

New isotropic and anisotropic sudden singularities

John D Barrow and Christos G Tsagas

DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road,
Cambridge CB3 0WA, UK

E-mail: j.d.barrow@damtp.cam.ac.uk and c.tsagas@damtp.cam.ac.uk

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Abstract

We show the existence of an infinite family of finite-time singularities in isotropically expanding universes which obey the weak, strong and dominant energy conditions. We show what new type of energy condition is needed to exclude them *ab initio*. We also determine the conditions under which finite-time future singularities can arise in a wide class of anisotropic cosmological models. New types of finite-time singularity are possible which are characterized by divergences in the time rate of change of the anisotropic-pressure tensor. We investigate the conditions for the formation of finite-time singularities in a Bianchi-type VII₀ universe with anisotropic pressures and construct specific examples of anisotropic sudden singularities in these universes.

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1. Introduction

It has recently been shown [1, 2] that the Einstein equations permit the development of finite-time singularities during the evolution of an expanding universe that obeys the strong energy condition. In isotropic and homogeneous cosmologies this singularity is characterized by a divergence in the pressure, p , and the acceleration of the expansion scale factor \ddot{a} as $t \rightarrow t_s < \infty$. The density ρ , expansion scale factor a and expansion rate \dot{a}/a all remain finite at t_s . In order to exclude this type of singularity, it is sufficient to introduce a pressure-boundedness condition, for example that $p < C\rho$ for some finite positive constant C , or to require that $dp/d\rho$ be continuous [1, 3]. Without such a condition linking the pressure to the density, a pressure singularity can occur independently of the density and leads to a violation of the dominant energy condition (expressed by $|p| \leq \rho$) as $t \rightarrow t_s$ [1, 4]. Note that the divergence of p with finite ρ leads to an unbounded quantity with dimensions of a velocity. Similar forms of singularity can be obtained in Friedmann universes arising as cosmological

solutions of higher-order gravity theories [5, 2], and other forms of finite-time singularity have been proposed in the presence of cosmological accelerations of fluid flow relative to the hypersurface-orthogonal fluid congruence in [6]. Sudden singularities are spacetime curvature singularities that are neither strong-curvature nor crushing [3], and their modest effects on geodesics have been studied in [7]. These mathematical features of simple cosmological models impinge upon investigations of the late-time behaviour of the so-called phantom matter models. The latter generally have unconventional behaviour leading to future ‘big rip’ singularities at finite time more severe than those discussed here [8]. It should be noted, however, that not all phantom cosmologies end in a big rip singularity [9].

We recall that, in the case of isotropic universes, sudden singular behaviour results from a scale factor of the form [1]

$$a(t) = \left(\frac{t}{t_s}\right)^q (a_s - 1) + 1 - \left(1 - \frac{t}{t_s}\right)^n, \quad (1)$$

with $a_s \equiv a(t_s)$. Hence, as $t \rightarrow t_s$ from below, we have

$$\ddot{a} \rightarrow q(q-1)Bt^{q-2} - \frac{n(n-1)}{t_s^2[1-(t/t_s)]^{2-n}} \rightarrow -\infty, \quad (2)$$

whenever $1 < n < 2$ and $0 < q < 1$. This solution of the Friedmann equations exists in the interval $0 < t < t_s$. More generally, a finite-time singularity will arise in a Friedmann universe when [2]

$$a(t) = \left(\frac{t}{t_s}\right)^q (a_s - 1) + 1 - (t_s - t)^n \left\{ \sum_{j=0}^{\infty} \sum_{k=0}^{N_j} a_{jk} (t_s - t)^{j/Q} (\log^k [t_s - t]) \right\}, \quad (3)$$

where a_{jk} are constants, $N_j \leq j$ are positive integers, $Q \in \mathbb{Q}^+$ and the quantity in $\{\dots\}$ brackets is a convergent double series. The latter tends to zero as $t \rightarrow t_s$.

In this paper we extend previous work on the subject by studying a new family of sudden singularities in isotropic Friedmann universes and the formation of sudden singularities in the presence of anisotropic expansion. In [10] an anisotropic and inhomogeneous cosmology of Stephani type was found to possess similar finite-time singularities, as well as spatial analogues that preserve the dominant energy condition within finite regions of space. We shall investigate whether new forms of anisotropic finite-time singularity can arise, possibly under weaker energy conditions than in the isotropic case.

We begin our discussion with a covariant formulation of the sudden singularity problem in section 2. This allows us to identify the various possibilities that may occur. In section 3 we describe a new infinite family of sudden singularities in isotropic universes which obey all the standard energy conditions. In section 4 we give an explicit example of some of the possibilities uncovered in section 2 by constructing anisotropic sudden singularities in a Bianchi-type VII₀ universe with anisotropic pressures. We summarize with our conclusions and a discussion of finite-time singularities in section 5.

2. Covariant formulation

We begin by investigating under what conditions new types of finite-time singularity can arise in an anisotropic, irrotational, spatially homogeneous, spatially-flat spacetime, containing matter with density ρ , isotropic pressure p and a trace-free anisotropic-pressure tensor π_{ab} . The evolution of this model is described by the following set of covariant equations (e.g. see [11]):

$$\dot{\rho} = -\Theta(\rho + p) - \sigma_{ab}\pi^{ab}, \tag{4}$$

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - \frac{1}{2}(\rho + 3p) - 2\sigma^2, \tag{5}$$

$$\dot{\sigma}_{ab} = -\frac{2}{3}\Theta\sigma_{ab} - \sigma_{c(a}\sigma^c_{b)} - E_{ab} + \frac{1}{2}\pi_{ab}, \tag{6}$$

$$\dot{E}_{ab} = -\Theta E_{ab} - \frac{1}{2}(\rho + p)\sigma_{ab} - \frac{1}{2}\dot{\pi}_{ab} - \frac{1}{6}\Theta\pi_{ab} + 3\sigma_{c(a}E^c_{b)} - \frac{1}{2}\sigma_{c(a}\pi^c_{b)}. \tag{7}$$

Here, Θ is the volume expansion scalar, σ_{ab} is the shear tensor, describing kinematical anisotropies and E_{ab} is the electric part of the Weyl tensor, which is associated with tidal forces. We proceed by assuming that p is independent of ρ and that π_{ab} is independent of both p and ρ . We will also leave the time evolution of π_{ab} unspecified by any constitutive relationships. These propagation equations must be supplemented by the constraint equations

$$\Theta^2 = 3(\rho + \sigma^2), \tag{8}$$

$$\dot{\sigma}_{ab} = -\Theta\sigma_{ab} + \pi_{ab}, \tag{9}$$

both imposed by the spatial flatness of the model. Note that the first of these is the generalized Friedmann equation and the latter is the residual Gauss–Codacci formula, which here provides an alternative description of the shear evolution. Also, expressions (6) and (9) combine to give

$$\pi_{ab} = \frac{2}{3}\Theta\sigma_{ab} - 2\sigma_{c(a}\sigma^c_{b)} - 2E_{ab}, \tag{10}$$

which directly relates all the sources of anisotropy.

Introducing non-zero spatial curvature leaves equations (4)–(6) the same, although generally it adds a curl H_{ab} term to the right-hand side of (7) (where H_{ab} is the magnetic Weyl tensor). In the presence of curvature, equations (8) and (9) also change into

$$\mathcal{R} = 2\left(\rho - \frac{1}{3}\Theta^2 + \sigma^2\right) \tag{11}$$

and

$$\mathcal{R}_{(ab)} = -\dot{\sigma}_{ab} - \Theta\sigma_{ab} + \pi_{ab}, \tag{12}$$

respectively. Here, $\mathcal{R}_{(ab)}$ is the symmetric and trace-free component of \mathcal{R}_{ab} , the spatial Ricci tensor, $\mathcal{R} = \mathcal{R}^a_a$ is the associated Ricci scalar and $\sigma^2 = \sigma_{ab}\sigma^{ab}/2$ is the magnitude of the shear tensor. Also, equation (12) combines with (6) to give

$$\mathcal{R}_{(ab)} = -\frac{1}{3}\Theta\sigma_{ab} + \sigma_{c(a}\sigma^c_{b)} + E_{ab} + \frac{1}{2}\pi_{ab}. \tag{13}$$

According to the system (4)–(10), we can have an *anisotropic-pressure singularity* where the quantities a , Θ , ρ , p and σ_{ab} are all finite but π_{ab} diverges as $t \rightarrow t_s < \infty$. Here the singular behaviour is manifested as a finite-time singularity in $\dot{\sigma}_{ab}$, the first time derivative of the shear tensor (see equations (6), (9) and (12))¹. Let us now look at this possibility from a different perspective.

Consider the scale factor a , defined by means of the isotropic volume expansion, according to $\dot{a}/a = \Theta/3$. Then, Raychaudhuri’s formula (see equation (5)) transforms into

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) - \frac{2}{3}\sigma^2. \tag{14}$$

Applied to a Bianchi I spacetime, the time derivative of the above gives

$$\frac{\ddot{a}}{a} = -\frac{1}{9}\Theta\rho - \frac{1}{2}\dot{p} + \frac{5}{9}\Theta\sigma_{ab}\sigma^{ab} - \frac{1}{2}\sigma_{ab}\pi^{ab}, \tag{15}$$

¹ One may also consider the vacuum case by setting $\rho = 0 = p = \pi_{ab}$. Then, the reduced system of equations (4)–(13) appears to allow for a finite-time singularity in $\dot{\sigma}_{ab}$ if $\mathcal{R}_{(ab)}$ or E_{ab} diverge. However, when Θ is bounded within the time interval in question, a series of known existence theorems due to Wald and Rendall prevents this from happening [12].

given the spatial flatness of the model. In an anisotropic cosmology with non-zero spatial curvature, however, this generalizes to

$$\frac{\ddot{a}}{a} = -\frac{1}{9}\Theta\rho - \frac{1}{2}\dot{p} + \frac{5}{9}\Theta\sigma_{ab}\sigma^{ab} - \frac{1}{2}\sigma_{ab}\pi^{ab} + \frac{2}{3}\sigma_{ab}\mathcal{R}^{(ab)}, \quad (16)$$

where $\mathcal{R}_{(ab)}$ satisfies expression (13). Following (14)–(16), we see that \ddot{a} diverges when π_{ab} is not bounded. At the same time the lower-order derivatives of the scale factor remain finite. It is worth noticing that, unless appropriate regularity conditions are imposed on the matter, a diverging \dot{p} will also lead to a singularity in \ddot{a} (see equation (16)).

When dealing with anisotropic expansion it helps to consider the scale factor defined by means of the generalized Hubble law. In covariant terms the latter reads [13]

$$\frac{\dot{\ell}}{\ell} = \frac{\dot{a}}{a} + \sigma_{ab}e^ae^b, \quad (17)$$

where e_a are unitary, linearly independent, spacelike vectors (i.e. $e_ae^a = 1$ and $e_a u^a = 0$). The shear term on the right-hand side of the above ensures that, in contrast with a , the new scale factor also contains information on the anisotropy of the expansion. The full role of the anisotropy is revealed by taking the time derivative of (17). On using (6) the latter leads to

$$\frac{\ddot{\ell}}{\ell} = \frac{\ddot{a}}{a} + (\sigma_{ab}e^ae^b)^2 - \left(\sigma_{c(a}\sigma^{c}_{b)} + E_{ab} - \frac{1}{2}\pi_{ab} \right) e^ae^b, \quad (18)$$

with \ddot{a}/a satisfying expression (14). In addition, one can use expression (17) to define the individual scale factors along the three linearly-independent directions determined by e_i (with $i = 1, 2, 3$). For example, contracting σ_{ab} twice along e_1 in (17) we obtain

$$\frac{\dot{\ell}_1}{\ell_1} = \frac{\dot{a}}{a} + \sigma_{11}, \quad (19)$$

which defines the scale factor in the e_1 -direction. Similarly, equation (18) leads to

$$\frac{\ddot{\ell}_1}{\ell_1} = \frac{\ddot{a}}{a} + \sigma_{11}^2 - \sigma_{c(1}\sigma^{c}_{1)} - E_{11} + \frac{1}{2}\pi_{11}, \quad (20)$$

with analogous expressions for ℓ_2 and ℓ_3 . Results (18) and (20) show that, when π_{ab} , E_{ab} or $\mathcal{R}_{(ab)}$ diverges, $\ddot{\ell}$ and $\ddot{\ell}_i$ will generally diverge as well, which is in distinct contrast with the behaviour of a . Recall that an unbounded anisotropic pressure leads to the singular behaviour of \ddot{a} instead of \ddot{a} (see equations (14), (15)). Note also that, unlike a diverging π_{ab} , a diverging \dot{p} manifests itself as a singularity in the third time derivative of ℓ and not in the second.

So far, our inspection of the self-consistent divergences in the sets (4)–(10), (14)–(16) and (17)–(20) has provided necessary rather than sufficient conditions for these new types of finite-time singularities to occur. Clearly, their detailed form must also comply with all of the Einstein field equations. In what follows we will construct explicit examples to show how some of the aforementioned situations may arise.

3. New isotropic examples

It is clear from equation (15) that in the isotropic case ($\sigma_{ab} = 0$) there exists an infinite family of Friedmann sudden singularities with the scale factor $a(t)$ chosen to have the form (1), or its generalization (3), which satisfy the weak energy condition ($\rho \geq 0$ and $\rho + p \geq 0$), the strong energy condition ($\rho + 3p \geq 0$ and $\rho + p \geq 0$) and the dominant energy condition ($\rho \geq 0$ and $\rho \pm p \geq 0$). If we choose n in equation (1) so that

$$N < n < N + 1,$$

where N is a positive integer, then there will be a finite-time singularity in the $(N + 1)$ st time derivative of the scale factor. In other words,

$$\frac{d^{N+1}a}{dt^{N+1}} \rightarrow \infty \quad \text{as } t \rightarrow t_s,$$

with all the lower-order derivatives remaining finite, namely

$$\frac{d^r a}{dt^{r+1}} \rightarrow L < \infty \quad \text{as } t \rightarrow t_s,$$

for all $r = 1, 2, \dots, N$. In this limit the $(N + 1)$ st time derivative of the scale factor produces an infinity in the $(N - 1)$ st time derivative of the pressure at t_s , but the density ρ and all lower derivatives of p remain finite as $t \rightarrow t_s$ since

$$\frac{1}{a} \frac{d^{N+1}a}{dt^{N+1}} \rightarrow -\frac{1}{2} \frac{d^{N-1}p}{dt^{N-1}} \rightarrow -\infty \quad \text{as } t \rightarrow t_s.$$

Specifically, for a scale factor of the form (1), we see that

$$\frac{d^{N+1}a}{dt^{N+1}} \rightarrow (-1)^N \frac{n(n-1)(n-2)\dots(n-N)}{t_s^N [1 - (t/t_s)]^{N+1-n}} + \mathcal{O}\left(\frac{t}{t_s}\right)^{q-1-N}.$$

If we choose $N = 1$, we create the sudden singularities given in [1, 2] and also by equation (2). Because they arise in \ddot{a} and p they lead to a violation of the dominant energy condition despite satisfying the weak and strong energy conditions. However, the infinite family of sudden singularities characterized by the choice $N \geq 2$ satisfies the weak, strong and dominant energy conditions and produces no divergent spacelike energy fluxes of the type discussed in [4]. Unbounded behaviour occurs in the time derivatives of the pressure and might suggest the need to define a family of new energy conditions if this behaviour is to be excluded by fiat. For example, we could exclude finite-time singularities of this type if we introduced a *generalized matter regularity condition that required*

$$\frac{d^r p}{dt^r} < C_s \frac{d^s \rho}{dt^s}, \quad \text{for some } s \leq r,$$

where C_s is a positive constant.

4. Anisotropic examples

Let us look in more detail at some of the possibilities revealed in the last section. Consider the spatially homogeneous, anisotropically expanding, Bianchi VII₀ universe with a line element given by [14–16]

$$ds^2 = -dt^2 + g_{\alpha\beta} dx^\alpha dx^\beta, \tag{21}$$

where t is the comoving proper time and

$$g_{\alpha\beta} = \begin{pmatrix} A^2[\cosh \mu + \sinh \mu \cos(kz)] & A^2 \sinh \mu \sin(kz) & 0 \\ A^2 \sinh \mu \sin(kz) & A^2[\cosh \mu - \sinh \mu \cos(kz)] & 0 \\ 0 & 0 & B^2 \end{pmatrix} \tag{22}$$

with $\alpha, \beta = 1, 2, 3$ and $z \equiv x^3$; also, $A = A(t)$, $B = B(t)$, $\mu = \mu(t)$ and k is a constant parameter. The above metric can be interpreted as the superposition of an axisymmetric Bianchi I spacetime and a circularly polarized gravitational wave propagating along the axis of symmetry (see [14, 15] and also [16]). The wavenumber and amplitude of this wave are given by k and μ respectively. Note that the above metric has anisotropic spatial curvature

as well as anisotropic expansion and reduces to the axisymmetric Bianchi I spacetime in the long-wavelength limit (i.e. as $k \rightarrow 0$). It is the most general spatially homogeneous generalization (or perturbation) of the zero-curvature Friedmann universe and has played an important role in discussions of the isotropy of the universe [17].

When matter is in the form of a fluid with anisotropic pressures, the Einstein equations associated with the metric (21), (22) lead to the evolution formulae [15, 16]:

$$\frac{\ddot{A}}{A} + \frac{\dot{A}}{A} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) = \frac{1}{2}(\rho - p_3), \quad (23)$$

$$\frac{\ddot{B}}{B} + 2\frac{\dot{A}}{A} \frac{\dot{B}}{B} = \frac{1}{2}(\rho + p_3 - p_1 - p_2) + \frac{1}{2} \left(\frac{k}{B} \right)^2 \sinh^2 \mu \quad (24)$$

and

$$\ddot{\mu} + \left[2 \left(\frac{\dot{A}}{A} \right) + \frac{\dot{B}}{B} \right] \dot{\mu} + \frac{1}{2} \left(\frac{k}{B} \right)^2 \sinh(2\mu) = p_1 - p_2. \quad (25)$$

These are supplemented by the following constraint,

$$\left(\frac{\dot{A}}{A} \right) \left[\frac{\dot{A}}{A} + 2 \left(\frac{\dot{B}}{B} \right) \right] - \frac{1}{4} \left[\dot{\mu}^2 + \left(\frac{k}{B} \right)^2 \sinh^2 \mu \right] = \rho, \quad (26)$$

which is the generalized Friedmann equation. The above reduces to the standard Friedmann equation for the flat FRW metric, where $A = B$ and $\mu = 0$. In equation (26) $A = A(t)$ and $B = B(t)$ represent the two scale factors that characterize the anisotropic expansion. Also recall that $\mu = \mu(t)$ and k are respectively the amplitude and the wavenumber of the superimposed gravitational wave of wavelength $2\pi B/k$.

Consider now the following power-law evolution equations for the two individual scale factors,

$$A(t) = A_s - 1 + \left(\frac{t}{t_s} \right)^q - \mathcal{A}(t_s - t)^n, \quad (27)$$

$$B(t) = B_s - 1 + \left(\frac{t}{t_s} \right)^m + \mathcal{B}(t_s - t)^r, \quad (28)$$

where t_s corresponds to a time in the late evolution of the model, $A_s = A(t_s)$, $B_s = B(t_s)$ and \mathcal{A}, \mathcal{B} are constants. Also, $q, m \in \mathbb{R}$ and, for reasons that will become clear next, we will assume that $2 < n$ and that $1 < r < 2$. Then, taking the first and second time derivatives of A we find

$$\dot{A} = \frac{q}{t_s} \left(\frac{t}{t_s} \right)^{q-1} + n\mathcal{A}(t_s - t)^{n-1}, \quad (29)$$

$$\ddot{A} = \frac{q(q-1)}{t_s^2} \left(\frac{t}{t_s} \right)^{q-2} - n(n-1)\mathcal{A}(t_s - t)^{n-2}, \quad (30)$$

while by successively differentiating B we obtain

$$\dot{B} = \frac{m}{t_s} \left(\frac{t}{t_s} \right)^{m-1} - r\mathcal{B}(t_s - t)^{r-1}, \quad (31)$$

$$\ddot{B} = \frac{m(m-1)}{t_s^2} \left(\frac{t}{t_s}\right)^{m-2} + r(r-1)\mathcal{B}(t_s-t)^{r-2}. \tag{32}$$

Given that $2 < n$, equations (29) and (30) guarantee that all of A , \dot{A} and \ddot{A} remain finite as t approaches t_s . On the other hand, since $1 < r < 2$, expressions (31), (32) imply that \ddot{B} diverges as $t \rightarrow t_s$, while B and \dot{B} remain finite. In addition, since $\ell = (AB^2)^{1/3}$ is the average scale factor of the anisotropic expansion, both ℓ and $\dot{\ell}$ are finite at the $t \rightarrow t_s$ limit but $\ddot{\ell} \rightarrow \infty$. All these properties ensure that at $t = t_s$ the model described by equations (27) and (28) experiences a sudden singularity at t_s of a sort discussed in section 2. Assuming the metric (21), (22), this sudden singularity is triggered by a divergent anisotropic pressure at t_s . For example, when one or both of the principal pressures p_1 and p_2 diverge as $t \rightarrow t_s$, expression (24) guarantees that \ddot{B} also diverges at t_s . Note that the singular behaviour of \ddot{B} is in agreement with the power-law evolution of (28). At the same time, equation (23) guarantees that \ddot{A} remains finite, which is compatible with definition (27). Also, according to (25), the divergence of either p_1 or p_2 immediately implies singular behaviour for $\ddot{\mu}$, unless $p_1 = p_2$. Clearly, the rest of the variables, namely ρ , p_3 , μ and $\dot{\mu}$, do not show any singular behaviour and all the terms in equation (26) are always finite at $t > 0$. Since ρ remains finite at t_s , while at least one of the principal pressures diverges there, leading also to a divergence of $\dot{\rho}$, we conclude that in our example the dominant energy condition is violated at the $t = t_s$ limit [1, 4].

Alternatively, we may assume that both p_1 and p_2 are finite and allow for p_3 to diverge as $t \rightarrow t_s$. Then, equations (23) and (24) ensure that both \ddot{A} and \ddot{B} diverge at t_s , while (25) shows that $\ddot{\mu}$ remains bounded at that point. This type of singular behaviour is also described by the scale factors (27) and (28), with the extra proviso that now n must also lie within the open interval (1, 2).

It is straightforward to generalize these examples in the same way that we did with isotropic singularities in the last section. By choosing n and r to lie in appropriate open intervals bounded by successive positive integers we can produce *mutatis mutandis* an infinite family of sudden singularities that occur in arbitrarily high derivatives of one or both of the anisotropic scale factors and in the associated time derivatives of π_{ab} .

Finally, we note that the anisotropic scale factors given in equations (27)–(28) can be generalized in the same way that the particular isotropic solution (1) was generalized using the logarithmic series to expression (3) in [2]. More specifically, we have

$$A(t) = A_s - 1 + \left(\frac{t}{t_s}\right)^q - \mathcal{A}(t_s-t)^n \left\{ \sum_{j=0}^{\infty} \sum_{k=0}^{N_j} a_{jk}(t_s-t)^{j/Q} (\log^k[t_s-t]) \right\},$$

$$B(t) = B_s - 1 + \left(\frac{t}{t_s}\right)^m + \mathcal{B}(t_s-t)^r \left\{ \sum_{j=0}^{\infty} \sum_{k=0}^{M_j} b_{jk}(t_s-t)^{j/S} (\log^k[t_s-t]) \right\},$$

where m, n, q and r are as before and a_{jk}, b_{jk} are constants, $N_j, M_j \leq j$ are positive integers and $Q, S \in \mathbb{Q}^+$.

5. Conclusions

We have shown how a new infinite family of finite-time singularities can arise in isotropically expanding universes which obey the strong, weak and dominant energy conditions. This is possible because they produce infinities in the time derivatives of the pressure of arbitrarily high

order. In particular, singularities in $d^{N+1}a/dt^{N+1}$ arise from singularities in $d^{N-1}p/dt^{N-1}$ for all positive integers, N . These can only be excluded by introducing new regularity conditions on the derivatives of the pressure.

We have also shown that finite-time singularities can arise in new ways when the expansion of the universe is anisotropic. The simple forms of sudden singular behaviour found in isotropic universes are not rendered unstable by expansion anisotropies. Moreover, we have found that new types of finite-time singularity can arise from the anisotropic part (π_{ab}) of the fluid pressure. We investigated the role of anisotropic pressure in detail. Beginning with a general discussion of the issue of anisotropic finite-time singularities, we identified a number of possible singular cases. We constructed a specific anisotropic model of Bianchi-type VII₀ to show how an unbounded anisotropic-pressure tensor can lead to finite-time singularities. The latter can occur whilst the matter density and isotropic pressure remain finite and can be avoided only by introducing appropriate restrictions on π_{ab} . For example, one might require that $\pi_{ab}\pi^{ab} < C\rho^2$ for some positive constant C . In summary, our investigations have shown that finite-time singularities are a feature of a wide class of cosmological solutions to the Einstein equations and are not simply a consequence of special symmetries. In order to exclude them from studies of the general behaviour of the Einstein equations at late times we need to introduce new restrictions on the allowed form of the energy–momentum tensor.

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