New Methods for Speeding up Computation of Newton Updates in Harmonic Balance

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Abstract

A new adaptive approach to solving large dimension harmonic balance (HB) problems is presented. The method is based on adjusting the order of the equation system according to the degree of nonlinearity of each node in the circuit. A block-diagonal preconditioner is used to construct an algorithm for order reducing during the iterative HB process.

1. Introduction

The Harmonic Balance (HB) method has proven to be very effective for many practical problems in RF circuit simulation [1, 2]. In contrast with traditional time domain circuit simulation, HB solves the steady state of a periodically driven system directly in the frequency domain. Each node voltage is presented by a truncated Fourier series, resulting in a system of equations of order $L=(2K+1)N$, where K is the number of terms in the Fourier series (or number of harmonics) and N is the number of nodes in the circuit. Highly nonlinear circuits require a large number of harmonics to accurately represent the waveforms, which may increase the size of the system to be solved beyond practical limits.

Historically the efficiency of HB analysis was achieved while applying the method to hybrid microwave circuits and MMIC [3], i.e. to such circuit types that contain a large linear subnetwork and a relatively small number of nonlinear elements. Since the linear and nonlinear parts can be easily identified beforehand, the piecewise HB technique (see for instance [4, 5]) was developed to improve the efficiency by eliminating the linear subnetworks from iterative process.

This situation has changed for the RFIC design problem, where a large number of transistors may preclude the use of the piecewise technique since most nodes are associated with an active device. Moreover the degree of linearity or nonlinearity depends on operation regime of element, which can change during the iterative process.

For this reason we have developed a new computational HB algorithm which adaptively reduces the problem size by not only linear but also weakly nonlinear contributions to the HB equations. This approach is a generalization of latent techniques [6,7].Section 2 contains a description of the foundation of the approach. Section 3 describes the computational algorithm, and Section 4 contains some experimental results.

2. Foundation

The system of equations of harmonic balance [1,2] for non-autonomous circuits can be represented in the following compact form:

$$H(X) = 0 \qquad (1)$$

Here $X = \{x_1, x_2, \dots, x_L\}$ is the vector of unknowns, i.e. the harmonics of electrical variables. Usually to find the solution of the nonlinear system (1) Newton's method is applied, and the following linear problem must be solved at each Newton iteration:

$$J \cdot \Delta X = b \qquad (2)$$

where J is the Jacobi matrix and b is the right hand side (RHS) vector.

The following features of the standard HB problem are important for the approach we propose:

1) The cost of solving of the linear problem is the dominant contribution to the total computational effort of the HB iterative cycle (approximately 75-95% of total efforts).

2) It has been shown that the relative tolerance to which the linear system (2) is solved need not be very stringent [8]. In fact, a loose relative tolerance of 0.1 can be used for the linear system solve without degradation of the convergence of the Newton iteration. For most circuits, this tolerance also leads to a near optimal overall run time. A much tighter tolerance increases the cost of the linear solve and a much looser tolerance prevents convergence of the Newton iteration.

3) The Jacobian matrix J in (2) has a block structure [1], and in particular, J is a block-diagonal matrix for networks with only linear elements.

Our goal is to construct an adaptive algorithm that reduces the order of the linear problem during the iterative HB process in accordance with the degree of nonlinearity at each iteration step. First we demonstrate applying a preconditioning transform to networks with linear elements, and then we present the extension to the general case of solving (2). This proceeds from the assumption that the distinction of the numerical properties of HB between linear problems and weakly nonlinear problems gradually increases with growth in the degree of nonlinearity.

We begin by introducing a block diagonal preconditioner, D, and re-writing the system (2) as:

$$(J - D) \cdot D^{-1}\ y + y = b \qquad (3)$$

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where $y = D \cdot \Delta X$, and the vector of unknowns *y* and right hand side vector *b* have components y_{ik} and b_{ik}, where *i* is the node index (*i = 1,...,N)*, and $k \in \widehat{K}$ is the multitone harmonic index. In the special case of single tone problems, *k = -K,...,0,...,K.*

The harmonics corresponding to linear nodes (nodes to which only linear circuit components are connected) have the following property: if the amplitude of any rhs harmonic of a linear node is equal to zero then in the solution of (3) the amplitude of this harmonic is also zero.

This property can be proved in the following way. Because a linear variable contributes to only block-diagonal entries of matrix *J* then:

1) all entries of the row of matrix $(J - D)$ corresponding to a linear variable are zero;

2) the product of matrix $(J - D)$ by any vector has zero components for all linear variables;

3) the equation of system (3) corresponding to the linear variable has the form: $y_{ik} = b_{ik}$;

4) therefore, if $b_{ik} = 0$ then $y_{ik} = 0$.

We can obtain the condition $b_{ik} = 0$ for all linear variables by introducing a new vector of unknowns:

$$y' = b - y \quad \text{or} \tag{4}$$

$$y = b - y' \tag{5}$$

Substituting (5) into (3) we obtain:

$$(J - D) \cdot D^{-1} \cdot y' + y' = b' \tag{6}$$

where

$$b' = (J - D) \cdot D^{-1} \cdot b \tag{7}$$

Components of vector b' corresponding to linear nodes will be zero, so the solution of system (6) contains zeroes for all harmonics of linear nodes, and these variables can be eliminated from processing. It is also expected that for weakly nonlinear nodes, vector *b'* will contain a smaller number of essential harmonics in comparison with vector *b.*

We propose an adaptive procedure based on the above discussion for linear circuit nodes and also on the following assumption: the preconditioning transform leads to a linear relationship between values for "residual" b_{ik} and the vector of unknowns ("solutions") y_{ik} such that "small" deviations from linear problems causes "small" values of harmonic residuals and respectively "small" values in solutions.

We performed a series of numerical experiments on a set of test problems to confirm this assumption. Fig. 1 contains the distribution of computed values of solution norms $\|y_i'\|$ versus RHS norm $\|b_i'\|$ for different nodal variables (*i=1,...,N*) during the iterative Newton process, where the norm is defined as $\|z_i\| = \sqrt{\sum_{k=1}^{K} |z_{ik}|^2}$. These results were

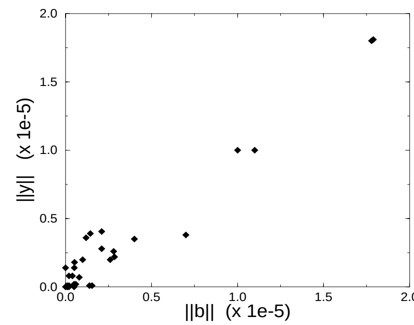

Fig 1. Dependence of solution norm on residual norm.

obtained for a ua-741 opamp, and they are typical for other types of circuits. The observed dependencies are in good agreement with the assumption of a linear function in quite wide region of distributions.The origin corresponds to purely linear nodes and increasing $\|b_i'\|$ can be interpreted as growth of the nonlinearity degree for different variables. Correlation coefficients can be extracted from distributions such as that of Fig. 1. The dependence of correlation coefficient on input amplitude (increasing amplitude increases the degree of nonlinearity of the circuit) for this example is shown in Fig. 2. The curves illustrate a strong correlation between estimates $\|y_i'\|$ and $\|b_i'\|$, and also the overall correctness of above assumption.

We conclude that $\|b_i'\|$ can be used as an estimate of the degree of nonlinearity of individual variables: zero $\|b_i'\|$ corresponds to linear networks, while the growth of $\|b_i'\|$ leads to increasing $\|y_i'\|$ with a subsequent increase in the required solution harmonics. Thus estimation of $\|b_i'\|$ for the *ith* node provides a criteria for reduction in the number of harmonics necessary to accurately represent the waveform at that node.

3. Computational Algorithm

The algorithm is based on Krylov subspace iterative techniques, which have proven to be particularly effective

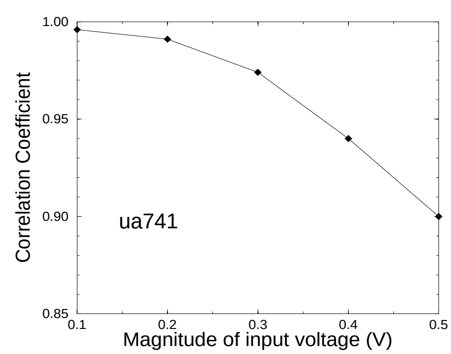

Fig 2. Correlation coefficient vs. the input signal

for harmonic balance [9,10]. An additional important advantage of using Krylov-subspace methods for our purpose is the natural application of block-diagonal preconditioners.

The aim of the algorithm is to reduce the order of the initial linear problem (6) and to establish an error control to avoid loss of accuracy. In the discussion that follows, we write (6) as $A \cdot y = b$, where $A = (J - D)D^{-1} + I$ and b is defined by (7). The subscripts are omitted for simplicity.

The RHS vector b is truncated by neglecting those components with small contribution into residual norm. The vector y is also truncated by eliminating the corresponding components:

$$\widehat{y} = \begin{bmatrix} \hat{y} \\ 0 \end{bmatrix} \qquad \widehat{b} = \begin{bmatrix} \hat{b} \\ 0 \end{bmatrix}$$

The computational algorithms of reducing the order are based on the assumption of a linear dependence of the error norm of the truncated solution on the truncation error of the RHS vector b:

$$\|A \cdot \widehat{y} - b\| = c \cdot \|b - \widehat{b}\| \tag{8}$$

Here c is an adjustable parameter whose value is predicted for the next Newton iteration. As a result a reduced linear problem

$$\hat{A} \cdot \hat{y} = \hat{b} \tag{9}$$

of lower dimension is solved at each Newton step.

Fig. 3 illustrates a numerical experiment which examines assumption (8). The experimental results are given for an iterative HB process of simulation of the ua-741 example. Each curve corresponds to solving the linear problem at the Newton iteration step by forming the Jacobian and RHS vector b. For different specified tolerance values $tol = \|b - \widehat{b}\| / \|b\|$ the truncation of harmonics was performed. Then the reduced system (9) was solved with respect to the reduced vector and the residual norm of the truncated error (8) was estimated. It can be seen from Fig. 3 that the relation (8) is correct with respect to the allowable error. Moreover the coefficient c changes slightly between Newton iterations, which allows us to consider c as an adjustable parameter that can be computed during the numerical process.

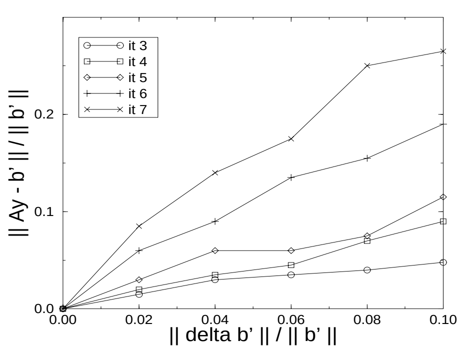

Fig 3. Error vs. truncated RHS norm.

The computational procedure can be described by the following way:

1. Guess initial value of adjustable parameter c.

2. Determine the truncated set of variables (number of harmonics for each node K_i, i=1,...,N) using the following inequality: $c \cdot \|b - \widehat{b}\| \le \|b\| \cdot tol$ *for the current value of c and for the given relative tolerance tol.*

3. Solve reduced system (9) by an iterative method

4. Compute a new value of parameter c using the formula:

$$c_{new} = c \cdot \frac{\sqrt{\|A \cdot \widehat{y} - b\|^2 - \|\hat{A} \cdot \hat{y} - \hat{b}\|^2}}{\|b\| \cdot tol}$$

Here the first term under radical is the error norm of the truncated solution and the second term corresponds to the final residual norm of the linear iterative process using the Krylov subspace techniques.

5. Check the residual: if $\|A \cdot \widehat{y} - b\| > \|b\|$ *go to step 2. This condition insures convergence of the Newton iteration because it provides a descent direction for the residual norm.*

To determine the truncated set of variables we sequentially determine and prune those harmonics with minimal amplitude keeping the norm of the neglected harmonics less than a predefined value.

The procedure terminates when we have achieved the minimal numbers of harmonics for all nodes under the condition

$$\sum_{i=1}^{N} \left(\sum_{k=K_i+1}^{K} |b_{ik}|^2 \right) < v^2$$

where $v = (tol \cdot \|b\|) / c$.

4. Experimental Results

Table 1 contains results of numerical experiments with some test circuit problems using the new approach and standard HB technique. The standard HB technique is the algorithm based on the GMRES method [9] for the linear problem and matrix-vector multiplication in time domain [9,10].

The order of the full linear system and the obtained order reduction factor are given in columns 4 and 5, respectively. The last column contains the speed up factor. which is the ratio of the time spent for standard HB and the proposed algorithm. The speed up factors are due to accelerating the orthogonalization stage of the GMRES method.

The specified error tolerance was the same for both techniques. The number of Newton iterations was approximately equal for both methods, with a difference of not more that 20%.

5. Conclusion

We have presented a new algorithm for harmonic balance circuit simulation. The algorithm applies a special-purpose transform at the linear level that is based on a block-diagonal preconditioner. This transform allows us to construct an adaptive computational algorithm which automatically adjusts the order of the system during the iterative process. This transform allows an estimation of the degree of nonlinearity of each circuit node using the individual component of the residual norm. Based on this estimation, we can eliminate harmonics for nodes which are weakly non-linear, and thus reduce computational expense and memory requirements while maintaining accuracy.

The new approach retains the convergence properties of standard HB and does not impact the final accuracy of the solution. In addition the transform (3) together with (7) can be used to eliminate linear nodes independent of the nonlinearity adaptation algorithm.

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Table 1: Results of order reduction algorithm and corresponding speedup

Circuit	#circuit variables	order of model	order reduction factor	speed up CPU
detector	6	494	6.1	2.2
amplifier	8	496	3.8	1.2
diffpair	12	504	5.0	2.0
amplifier class C	12	624	5.9	2.9
mosamp	24	1488	3.2	1.06
rectifier	10	1720	1.6	1.5
ua741	29	4118	5.6	2.9
filter	43	6966	14.3	2.5
Six-pole filter	183	7686	4.2	2.4
MOS opamp	159	38478	6.3	3.3
BJT mixer	10	2900	6.16	1.9