Osaka University Knowledge Archive

Title	New Measuring Method of Three Dimensional Residual Stresses Based on Theory of Inherent Strain(Welding Mechanics, Strength & Design)
Author(s)	Ueda, Yukio; Fukuda, Keiji; Tanigawa, Masayuki
Citation	Transactions of JWRI. 8(2) P.249-P.256
Issue Date	1979-12
Text Version	publisher
URL	http://hdl.handle.net/11094/4808
DOI	
rights	本文データはCiNiiから複製したものである
Note	

Osaka University Knowledge Archive : OUKA

https://ir.library.osaka-u.ac.jp/

Osaka University

New Measuring Method of Three Dimensional Residual Stresses Based on Theory of Inherent Strain†

Yukio UEDA*, Keiji FUKUDA** and Masayuki TANIGAWA***

Abstract

The authors have already proposed the general principles in measurement of residual stresses. In this paper, the authors present a new measuring method of three dimensional residual stresses based on the principle which is simplified by utilizing the characteristics of the distribution of inherent strains induced in a long welded joint. According to this theory, three dimensional inherent strains can be divided into two sets of the components; one set of components is contained in a cross section parpendicular to the weld line and the other is the longitudinal component which is parallel to the weld line. Taking advantage of this characteristics, stresses induced by the two sets of components of the inherent strains can be estimated separately and as the summation of these stresses the measurement of three dimensional welding residual stresses has become practically possible.

With the support of the rational theory, the distributions of residual stresses and longitudinal inherent strains in a multipass welded joint have been measured for the first time. And the estimated residual stresses show a good coincidence with the directly observed stresses on the surfaces of the joint.

This implies that the present theory is reliable and applicable to measure such complex internal residual stresses.

KEY WORDS: (Measuring Theory) (Residual Stresses) (Inherent Strains) (Welding Stresses) (Three Dimensional Stresses)

1. Introduction

It is very difficult to observe directly three dimensional residual stresses produced in the interior of a body. Accordingly, internal stresses must be estimated by strains observed on the surfaces of the body and/or on the sectioned surfaces after cutting the body.

In connection with this, the authors have already proposed the general principles in measurement of residual stresses and shown that there are two measuring theories for the application of the principles, (1) theory of inherent strain in which inherent strains are dealt as parameters of measurement (2) theory of sectioned-force in which sectioned-forces are dealt as its parameters. These measuring theories have been formulated with the aid of the finite element method, and generalized by a statistic approach in order to investigate reliability of estimated residual stresses.[1)]

In this paper, the authors present a new measuring method of three dimensional residual stresses which is simplified by taking advantage of the characteristics of the distribution of its inherent strains produced in a long welded joint. Applying this method, the distributions of three dimensional residual stresses in a multipass welded joint are measured. In addition to the measurement, stresses on the surfaces of the joint are directly observed and compared with the former in order to demonstrate reliability and applicability of the new measuring method.

2. Basic Formulae of Measurement of Residual Stresses Based On Theory of Inherent Strain

In a welded joint, residual inherent strains which include dislocations are generally produced at the weld and in its vicinity by thermal elastic plastic strain history due to welding. Some portion of these residual inherent strains results in free expansion-contraction of the welded joint, and does not produce residual stresses. Then, the remaining portion of the residual inherent strains causes residual stresses. These inherent strains are called effective inherent strains (sometimes called inherent strains simply).

If the distribution of effective inherent strains is represented by an equation with q parameters $\{\varepsilon^*\}$, the resulting residual elastic strains $\{\varepsilon\}$ at any point of the body produced by the inherent strains are obtained in the

† Received on September 17, 1979

* Professor

** Research Associate

*** Graduate Student, Osaka University (Presently, Researcher, Technical Research Institute, Hitachi Shipbuilding and Engineering Co., Ltd.)

Transactions of JWRI is published by Welding Research Institute of Osaka University, Suita, Osaka, Japan

following form.

$$\{\varepsilon\}=[\bar{H}^*]\{\varepsilon^*\},\ [\bar{H}^*]=(n\times q) \quad (1)$$

where n : the total number of components of elastic strains

q : the total number of parameters of inherent strains

In Eq. (1), the components of the jth row of elastic response matrix $[\bar{H}^*]$ correspond to elastic strains produced in the body when only jth parameter ε_j^* of the inherent strain distribution being unit is imposed.

If m number of elastic strains can be observed, m equations can be taken out of Eq. (1) and the observation equation is constituted as follows.

$$\{{}_m\varepsilon\}-[H^*]\{\hat{\varepsilon}^*\}=\{v\},\ [H^*]=(m\times q) \quad (2)$$

where $\{{}_m\varepsilon\}$: observed strains

$\{v\}$: residuals

In the case of rank $[H^*]=q$, the most probable values $\{\hat{\varepsilon}^*\}$ of parameters of the inherent strains are decided so as to minimize the sum of squares of the residuals.

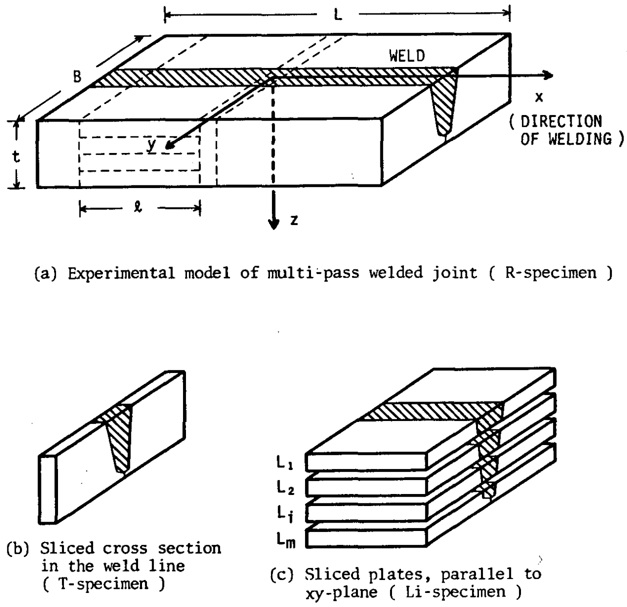
$$\{\hat{\varepsilon}^*\}=([H^*]^T[H^*])^{-1}[H^*]^T\{{}_m\varepsilon\} \quad (3)$$

Then, the elastic strain distribution in the whole object can be calculated by substituting Eq. (3) into Eq. (1).

Although stresses and strains at any point are relaxed by cutting an object, the inherent strains are invariable in the case of its change of stress state being elastic. According to the theory of inherent strain, the original stress state can be easily calculated by elastic analysis with the aid of the finite element method if these invariable inherent strains can be estimated. Then, in the case where the inherent strains are also effective after cutting the object, Eq. (1) can be constitute for the object after cutting to estimate the original effective inherent strains. This feature of the measuring theory is very important to obtain a good elastic response matrix and prevent propagation of observation errors.

3. Measurement of Three Dimensional Residual Stresses in Multipass Welded Joint

When the general measuring theory which was proposed before is applied to measure three dimensional residual stresses in the middle of the weld line of a multipass welded joint as shown in **Fig. 1**, the theory can be simplified by taking advantage of the characteristics of the distribution of inherent strains induced in the welded joint. Measurement of three dimensional residual stresses produced in an actual welded joint will be conducted by the simplified theory.



(a) Experimental model of multi-pass welded joint (R-specimen)

(b) Sliced cross section in the weld line (T-specimen)

(c) Sliced plates, parallel to xy-plane (Li-specimen)

Fig. 1 Experimental model and procedure of slicing T and Li specimens

3.1 Simplification of the measuring theory

In general, inherent strains in a welded joint are produced by the result of thermal elastic-plastic behavior due to non-steady temperature distribution and restraint of the welded joint. In such a case as this specimen, restrainting conditions and temperature distributions of the welded joint are nearly uniform and guasi-steady states except in the vicinity of both ends of the weld line. Then, when the weld line is very long, the inherent strain distribution can be considered to be uniform along the weld line except in the vicinity of both ends. And it is well known that welding residual stresses are almost symmetric with respect to a cross section at the middle. This implies that the components γ_{xy}^* and γ_{zx}^* of the inherent strains which produce non symmetric residual stress with respect to that cross section can be ignored.

Here, the following assumptions may be introduced for the measurement of the three dimensional welding residual stresses.

1) Strains in the specimen change elastically due to cutting.
2) The remaining stresses in sliced thin plates are in the state of plane stress (to be sliced thin enough).
3) Each component of inherent strains do not change along the weld line (x-axis) and the components ε_x^*, ε_y^*, ε_z^*, γ_{yz}^* are functions of y and z co-ordinates.

The assumption 3) is only for simplification of the measuring theory based on the characteristics of the inherent strain distribution in a long welded joint and not indispensable to theory of inherent strain.

3.1.1 Separation of components of three dimensional inherent strains

As the cutting lines are shown in Fig. 1, T-specimen and $L_1 \sim L_m$ specimens are taken out of the original welded joint, which is named as R-specimen. The same magnitude of inherent strains as exist in R-specimen remain in T and L_i specimens because the inherent strains do not change without production of plastic strains by these slicings. These plates are sliced so thin that the inherent strains in the normal direction to the surfaces of these plates do not contribute to the remaining stress distribution of the plates. Then, the remaining stresses in T-specimen are produced only by the inherent strains (ε_y^*, ε_z^* γ_{yz}^*) in the cross section and the remaining stresses in L_i-specimen only by the longitudinal inherent strain (ε_x^*) (ε_y^* does not contribute to these stresses in L_i-specimen because it is constant along x-axis).

As the result, three dimensional inherent strains can be divided into the inherent strains in the cross section and the inherent strains in the longitudinal direction. The three dimensional residual stresses $\{\sigma\}$ of R-specimen can be expressed by the sum of the stresses $\{\sigma^A\}$ which are produced in R-specimen only by the inherent strains in the cross section, and the stresses $\{\sigma^B\}$ which are produced in R-specimen only by the inherent strain in the longitudinal direction, that is,

$$\{\sigma\} = \{\sigma^A\} + \{\sigma^B\} \tag{4}$$

3.1.2 Stresses $\{\sigma^A\}$ produced by the inherent strains in the cross section

The residual stresses $\{\sigma^{A0}\} = \{0, \sigma_y^{A0}, \sigma_z^{A0}, \tau_{yz}^{A0}, 0, 0\}^T$ in T-specimen are in the state of plane stress and can be observed directly. Then, the inherent strains (ε_y^*, ε_z^*, γ_{yz}^*) in the cross section can be estimated by the observed stresses (or strains) and the three dimensional stresses $\{\sigma^A\}$ in R-specimen may be calculated by these resulting inherent strains. On the other hand, there is a clear relation between $\{\sigma^{A0}\}$ and $\{\sigma^A\}$, which makes determination of the stresses $\{\sigma^A\}$ simplified by the stresses $\{\sigma^{A0}\}$.

As the inherent strains (ε_y^*, ε_z^*, γ_{yz}^*) in the cross section are uniform along the weld line, it can be considered that the cross sections remain plane (so called plane deformation) in the middle portion which are away approximately by its thickness from the ends of weld line, after R-specimen is subjected to these inherent strains. Further, as the stresses in the plane of T-specimen and in the state of plane strain which are produced by these inherent strains are balanced in the cross section (yz-plane), the stresses parpendicular to its cross section (in the direction of x-axis) which are produced by constrainning the deformation in its direction are also self-balance in the cross section. As the result, the state of the above mentioned plane deformation is equivalent to that of plane strain (The detail proof is shown in Appendices 1 and 2.).

In such a case, the stresses $\{\sigma^A\}$ can be determined by the observed values $\{\sigma^{A0}\}$ in the state of plane stress, using the relation between plane strain and plane stress without knowing the inherent strains in the cross section.

$$\left.\begin{aligned} &\sigma_x^A = \nu(\sigma_y^{A0} + \sigma_z^{A0})/(1-\nu^2), \ \nu: \text{Poisson's ratio} \\ &\sigma_y^A = \sigma_y^{A0}/(1-\nu^2), \sigma_z^A = \sigma_z^{A0}/(1-\nu^2) \\ &\tau_{yz}^A = \tau_{yz}^{A0}/(1-\nu^2), \tau_{xy}^A = \tau_{zx}^A = 0 \end{aligned}\right\} \tag{5}$$

Judging from this fact, it is evident that the longitudinal elastic strain ε_x of R-specimen at a certain distance (approximately its thickness) away from the ends of weld line is produced only by the inherent strain ε_x^* in its direction because the inherent strains in the plane of a cross section make the state of plane strain and does not produce any longitudinal elastic strain but stresses.

In connection to $\{\sigma^{A0}\}$ there are many reports[2),3)] on the results of observation of residual stresses remained in the planes of thin plates which are sliced in the prependicular direction to the weld line. According to the above mentioned fact, these observations can be considered nearly to measure the stresses due to the inherent strains in the cross section.

3.1.3 Stresses produced by the inherent strain in the longitudinal direction $\{\varepsilon_x^*\}$

Residual elastic strains in L_i-specimen will be observed. If it is neccessary to increase the number of observation in order to obtain more accurate solutions of the observation equations, it becomes possible to observe more strains to be changed by cutting L_i-specimen along several lines parallel to x-axis.

If the measurements of change of strains in L_i-specimens after each cutting are conducted, the range where the inherent strains exist can be determined. Eqs. (1) and (2) of each L_i-specimen are calculated with the aid of the finite element method and the most probable values $\hat{\varepsilon}_x^*$ of inherent strains are obtained by substituting the observed values into Eq. (3). Then, the longitudinal inherent strain distribution of R-specimen can be decided by applying smooth interpolation to the estimated values of the inherent strains in each L_i-specimen.

The stresses $\{\sigma^B\}$ of R-specimen due to the longitudinal inherent strains can be calculated by three dimensional stress analysis with the aid of the finite element method. Then, the three dimensional residual stresses $\{\sigma\}$ can be estimated as the sum of these stresses $\{\sigma^B\}$ and the stresses $\{\sigma^A\}$ produced by the inherent strains in the cross section, which are already observed.

3.2 Experiment

According to the proposed measuring theory, an experiment is conducted to measure three dimensional residual stresses in a multipass welded joint and demonstrate the effectiveness of the method.

3.2.1 Specimen and procedure of measurement

The material used in this experiment is mild steel and its initial residual stresses before welding are released by stress relief annealing. R-specimen was made by multipass butt welding and the length, the width and the thickness of the specimen are L=200 mm, B=200 mm, t=50 mm, respectively. For welding, submerged arc welding was adopted (current 650 A, voltage 35 V, welding velocity 42 cm/min.) and the passes of weld metal were accumulated from the bottom of the specimen to the top. The gage length of SR-4 strain gages used in this experiment is 2 mm. They were attached in pair on both faces of the sliced plates and the mean value of each pair of the observed strains was considered the observed strain at the point.

At the first stage of the experiment, one T-specimen and four L_i-specimens were cut off from R-specimen. The thickness of these sliced plates is about 10 mm. The length of L_i-specimen, l is about 70 mm.

At the second stage, strain gages were attached on the top and bottom surfaces (z=0, z=50 mm) and on the sliced cross sections as shown in **Fig. 2**. After attaching gages, the residual stresses in T-specimen were observed by cutting T-specimen to small pieces.

At the third stage, strain gages were attached on

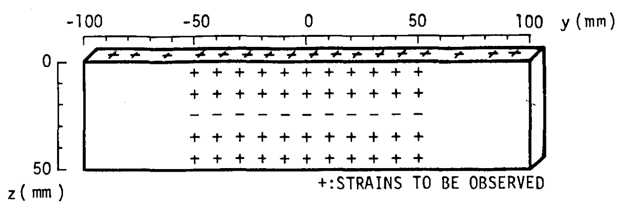


Fig. 2 Locations of observing positions (T-specimen)

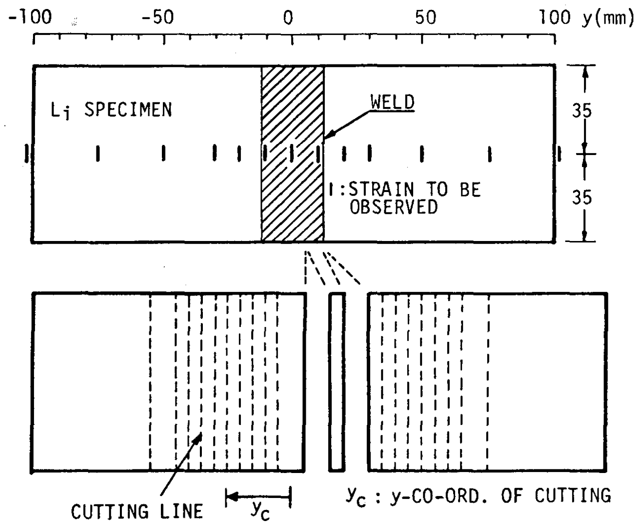


Fig. 3 Locations of observing positions and procedure of cuttings ($L_1 \sim L_4$ specimens)

$L_1 \sim L_4$-specimens which were for estimations of the longitudinal inherent strains **(Fig. 3)**. After then, every L_i-specimen was splitted along the center line of the weld parallel to x-axis and sliced into several pieces with a certain width from the center line to both sides as shown in Fig. 3. The variations of strains were observed at every step of these cuttings.

3.2.2 Estimation of the longitudinal inherent strain distribution

Typical observed variations of strains due to the above splitting and slicings of L_i-specimens are shown in **Fig. 4**. It is seen that variation of strains almost ceases when the co-ordinate of cutting line reaches $\pm$50 mm. This fact

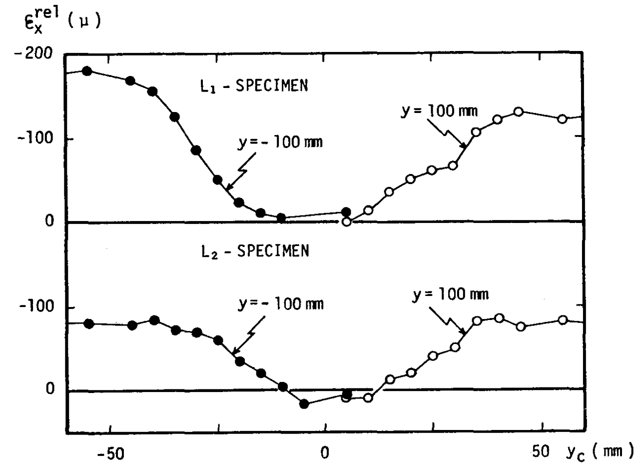


Fig. 4 Relaxed strains ϵ_x^{rel} due to splitting and slicing, parallel to x-axis (yc ; y-co-ord. of splitting and slicings)

implies that the range of the distribution of inherent strains ε_x* is about $|y| \leqq 50$ mm because any change in strains can not be observed due to cutting if there exists no inherent strain in a specimen.

The inherent strains ε_x* in L_i-specimen are estimated by using the observed values of remaining elastic strains at every observing position and variations of these elastic strains due to cuttings. If it is assumed that the inherent strains distribute continuously and linearly in a narrow width of L_i-specimens, the shape of the distribution in the entire width would be as shown in **Fig. 5** and seven unknown components of inherent strains should be determined to represent the distribution, that is, q=7. Finite elements used in these estimations are a rectangular one.

The estimated values of the longitudinal inherent strains in each specimen are shown in **Fig. 6**. These values obtained in $L_1 \sim L_4$-specimens can be considered to represent the distributions at the center of thickness of these thin plates, which correspond to ones at z=5, 18, 32, 45 mm of R-specimen, respectively.

Reversely, the remaining elastic strains in L_i-specimen and variation of elastic strains due to cuttings can be reproduced numerically with these calculated most probable values of inherent strains. Such an example on L_1-specimen is shown in **Fig. 7** The deviation of the measurement is about $\hat{s} \doteqdot 40\mu$ in any L_i-specimen and the reproduced elastic strain distribution in L_i-specimen can be considered to be reliable.

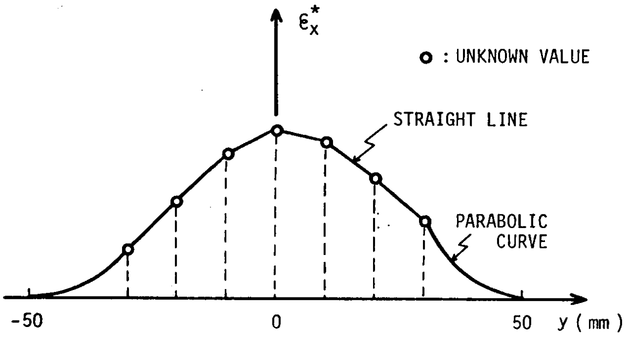


Fig. 5 Assumed distribution of longitudinal inherent strains (The symmetry of its distibution with respect to x-axis is not assumed.)

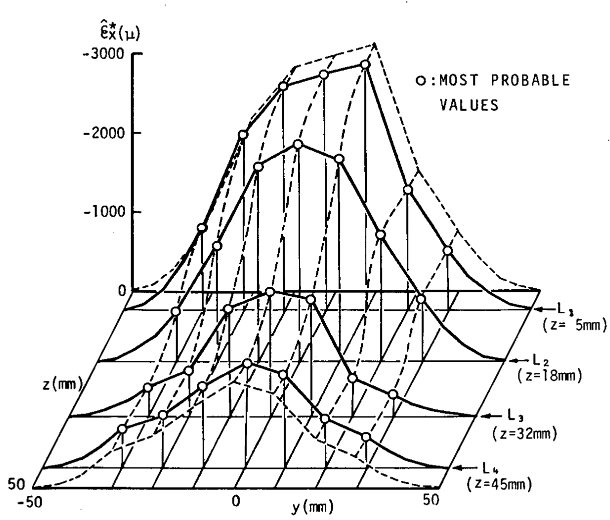


Fig. 6 Most probable values of longitudinal inherent strains

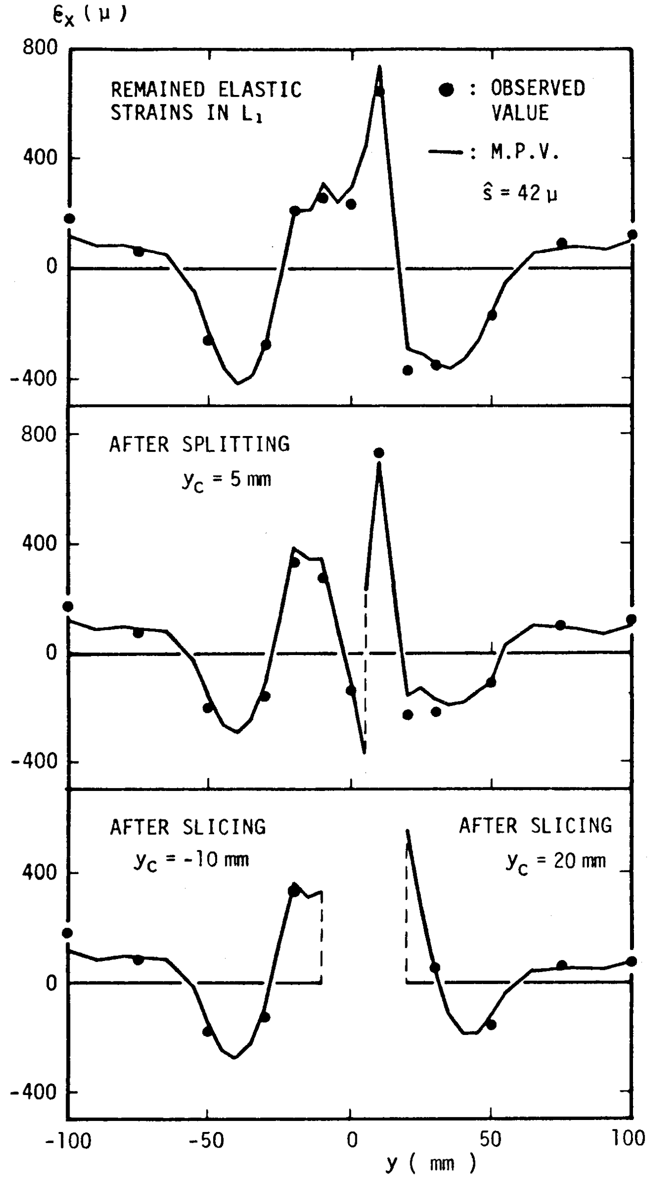


Fig. 7 Variation of elastic strains in L_1-specimen by cuttings ($\hat{s}$; Deviation of measurement, M.P.V. : Most probable value)

3.2.3 Three dimensional residual stresses produced in multipass weld joint

The inherent strain distribution ε_x^* in the direction of z-axis can be expressed by applying smooth interpolotion to the inherent strains obtained in $L_1 \sim L_4$-specimens (Fig. 6). The three dimensional stresses $\{\sigma^B\}$ in R-specimen produced by this inherent strain distribution were analyzed by the finite element method. Finite elements used in calculation are one kind of rectangular prismatic element. The mesh division of R-specimen is indicated in **Fig. 8** (Young's modulus E=21,000 kg/mm^2, Poisson's

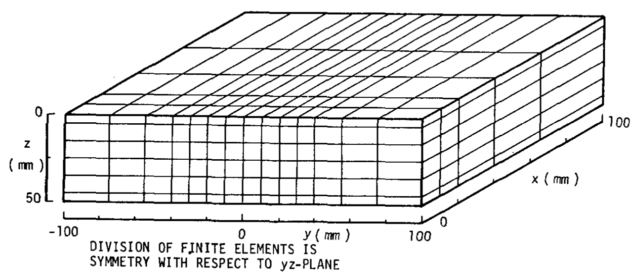


Fig. 8 Mesh division for stress analysis due to longitudinal inherent strains

ratio ν=0.3, the numbers of elements, nodal points and unknown nodal displacements are 420, 630 and 1784, respectively). C.P.U. time used in this computation was 285 seconds (ACOS series 77 NEAC system 900).

The three dimensional residual stresses $\{\sigma\}$ were obtained as the sum of these stresses $\{\sigma^B\}$ and the stresses $\{\sigma^A\}$ which were observed directly.

Estimated residual stress distributions on the top and bottom surfaces in the middle of the weld line are shown in **Fig. 9**. In the same figure, the direct measurements of the surface stresses are indicated in order to compare with the estimated values. These directly observed values are not used for estimation of the three dimensional residual stresses by the present method. Then, it can be considered that the estimated residual stresses show a good coincidence with the directly observed stresses on its surfaces. Especially, the accuracy of the estimated transverse stresses σ_y is very high. But there is some difference in some portions between the estimated and directly observed values of the longitudinal stress σ_x. The reasons for these differences can be considered that the length of the weld of this specimen is not long enough and the longitudinal inherent strain distribution in the direction of z-axis was decided by interpolation of only four L_i-specimens. Then, these differences will decrease if a weld length of the object to be measured is long enough to satisfy the assumption 3) and the number of L_i-specimens are increased.

Next, **Fig. 10** shows the estimated residual stresses in

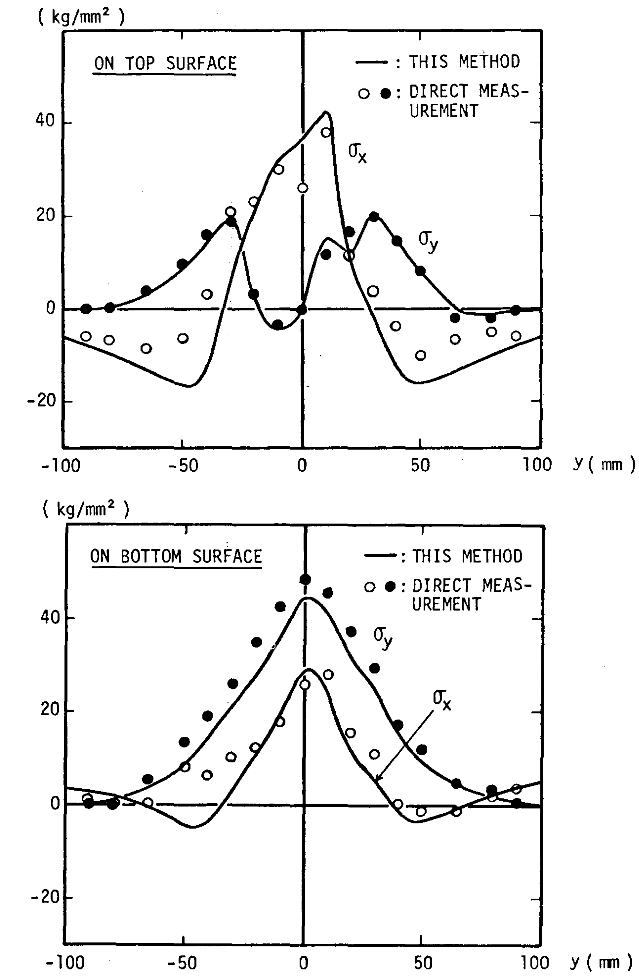


Fig. 9 Welding residual stresses on the top and bottom surfaces in the middle of weld line

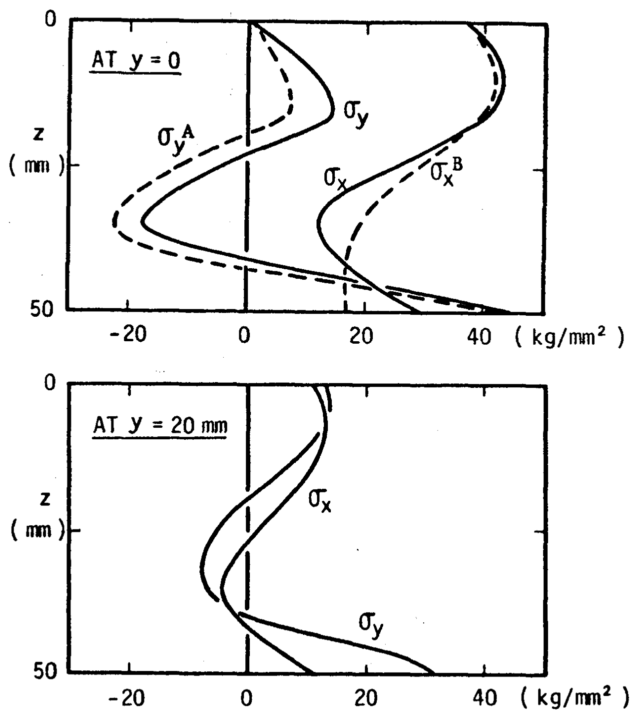


Fig. 10 Estimated welding residual stresses on the cross section (x = 0)

the object which can not be observed directly. The accuracy of the estimated internal stresses can be considered almost the same as that of the estimated surface stresses, because in the present theory there is no difference in the procedure of estimation of the surface and internal stresses. The stress distribution σ_y along z-axis at the weld shows a small tension below the finishing bead, a compression at its following part and large tension at the bottom surface, which explains the fact that root cracking in multipass welded joints occasionally occurs in the case of the bending restraint being small.[4] For a reference, the stress components $\sigma_x{}^B$ and $\sigma_y{}^A$ are shown in the same figure.

From the above results, it is considered that the proposed measuring method of three dimensional welding residual stresses based on the theory of inherent strain is reliable and applicable.

4. Conclusion

In this paper, based on the theory of inherent strain which is one of the measuring principles of three dimensional residual stresses proposed by the authors, a measuring theory of three dimensional residual stresses induced in a multipass welded joint was developed and the actual residual stresses were measured to demonstrate the practical procedure of measurement and show the validity and applicability of the theory.

A summary of the results obtained is shown below.

1) In the case where the length of a welded joint is long and inherent strain distribution can be considered uniform along the weld line, the proposed measuring method of three dimensional residual stresses can be simplified utilizing the characteristics of its distribution. According to this new theory, three dimensional inherent strains ($\varepsilon_x{}^*$, $\varepsilon_y{}^*$, $\varepsilon_z{}^*$, $\gamma_{yz}{}^*$) can be divided into the inherent strains ($\varepsilon_y{}^*$, $\varepsilon_z{}^*$, $\gamma_{yz}{}^*$) in the cross section and inherent strain ($\varepsilon_x{}^*$) in the longitudinal direction, and the measurement of three dimensional welding residual stresses has become practically possible. The stresses due to the former components can be directly measured by substituting the observed stresses into Eq. (5), which are remained in the sliced thin cross section in the middle portion of the weld line. The longitudinal inherent strain distribution and the stresses due to this component can be estimated by observed strains which are remained in the sliced thin plates parallel to the top surface of its joint.
2) By this theory, the distributions of residual stresses and longitudinal inherent strains in a multipass welded joint have measured for the first time.
3) And the estimated residual stresses show a good coincidence with the directly observed stresses on the surfaces of the specimen. This implies that the present theory is reliable and applicable to measure such complex three dimensional residual stresses.
4) A practical procedure of measurement of three dimensional residual stresses has been shown for application of the present theory. On the other hand, some other procedures of measurement than this may be developed based on the theory of inherent strain.

Acknowledgement

The authors would like to express their thanks to Messrs. I. Nishimura, Manager, and N. Chiba, Researcher of Kurimoto Iron Works, Ltd. for the welded joint which they prepared for this study. And sincere appreciation is expressed to the members of the welding research committee of the Society of Naval Architects of Japan and the welded structure research committee of the Japan Welding Society for their valuable discussions.

References

1) Y. Ueda, K. Fukuda, K. Nakacho and S. Endo; A New Measuring Method of Residual Stresses with the Aid of Finite Element Method and Reliability of Estimated Values, Trans. JWRI (Welding Research Institute of Osaka Univ., Japan), Vol. 4 (1975), No. 2, pp.123-131, J. Soc. Naval Arch. of Japan, Vol. 138 (1975), pp.525-533 (in Japanese) and THEORETICAL AND APPLIED MECHANICS, University of Tokyo Press, Vol. 25 (1977), pp.539-554.
2) Y. Ueda, E. Takahashi, K. Fukuda and K. Nakacho; Transient and Residual in Multipass Welds, Trans. JWRI, Vol. 3 (1974), No. 1, pp.59-67 and J. Japan Welding Soc., Vol. 44 (1975), No. 6, pp.466-474 (in Japanese).
3) Y. Ueda, E. Takahashi, K. Fukuda, K. Sakamoto and K. Nakacho; Multipass Welding Stresses in Very Thick Plates and Their Reduction from Stress Relief Annealing, Trans. JWRI, Vol. 5 (1976), No. 2, pp.179-189 and J. Japan Welding Soc., Vol. 47 (1978), No. 8, pp.500-506.
4) Y. Ueda, K. Fukuda, I. Nishimura, H. Iiyama and N. Chiba; Dynamical Characteristics of Weld Cracking in Multipass Welded Corner Joint, Trans. Japan Welding Soc., Vol. 8 (1977), No. 2, pp.138-142 and J. Japan Welding Soc., Vol. 48 (1979), No. 1, pp.34-39.

Appendix 1

It will be varified in the following that the stresses $\{\sigma^A\}$ in the state of plane deformation produced only by the inherent strains ($\varepsilon_y{}^*$, $\varepsilon_z{}^*$, $\gamma_{yz}{}^*$) in the cross section are equal to the stresses $\{\sigma^{A1}\}$ in the state of plane strain and are related to the stresses $\{\sigma^{A0}\}$ in the state of plane stress, as shown in Eq. (5).

When the same inherent strains ($\varepsilon_y{}^*$, $\varepsilon_z{}^*$, $\gamma_{yz}{}^*$) exist in the states of plane stress (σ_x=0) and plane strain (ε_x=0), and these stress functions of T-specimen under the same boundary condition are denoted by φ_0 and φ_1, respectively, the following equations are obtained.

$$\varphi_1 = \varphi_0 / (1-\nu^2), \qquad \nu: \text{ Poisson's ratio} \tag{A-1}$$

$$\left.\begin{aligned} \sigma_x^{A1} &= \nu(\sigma_y^{A0} + \sigma_z^{A0})/(1-\nu^2), \\ \sigma_y^{A1} &= \sigma_y^{A0}/(1-\nu^2), \\ \sigma_z^{A1} &= \sigma_z^{A0}/(1-\nu^2), \\ \tau_{yz}^{A1} &= \tau_{yz}^{A0}/(1-\nu^2) \end{aligned}\right\} \tag{A-2}$$

On the other hand, in order to maintain the state of plane strain, the constraining force P_x and the constraining moments M_y and M_z must apply to the cross section, which are the resulting force and moments of the stress σ_x^{A1} in the normal direction of the cross section. These are calculated as follows.

$$P_x = \int \sigma_x^{A1} dydz = \{\int(\int \sigma_y^{A0} dz)dy + \int(\int \sigma_z^{A0} dy)dz\} \nu/(1-\nu^2) = 0$$

$$M_y = \int \sigma_x^{A1} zdydz = \{\int(\int \sigma_y^{A0} zdz)dy + \int(\int \sigma_z^{A0} dy)zdz\} \nu/(1-\nu^2) = 0$$

$$M_z = \int \sigma_x^{A1} ydydz = 0$$

The state of plane deformation is equivalent to the state where the constraint forces (P_x, M_y, M_z) to maintain the state of plane strain are relaxed. But none of additional stresses is produced by relieving these forces because all of them are zero.
Then,

$$\{\sigma^A\} = \{\sigma^{A1}\} \tag{A-3}$$

Eqs. (A-2) and (A-3) yield Eq. (5).

Appendix 2

It was shown in the text that the stresses near the middle of the weld line due to the inherent strains (ε_y*, ε_z*, γ_{yz}*) in the cross section are in the state of plane strain if a weld line is sufficiently long. Here, the weld length to satisfy this stress state will be considered.

In the case where a constant inherent strain ε_z* in the direction of z-axis exists in the neighborhood of the weld as shown in Fig. A, three dimensional stresses induced by the inherent strains were analysed by the finite element method. The result indicates that the maximum value of the longitudinal elastic strain, ε_x^{max}, on the cross section is produced at the portion (y=0, z=t/2). The distributions of ε_x^{max} along x-axis are shown in the same figure. It is seen that the strain ε_x^{max} decreases rapidly as the distance from the end of the weld line increases and converges to zero in the case of t/L being small enough. Furthermore, at a distance away about the plate thickness from the end of the weld line, the strain ε_x is less than 0.2% of ε_z*. Then, the stresses due to the inherent strains in the cross section can be considered in the state of plane strain. Other analyses indicate that the width B scarsely influences the distribution ε_x^{max}. And it had been confirmed that the stress state due to the other components of the inherent strains contained in the cross section is similar in the distribution to the above one.

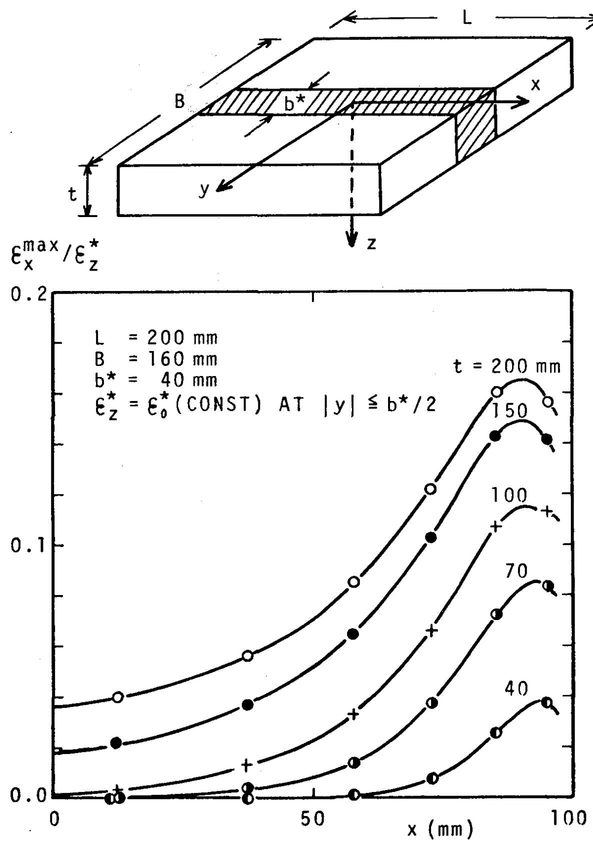


Fig. A Maximum longitudinal strains on the cross section along x-axis

Judging from these results, it can be considered that the stresses $\{\sigma^A\}$ at a distance about the thickness from the ends of the weld line, which are produced by the inherent strains (ε_y*, ε_z*, γ_{yz}*), are in the state of plane strain.