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New Product Innovation with Multiple Features and Technology Constraints

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We model a firm's decisions about product innovation, focusing on the extent to which features should be improved or changed in the succession of models that comprise a life cycle. We show that the structure of the internal and external environment in which a firm operates suggests when to innovate to the technology frontier. The criterion is maximization of the expected present value of profits during the life cycle. Computational studies complement the theoretical results and lead to insights about when to bundle innovations across features. The formalization was influenced by extensive interviews with managers in a high-technology firm that dominates its industry.

(*Product Development; Innovation; Technology Management*)

1. Introduction

1.1. Background

The accelerating pace of technological advancement magnifies the competitive importance of the management of product and process innovation for manufacturing firms. Their profitability and viability depend on decisions about when to initiate product development and what development goals to set. As an example of the intellectual puzzle, consider the different innovation strategies pursued within the automotive industry. At BMW, model redesign occurs relatively infrequently, with a completely new model being introduced approximately every eight years. Japanese companies, in contrast, introduce redesigned or new models approximately every four years (Clark and Fujimoto 1991, Pisano 1992). Clearly, these automobile manufacturers have developed different design strategies. BMW may be viewed as a *frontier innovator*, choosing not to introduce a new model until it is very different from the previous models and is at

the leading edge of the technology frontier. In comparison, Japanese automobile manufacturers may be viewed as *incremental innovators*, frequently introducing new models that are only slightly different from the previous ones and do not incorporate all possible technological advances. This paper examines how cost structures and market and industry conditions influence the amount of innovation that a firm builds into a new product. The characterization of product innovation as frontier or incremental may be further complicated in a scenario in which multiple products share some technologies and features, and when various firms stand at different places on the continuum of product innovation. Thus, a given innovation strategy may be viewed as either frontier or incremental depending on the context of the firm and its history of innovation.

The specific model of innovation developed in this paper was motivated by our interactions with managers who were directly involved in NPD (new

product development) in a high-technology firm that manufactures bar code scanners and related equipment. From interviews with marketing, engineering, operations, and accounting managers, we learned about the decision-making process involved in NPD. Coupled with cost and sales data, these discussions allowed us to create a model which captures the complexities of the innovation decision faced by managers in this industry. More importantly, our results apply to a large number of firms which integrate components whose basic technology is developed outside the firm itself. Some industries with such firms include computers, computer peripheral equipment, surgical and medical instruments, and automobiles. The key market and industry characteristics captured by our model and computational study follow.

Industry Driven by Technological Advances. Future growth of a firm in the industry depends to a great extent on its ability to apply technology to develop new products and improve existing products, as well as to expand market applications for its products. This emphasis on technology-driven product differentiation is partially the result of the highly competitive nature of the industry. Numerous competitors fighting for market share attempt to soften price competition and gain a competitive advantage by being the first to introduce a new product or an improved version of an existing product. Depending on the nature of demand, such product differentiation may allow a firm to increase its price or merely maintain its current price. Evidence of the latter is presented by Adner and Levinthal (2001) for the personal computer and VCR markets in the 1990s.

Consistent with Adner and Levinthal (2001), the NPD team and finance and accounting managers at the scanner firm stressed that there is a constant downward pressure on price, which obliges cost reductions if there are no improvements in product features. Taking into account the fact that the scanner firm we studied has potential competitors in the computer peripherals field with far greater financial, marketing, and technical resources, the company's strategy (as delineated in its 2000 annual report) is to compete principally on the basis of performance and quality of its products and services. In particular, R&D projects were aimed at improving the size,

weight, reliability, quality, and readability of scanners at increased distances, faster speeds, and higher-density codes.

An emphasis on market expansion through NPD typifies a number of other industries as well, including the computer industry (smaller, faster machines with greater memory), the surgical and medical instruments industry (miniaturization in the form of minimally invasive surgical instruments such as laparoscopic and endoscopic devices and angioplasty catheters), the telecommunications industry (new data transmission technologies and fiber optic networks) and the automobile industry (improving fuel efficiency, developing alternative fuels, and reducing vehicle emissions)(Hell and Peck 1998, Tardiff 1998, Bossong-Martines 2000).

The use of product differentiation to expand demand and soften price competition is captured in our model's revenue function. Under such a strategy of market expansion, there is a significant risk that NPD projects may not reach fruition. The time to completion may also depend on the scope of the project. These factors are incorporated in our model as well. In addition, to reflect the costs of adopting a strategy of product differentiation, we assume that the direct cost of manufacturing is higher with greater innovation, because new production processes need to be set up and perfected. We also assume that the costs of adverse quality are higher when new product innovation is more aggressive. Members of the quality team at the scanner firm cited high costs of auditing and control, inspection of raw materials, final product inspection and qualification, and costs of repairing and replacing defective units. This is reflected in the analytical example in §5.2 and in the numerical study in §6.

Firm Is a Technology Taker and Is Affected by Exogenous Rates of Technology Change. The scanner firm we studied is a *technology taker*; i.e., it does not do basic research on lasers or motors, but integrates "off the shelf" subcomponents to build components for its own product. In addition, heterogeneous rates of technological change for different product features motivate the scanner firm to differentiate among features in terms of innovation. For example, one engineering

manager told us that the new product that we studied would not have been possible without advances in the miniaturization of motors that are used in this type of scanner.

Similarly, many of the technological innovations adopted in the computer industry were developed in other industries (e.g., microprocessors developed by the semiconductor industry, increased memory storage developed by the computer storage device industry, and faster communication capabilities developed by the telecommunications equipment industry). The development of new surgical instruments was made possible by the discovery of new shape-memory polymers (Tardiff 1998). Automobile manufacturers are becoming more and more dependent on suppliers to assume greater design and engineering responsibilities in creating new parts and systems (Bossong-Martines 2000).

We incorporate the idea of the firm as a technology taker by modeling technology change as an exogenous process in §3.1. In the numerical study, we find that the speed of technology affects innovation decisions.

Economies of Scale in R&D. R&D costs of innovative products exhibit economies of scale. That is, the cost function is concave in the amount of innovation. Concavity may be observed when the initial R&D investment allows not only for a particular improvement to be incorporated in a product, but also suggests the exploration of new, previously unanticipated improvements. These improvements may further advance the same feature, allow superior performance on another related product dimension, or provide additional functionality. For example, Hewlett-Packard originally designed the HP85 to function solely as a personal computer, but discovered that its functionality could be expanded such that the HP85 could also be used as an equipment controller (Lynn 1998). In this paper, the idea of synergies in R&D is reflected by economies of scale in the R&D cost function in Theorem 1 and in the numerical study in §6.

In some instances a feature may be “regressed” to improve the product on another dimension. For example, the addition of photographic capability to the new scanner product would necessitate an

increase in unit bulk and weight. An automotive example is a reduction in gasoline mileage to facilitate a reduction in NO₂ emissions. We allow for this possibility in the model presented in §3.1, and we find regression in the numerical example in §6.

Increasing Returns to Successful Advances in Product Technology. Some business purchases of evolving moderately durable products, such as bar code scanners, are replacements of damaged or deteriorating items, and other purchases are driven by expanding uses which have been made possible by product innovations. Sales to new customers and for new uses depend significantly on the amount of innovation in the product. For example, the scanner firm told us that the innovation effect displayed increasing returns to scale. That is, small innovations had negligible market impacts and customers would not tolerate any price increases for slightly improved products. Larger innovations, in contrast, had disproportionately great market impacts and could sustain higher prices. So net revenue, not including costs of R&D, showed increasing returns to the scale of product innovation.

Consistent with the market and industry characteristics outlined above, in this paper we model a single firm’s innovation decisions within the life cycle of one product with multiple features (which may have different innovation characteristics). We use “features” for the product dimensions on which innovation may occur. For example, a notebook computer’s features might include its weight, hard drive storage, screen brightness, etc. A product’s “bundle” specifies the levels of its features. We consider a number of factors: the bundle of features incorporated in the previous version, how close the current product is to the frontier of technology, the costs of setting up production for the new or enhanced product, and the risk that the development effort may fail.

The model combines attributes that have only been analyzed in isolation previously, including multiple product features, costs of R&D, and revenue structures. Most of the literature on innovation focuses on either the movement of the technology frontier or the decision by individual firms to move directly to the technology frontier in terms of a single product feature. In contrast, we develop a model that analyzes the product development process once a discovery

has been made, emphasizing the interrelated nature of the innovation decision across product features.

One of the reasons why these disparate factors have not previously been integrated is that the resulting optimization model, the dynamic program in §4, suffers from the “curse of dimensionality.” It becomes impractical to solve it as the number of features increases and the discretization becomes finer. Our model has properties that permit a dramatic reduction of dimensionality and acceleration of computation (Proposition 1 and Theorem 1).

We employ the dynamic program to analyze the structure of optimal decisions. Frontier innovation (here meaning a jump to the technology frontier if there is any innovation at all) is optimal when there are increasing returns to successful advances in product technology and R&D cost has increasing returns to scale with respect to the magnitude of innovation (Theorem 1). The assumptions underlying these conclusions are satisfied by reasonably realistic specific models, as is illustrated by an analytical example (§5.2). A numerical example (§6) with two features has a frontier innovation policy under more general conditions than are assumed in Theorem 1, and provides additional insights as to when it is advisable to bundle innovative features (i.e., one feature is improved only when the other feature can be improved at the same time). The model in the example is too large to solve without exploiting Proposition 1.

The remainder of the paper is structured as follows. Section 2 reviews related research on product development and innovation. In §3 we formulate the model, and in §4 we formulate and streamline a dynamic program that is utilized in §5 to investigate conditions for frontier innovation. In §6 we describe the numerical study, and §7 presents our conclusions.

2. Related Work

Product and process innovation, time to market, and the quality implications of product development and enhancement have been modeled in a number of settings. The literature on technological innovation is reviewed by Bayus (1995), Kamien and Schwartz (1982), and Reinganum (1989). We summarize below those areas of the literature that are

most closely related to our model, indicating how our approach differs.

Analytical studies of innovation can be divided into those that focus on discovery—i.e., the initial breakthrough in the research phase of R&D—and those that focus on process or product development. Discovery models are typically presented as one-shot patent races in which competing firms try to be the first to reach the technological frontier. These models often focus on the trade-off between time to market and total resources spent as well as the effect of competition on the supply of new technology. See, for example, Reinganum (1982), Fudenberg and Tirole (1984), Tirole (1990), and Hendricks (1992). Our model differs from traditional supply-of-innovation models in that we focus on how product development progresses after a technological breakthrough has occurred.

Innovation may also be viewed from the demand side in terms of the diffusion of new technology. This innovation diffusion process can be modeled from the perspective of the time at which a customer purchases a new product (Chatterjee and Eliashberg 1990) or from the perspective of the firm in terms of when an innovation is adopted within the organization (Reinganum 1981, McCardle 1985). Our framework differs from these demand-for-innovation models in that we allow a firm to adopt multiple innovations over time and to operate on a continuum bounded by the technological frontier. Adner and Levinthal (2001) explicitly consider the interaction between technology change and demand; our model differs in that we consider multiple features and include an explicit cost structure. In the literature on technology adoption, Balcer and Lippman (1984) is particularly germane to our work because they too have a model with multiple sequential innovations. However, our model differs significantly from theirs in that (i) our exogenous technology and R&D goals are vector-valued to reflect multiple product features, (ii) net revenue in our model is a function of more than the level of exogenous technology, and (iii) our model allows for (vector-valued) incremental innovation as well as frontier innovation.

The discovery models discussed above focus primarily on the adoption of a single innovation. In reality, however, innovation is induced by a sequence

of discoveries. The repeated-innovation strand of the R&D literature focuses on this modification of the traditional discovery model. See, for example, Grossman and Helpman (1991b, 1991a), Aghion and Howitt (1992), and Segerstrom et al. (1991). Like the models of repeated innovation, we focus on multiple product or process improvements which occur over a period of time. However, we assume that the current state of technology is exogenous and applies to all firms within the industry. Instead of focusing only on the movement of the technological frontier, we also consider movements of products or processes toward the frontier. In this regard, our model is similar to the analytical models of innovation which deal primarily with product or process development.

The product development models are based on the idea that after a breakthrough in R&D, a new technology must be further improved before it can be brought to market. In this context, the central issue is determining the length of the development stage. The longer the development stage, the higher the quality of the product and the greater the returns. However, a longer development stage delays the commencement of these higher profits. For examples of these models, see Dutta et al. (1995), Reinganum (1982), Cohen et al. (1996b, 1996a), Bayus et al. (1997), and Bayus (1998). Although these models focus on the development and marketing aspects of innovation, they incompletely address the dynamic nature of this process. Our work builds on the previous development models in that we incorporate repeated product developments which move the firm closer to the evolving technological frontier. Moreover, we allow for the fact that different features can be developed simultaneously.

There are a few papers that explicitly consider different types of innovation. Lynn et al. (1996) find from their case-study interviews that one difference between incremental and frontier (here called discontinuous) innovation is that market research is experimental ("probe and learn") rather than analytical. Lambe and Spekman (1997) use historical sources to examine the correlation between discontinuous technological change and alliances between firms, finding that alliances are more likely at the beginning of the innovation life cycle. Veryzer (1998) explores

the differences between the incremental and discontinuous innovation in the NPD process using eight in-depth case studies. Chandy and Tellis (1998) use survey results to examine the importance of the willingness to "cannibalize" its own products in order to introduce more innovations. These papers provide a backdrop for our work by defining the two types of innovative processes and by sketching the external (market forces, alliances) and internal (NPD process, cannibalization) aspects. We go on to specify quantifiable characteristics (demand and cost functions) in order to understand their influence on innovation decisions.

3. The Model

3.1. Formulation

Suppose that a product line has J features which are relevant for purposes of R&D. Let K be an integer upper bound on the number of models which could be developed during the life of the product. Development is initiated at an epoch (moment in chronological time) T_k and continues for an elapsed time of τ_k between the k th and $(k+1)$ st epochs, where $k = 1, 2, \dots, K$ indexes the development epochs. Therefore, the clock time of the k th development epoch is $T_k = \sum_{j=1}^{k-1} \tau_j$, and $T_0 = 0$. The subsequent development epoch occurs when the product being developed either reaches market or development terminates prematurely.

The state of technology during development and production influences the resulting costs. Let $\chi(t) \in \mathcal{R}^J$ be the state of technology at epoch t ; we assume that $\{\chi(t), t \geq 0\}$ is a J -dimensional continuous-time Markov chain with nonnegative increments on each dimension. We consider a firm with a product line in which it is a *technology taker*; i.e., the firm's own R&D does not significantly advance the technologies relevant to the product line, so we assume that $\{\chi(t)\}$ is not affected by the firm's decisions. Let χ_{jk} be the state of technology in feature j at epoch T_k ($j = 1, \dots, J$), and let $\hat{\chi}_k = (\chi_{jk}, j = 1, \dots, J)$ be the vector of technology levels at epoch T_k . Let I_k be the increment to the technology between epochs T_k and T_{k+1} , and so $\hat{\chi}_{k+1} = \chi(T_{k+1}) = \chi(T_k) + [\chi(T_{k+1}) - \chi(T_k)] = \hat{\chi}_k + I_k$.

At epoch T_{k-1} the firm chooses the k th bundle of product features $B_k = (B_{jk}, j = 1, \dots, J)$ for the next generation of the product, where B_{jk} is the level of feature j selected for the k th bundle. The k th bundle of features reaches market or is shelved at T_k . The improved bundle is constrained to lie within the current limits of technology; i.e., $0 \leq B_k \leq \hat{\chi}_{k-1}$ (i.e., $0 \leq B_{jk} \leq \chi_{j,k-1}, j = 1, \dots, J$). The lower vector inequality is $0 \leq B_k$ instead of $B_{k-1} \leq B_k$ in order to admit trade-offs among features which lead to the diminution of one feature to facilitate an increase in another. However, in the case of a single feature, i.e., $J = 1$, this trade-off cannot occur; so we constrain $B_{k-1} \leq B_k \leq \hat{\chi}_{k-1}$.

The random variable $\xi_k = 1$ if the k th bundle B_k eventually reaches market, and $\xi_k = 0$ if the k th bundle is shelved. Let $\hat{\beta}_{k-1}$ and $\hat{\beta}_{k-2}$ denote the vectors of feature levels which are *marketed* starting at epochs T_k and T_{k-1} , respectively. Then $\hat{\beta}_{k-1} = B_k$ if $\xi_k = 1$, and $\hat{\beta}_{k-1} = \hat{\beta}_{k-2}$ if $\xi_k = 0$. So $\hat{\beta}_{k-1} = \hat{\beta}_{k-2} + \xi_k(B_k - \hat{\beta}_{k-2})$. We assume that the time span τ_k until the next generation can be brought to market (or shelved) is a random variable. Except in §5 we let $P\{\xi_k = 1\}$ and the probability distribution of τ_{k-1} depend on $k, B_k, \hat{\beta}_{k-2}$, and $\hat{\chi}_{k-1}$. So, the probability that a product reaches market, and the elapsed time until it does, may depend on timing, the current state of technology, and the previous and current choices of bundle (Cohen et al. 1996a, 1996b; Griffin 1997).

3.2. Costs and Revenues

We model the firm's costs and revenues with three terms. Let $c(B_{k+1}, \hat{\beta}_{k-1})$ be the expected present value of the cost of R&D for product and process development which is charged to epoch T_k and incurred during $[T_k, T_{k+1})$. This cost depends on the current and previous bundles, since presumably a larger "jump" in features would incur a larger R&D expenditure.

Let $r_{k+1}(\hat{\beta}_{k-1}, \hat{\beta}_{k-2}, \hat{\chi}_{k-1})$ be the expected present value of the firm's net revenue during $[T_k, T_{k+1})$, exclusive of R&D costs, and credited to epoch T_k . In the numerical example in §6, $r_{k+1}(\cdot)$ is the value function of a pricing optimization that includes revenues from market demand offset by costs of production and adverse quality.

Let $s(\hat{\beta}_K)$ be the salvage value of the product line, evaluated at epoch T_K , where $\hat{\beta}_K$ is the bundle being marketed during $[T_K, T_{K+1})$. Let $\alpha > 0$ be the instantaneous (continuous-time) discount factor, and let Π be the present value of net profits over the life cycle of the product.

Let R_k and C_k be the respective present values of the net revenue and R&D cost during $[T_k, T_{k+1})$. Then the expected present value of net profits over the life cycle of the product is

$$E(\Pi) = E \left[\sum_{k=1}^K e^{-\alpha T_k} (R_k - C_k) + e^{-\alpha T_K} s(\hat{\beta}_K) \right]. \quad (1)$$

We maximize the expected present value of the net profits, that is, maximize $E(\Pi)$, given the initial values: $\hat{\chi}_0, \hat{\chi}_1, \hat{\beta}_0$ and $\hat{\beta}_{-1}$.

In summary, for accounting purposes the following sequence of events occurs "at" epoch T_k : Observe the new state of technology ($\hat{\chi}_k$) and the bundle being marketed now ($\hat{\beta}_{k-1}$), choose the bundle to develop now (B_{k+1}), incur cost $c(B_{k+1}, \hat{\beta}_{k-1})$, and receive net revenue $r_{k+1}(\hat{\beta}_{k-1}, \hat{\beta}_{k-2}, \hat{\chi}_{k-1})$. The probability distribution of the duration of the development effort (τ_k) and the probabilities that the effort succeeds or fails ($P\{\xi_{k+1} = 1\}$ and $P\{\xi_{k+1} = 0\}$) depend on the bundle being developed (B_{k+1}), the bundle being marketed ($\hat{\beta}_{k-1}$), and the current technology frontier ($\hat{\chi}_k$).

4. Dynamic Programming

In this section we formulate and analyze a dynamic program that corresponds to maximizing the expected present value of net profit and we begin the investigation of optimal feature selection policies that continues in §5.

The dynamic program corresponds to maximizing the expected present value of net profits (1). In the argument of the following dynamic program value function, β_{k-1} and β_{k-2} are respective bundles which will be marketed during $[T_k, T_{k+1})$ and were marketed during $[T_{k-2}, T_{k-1})$, respectively; i.e., they are the potential values of $\hat{\beta}_{k-1}$ and $\hat{\beta}_{k-2}$. Similarly, χ_k and χ_{k-1} are the potential technology frontier vectors at T_k and T_{k-1} ; i.e., they are potential values of $\hat{\chi}_k$ and $\hat{\chi}_{k-1}$, respectively. For each $k = 1, \dots, K$ the value

function for epoch T_k , denoted $v_k(\cdot)$, satisfies the following recursion with $v_{K+1}(\beta_{k-1}, \dots) \equiv s(\beta_{k-1})$:

$$\begin{aligned} &v_k(\beta_{k-1}, \beta_{k-2}, \chi_k, \chi_{k-1}) \\ &= \max\{r_k(\beta_{k-1}, \beta_{k-2}, \chi_{k-1}) - c(B, \beta_{k-1}) \\ &\quad + E(e^{-\alpha\tau(B, \beta_{k-1}, \chi_k)} v_{k+1}[\beta_{k-1} + \xi_{k+1}(B - \beta_{k-1}), \\ &\quad \beta_{k-1}, \chi_k + I_k, \chi_k])\} : 0 \leq B \leq \chi_k. \end{aligned} \quad (2)$$

It is apparent in (2) that the value function (v_k) depends on the previous development cycle's bundle and technology frontier (β_{k-2} and χ_{k-1} , respectively) because the expected net revenue term r_k depends on them. Because that term is completely determined by previous choices and previous state variables, it does not depend on the current choice of bundle (B) and it can be transferred out of the maximization operation. That is, the controllable portion of the net present value of the time stream of revenues and costs depends only on the state variables β_{k-1} and χ_k . This observation is intuitive and is the basis for the following property, which leads in §5 to sufficient conditions for the optimality of innovations that go to the frontier. Also, the proposition accelerates the numerical solution of the dynamic program. In (3), $w_k(\beta_{k-1}, \chi_k)$ is a controllable component of the maximal expected present value of the net profits. Let $r_{K+1}(\cdot, \cdot, \cdot) \equiv 0$ and $w_{K+1}(\beta, \cdot) \equiv s(\beta)$.

PROPOSITION 1. *The dynamic program value function satisfies*

$$\begin{aligned} &v_k(\beta_{k-1}, \beta_{k-2}, \chi_k, \chi_{k-1}) \\ &= r_k(\beta_{k-1}, \beta_{k-2}, \chi_{k-1}) + w_k(\beta_{k-1}, \chi_k), \end{aligned} \quad (3)$$

where

$$\begin{aligned} &w_k(\beta, \chi) = \max\{-c(B, \beta) \\ &\quad + E[e^{-\alpha\tau_k(B, \beta, \chi)} (r_{k+1}[\beta + \xi_{k+1}(B - \beta), \beta, \chi] \\ &\quad + w_{k+1}[\beta + \xi_{k+1}(B - \beta), \chi + I_k])]\} : \\ &\quad 0 \leq B \leq \chi. \end{aligned} \quad (4)$$

PROOF. To initiate an inductive proof, $v_{K+1}(\beta_{k-1}, \cdot, \cdot, \cdot) \equiv s(\beta_{k-1})$ in (2) yields (3) at $k = K$. If $v_{k+1}(\beta_{k-1}, \beta_{k-2}, \chi_k, \chi_{k-1}) = r_{k+1}(\beta_{k-1}, \beta_{k-2}, \chi_{k-1}) + w_{k+1}(\beta_{k-1}, \chi_k)$ for

all arguments; i.e., if (3) is valid at $k + 1$, then a substitution in (2) yields

$$\begin{aligned} &v_k(\beta_{k-1}, \beta_{k-2}, \chi_k, \chi_{k-1}) \\ &= r_k(\beta_{k-1}, \beta_{k-2}, \chi_{k-1}) + \max\{-c(B, \beta_{k-1}) \\ &\quad + E[e^{-\alpha\tau_k(B, \beta_{k-1}, \chi_k)} (r_{k+1}[\beta_{k-1} + \xi_k(B - \beta_{k-1}), \beta_{k-1}, \chi_k] \\ &\quad + w_{k+1}[\beta_{k-1} + \xi_k(B - \beta_{k-1}), \chi_k + I_k])]\} : 0 \leq B \leq \chi_k \\ &= r_k(\beta_{k-1}, \beta_{k-2}, \chi_{k-1}) + w_k(\beta_{k-1}, \chi_k), \end{aligned}$$

where $w_k(\beta_{k-1}, \chi_k)$ satisfies (4). \square

If $c(\cdot, \beta)$ is nondecreasing and $s(\cdot) \equiv 0$, then it is optimal not to engage in any product development at the end of the planning horizon, so $w_K(\beta, \chi) = -c(\beta, \beta)$.

We exploit Proposition 1 in the remainder of the paper and in §6 we comment on its role in dramatically reducing the computational effort in Dynamic Program (2).

5. Frontier Innovation

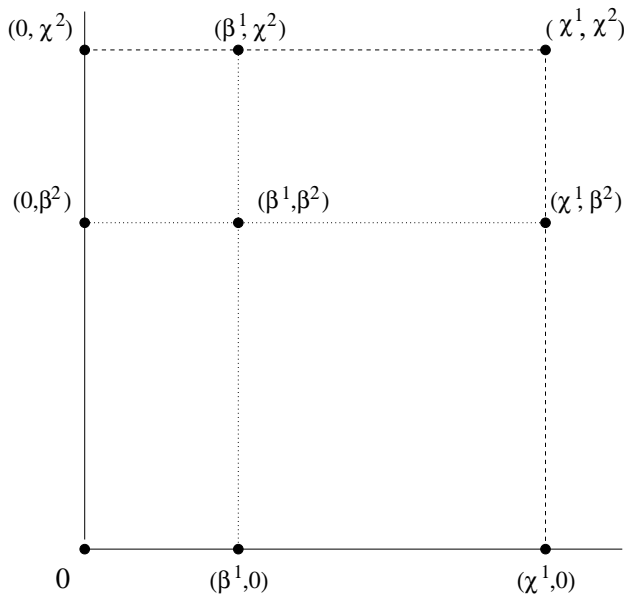
5.1. Sufficient Conditions for Optimality of Frontier Innovation

In this section we identify sufficient conditions which make it optimal to innovate to the frontier. That is, a feature is improved maximally if it is improved at all.

The feasibility set for the dynamic program in (4) is the multidimensional rectangle $[0, \chi]$. The forthcoming Theorem 1 gives sufficient conditions for the optimization to be restricted to a small discrete subset, namely the extreme points of a collection of rectangles that cover $[0, \chi]$. These extreme points have the property that each coordinate is at the technology frontier if that coordinate exceeds the level of the currently marketed feature. Therefore, an innovation policy moves to the frontier if it is based only on these extreme points. Also, each iteration of (4) is greatly accelerated by restricting the optimization to extreme points.

It is convenient to regard $[0, \chi]$ as the union of J -dimensional rectangles R_i constructed as follows. Let $R_i = [a, b]$, and let the respective j th components of a, b, β , and χ be a^j, b^j, β^j , and χ^j . For $j = 1, \dots, J$, a^j and b^j are elements of $\{0, \beta^j, \chi^j\}$ with $a^j \leq b^j$. Let

Figure 1 Feasibility Set for $J = 2$



$\mathcal{E}(\beta, \chi)$ be the set of the extreme points of $\{R_k\}$. For example, if $J = 2$ and $0 < \beta^j < \chi^j$ for $j = 1, 2$, Figure 1 shows that there are four rectangles and nine extreme points. Generally, if $0 < \beta^j < \chi^j$ for all j , then $\mathcal{E}(\beta, \chi)$ has 3^J extreme points. We represent each $e \in \mathcal{E}(\beta, \chi)$ as the following linear combination. For $j = 1, \dots, J$, let γ_{j1} and γ_{j2} be zero or unity, with $\gamma_{j1}\gamma_{j2} = 0$, and let e_j be the j th unit vector. Let $\gamma = \{(\gamma_{j1}, \gamma_{j2})\}$ and let $\mathbf{e}(\gamma, \beta, \chi) = \sum_{j=1}^J e_j(\gamma_{j1}\beta^j + \gamma_{j2}\chi^j)$. For each $e \in \mathcal{E}(\beta, \chi)$ there exists γ such that $e = \mathbf{e}(\gamma, \beta, \chi)$.

In this section we exploit the fact that a convex function achieves its maximum at an extreme point of its convex domain (if it achieves the maximum at all). Let $Y_k(B, \beta, \chi)$ denote the maximand in (4):

$$Y_k(B, \beta, \chi) = -c(B, \beta) + E\{e^{-\alpha\tau_k(B, \beta, \chi)}(r_{k+1}[\beta + \xi_{k+1}(B - \beta), \beta, \chi] + w_{k+1}[\beta + \xi_{k+1}(B - \beta), \chi + I_k])\}. \quad (5)$$

It follows from Proposition (1) and the preceding discussion that frontier innovation would be implied by convexity (with respect to B) in (5) on each R_i . If the distribution of τ does not depend on B , it is apparent that a sufficient condition for (5) to be convex with respect to B is convexity of $r_{k+1}(\cdot, \beta, \chi)$, concavity of $c(\cdot, \beta)$, and convexity of $w_{k+1}(\cdot, \chi)$ (on each R_i).

In a multifeature model, suppose $0 < \beta^j < \chi$ for each j . If the assumptions merely yielded convexity of $Y_k(\cdot, \beta, \chi)$ on $[0, \chi]$, then the 2^J extreme points of this set would *not* include β . However, $B = \beta$ occurs in reality in conjunction with frontier innovation. So the convexity of $Y_k(\cdot, \beta, \chi)$ on $[0, \chi]$ is *too weak* to yield solutions that are consistent with salient features of reality. On the other hand, it is violated by the plausible numerical example in §6, so $Y_k(\cdot, \beta, \chi)$ on $[0, \chi]$ is *too strong*. Theorem 1 illuminates real NPD strategies partly because it resolves this paradox. The 3^J elements of $\mathcal{E}(\beta, \chi)$ are a strategically richer set of elements than the extreme points of $[0, \chi]$. Not only does $3^J/2^J$ grow rapidly with J (e.g., $(3/2)^4 = 5.0625$), but $\beta \in \mathcal{E}(\beta, \chi)$. The computational value of Theorem 1 is that 3^J is minuscule compared to realistic discretizations of $[0, \chi]$ for most χ . So it dramatically reduces the effort to solve (2), which rapidly becomes prohibitive as J grows.

In the notation of (4) and (5),

$$w_k(\beta, \chi) = \max\{Y_k(B, \beta, \chi) : 0 \leq B \leq \chi\}, \quad k = 1, \dots, K. \quad (6)$$

THEOREM 1. Suppose $s(\cdot)$ is a convex function on its domain and that the following assumptions are valid for each k and i :

$$P\{\xi_{k+1} = 1\} \text{ and the distribution of } \tau_k \text{ depend only on } \beta_{k-1} \text{ and } \chi_k \text{ (but not on } B_{k+1}); \quad (7)$$

$$\text{for all } \beta, c(\cdot, \beta) \text{ is concave on } R_i; \quad (8)$$

$$\text{for all } \gamma \text{ and } \chi, c[\mathbf{e}(\gamma, \beta, \chi), \beta] \text{ is concave with respect to } \beta \in [0, \chi]; \quad (9)$$

$$\text{for all } \beta, \chi, \text{ and } k, r_{k+1}(\cdot, \beta, \chi) \text{ is convex on } R_i; \quad (10)$$

$$\text{for all } \gamma \text{ and } \chi, r_{k+1}[\mathbf{e}(\gamma, \beta, \chi), \beta, \chi] \text{ is convex with respect to } \beta \in [0, \chi]. \quad (11)$$

Then for each k, β , and χ ,

$$w_k(\beta, \chi) = \max\{Y_k(B, \beta, \chi) : B \in \mathcal{E}(\beta, \chi)\}. \quad (12)$$

PROOF. To begin an inductive proof, the convexity of $s(\cdot)$ and the definition $w_{K+1}(\beta, \chi) \equiv s(\beta)$ imply convexity of $w_{K+1}(\cdot, \chi)$ on $[0, \chi]$ for all $\chi \geq 0$. Suppose for

some $k \leq K$ that $w_{k+1}(\cdot, \chi)$ is convex on $[0, \chi]$ for all χ . Then (7) implies that the third term of Y_k in (5) is convex with respect to $B \in [0, \chi]$. Also, (8) and (10) imply that the first and second terms in (5) are convex in B on each R_j . Therefore, $Y_k(\cdot, \beta, \chi)$ is convex on each R_j and

$$\begin{aligned} w_k(\beta, \chi) &= \max_i \max_{B \in R_i} Y_k(B, \beta, \chi) \\ &= \max_{e \in \mathcal{E}(\beta, \chi)} Y_k(e, \beta, \chi) \end{aligned}$$

Since the maximum of convex functions is a convex function, $w_k(\cdot, \chi)$ is convex on $[0, \chi]$ if $Y_k(e, \beta, \chi)$ is a convex function of β for each $e \in \mathcal{E}(\beta, \chi)$. This is true if and only if $Y_k[\mathbf{e}(\gamma, \beta, \chi), \beta, \chi]$ is a convex function of $\beta \in [0, \chi]$ for each γ and χ . This property follows from (9), (11), and the inductive assumption. So, for all k and χ , $w_k(\cdot, \chi)$ is convex and there exists $B \in \mathcal{E}(\beta, \chi)$, which is optimal in (2) and (4). \square

Theorem 1 states that frontier innovation (maximal improvement of a feature if it is improved at all) is optimal under the following conditions. First, the salvage value function is convex. Second, the duration of R&D and the likelihood that the model with the bundle under development reaches market, i.e., the probability distribution of τ_k and $P\{\xi_k = 1\}$, do not depend on the bundle under development. However, in our numerical study we relax this assumption, and still find that frontier innovation is always optimal.

Third, the R&D cost function is concave in the sense specified by (8) and (9). Economies of scale in R&D costs can occur for various reasons mentioned in §1. The R&D cost in the numerical example in §6 satisfies (8) and (9). There are two features, and $c(B, \beta) = \sum_{j=1}^2 C_1^j \ln[1 + (B^j - \beta^j)^+]$. In Figure 1, in the rectangle that is furthest northeast, for example, $c(B, \beta) = \sum_{j=1}^2 C_1^j \ln[1 + (B^j - \beta^j)]$ (because $\beta^j \leq B^j$ for each j) which satisfies (9). However, this function would not satisfy a more stringent version of (9) in which $c(\cdot, \beta)$ is obliged to be concave on $[0, \chi]$. Although $\ln[1 + (B^j - \beta^j)^+]$ is concave with respect to B^j on $[0, \beta^j]$ and $[\beta^j, \chi^j]$, it is not concave on $[0, \chi^j] = [0, \beta^j] \cup [\beta^j, \chi^j]$.

Fourth, the revenue function is convex in the sense specified by (10) and (11). In the analytical example in §5.2 and the numerical examples with two features in §6.1, the revenue functions (15) and (17) satisfy (10) and (11).

5.2. Analytical Example with One Feature

In this example, we demonstrate that there are reasonably realistic revenue functions that satisfy the assumptions in Theorem 1. Suppose that (a) the volume demanded is a linear function of price plus the squared difference between β_{k-1} and β_{k-2} , (b) production cost is directly proportional to the level of technology in the bundle now going to market, and (c) the cost of adverse quality rises as the marketed level of technology gets closer to the technology frontier.

The revenue during $[T_k, T_{k+1})$, exclusive of R&D costs, consists of sales at price p , a decision variable, offset by costs of production and adverse quality. A more detailed discussion of the relevant pricing results is available from the authors. Suppose that the expected present value of the number of units sold during $[T_k, T_{k+1})$ is $a - bp + (\beta_{k-1} - \beta_{k-2})^2$, where the factor $a - bp$ describes consumer response to price, and the factor $(\beta_{k-1} - \beta_{k-2})^2$ describes consumer response to product innovation. Let the unit costs of production and adverse quality, respectively, be $c_{22}\beta_{k-1} + c_{22}(\beta_{k-1} - \beta_{k-2})$ and $d - e(\chi_{k-1} - \beta_{k-1})$. All parameters are assumed to be nonnegative. The unit production cost $c_{22}\beta_{k-1} + c_{22}(\beta_{k-1} - \beta_{k-2})$ is proportional to the level of technology in the bundle now going to market (β_{k-1}), and proportional to the technology improvement ($\beta_{k-1} - \beta_{k-2}$).

Therefore, the maximal net profit is

$$\begin{aligned} r_k(\beta_{k-1}, \beta_{k-2}, \chi_{k-1}) &= \max_{p \geq 0} \{ [a - bp + (\beta_{k-1} - \beta_{k-2})^2] \\ &\quad \times [p - d + e\chi_{k-1} + c_{22}\beta_{k-2} - (c_{21} + c_{22} + e)\beta_{k-1}] \}. \end{aligned} \quad (13)$$

Straightforward calculus shows that the optimal price is

$$\begin{aligned} p &= \frac{1}{2b} [a + (\beta_{k-1} - \beta_{k-2})^2] + \frac{1}{2} [d - e\chi_{k-1} - c_{22}\beta_{k-2} \\ &\quad + (c_{21} + c_{22} + e)\beta_{k-1}], \end{aligned} \quad (14)$$

and that the maximal net profit is

$$\begin{aligned} r_k(\beta_{k-1}, \beta_{k-2}, \chi_{k-1}) &= \{ a + (\beta_{k-1} - \beta_{k-2})^2 + b[d - e\chi_{k-1} - c_{22}\beta_{k-2} \\ &\quad + (c_{21} + c_{22} + e)\beta_{k-1}] \}^2 / (4b). \end{aligned} \quad (15)$$

Constraining $0 \leq \beta_{k-2} \leq \beta_{k-1} \leq \chi_{k-1} \leq U$ for all $U < \infty$,

$$\begin{aligned} & 2b \frac{\partial^2 r_k(\beta_{k-1}, \beta_{k-2}, \chi_{k-1})}{\partial \beta_{k-1}^2} \\ &= [2(\beta_{k-1} - \beta_{k-2}) - b(c_{21} + c_{22} + e)]^2 + 2(\beta_{k-1} - \beta_{k-2})^2 \\ &\quad + 2b[a/b - d + e\chi_{k-1} - c_{22}\beta_{k-2} - (c_{21} + c_{22} + e)\beta_{k-1}] \\ &\geq 2b[a/b - d - (c_{21} + c_{22} + e)U] \end{aligned}$$

which is nonnegative, hence $r_k(\cdot, \beta_{k-1}, \chi_{k-1})$ convex, for sufficiently large values of a . Also, the maximal net profit is convex in β_{k-2} because

$$\begin{aligned} & 2b \frac{\partial^2 r_k(\beta_{k-1}, \beta_{k-2}, \chi_{k-1})}{\partial \beta_{k-2}^2} \\ &= [bc_{22} - 2(\beta_{k-1} - \beta_{k-2})]^2 \\ &\quad + 2b[a/b - d + e\chi_{k-1} - c_{22}\beta_{k-2} - (c_{21} + c_{22} + e)\beta_{k-1}] \\ &\quad + 2(\beta_{k-1} - \beta_{k-2})^2 \leq 2b[a/b - d - (c_{21} + c_{22})U]. \end{aligned}$$

Why is r_k a convex function of the new feature level β_{k-1} ? A technical reason is that the maximand in (13) is the product of two factors, and each one is convex in β_{k-1} , so the product is convex for appropriate parameters. Since the pointwise maximum of a collection of convex functions is itself convex, $r_k(\cdot, \beta_{k-2}, \chi_{k-1})$ is convex. An intuitive explanation of convexity stems from the factor $(\beta_{k-1} - \beta_{k-2})^2$ in (13) which models consumer response to product innovation. This factor is consistent with a negligible response to a small innovation but a disproportionately large response to a greater innovation. For example, in high-tech industries such as personal computers, a product must meet a certain level of performance to be competitive, and so there is a substantial penalty associated with falling slightly short of the technology frontier, while the marginal impact of falling even further behind the frontier is proportionately smaller (Ward et al. 1999).

5.3. Lower R&D Costs Accelerate Innovation

The next result shows that for a univariate technology, lower costs of R&D or process development induce more aggressive product innovation because the gain from frontier innovation is relatively larger. *Although this result is intuitive, there are counterexamples if the*

assumptions of Theorem 1 are relaxed, or if there are two or more features in the model. We consider the effects during $[T_k, T_{k+1})$ of replacing the R&D cost function $c(\cdot, \cdot)$ with $c^\#(\cdot, \cdot)$; let $B^\#$ and $Y_k^\#$ and denote a corresponding optimal bundle and optimand defined by (5). Recall that we constrain $B_{k+1} \in [B_k^p, x_k]$ when $J = 1$.

COROLLARY 1. *Under the assumptions of Theorem 1 with $J = 1$ (so that $\beta \leq B \leq \chi$ in (4)), suppose that $c(\beta, \beta) = c^\#(\beta, \beta) = 0$ and $c^\#(B, \beta) \leq c(B, \beta)$ for all $\beta \leq B$. If $B = \chi$ is optimal, then $B^\# = \chi$ is optimal in (2) and (4), and $Y_k(\chi, \beta, \chi) - Y_k(\beta, \beta, \chi) \leq Y_k^\#(\chi, \beta, \chi) - Y_k^\#(\beta, \beta, \chi)$.*

PROOF. If $B = \chi_k$ is optimal, then Theorem 1 implies $0 \leq Y_k(\chi, \beta, \chi) - Y_k(\beta, \beta, \chi) = -c(\chi, \beta) + E\{e^{-\alpha\tau_k(-, \beta, \chi)}(r_{k+1}[\beta + \xi_{k+1}(\chi - \beta), \beta, \chi] - r_{k+1}(\beta, \beta, \chi) + w_{k+1}[\beta + \xi_{k+1}(\chi - \beta), \chi + I_k] - w_{k+1}(\beta, \chi + I_k))\} \leq -c^\#(\chi, \beta) + E\{e^{-\alpha\tau_k(-, \beta, \chi)}(r_{k+1}[\beta + \xi_{k+1}(\chi - \beta), \beta, \chi] - r_{k+1}(\beta, \beta, \chi) + w_{k+1}[\beta + \xi_{k+1}(\chi - \beta), \chi + I_k] - w_{k+1}(\beta, \chi + I_k))\} = Y_k^\#(\chi, \beta, \chi) - Y_k^\#(\beta, \beta, \chi)$. \square

6. Numerical Example with Two Features

In this section we describe the optimization and sensitivity analysis of a numerical model with two features. We find that (i) a frontier improvement policy is optimal under more general conditions than are assumed in Theorem 1, including a dependence of the probability of success and development time on the degree of innovation; (ii) the general model that is formulated in §3 can yield policies in which it is optimal to bundle improvements (i.e., coordinate the improvements of separate features); and (iii) Proposition 1 enables the numerical solution of models which would otherwise be extremely difficult or impossible to solve. We also identify conditions under which a firm is more or less likely to innovate and to bundle improvements in product features. The computational study entails the solution of 729 dynamic programs, each having a 10-period planning horizon. Each dynamic program roughly corresponds to a Markov decision process with 60,000 states and 130 feasible actions in each state. The study might not have been computationally feasible without the acceleration provided by Proposition 1.

6.1. Model and Parameters

The model has two features ($J = 2$), and the two coordinates of the technology frontier $\{\chi(t) : t > 0\}$ are independent Poisson processes with intensities λ_1 and λ_2 (for Features 1 and 2, respectively). The salvage value function was $s \equiv 0$, and each coordinate of χ was truncated at 5. The model was discretized by confining each coordinate of B and β to be a nonnegative integer no greater than the corresponding component of χ . The model was solved for a planning horizon of $K = 10$ product versions in the product life cycle, so in §6.2 we discuss the characteristics of optimal decisions in period 1, i.e., when there are nine subsequent innovation opportunities.

The unit cost rate is similar to expressions in §5.2. Let the superscript designate the feature; so $B = (B^1, B^2)$, $\beta_{k-1} = (\beta_{k-1}^1, \beta_{k-1}^2)$, $\beta_{k-2} = (\beta_{k-2}^1, \beta_{k-2}^2)$, and $\chi = (\chi^1, \chi^2)$. The demand function is $(100 - Dp) \cdot f(\beta_{k-1}, \beta_{k-2})$ where D parameterizes consumer sensitivity to price and

$$f(\beta_{k-1}, \beta_{k-2}) = \left\{ 1 + (\beta_{k-1}^1 \beta_{k-2}^2 + \beta_{k-1}^2 \beta_{k-2}^1) / \phi + \sum_{j=1}^2 [(\beta_{k-1}^j - \beta_{k-2}^j) - \Delta(|\beta_{k-2}^j - \beta_{k-1}^j|)] \right\}^+, \quad (16)$$

where $\Delta(z) = 1$ if $z > 0$, and $\Delta(z) = 0$ if $z \leq 0$. The term $-\Delta(|\beta_{k-2}^j - \beta_{k-1}^j|)$ satisfies (10) and (11) but would fail to satisfy a stronger version of (10) where $r_{k+1}(\cdot, \beta, \xi)$ is obliged to be convex on $[0, \chi]$. The term $(\beta_{k-1}^1 \beta_{k-2}^2 + \beta_{k-1}^2 \beta_{k-2}^1) / \phi$ satisfies (10).

The R&D cost function is $c(B, \beta) = \sum_{j=1}^2 C_1^j \ln[1 + (B^j - \beta^j)^+]$, the unit production cost is $C_2^1 * \beta_{k-1}^1 + C_2^2 * \beta_{k-1}^2$, and the unit cost of adverse quality is $3 - C_3^1(\chi^1 - \beta_{k-1}^1) - C_3^2(\chi^2 - \beta_{k-1}^2)$. The continuous-time discount factor is $\alpha = 0.8$, and a net revenue rate function $S(\cdot)$ can be specified analytically as is $r_k(\cdot)$ in §5.2 by maximizing intra-period profit not including R&D cost:

$$S(\beta_{k-1}, \beta_{k-2}, \chi) = \max\{(100 - Dp)f(\beta_{k-1}, \beta_{k-2})[p - c_2(\beta_{k-1}) - c_3(\beta_{k-1}, \chi)] : p \geq 0\} \quad (17)$$

One of our goals is to investigate numerically whether the conclusions of Theorem 1 are valid if

some of its assumptions are violated. In order to accommodate the greater generality of the numerical model in this section, we alter (3) as follows (where \mathcal{T} denotes the expected present value of the duration of R&D):

$$v_k(\beta_{k-1}, \beta_{k-2}, \chi_k, \chi_{k-1}) = \max\{S(\beta_{k-1}, \beta_{k-2}, \chi_{k-1})\mathcal{T}(B, \beta_{k-1}, \chi_k) - c_1(B, \beta_{k-1}) + E(e^{-\alpha\tau(B, \beta_{k-1}, \chi_k)} \times v_{k+1}[\beta_{k-1} + \xi_k(B - \beta_{k-1}), \beta_{k-1}, \chi_k + I_k, \chi_k]) : 0 \leq B \leq \chi_k\} \quad (18)$$

If $B = \beta$, we let $\tau(\beta, \beta, \chi)$ be exponentially distributed with mean $1/(\lambda_1 + \lambda_2)$. In words, if there is no innovation, then the length of time until the next R&D opportunity is the elapsed time until the next jump in either coordinate of the technology frontier. If $B \neq \beta$, we let the time for development $\tau(B, \beta, \chi)$ be exponentially distributed with mean ν (specified below), and so $\mathcal{T}(B, \beta, \chi) = E[\int_0^{\tau(B, \beta, \chi)} e^{-\alpha y} dy] = \nu/(\alpha\nu + 1)$. The probability of successful completion is $P\{\xi = 1\} = q + (1 - q)e^{-\nu}$, $0 < q < 1$, and the probability of premature termination is $P\{\xi = 0\} = (1 - q)(1 - e^{-\nu})$. We define

$$\nu = \mu \frac{(\chi_k^1 - \beta_{k-1}^1) + (\chi_k^2 - \beta_{k-1}^2)}{(\chi_k^1 - B^1) + (\chi_k^2 - B^2) + \epsilon'}$$

which captures the property that the development time stochastically increases and the probability of success decreases as the ambition of the product version increases. In other words, lower B (or higher β_{k-1}) decreases ν , so that mean development time decreases and probability of success increases. The ϵ term precludes division by 0. For the computational study, we set $q = 0.3$, $\epsilon = 3$, and varied μ as described below.

There is an economic rationale for each component of the revenue and cost functions. In (16), the coefficient ϕ in the term $(\beta_{k-1}^1 \beta_{k-2}^2 + \beta_{k-1}^2 \beta_{k-2}^1) / \phi$ parameterizes the effect on demand of the interactions among features across time. We refer to ϕ as the synergy parameter. The difference between the β_{k-2}^j s and β_{k-1}^j s reflects the magnitude of the innovative step. So, $\sum_{j=1}^2 (\beta_{k-1}^j - \beta_{k-2}^j)$ scales the benefit gained from innovation, and the last factor reflects fixed costs associated with regressing and advancing a feature.

The current bundle β_{k-1} should clearly be a factor in the direct cost of manufacturing per unit made and

Table 1 Parameter Values for Computational Study

Technology frontier		Development time & risk	R&D cost		Production cost		Consumer price sensitivity	Synergy
λ_1	λ_2	μ	C_1^1	C_1^2	C_2^1	C_2^2	D	ϕ
0.4	0.4	0.5	50.0	20.0	10.0	5.0	5.0	0.5
0.4	0.8	1.0	50.0	50.0	10.0	10.0	10.0	5
0.8	0.8	1.5	50.0	80.0	10.0	15.0	15.0	50

sold because it is reasonable to expect that a more aggressive current bundle would be more expensive to produce. We could include a learning curve effect by adding a dependence on β_{k-2} (the more advanced the features in the previous period, the better the chance that process improvements would carry over).

The unit cost of adverse quality includes the costs of returns, repairs, and replacements. More aggressive innovation (β_{k-1} larger) results in higher expected quality costs; in addition, the closer that the current bundle of features was to the technology frontier at the time that it was developed, the higher the risk of quality problems due to leading-edge features.

To consider the effects of different cost and demand scenarios on innovation decisions, we systematically varied the speed of technology change for both features (λ_1 and λ_2), probability distribution of development time and risk (μ), two cost coefficients for feature 2 (C_1^2 and C_2^2 , considering the two features to be symmetric), consumer sensitivity to price (D), and the degree of synergy (ϕ). Since Feature 2's variable cost of production C_2^2 and variable cost of quality C_3^2 are both linear in the example, the marginal total cost can be varied with either one; we only varied the former. Varying the first two parameters over three combinations of values and the remaining parameters over three values generated 729 different scenarios. See Table 1 for the parameter values. The resulting problem with $K = 10$ product versions in the life cycle took a little more than eight minutes per scenario (for 729 scenarios, about 97 hours) to solve coded in FORTRAN 77 on a SUN Ultra 10 (Solaris O.S. 2.8).

Since there are no studies linking the rate of technology change and cost factors to different industrial phenomena, we determined the range of parameters through pilot tests in which we ascertained values that produced interesting results, i.e., varying a

parameter caused different policies in terms of innovation and bundling behavior.

6.2. General Discussion of Results

In the 729 scenarios, the following outcomes were observed (in order of frequency from most to least frequent) in Period 1: (1) *stays* (B^j stays at β^j); (2) *jumps* (B^j jumps from β^j to χ^j , i.e., innovation to the frontier); (3) *back to 0* (B^j regresses from β^j to 0); (4) *back > 0* (B^j regresses from β^j but does not regress to 0 [note that this last is an outcome that would *not* be generated if the revenue function were actually convex]). *There were no cases of incremental innovation in more than 87×10^6 opportunities.* See Table 2. Regressing a feature would mean, for example, increasing the bulk and weight of a scanner in order to add photographic capability. The cost/revenue trade-off for regression comes from the decrease in unit production and adverse quality costs when β_{k-1} regresses and there is a concomitant decrease in revenue (the last term in (16) is positive when $\beta_{k-2}^i > \beta_{k-1}^i$).

The assumptions underlying Theorem 1 are sufficient but not necessary. Therefore, the computational results that we present below suggest managerial insights related to the effects of costs (R&D, production, adverse quality), demand (price elasticity), development time and risk, speed of technology change, and degree of demand synergy on optimal innovation and bundling behavior. In the following discussion, a "feature jump" is the advance of a single feature to the technology frontier. A "bundling event"

Table 2 Innovation: General Results

	Stays	Jumps	Back to 0	Back > 0
Total	57,361,299	24,746,209	5,215,358	193,583
% Total	65.54	28.28	5.96	0.22

is a combination of circumstances that demonstrate that one feature moves forward only if the other feature can move forward too.

Within the set of 729 scenarios, we can make pairwise comparisons between two specific scenarios that differ in terms of only one parameter value, in order to determine how changes in the parameter affect innovation and bundling behavior. For example, there are a total of 243 scenarios in which D is equal to 5 and 243 scenarios in which D is equal to 10. Pairing these scenarios enables us to observe the effect of an increase in D from 5 to 10 under 243 different sets of conditions. The same comparison can be made as D increases from 10 to 15. Thus, our set of 729 parameters allows us to make a total of 486 pairwise comparisons to determine how an increase in D affects innovation and bundling under 486 different sets of conditions.

6.3. Innovation Results

The frequency of jumps to the technology frontier is affected by both the internal and external environment in which a firm operates. The internal environment includes those factors over which the firm has some degree of control, such as the production process. In contrast, elements of the external environment, such as the type of market in which it operates, are more difficult to control. We find that the frequency of innovation is determined primarily by the main and interaction effects of three internal parameters and three external parameters.

Internal Conditions Encouraging Innovation to the Technology Frontier:

Long Product Development Time and High Risk of Failure. As indicated in Table 3 (and in all pairwise comparisons), longer product development times and

lower chances of success (larger μ), result in innovation in more circumstances. This is the most pronounced effect in terms of magnitude of increase as the parameter in question increases. Intuitively, the firm takes more of the available opportunities to innovate when it will have to wait longer to realize results, and there is a higher risk of failure.

Low Cost of R&D. As indicated in Table 4 (and in 99% of all pairwise comparisons), B^1 and B^2 jump more frequently to the technology frontier as C_1^2 decreases. This is the parameter that scales the R&D cost for B^2 . When a firm faces higher R&D costs for a feature, the economies of scale are more pronounced; so it would be inclined to batch improvements, scheduling them less often, in order to take advantage of economies of scale. In contrast, when the cost of R&D is relatively low, it is not as critical to achieve further reductions in this cost, so more frequent innovation is likely.

High Unit Cost of Production and/or Adverse Quality. As indicated in Table 4, the main effect of an increase in C_2^2 , the unit cost of production (or adverse quality) for B^2 , is an increase in the frequency with which B^1 and B^2 jump to the technology frontier. When a firm faces higher unit costs of production or adverse quality, it is particularly important to exploit measures such as product differentiation in order to increase per-unit revenue. The magnitude of this effect, however, is influenced by two other factors. First, a longer product development time increases the magnitude of this effect. At the shortest product development time ($\mu = 0.5$), an increase in the unit cost results in more frequent innovation for both features in 28% of the pairwise comparisons. This percentage increase rises to 88% when $\mu = 1$ and to 94% when $\mu = 1.5$. As discussed previously, when there is considerable time between innovation opportunities and the probability of success is low, a firm cannot afford to wait too long to innovate nor be too cautious in terms of the extent of innovation. Second, this effect is accelerated by faster movement of the technology frontier. At the slowest technology speeds ($\lambda_1 = \lambda_2 = 0.4$), an increase in the unit cost results in more frequent innovation for both features in 56% of the pairwise comparisons. However, the percentage increase rises to 65% when $\lambda_1 = 0.4$ and $\lambda_2 = 0.8$ and to 88% when $\lambda_1 = \lambda_2 =$

Table 3 Innovation and Bundling as Mean Development Time and Risk of Failure Change

	$\mu = 0.5$	$\mu = 1.0$	$\mu = 1.5$
Total jumps	3,145,174	9,720,846	11,880,189
% total jumps	12.71	39.28	48.01
Total bundles	144,151	349,070	343,810
% total bundles	17.22	41.70	41.08

Table 4 Innovation and Bundling as R&D and Production Costs Change

	$C_1^2 = 20$	$C_1^2 = 50$	$C_1^2 = 80$	$C_2^2 = 5$	$C_2^2 = 10$	$C_2^2 = 15$
Total jumps	8,483,620	8,234,647	8,027,942	8,060,178	8,190,908	8,495,123
% total jumps	34.28	33.28	32.44	32.57	33.10	34.33
Total bundles	277,576	279,587	279,868	270,647	279,724	286,660
% total bundles	33.16	33.40	33.44	32.33	33.42	34.25

0.8. We see the pull of the technology frontier, with the additional advantage that adverse quality costs decline when the frontier moves out from the current state of the product. Conversely, a higher risk of adverse quality is experienced on the “bleeding edge” of the frontier, as described to us by the quality team of the scanner manufacturer.

External Conditions Encouraging Innovation to the Technology Frontier:

Low Synergy Between Product Features (as Valued by the Customer). As indicated in Table 5 (and all pairwise comparisons), as synergy between features increases (the parameter ϕ increases), there are fewer jumps to the technology frontier. Recall that high synergy between product features means that an advance in one feature now increases the return to advancing another feature. When this is *not* the case—i.e., synergy is relatively low—although customers value advances in all features, these advances need not be simultaneous. Since there is less incentive to wait for another feature to be ready to innovate, there are more jumps of a feature by itself. We elaborate on this point in §6.4.

High Degree of Customer Price Sensitivity. As indicated in Table 5, innovation occurs in more circumstances when customers are relatively sensitive to price. Therefore, product differentiation as a means of stimulating demand becomes increasingly important (cf. Adner and Levinthal 2001). The magnitude of

this effect, however, is influenced by the speed of the exogenous technology frontier. When $\lambda_1 = \lambda_2 = 0.4$, an increase in price sensitivity results in more frequent innovation for both features in 78% of the pairwise comparisons. This percentage increases to 85% when $\lambda_1 = 0.4$ and $\lambda_2 = 0.8$ and to 98% when $\lambda_1 = \lambda_2 = 0.8$. In short, a fast-moving technology frontier increases the opportunities for innovation.

Fast-Moving Exogenous Technology Frontier. As indicated in Table 6 (and over 99% of all pairwise comparisons), faster-moving technology frontiers result in innovation in more circumstances. In short, when technology frontiers move forward at a relatively faster rate, a firm has more to lose from missing innovation opportunities. It is interesting to note that an increase in the speed of the technology frontier for one feature results in more innovation for both features. In other words, coordination of advances across features exists. For example, when $\lambda_1 < \lambda_2$, advances in B^2 are constrained by the now relatively slow-moving technology frontier for B^1 , as B^2 “waits” for B^1 so that both can jump to a new frontier. An increase in λ_1 eliminates this bottleneck and allows both features to jump to their respective frontiers. We discuss coordination and this bottleneck phenomenon more extensively in §6.4.

Diminishing returns is an attribute of some R&D settings that we do not model in this paper. If the R&D cost function were convex instead of concave,

Table 5 Innovation and Bundling as Synergy and Price Elasticity Changes

	$\phi = 0.5$	$\phi = 5$	$\phi = 50$	$D = 5$	$D = 10$	$D = 15$
Total jumps	12,024,726	7,025,814	5,695,669	7,485,563	8,455,382	8,805,264
% total jumps	48.59	28.39	23.02	30.25	34.17	35.58
Total bundles	261,167	284,359	291,001	259,791	283,877	293,363
% total bundles	31.26	33.97	34.77	31.03	33.92	35.05

Table 6 Innovation and Bundling as Speed of Technology Frontier Changes

	$\lambda_1 = 0.4,$ $\lambda_2 = 0.4$	$\lambda_1 = 0.4,$ $\lambda_2 = 0.8$	$\lambda_1 = 0.8,$ $\lambda_2 = 0.8$
Jumps/Feature 1	2,927,195	4,227,330	5,209,836
Jumps/Feature 2	2,923,455	4,236,492	5,221,901
Total jumps	5,850,650	8,463,822	10,431,737
% total jumps	23.64	34.20	42.15
Total bundles	236,724	276,725	323,582
% total bundles	28.28	33.06	38.66

both incremental innovation and innovation to the frontier would occur. Details are available from the authors.

6.4. Bundling Results

Just as with innovation to the technology frontier, the frequency of bundling is influenced by internal and external factors; there are primarily three model parameters that drive this behavior. In particular, the main effects of the speed of the technology frontier and the synergy between product features, and interaction effects with product development time and risk, together suggest when coordination of improvements among features is advantageous. Recall that when improvements in product features are bundled, one feature moves forward only if the other feature can move forward as well. For bundling to take place, the return to coordinating advances across features must outweigh the opportunity cost associated with waiting. When this is the case, we may think of the feature with the slower-moving technology as the bottleneck (as discussed previously). The main effects are:

Significant Synergies Between Product Features (as Valued by the Customer). When there are significant synergies between product features as valued by the customer, the return to coordinating advances between features (and delaying advancement of individual features) is greater. We find that as synergy between features increases (ϕ increases), the frequency of bundling behavior over all other parameter values increases from 31% ($\phi = 0.5$) to 34% ($\phi = 5$) to 35% ($\phi = 50$). This result is also consistent with the discussion in §6.3 (see Table 5).

Fast-Moving Exogenous Technology Frontier.

When technology advances rapidly, coordinating advances between features becomes less costly since the time spent waiting for features to catch up is reduced. Looking at the frequency of bundling behavior over all other parameter values as λ_1 and/or λ_2 rise, it increases from 28% ($\lambda_1 = \lambda_2 = 0.4$) to 33% ($\lambda_1 = 0.4$ and $\lambda_2 = 0.8$) to 39% ($\lambda_1 = \lambda_2 = 0.8$). See also Table 6.

The magnitude of each of the main effects is mitigated by interaction effects. Consider increases in the speed of the technology frontier, for example, which typically increase the frequency of bundling. This effect holds as long as the product development time is sufficiently short. In particular, at the stochastically shortest product development time ($\mu = 0.5$), an increase in the speed of the technology frontier results in more frequent bundling behavior in 100% of the pairwise comparisons. This percentage decreases to 60% when $\mu = 1$ and to 2% when $\mu = 1.5$. In this case, even if technology is advancing rapidly, bundling behavior is less likely because coordination of advances simply adds more time to an already long and risky product development process.

Similarly, the main effect of the speed of the technology frontier is also reduced if the level of synergies between features is sufficiently low. At the greatest level of synergy ($\phi = 50$), an increase in the speed of the technology frontier results in more frequent bundling behavior in 58% of the pairwise comparisons. This percentage decreases to 54% when $\phi = 5$ and to 49% when $\phi = 0.5$. This example illustrates how the return from waiting diminishes as the degree of synergy decreases. As a result, a firm “chases” the technology frontier to increase demand instead of waiting to coordinate advances across features.

7. Conclusion

Our model of product innovation shows that decisions about enhancing product features should be influenced by both the internal and external environment in which a firm operates. This study was motivated by differential innovation strategies in a number of industries, and was influenced by interviews with decision makers in a scanner

manufacturing firm. The model considers interactions among revenues and costs, stage of product life cycle, features of the previous product release, and the externally propelled frontier of technology.

We begin by considering a product with multiple features, each of which may be enhanced by the current development effort. The decision about which bundle of features to include in the version of the product now being prepared for market depends on costs, projected revenues, and the external frontier of technology. The results include characteristics of optimal feature selection and dynamic programs to compute optima. We use a dynamic program to deduce the structure of an optimal policy for innovation, present conditions that permit the dynamic program to be streamlined dramatically, and employ the streamlined program to solve a large numerical example with two features under more general conditions.

In the computational study the development time and probability of success depend on the scope of product improvement. We find additional insights into the impact of specific internal and external conditions on both the frequency of innovation to the technology frontier and the degree of coordination of advancement activities across features. Innovation to the frontier is more likely when the internal environment of a firm is characterized by long product development times, low cost of R&D, and high unit cost of production and/or adverse quality. Such innovation may also be driven externally by price-sensitive customers who value innovation, and the innovation need not be coordinated across features. We also find that a fast-moving exogenous technology frontier increases the frequency of innovation.

Also, bundling behavior is more likely when technology frontiers advance rapidly, particularly when development times are relatively short and risks low. We also find more bundling when customers value features that advance together. In these instances, the relative return to waiting increases, so the frequency of coordination increases. It is important to note that there are significant interaction effects among the internal and external parameters driving innovation and bundling activity.

These structural and computational results reinforce and elucidate what we learned from the scanner

firm. First, aggressive innovation is most favorable when there is a good chance of expanding the market with the new product, and when costs of research and development yield increasing returns to successful advances in product technology, for example, when larger innovations (such as a drastically miniaturized scanner system) have the potential to open a new market, i.e., innovation is expected to expand demand. Smaller innovations are not as interesting to customers, and in fact there are annual price decreases on products that do not have significant innovation. This demonstrates a firm's sensitivity to the trade-off between demand elasticity and customer response to new product features. Innovations are likely to be combined in new versions or products when the state of technology is progressing relatively rapidly, and less likely to be bundled when there is less return to such synergies, compared to the time and risk of long development periods.

These results apply to firms with the following characteristics: technology driven, technology takers, integrators of components, competitive markets where price or feature improvement is needed to expand the market, quality affects the bottom line because of commitment to customer satisfaction via warranties as well as internal quality control, and ability to absorb high R&D costs in order to remain market leaders.

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