the wavelet subbands. In a nonlinear approximation experiment for natural images, we will show that our proposed HWD scheme is capable of retaining textures and fine details in the results when compared to the wavelets, while the amount of introduced artifacts in smooth regions is comparable to that of wavelets. Owing to the similarity of the WBCT to HWD, one can consider the WBCT as a member of the HWD family.

The paper is organized as follows. In Section 2, we develop the modified versions of the DFB. In Section 3, we construct the proposed HWD, and we provide the experimental results in Section 4. Lastly, our main conclusions are presented in Section 5.

II. HORIZONTAL AND VERTICAL DIRECTIONAL FILTER BANKS

Directional filter banks (DFB) [11] decompose the frequency space into wedge-shaped partitions as illustrated in Fig. 1. In this example, eight directions are used, where directional subbands of 1, 2, 3, and 4 represent *horizontal* directions (directions between -45° and +45°) and the rest stand for the *vertical* directions (directions between 45° and 135°). The DFB is realized using iterated quincunx filter banks.

For the proposed HWD family, we are required to decompose the input into either horizontal directions or vertical directions or both. Hence, we propose *Vertical DFB* (VDFB) and *Horizontal DFB* (HDFB), where one can achieve either vertical or horizontal directional decompositions, respectively. Fig. 2 shows the frequency space partitioned by the VDFB and HDFB. The implementation of these schemes is

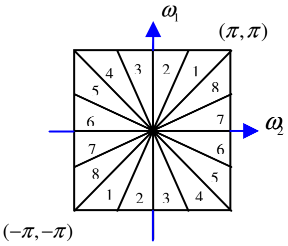
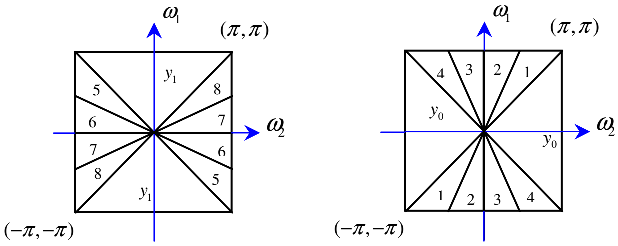


Fig. 1. Directional filter bank frequency partitioning using 8 directions.



(a) (b)

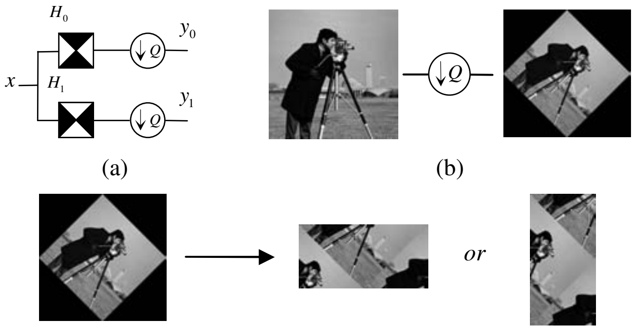
Fig. 2. (a) An example of the *vertical* directional filter banks.
(b) An example of the *horizontal* directional filter banks.

straightforward when we use the iterated tree-structured filter banks [11] to realize the DFB. At the first level of the DFB, we employ a quincunx filter bank (QFB) as depicted in Fig. 3(a). The quincunx sampling matrix that we use is

$$Q = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.$$

Fig. 3(b) shows how downsampling by Q affects the input image. The image is rotated +45° clockwise. So in the DFB, since this is not a rectangular output, we decompose the image further by using two other QFBs at the outputs y_0 and y_1. As a result, we obtain four outputs corresponding to the four directions of the DFB. At level three and higher we employ QFBs in conjunction with some resampling matrices to further decompose the DFB [11]. In the proposed VDFB (HDFB), however, we stop at y_1 (y_0) and just decompose the other channel (y_0 in VDFB and y_1 in HDFB) in a similar manner as we decompose the DFB. Therefore, since we keep y_1 or y_0, we have to find a way to represent these outputs in a rectangular form.

Assume one uses periodic filters; one can select a rectangular strip of these outputs (Fig. 3(c)). However, for better visualization and possible further processing of the coefficients in image processing applications such as coding, we need a better representation. A solution to this issue is the use of a resampling matrix. During resampling, the sampling rate of the input image does not change and the samples are merely reordered. In particular, we find resampling matrices to reorder the samples of y_1 or y_0 from a diamond shape to a shape of parallelogram. Remarkably, there exists no resampling matrix with integer elements to change those outputs to a rectangular form. We propose using the following resampling matrices:



(c)

Fig. 3. (a) Quincunx filter bank. H_0 and H_1 are fan filters and Q is the sampling matrix. Pass bands are shown by white color in the fan filters. (b) An image downsampled by Q.
(c) A horizontal or vertical strip of the downsampled image.

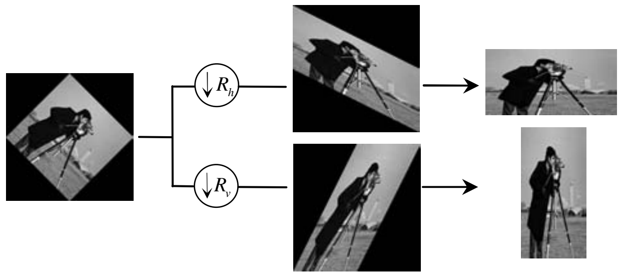


Fig. 4. Applying resampling operations R_h and R_v to an image downsampled by Q. The right side images show the resulting outputs after shifting the coefficients into a rectangular box.

$$R_h = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } R_v = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}.$$

Applying these resampling operations to the outputs of the QFB, we obtain parallelogram-shaped outputs as illustrated in Fig. 4. Now we simply shift the resulting coefficients (column-wise in the case of R_h and row-wise in the case of R_v) to obtain rectangular outputs. Thus, the resulting overall sampling matrix for representing y_1 and y_0 is $Q_h = QR_h$, or $Q_v = QR_v$, where Q_h (Q_v) in conjunction with a shifting operation results in a horizontal (vertical) rectangular output.

III. Hybrid Wavelets and Directional Filter Banks (HWD)

Here we develop the image transform family of *Hybrid Wavelets and Directional filter banks* (HWD). For HWD, similar to the WBCT, we consider the wavelet transform as the multiresolution subband decomposition. The rationale for this is as follows: 1) wavelets have already shown their good nonlinear approximation property for piece-wise smooth signals [8]; thus, we expect that by adding the feature of directionality in an appropriate manner we could improve the nonlinear approximation results yielded from wavelets, 2) there are efficient algorithms developed for image processing applications such as image coding; therefore, one could properly adapt these algorithm to HWD, 3) similar adaptive schemes such as those used for wavelet packets can be developed for this new family.

For the WBCT scheme, we apply the DFB to all wavelet detail subbands in such a way to comply the anisotropy scaling law [4]. Because the WBCT coding scheme introduces visible artifacts in the smooth regions of images, we merely used it for pure texture images. Texture areas, in fact, could *hide* the pseudo-Gibbs phenomena artifacts; consequently, we achieved better results when compared with the wavelet coder [4]. These artifacts are mainly introduced by the DFB when we set some transform coefficients to zero. Regarding the human visual system, eyes are more sensitive to low-frequency portions of an image. To reduce artifacts, therefore, we just apply the (modified) DFB to m_d, ($m_d < L$, L is the number of wavelet levels) finest scales of the wavelet subbands. We propose the following two types of the HWD family basis functions:

1. HWD type 1

   a. apply the DFB to the m_d finest *diagonal* wavelet subbands (HH_i, $(1 \le i \le m_d)$),

   b. apply the VDFB to the m_d finest *vertical* wavelet subbands (HL_i, $(1 \le i \le m_d)$),

   c. apply the HDFB to the m_d finest *horizontal* wavelet subbands (LH_i, $(1 \le i \le m_d)$).

2. HWD type 2

   a. apply the DFB to the m_d finest *diagonal* wavelet subbands (HH_i, $(1 \le i \le m_d)$),

   b. apply the VDFB to the m_d finest *horizontal* wavelet subbands (LH_i, $(1 \le i \le m_d)$),

   c. apply the HDFB to the m_d finest *vertical* wavelet subbands (HL_i, $(1 \le i \le m_d)$).

In HWD1, we further directionally decompose the vertical and horizontal coefficients already obtained through wavelet filtering. We use the proposed modified versions of the DFB to lower the complexity and to further reduce the artifacts. In HWD2, however, we decompose the horizontal subbands vertically and the vertical subbands horizontally. Indeed, there are still horizontal (vertical) coefficients with low magnitude in the vertical (horizontal) subbands. One can boost these coefficients by applying the HDFB (VDFB) to these subbands and improve the directionality of wavelets. Since we apply the HDFB or VDFB to the coefficients with low magnitude, we expect fewer artifacts during nonlinear approximation. In both HWD1 and HWD2 we use DFB with D ($d = D/2$ for VDFB and HDFB) directions at $i = 1$, then we decrement the number of directions at every other levels.

Fig. 5 shows some basis functions of the HWD family as well as the wavelet transform and the WBCT. As seen, the wavelet basis functions are point-wise while those of the HWD family are both directional and point-wise. Note that the *non-directional* basis functions of HWD2 (that happen at y_0 or y_1-the two at the lower left corner) are more similar to those of wavelets when compared with the HWD1 (the two at the upper right corner). In the WBCT all basis functions are directional. The center basis function in these schemes is an instance from coarser scales, which is the same for wavelets and also HWD type 1 and 2. In contrast, for the WBCT it appears as a *scattered* directional basis function, which is a source of artifacts in this type.

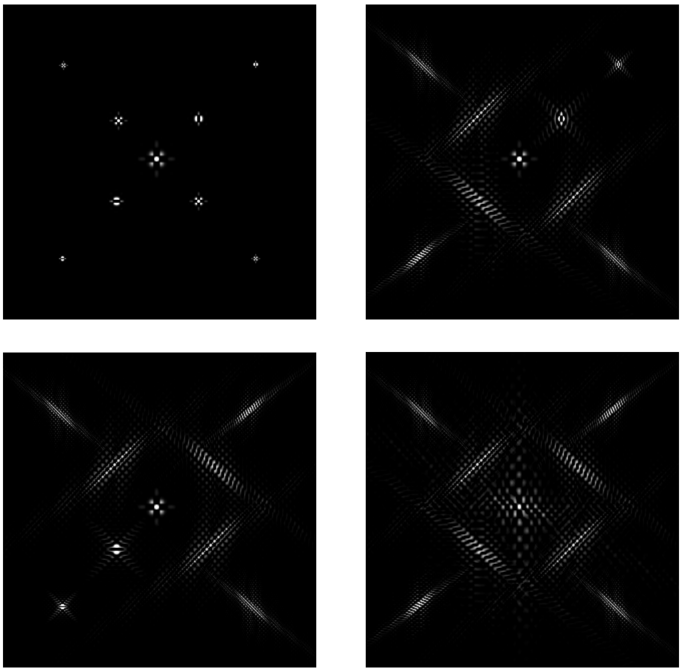


Fig. 5. Some basis functions of the wavelets and HWD family.
From *left* to *right*, *top* to *bottom*: Wavelets, HWD1, HWD2, and WBCT.

Fig. 6 illustrates an example of the HWD1 transform of the *Barbara* image. At the finest wavelet level, the upper right (lower left) subband is HL_1 (LH_1), which is decomposed into 16/2 vertical (horizontal) directions, and the lower right subband is HH_1 decomposed into 16 directions.

IV. Nonlinear Approximation

To evaluate our proposed HWD transform, we performed a nonlinear approximation experiment in which one keeps some transform coefficients with the largest magnitudes and set the rest to zero and then reconstruct the image. We also compared our scheme with wavelets, WBCT, and contourlets. For the experiment, we used $L = 6$ wavelet levels, $m_d = 2$ and $D = 16$ for both directional levels at HWD1 and 2. The WBCT and contourlet transforms are the same as those in [4]. The fan filters designed in [12] are used for the directional filters. Fig. 7 demonstrates the visual results as well as the PSNR values[1] for the *Barbara* image when 4096 coefficients are retained. As seen, the contourlet and WBCT results show a lot of artifacts in the smooth regions. That is because the DFB is applied to the low-frequency scales of these schemes as

[1] The result of the contourlet transform is also justified using the software provided at http://www.ifp.uiuc.edu/~minhdo/software/

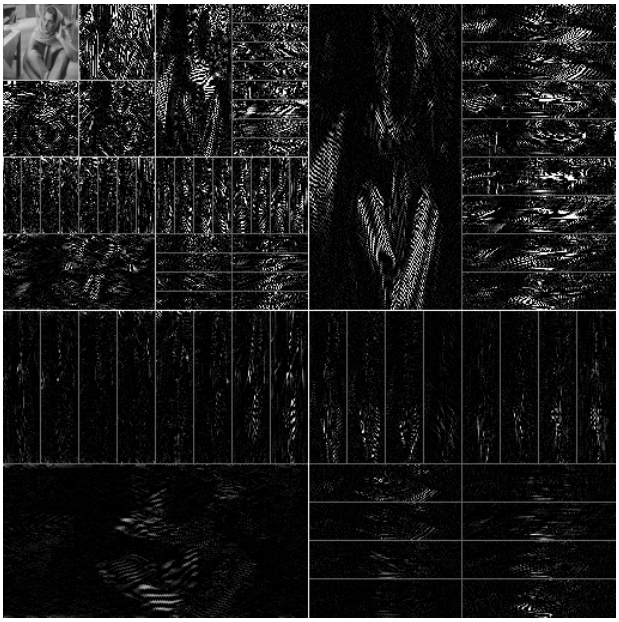


Fig. 6. HWD1 transform of the Barbara image. Here $L = 3$, $m_d = 2$ and $D = 16$ directions for both levels are used. Detail coefficients are clipped for better visualization.

well. The artifacts, however, are significantly reduced in the HWD type 1 and 2 results while they still provide better recovering of textures and fine details and better PSNR values when compared with the wavelet scheme. Comparing the HWD1 with the HWD2 results, we see that textures are better retrieved in HWD1 at the expense of introducing more artifacts. Despite the many artifacts introduced by the WBCT, we showed its potential in coding of pure texture images [4].

V. CONCLUSIONS

In this paper we proposed a new family of non-redundant directional image transforms where we applied the DFB and modified versions of the DFB to the subbands of a few finest levels of wavelets. This way we could significantly reduce the artifacts introduced in nonlinear approximation. As a result, we expect a good coding performance when we use this family. In a later work, we will explain the application of the proposed schemes in image coding.

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Wavelet – PSNR = 24.64 | Contourlet – PSNR = 25.07

HWD1 – PSNR = 25.38 | HWD2 – PSNR = 25.23

WBCT – PSNR = 25.04

Fig. 7. Nonlinear approximation results of the *Barbara* image using 4096 coefficients. Part of the image is shown.

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