New Product Launch Decisions with Robust Optimization

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March 2014, revised November 2014

Abstract

We consider a problem where a company must decide the order in which to launch new products within a given time horizon and budget constraints, and where the parameters of the adoption rate of these new products are subject to uncertainty. This uncertainty can bring significant change to the optimal launch sequence. We present a robust optimization approach that incorporates such uncertainty on the Bass diffusion model for new products as well as on the price response function of partners that collaborate with the company in order to bring its products to market. The decision-maker optimizes his worst-case profit over an uncertainty set where nature chooses the time periods in which (integer) units of the budgets of uncertainty are used for worst impact. This leads to uncertainty sets with binary variables. We show that the robust problem can nonetheless be reformulated as a mixed integer linear programming problem, is therefore of the same structure as the deterministic problem and can be solved in a tractable manner. Finally, we illustrate our approach on numerical experiments. Our model also incorporates contracts with potential commercialization partners. The key output of our work is a sequence of product launch times that protects the decision-maker against parameter uncertainty for the adoption rates of the new products and the response of potential partners to partnership offers.

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1 Introduction

We consider the introduction of a set of innovative products or services in a national market within a finite time horizon. Our goal is to determine the optimal launch time for each product when the parameters of the products' adoption curves are not known precisely and the company is assumed to have a limited marketing budget at each period. The products or services under consideration can be durable goods with different purposes, drugs for different diseases or a single product or service launched sequentially in multiple geographic markets. We neglect the substitution effect among the products and assume that the adoption processes of the products are not affected by each other. Examples include a car-sharing company launching its service in certain cities, an innovative retail kiosk service aiming at providing customers with health care information or an express courier service launching new pickup or delivery options in various metropolitan areas.

We use the product growth model introduced by Bass (1969) to estimate the adoption rate of the customers for each product. The Bass diffusion model and its revised versions have been used for forecasting the diffusion of innovation in durable goods, pharmaceutical, and industrial technology markets, among others. It considers two types of potential adopters, namely innovators and imitators, and assumes that two communication channels are used to influence the potential adopters: mass media and word of mouth. The innovators are affected by the external influence (mass media), whereas the imitators' motivation to adopt the innovation comes from the internal influence of the customers who have already adopted the innovation. The Bass model can estimate the long term sales patterns of an innovative product in the following two cases (Lilien et al, 2007):

- The new product has already been introduced to the market and the first few periods' sales amounts have been observed,
- The new product has not been introduced to the market; however, an existing product's diffusion process can be used as a proxy for the product of interest.

Using the basic Bass model requires estimating three parameters for each product: $m,\ p,$ and q for each product, which stand for the potential number of ultimate adopters in the market, the coefficient of external influence, and the coefficient of internal influence, respectively.

Srinivasan and Mason (1986) show that reliable estimates for the parameters can be obtained when the available data set is large enough to cover the rate peak of the product's adoption curve. The estimates for the coefficients of internal and external influence are subject to uncertainty and might depend on time. On the other hand, the parameter m_i (the ultimate number of adopters of product i, or steady-state market size) is not expected to be time-dependent; however, it is subject to estimation errors, and it can be forecast more accurately after the first few periods' sales amounts are revealed and analyzed.

For each new product, the company seeks a partner in commercialization, whose willingness to enter a partnership depends on the proposed unit payments for the service it will provide. The partner might help the innovative company establish the infrastructure to disseminate the new product, such as providing space to park the cars in the car-sharing service, space to install the self-service kiosks, or the provision of services such as shipping and handling or customer service. For instance, a bank may seek partnerships with information technology firms or a manufacturer with brick-and-mortar retailers for product placement. A potential partner's probability to accept an offer is modeled as a logit model with the unit payment amount as the main variable. In other words, the innovative company offers the partner company a specific payment amount per new adopter for its collaboration starting from the period when the product is launched until the end of the time horizon. For a potential partner j, given the logit model parameters a_j and b_j, the probability of this potential partner agreeing to collaborate with the innovative company when the unit payment offered by the company is R_j is represented as:

$$P_j(\text{Yes}) = \frac{e^{(a_j+b_jR_j)}}{1+e^{(a_j+b_jR_j)}} = \frac{1}{1+e^{-(a_j+b_jR_j)}}$$

The parameters a_j and b_j of the logit functions are estimated based on available data or managerial judgments. Therefore, they are subject to uncertainty as well.

Our main goal in this paper is to propose a tractable mathematical framework that incorporates parameter uncertainty in the new product introduction problem with partnership and answers the following questions for the innovative company:

- How should it schedule the launch of each innovative product/service?
- Which partner should it select for each product so that the total expected profit over a specific time horizon is maximized?

The main example we use to motivate our approach is that of a new service launched sequentially in multiple geographic areas, when the launch requires partnerships with other companies, providing for instance warehouse space or retail capabilities that the innovative company does not have. Those partnership contracts provide a fee to the partner in exchange for agreed-upon services, such as providing the infrastructure for the launch.

We assume that the unit price of the product is determined and constant. Therefore, we calculate the present value of the revenue obtained from new products' sales by discounting the number of adopters in each period by a constant discount factor.

1.1 Literature Review

1.1.1 Diffusion of Innovations and the Bass Model

The diffusion of innovation is achieved by propagation through certain communication channels over time in a social system (Rogers, 1983). The adoption of new technologies over time usually follows an S-curve, with four main classes of models: epidemic models, probit models, models of density dependence, and models of information cascades (Geroski, 2000). Epidemic models generally assume that the diffusion of innovation occurs by means of direct contact with the previous adopters or by imitating them. In addition, they depend on the premise that the potential adopters form a homogeneous population in terms of their needs and willingness to adopt innovation. The probit models address the fact that different potential adopters have heterogeneous preferences and abilities to adopt the new technologies at different times. The density dependence models capture the balance of the impacts of legitimation and competition in adopting innovation. Finally, the main idea behind the information cascade models is that the adopters make sequential decisions rationally based on the information they have. In addition, the subsequent speed of the diffusion of the new technology depends on the initial choice of the adopter.

The diffusion of innovation was first introduced in the marketing community in the 1960s (Frank et al (1964), Arndt (1967) and Bass (1969)). A review of new product diffusion models up to 1990 is provided in Mahajan et al (1990), which reviews the Bass (1969) model and its extensions to various markets including the retail services, pharmaceutical industry, consumer durables market, and industrial technology. As explained earlier, potential adopters are divided between innovators and imitators. Tidd (2006) suggests that the diffusion process occurs in an epidemic form for imitators; however, the innovators are not subject to social emulation. Therefore, the adoption of the innovators in early periods is followed by that of the imitators in later periods. This leads to a skewed S-curve for the adoption rate for the whole population. The Bass model also assumes that a member of the population can adopt the product only once and that the probability of an adoption at time t can be modeled as a hazard rate. Specifically, let us denote the density function of time to adoption as $f(t)$ and the cumulative fraction of adopters at time t as $F(t)$. Then the hazard function leads to the following equality:

$$\frac{f(t)}{1-F(t)} = p + qF(t),$$

where the parameter p stands for the external influence and the parameter q reflects the internal influence resulting from earlier adopters. The function $F(t)$ is assumed to be a non-decreasing function and approaches 1 as t gets larger. In addition, it assumes that the process starts with no initial adopters ($F(0) = 0$ and $f(0) = 0$.)

If q is zero, $f(t)$ follows a negative exponential distribution (Mahajan et al, 1990). Lilien et al (2007) provide the following additional insights: if $q \geq p$, then the innovation influence is dominated by the imitation influence and the plot of $f(t)$ versus time has an inverted U shape. Otherwise, the innovation influence prevails over the imitation influence, the highest sales are observed at the onset and the rate of adoption decreases as time passes. In addition, a decrease in p leads to a longer time to realize the sales growth for the innovation. Furthermore, if both p and q are large, the adoption rate takes off rapidly and falls off quickly after reaching its peak point (Mahajan et al, 1990).

With the parameter m denoting the potential number of ultimate adopters, the number of (new) adopters at time t, $S(t)$, and the cumulative number of adopters at time t, $C(t)$, are represented as:

$$S(t) = mf(t), \text{ and } C(t) = mF(t).$$

It can be shown that the following expressions hold for $f(t),\ F(t),\ S(t),$ and $C(t)$:

$$\begin{aligned} f(t) &= \frac{(m+q)^2}{p}\frac{e^{-(p+q)t}}{\left(1+\frac{q}{p}e^{-(p+q)t}\right)^2}, \\ S(t) &= m\frac{(m+q)^2}{p}\frac{e^{-(p+q)t}}{\left(1+\frac{q}{p}e^{-(p+q)t}\right)^2}, \\ F(t) &= \frac{1-e^{-(p+q)t}}{1+\frac{q}{p}e^{-(p+q)t}}, \\ C(t) &= m\frac{1-e^{-(p+q)t}}{1+\frac{q}{p}e^{-(p+q)t}}. \end{aligned} \tag{1}$$

In addition, the period when the sales amount peaks (T^*) and the marginal sales at peak time are given by:

$$T^* = \frac{1}{p+q}\ln\left(\frac{q}{p}\right) \text{ and } S(T^*) = \frac{m}{2}\left(1-\frac{p}{q}\right) \tag{2}$$

In our paper, we will use these equations in a discrete-time setting for notational convenience and clarity.

This model has been the focus of multiple extensions. In particular, Kalish and Lilien (1986) address the impacts of perceived product quality and information level in the market place (advertisement) in a period on the number of new adopters in that period. Bass et al (1994) incorporate pricing and advertising decisions in their formulation. Kamrad et al (2005) propose a stochastic model of innovation diffusion and determine the optimal advertisement and pricing policies using a stochastic dynamic programming approach. Finally, Kumar and Krishnan (2002) reformulate the adoption rate so that one country's diffusion process impacts the other, and capture the lag-lead, lead-lag, and lag-lag (simultaneous) impacts of inter-country interactions on the diffusion processes.

1.1.2 Parameter Estimation

Using the Bass model requires the estimation of the three parameters $m,\ p,$ and q. If the product has already been introduced to the market and some sales observations are available, historical data sets are used for estimating the parameters with ordinary least squares (OLS) methods (Young and Ord, 1985) and maximum likelihood estimation procedures (Schmittlein and Mahajan, 1982). The quality of the estimation results depends on the number of data points available (Hyman, 1988). Srinivasan and Mason (1986) show that reliable estimates for the parameters can be obtained when the available data set is large enough to cover the rate peak of the adoption curve. Time-varying estimation procedures have also been proposed including Bayesian estimation and adaptive-filtering methods. Sultan et al (1990) update the initial estimates of the parameters p and q after obtaining new estimates by taking the weighted sum of these two estimates. Bretschneider and Mahajan (1980) propose a time-varying parameter estimation method based on a feedback filter. When no historical data is available, parameters can be estimated by expert judgments or using historical observations of the diffusion process of a similar product. In both settings, though, estimation errors are a concern and we will discuss their impact below. Uncertainty on the values of the p and q parameters, which we will assume to be time-dependent, will be approached using robust optimization with budgets-of-uncertainty-type uncertainty sets because it is unlikely that all the parameters will reach their worst-case values over the whole time horizon. The uncertainty on the m parameter will best be approached using a real options approach, since there is only a single m per product. Future work beyond the scope of this paper includes using robust optimization to mitigate uncertainty on the market-size m parameters when multiple products are considered. We first discuss robust optimization in Section 1.2 and then real options in Section 1.3.

1.2 Robust Optimization

1.2.1 Motivation for Robust Optimization with the Bass Model

In this section, we show on a small example how estimation errors affect the optimal product launch schedule, to make the case for a robust optimization approach. Let us consider a case where a company plans to launch five new products so that only one product is introduced in each period during five consecutive periods. In this setting, one can compute 120 different launch sequences ($5 \cdot 4 \cdot 3 \cdot 2 = 120$), numbered 1 through 120. (Knowing the specific strategy corresponding to a given index is not particularly insightful for this example and we omit this information here, but the reader can re-derive the list of strategies by looping on the index of the products launched first, second, third and fourth.) We will use the Bass model parameter values provided in Table 1 and assume that the discount rate is 0.001. For this small example, we assume that each product is identical in terms of potential partners and their choice models, in order to show the impact of the parameter uncertainty on the optimal strategy.

In the context of innovation adoption, we are particularly interested in robust *solutions*, because the choice of a specific metropolitan area to be first (or among the first) to have a new service, for instance, has an impact in the public's perception of this metropolitan area as an innovation hotbed. Hence, in this paper we will highlight changes in the optimal launch sequence when the parameters change. In practice, the decision-maker may also consider a more classical approach, comparing the objective achieved in the true model with uncertainty by the optimal launch sequence in the nominal model and that in the robust model.

Table 1: Nominal values of the Bass model parameters for each product

p	q	m
0.045	0.44	1000
0.044	0.42	1000
0.047	0.4	1000
0.043	0.43	1000
0.042	0.44	1000

Table 2 provides the optimal strategies for the cases where the q parameter of only one product changes (its new value is the nominal value multiplied by the coefficient in the first column, which varies from 0.6 to 1.4 and thus represents a change of at most $\pm 40\%$) while the other parameters take their nominal values. Table 3 summarizes the outcomes of the same analysis repeated for the p parameters. Note that the strategy shown when the coefficient is 1 represents the optimal strategy in the nominal case, which is Strategy 7 or 1-3-2-4-5. Both Products 1 and 5 have the same q value; Product 1 has the highest p value and Product 5 the smallest. We observe for instance that a change of $\pm 4\%$ in q_1 or q_4 is sufficient to make the optimal strategy change. This is also true of upward changes by 4% in p_1 and downward changes by 4% in p_5. More generally, Tables 2 and 3 show the sensitivity of the optimal strategy to the parameters q and p of each product, respectively; the results show that small changes in the parameters can change the strategy substantially. As an example, Strategy 112 represents the launch order 5-3-2-4-1, launching last the product that was initially launched first and first the product that was initially launched last, and is optimal for upward changes in q_1, downward changes in p_1 and upward changes in p_5 as small as 4%. (As further examples, Strategy 107 corresponds to 5-2-4-1-3, 111 to 5-3-2-1-4, 112 to 5-3-2-4-1 and 113 to 5-3-4-1-2.) We thus observe that when the coefficient q_i becomes slightly higher than the nominal case of 1, the corresponding product i launches last. While there are only a dozen different optimal strategies in each case, the precise launch sequence can vary a lot and there is no discernible commonalities among the sequences, which highlights the benefit of an analytical approach to determine the order launch.

Therefore, we believe that it is important for the decision maker to address uncertainty on the parameter estimates in the Bass diffusion model with partnership. In what follows, we present robust optimization as a technique to achieve that goal.

1.2.2 Mathematical Modeling of Uncertainty

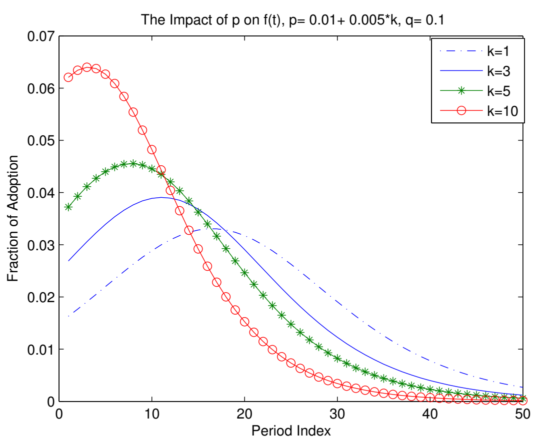
The parameters of the Bass model and the logit choice model are estimated through different processes, and are subject to different sources of uncertainty. For instance, the uncertainty affecting the logit choice parameters mostly results from the difficulty in predicting potential partners' responses to the company's offer. On the other hand, the uncertainty involved in the parameters of the Bass diffusion model is generally caused by the uncertainty in the potential customers' willingness to adopt the innovative product through mass media or word-of-mouth. Therefore, we model uncertainty for those two groups of parameters separately.

Revenue management under demand parameter uncertainty has for instance been studied in Rusmevichientong and Topaloglu (2012). Their setting focuses on assortment decisions given uncertainty in the multinomial logit model, which determines customers' preferences for products. The authors find an assortment that maximizes the worst-case expected revenue over a compact uncertainty set, both in the static and dynamic cases, leading to a 10% improvement in worst-case revenue over benchmark approaches. While this case is not directly applicable to our problem because (i) we do not decide *whether* to launch products or enter a geographic area (the equivalent of including a product in the assortment) but *when* in a finite horizon context, and (ii) our products are not substitutable (in the example of a new product being introduced successively in a series of geographic markets, the customer cannot change the city she resides in), Rusmevichientong and Topaloglu (2012) represents one of the earlier examples of robust revenue management applied to *nonlinear* optimization models, an area that the present paper also falls into.

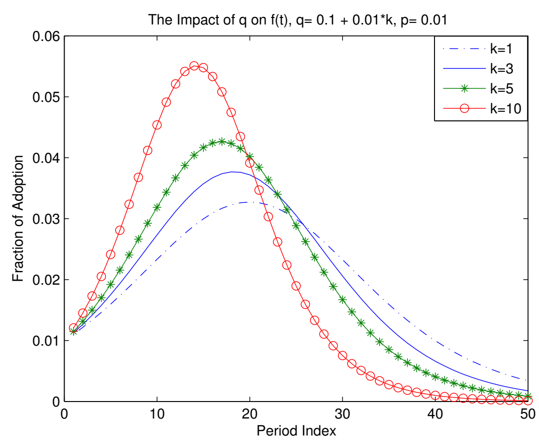
The key methodological tools we will use in modeling uncertain parameters consist of (i) range forecasts (or confidence intervals) centered at their nominal values and (ii) a budget-of-uncertainty constraint that limits the number of parameters that can deviate from their nominal value (also known as the Bertsimas-Sim model of uncertainty in robust optimization, first presented in Bertsimas and Sim (2004)). Due to the nonlinearities of the problem, we will constrain the parameters to be either equal to their nominal values or to one of the extremities of their confidence intervals. This use of a discrete budget of uncertainty is in line with the description of uncertainty in Bienstock and Ozbay Bienstock and Ozbay (2008) and is motivated by the need to balance a meaningful description of uncertainty with computational tractability. Classical robust optimization is preferred here to distributionally robust optimization due to the difficulty in estimating probabilities for model coefficients and tractability issues.

Bass model parameters

To mitigate the risk of over-conservatism in the approach, we make the following key modeling decisions. First, we allow the p's and q's to vary *with time* and by product. At each time period, each parameter p_{it} and q_{it} (driving the marginal adoption rate at that time period) belong to an interval $[p_i - \hat{p}_i, p_i + \hat{p}_i]$ and $[q_i - \hat{q}_i, q_i + \hat{q}_i]$, respectively. This is motivated as follows. As the two graphs in Figure 1 indicate, the diffusion process with the *smallest* p parameter value results in the lowest adoption rates from the beginning of the process until the 14^{th} period in the example, but the process with the *highest* p parameter value leads to the smallest adoption rates after the 14^{th} period. Similarly, the diffusion process with the *smallest* q parameter value provides the smallest adoption rates until the 21^{st} period; however, the one with the *largest* q parameter value leads to the smallest adoption rates thereafter. Therefore, the worst-case values of parameters p_i and q_i can be their smallest or highest values depending on the time elapsed since the start of the diffusion process.



(a) Varying p, constant q



(b) Varying q, constant p

Figure 1: The impact of q and p on marginal adoption rates

The model we devise will identify the worst-case values of the parameters at each time period among three possible choices (high/nominal/low) for each p and q. In particular, in this model, nature as a "malicious adversary to the decision-maker" will decide whether two units of the budget of uncertainty (one for p and one for q) should be spent to protect against uncertainty at a given time period for a given product, or only one (and if so, which parameter it should be spent on), and whether the unit of uncertainty budget should be spent on the parameter being higher or smaller than expected in that time interval. Note that at each time period and for each product, this leads to 9 possible configurations: 1 for the all-nominal case, 4 for the 1-unit case (either on p or on q, either to the maximum value or the minimum), 4 for the 2-unit cases (4 combinations of maximum or minimum values for the two parameters).

In mathematical terms, the marginal adoption rate for product i at time period t takes one of the following 9 values:

- $f_i^0(t) = f(t, \bar{q}_i, \bar{p}_i)$
- $f_i^1(t) = f(t, \bar{q}_i - \hat{q}_i, \bar{p}_i)$
- $f_i^2(t) = f(t, \bar{q}_i + \hat{q}_i, \bar{p}_i)$
- $f_i^3(t) = f(t, \bar{q}_i, \bar{p}_i - \hat{p}_i)$
- $f_i^4(t) = f(t, \bar{q}_i, \bar{p}_i + \hat{p}_i)$
- $f_i^5(t) = f(t, \bar{q}_i - \hat{q}_i, \bar{p}_i + \hat{p}_i)$
- $f_i^6(t) = f(t, \bar{q}_i + \hat{q}_i, \bar{p}_i + \hat{p}_i)$
- $f_i^7(t) = f(t, \bar{q}_i - \hat{q}_i, \bar{p}_i - \hat{p}_i)$
- $f_i^8(t) = f(t, \bar{q}_i + \hat{q}_i, \bar{p}_i - \hat{p}_i)$

Note that this means that the Bass parameters p and q will not remain constant over the whole time horizon in this setting. Hence, the robust model will not be equivalent to a deterministic model with modified parameters, in contrast with other applications of the classical robust optimization methodology.

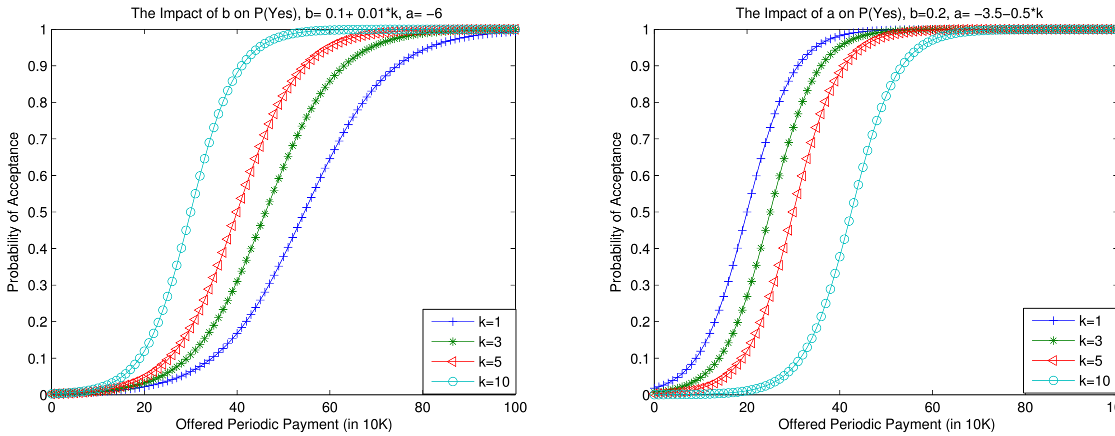
Continuing our modeling of uncertain parameters, we observe that the m's, which represent the *final* or *steady-state* market size of the new products, do not lend themselves to such a time-varying representation: in the worst case, the m_i's, when assumed to lie in a given range or confidence interval, will take their smallest possible value independently from the values of p_i's and q_i and this value will remain constant throughout the time horizon. Therefore, to avoid over-conservatism in the number of product adopters, we address the uncertainty involved in the parameters p_i and q_i of each product i using robust optimization techniques but initially assume that the parameter m_i of each product is constant, and later extend our original framework to handle the uncertainty in the m's using a real options approach.

Logit choice model parameters

We assume that the parameters a_{ij} and b_{ij} defining the response function of the potential partner j for the product i belong to the intervals $[\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij}]$ and $[\bar{b}_{ij} - \hat{b}_{ij}, \bar{b}_{ij}]$, respectively. For the logit choice model we do not need to consider the case where the parameters are higher than their nominal values because (it is easy to show using simple mathematical computations that) this always results in improved acceptance probabilities, so the worst case is never attained for the parameters being equal to their upper bound. Figure 2 shows an example of acceptance probability varying with the parameters.

Allowing again only deviations to the bounds of the interval (so a parameter takes either its nominal value or its lower bound, for the reasons above), we find that the probability of the potential partner j accepting the collaboration offer when the offered unit payment is R_j can take one of the following four values:

- $P^0_{ij}(R_j) \;=\; P(\bar{a}_{ij}, \bar{b}_{ij}, R_j)$
- $P^1_{ij}(R_j) \;=\; P(\bar{a}_{ij} - \hat{a}_{ij}, \bar{b}_{ij}, R_j)$



(a) Varying b_j, constant a_j

(b) Varying a_j, constant b_j

Figure 2: The impact of a_j and b_j on the probability of acceptance

- $P_{ij}^2(R_j) \;=\; P(\bar{a}_{ij}, \bar{b}_{ij} - \hat{b}_{ij}, R_j)$
- $P_{ij}^3(R_j) \;=\; P(\bar{a}_{ij} - \hat{a}_{ij}, \bar{b}_{ij} - \hat{b}_{ij}, R_j)$

The company seeks a partner whose probability to accept the collaboration offer exceeds a specified probability level, α, for the payment offer R_j.

1.3 Real options

Cetinkaya and Thiele (2014) provide a general introduction to real options. In the problem considered here, the size parameter m_i does not change during the diffusion process in contrast with the other parameters of the Bass model, p_i and q_i for product i. It is estimated before the process starts by using similar products that have already been brought to market or based on managerial judgment. However, the company will have a more educated estimate for the parameter m_i once it has observed the adoption rates or sales in the first few periods after the product is launched. The real option considered here provides the company with the right to reduce the size of the contract (reserved capacity) with the partner by a given fraction at a certain time period. In the absence of the real option, the company sets the size of the contract according to the highest possible new adoption rates considering the parameters $\bar{p}_i,\ \bar{q}_i,\ \hat{p}_i,$ and, $\hat{q}_i$ and the estimate for m_i. For the period that is s periods after the process starts, $f_i^+(s)m_i$ units of capacity are reserved at the partner company. This corresponds to a case where the partner company of a new shared-car service has to agree to have a certain number of parking spots available for users of the car service. In other possible examples, the retailer must develop a specific infrastructure to house the new products, such as storage space for parcel delivery, and is compensated for the electrical bill incurred in operating the storage space. Although this strategy protects against underestimated m_i, it might result in extra payments made to the partners in the case of overestimated m_i. The option to update the contract size mitigates the risk of over-paying the retailer. This real option can be viewed as a European type option to shrink the size of the contract while keeping the payment per unit constant.

The partner firm (the writer of the option) has to honor the innovative company's (the buyer of the option) wish to decrease the contract size by a fraction $1 - \kappa_i$ as stated in the contract at the specified date for product i. This specified date is the option's expiration date, which is η_i periods after the product is launched. Therefore, the innovative firm pays an option premium Ω_i to the partner in exchange for his commitment to the terms of the real option for the product i. If the innovative company exercises the real option for product i, the selected partner for this product has to give up the profit that it could have gained by conducting business for $1 - \kappa_i$ units of product.

The real options formulation will combine expected value optimization and robust optimization because stochasticity is required in order for the problem not to be trivial. (If there is no stochasticity, there is no need for a real option to begin with.)

2 New Product Launch Decisions with Robust Optimization

2.1 Problem Setup

In this section, we provide a tractable robust optimization formulation for an innovative company that:

- maximizes its total (Net Present Value of) profit considering:

- · the worst-case total number of adopters of each product,
- · the minimum unit payments to partners that ensure that the worst-case probability of accepting the offer is no less than a specific target,
- · the present value of each product's cash flow,
- · product-specific sets of potential partners with different choice model parameters, and
- · product-specific setup cost and available investment budget limits per time period,

- decides on:
  - · the sequence of the products to be launched,
  - · the product-specific potential partner to launch the product with,
- by incorporating the uncertainty structure presented in Section 1.2.2 with:
  - · the uncertainty budget for the Bass model parameters,
  - · the uncertainty budget for the logit choice model parameters, and
  - · the estimation errors for the market sizes of each product.

We assume launch decisions are static or here-and-now, i.e., all determined at the beginning of the time horizon by the decision-maker. This is because product demands are independent from each other. If product demands are correlated, so that information from a launch can be used to narrow down uncertainty regarding subsequent launches, we recommend an open-loop approach combined with the updating of the confidence intervals for the uncertain parameters. Such an approach, however, is outside the scope of the present paper.

We will use the following notation:

General Parameters

N : the total number of new products,

K_i : the maximum number of time periods (over all the values of p_i and q_i considered) until adoption of product i reaches steady-state,

T : the end of the time horizon considered,

S : the last time period when a product can be launched,

r : the discount rate,

μ_i : the price of new product i,

m_i : the estimated number of ultimate adopters of product i,

A^i : the set of potential partners for product i,

α : the specified probability level for the partner selection process,

B_t : the available investment budget for the time period t,

D_i : the setup cost of launching the product i.

Further, when we introduce the real options framework to update the parameter m_i, we will use:

η_i : the number of periods between the time period when product i is launched and the time period when the real option on the product i can be exercised,

κ_i : the fraction by which the capacity at the partner can be decreased if the real option on product i is exercised η_i periods after the start of product i's diffusion process.

The Bass Model and Logit Choice Model Parameters

$f_i^k(t)$: adoption rate of product i, t periods after the start of diffusion process when the parameters of the Bass model belong to case $k,\ k \in \{0, .., 8\}$

$f_i^+(t)$: the maximum possible adoption rate of the product i over $\{0, \ldots, t\}$,

$P_{ij}^k(R)$: the probability of potential partner j for product i accepting the unit payment offer R when the parameters of the logit choice model belong to case $k,\ k \in \{0, .., 3\}$,

$Q_{ij}^k(\alpha)$: the minimum payment accepted (inverse logit probability function) by potential partner j for product i and probability level α when parameters of the logit choice model belong to case k,

$\tilde{m}_i^l$: (in the real options model) the updated market-size estimate for product i in scenario l when η_i periods have passed since product i was launched,

π_i^l : (in the real options model) the probability that the updated market-size estimate for product i is $\tilde{m}_i^l$, in scenario l.

Robust Optimization Parameters and Decision Variables

Γ_B : the uncertainty budget (*parameter*) for the Bass model parameters restricting the number of parameters whose values deviate from the nominal value,

Γ_L : the uncertainty budget (*parameter*) for the logit choice model parameters restricting the number of parameters whose values deviate from the nominal value,

$x_{i\tau}$: a binary variable (*chosen by decision-maker (DM)*) =1 if product i is launched at time τ, 0 o.w.,

y_{ij}: a binary variable (*chosen by DM*) =1 if the potential partner j is selected for product i, 0 o.w.,

v_{is}^k: a binary variable (*chosen by nature*) =1 if the adoption rate of product i is $f_i^k(s)$ s time periods after the diffusion process starts, 0 o.w.,

w_{ij}^k: a binary variable (*chosen by nature*) =1 if the probability of acceptance by potential partner j for product i is P_{ij}^k, 0 o.w.

Note that the budgets of uncertainty Γ_B and Γ_L are selected by the decision maker to reflect his aversion to risk, rather than to guarantee probabilities of constraint violation as in Bertsimas and Sim (2004). This is because the problem structure, with uncertain parameters inside exponentials, does not satisfy the assumptions required to use the results in Bertsimas and Sim (2004).

Let us now consider the deterministic problem for an instant. Let $\Phi_i(\alpha)$ be the optimal objective in the partner selection problem for product i, i.e., the smallest payment the company must offer the partner for its partnership offer to be accepted with probability α. We have:

$$\Phi_i(\alpha) = \min_{j \in A^i,\, R_{i,j}} \left\{ R_{i,j} : \; P_{ij}^0(R_{ij}) \geq \alpha \right\}. \tag{3}$$

Equivalently, we have, using the definition of the inverse probability function:

$$\Phi_i(\alpha) = \min_{j \in A^i} Q^0_{ij}(\alpha),\ \forall i.$$

It is easy to check that the following holds.

Lemma 1 (Inverse Logit Probability Function) *For a given probability level α, the optimal payment offer by the company is given by:*

$$\Phi_i(\alpha) = \min_{j \in A^i} \left\{ \frac{1}{\bar{b}_{ij}} \left(-\bar{a}_{ij} + \ln \left(\frac{\alpha}{1-\alpha} \right) \right) \right\},\ \forall i. \tag{4}$$

Proof. Follows directly from the fact that $P^0_{ij}(R_{ij}) = \frac{1}{1+e^{-(\bar{a}_j - \bar{b}_j R_{ij})}}$ so that $Q^0_{ij}(\alpha) = \frac{1}{\bar{b}_{ij}} \left(-\bar{a}_{ij} + ln \left(\frac{\alpha}{1-\alpha} \right) \right)$.
□

The deterministic product launch problem where each product can be launched at most once and the partner is paid per unit of total capacity he has to install is formulated as:

$$\begin{aligned}
\max_x \quad & \sum_{i=1}^{N} \sum_{\tau=1}^{S} \frac{x_{i\tau}}{(1+r)^{\tau-1}} \left[\sum_{s=1}^{K_i} \left(\frac{m_i}{(1+r)^s} [\mu_i f_i^0(s) - f_i^+(s) \Phi_i(\alpha)] \right) \right] \\
s.t. \quad & \sum_{\tau=1}^{S} x_{i\tau} \leq 1,\ \ \forall i, \\
& \sum_{i=1}^{N} x_{i\tau} D_i \leq \frac{B_\tau}{(1+r)^{\tau-1}},\ \ \forall \tau, \\
& x_{i\tau} \in \{0,1\},\ \ \forall i,\ \forall \tau.
\end{aligned} \tag{5}$$

The available budget in time period τ is divided by $(1+r)^{\tau-1}$ to capture that a budget representing the same marketing power at each time period would need to grow by $1 + r$ at each time period. Alternatively, this could be incorporated into the definition of B_τ.

Note that in this model, the innovative company pays the partner for the maximum adoption rate summed over the time horizon (equal to the maximum number of products sold) because this determines the capacity that must be available at the partner. The partner is not allowed to change capacity over time except through the real options framework described later. For instance, in the case of a new shared-car service, the partner must agree to have space to park the maximum number of cars. While the specifics of the payment model to the partners in Problem (5) are motivated by the company's desire not to sell out of its new products before their full sales potential has been realized, other payment schemes can easily be incorporated by changing the objective coefficients appropriately. In particular, the extension to a model where the partner must pay the actual (rather than maximum) adoption rate is immediate.

Therefore, the deterministic product launch problem can be solved by first determining $\Phi_i(\alpha)$ using Eq.(4) for each product i and then solving Problem (5).

2.2 Robust Product Launch

2.2.1 Partner Selection Subproblem

As explained in Section 1.2.2, the uncertainty involved in the logit choice parameters results in four possible acceptance probability values, for each potential partner, each product and a given periodic payment offer. This is under the budget-of-uncertainty constraint restricting the total number of logit choice parameters taking their worst-case values. The company selects partners with the aim of minimizing its worst-case total payment across all products. We will denote y a binary decision matrix capturing the partner choice for each product and w a parameter matrix indicating the worst-case uncertainty outcome. Let Y be the feasible set for the partner selection vector:

$$Y = \left\{ y \mid \sum_{j \in A^i} y_{ij} = 1, \ \forall i, \ y_{ij} \in \{0,1\}, \ \forall i, \ j \right\}.$$

Let W be the uncertainty set for the payoffs:

$$W = \left\{ w \mid \sum_{k=1}^{3} w_{ij}^k \leq 1, \forall i, \ \forall j, \ \sum_{i=1}^{N} \sum_{j \in A^i} \left(\sum_{k=1}^{2} w_{ij}^k + 2w_{ij}^3 \right) \leq \Gamma_L, \ w_{ij}^k \in \{0,1\}, \ \forall i, \ j, \ k \right\}.$$

The problem becomes:

$$\min_{y \in Y} \quad \max_{w \in W} \quad \sum_{i=1}^{N} \sum_{j \in A^i} y_{ij} \left(Q_{ij}^0 + \sum_{k=1}^{3} \left(w_{ij}^k [Q_{ij}^k - Q_{ij}^0] \right) \right) \tag{6}$$

where we have dropped, for notational convenience, the argument in α for the Q_{ij}^k (minimum payment the innovative company would pay retailer j in uncertainty scenario k in order for that retailer to accept the partnership offer for product i with probability at least α).

The constraints in Problem (6) can be explained as follows. For each product and retailer, the worst-case uncertainty can be either case 0, or 1, or 2, or 3. This is modeled by stating that at most one among the cases 1, 2, 3 can be selected for each (product, retailer) pair (i, j). Recall that case 0 is the benchmark using zero unit of the uncertainty budget, cases 1 and 2 each use one unit and case 3 uses two units, because both parameters a and b are equal to their lowest values in that case. Hence, each (product, retailer) pair (i, j) uses $w_{ij}^1 + w_{ij}^2 + 2w_{ij}^3$ units of the budget of uncertainty, taking into account that at most one of the w_{ij}^k among $k = 1, 2, 3$ will be 1 and the others 0. The total amount of budget used cannot exceed Γ_L, which is selected by the manager and measures his degree of aversion to uncertainty. An intuition behind this budget-of-uncertainty constraint is that, if unfavorable independent parameter values have been realized for many periods, favorable parameters will materialize in the future. In other words, it is unlikely that the decision-maker will observe unfavorable parameter values throughout the *whole* time horizon. The constraint $\sum_{j \in A^i} y_{ij} = 1, \ \forall i$ ensures that exactly one partner is selected per new product.

The objective is calculated, for each (product, retailer) pair (i, j), by first considering the benchmark payment Q_{ij}^0 and adding to it the complement $Q_{ij}^k - Q_{ij}^0$ if (i, j) is in uncertainty scenario $k = 1, 2, 3$, of which 1 at most can be chosen. The part of the payment about product i, $\sum_{j \in A^i} y_{ij}(Q_{ij}^0 + \sum_{k=1}^{3}(w_{ij}^k[Q_{ij}^k - Q_{ij}^0]))$, is then denoted $\Theta_i^*(\alpha, \Gamma_L)$. Our approach for the partner payment subproblem remains, as in the deterministic case, to compute the payments offline in a tractable manner before incorporating the resulting values into the optimization problem that determines the launch sequence.

2.2.2 Robust Launch Master Problem

As explained above, the uncertainty involved in the Bass model's parameters leads to nine possible adoption rates for each product and each time period under the budget-of-uncertainty constraint. We model again the changes in objective by isolating the nominal rate $f_i^0(s)$ as benchmark and adding the changes in marginal adoption rate from that benchmark $f_i^k(s) - f_i^0(s)$ using binary variables v_{is}^k, equal to 1 if the adoption rate of product i is $f_i^k(s)$ s time periods after the diffusion process starts and 0 otherwise. We also subtract the payment to the chosen retailer for that time period, for each product. Recall that one unit of uncertainty is used from the budget when selecting $k = 1, \ldots, 4$ and two units when selecting $k = 5, \ldots, 8$. The budget of uncertainty Γ_B is again selected by the decision-maker to reflect his degree of aversion to uncertainty. An intuition behind this budget-of-uncertainty constraint is that it is unlikely that the decision-maker will observe unfavorable values for all parameters throughout the whole time horizon.

Let X be the feasible set for the launch decision vector:

$$X = \left\{ \sum_{\tau=1}^{S} x_{i\tau} \leq 1,\ \forall i,\ \sum_{i=1}^{N} D_i x_{i\tau} \leq \frac{B_\tau}{(1+r)^{\tau-1}},\ \forall \tau,\ x_{i\tau} \in \{0,1\},\ \forall i,\ \tau \right\}.$$

Let V be the uncertainty set vector for the Bass curves:

$$V = \left\{ \sum_{k=1}^{8} v_{is}^k \leq 1, \forall i,\ \forall s,\ \sum_{i=1}^{N} \sum_{s=1}^{K_i} \left(\sum_{k=1}^{4} v_{is}^k + 2 \sum_{k=5}^{8} v_{is}^k \right) \leq \Gamma_B,\ v_{is}^k \in \{0,1\},\ \forall i,\ s,\ k \right\}.$$

The robust product launch problem is then formulated as:

$$\max_{x \in X} \ \min_{v \in V} \ \sum_{i=1}^{N} \sum_{\tau=1}^{S} \frac{x_{i\tau}}{(1+r)^{\tau-1}} \left[\sum_{s=1}^{K_i} \left(\frac{m_i}{(1+r)^s} \left[\mu_i \left(f_i^0(s) + \sum_{k=1}^{8} v_{is}^k [f_i^k(s) - f_i^0(s)] \right) - f_i^+(s) \Theta_i^*(\alpha, \Gamma_L) \right] \right) \right] \tag{7}$$

2.3 Problem Solution Approach

The tractability of Problem (6) (respectively, Problem (7)) above depends on the decision maker's ability to transform the inner maximization (minimization) problem into a minimization (maximization) problem that can then be incorporated into the outer problem of the same type. Bertsimas and Sim (2004) use strong duality to obtain a tractable formulation for the robust optimization model when the inner problem is a linear program. However, the inner problems of Problem (6) and Problem (7) have integer variables, which complicates the use of strong duality. Therefore, we seek ways of expressing the inner problems as linear programming models and only then using strong duality for those new problems. Specifically, we follow Duzgun and Thiele (2010) and investigate total unimodularity of the constraint matrices of the inner problems. While neither constraint matrix, in the problems above, satisfies the total unimodularity property, we show that we can reformulate the problems as a series of problems with totally unimodular constraint matrices, which will allow us to conclude.

Let us define the relevant inner problems as follows. For a given feasible decision vector y, the inner maximization problem of Problem (6), which addresses the uncertainty in the logit choice model parameters, is given by:

$$\max_{w \in W} \quad \sum_{i=1}^{N} \sum_{j \in A^i} y_{ij} \left(Q_{ij}^0 + \sum_{k=1}^{3} \left(w_{ij}^k [Q_{ij}^k - Q_{ij}^0] \right) \right). \tag{8}$$

For a given feasible x decision, the inner minimization problem of Problem (7), which addresses the uncertainty in the Bass model parameters, is given by:

$$\min_{v \in V} \quad \sum_{i=1}^{N} \sum_{\tau=1}^{S} \frac{x_{i\tau}}{(1+r)^{\tau-1}} \left[\sum_{s=1}^{K_i} \left(\frac{m_i}{(1+r)^s} \left[\mu_i \left(f_i^0(s) + \sum_{k=1}^{8} v_{is}^k [f_i^k(s) - f_i^0(s)] \right) - f_i^+(s) \Theta_i^*(\alpha, \Gamma_L) \right] \right) \right]. \tag{9}$$

Neither Problem (8) nor Problem (9) has a totally unimodular constraint matrix. However, they have a similar structure allowing us to reformulate the budget-of-uncertainty constraint by introducing two new integer parameters Γ_L' (to be enumerated between 0 and $\lfloor 0.5\Gamma_L \rfloor$) and Γ_B'

(to be enumerated between 0 and $\lfloor 0.5\Gamma_B \rfloor$). We then decompose the original uncertainty budget constraints so that the constraint matrices of both problems become totally unimodular, we can relax the integrality constraints and then use strong duality to reformulate the master problems in a tractable manner.

Lemma 2 (Inner robust problems) *(i) Problem (8) is equivalent to solving the series of problems:*

$$
\begin{aligned}
\max_{w} \quad & \sum_{i=1}^{N} \sum_{j \in A^i} y_{ij} \left(Q_{ij}^0 + \sum_{k=1}^{3} \left(w_{ij}^k [Q_{ij}^k - Q_{ij}^0] \right) \right) \\
s.t. \quad & \sum_{k=1}^{3} w_{ij}^k \leq 1, \forall i, \forall j \in A_i, \\
& \sum_{i=1}^{N} \sum_{j \in A^i} \sum_{k=1}^{2} w_{ij}^k \leq \Gamma_L - 2\Gamma'_L, \\
& \sum_{i=1}^{N} \sum_{j \in A^i} w_{ij}^3 \leq \Gamma'_L, \\
& w_{ij}^k \in \{0, 1\}, \ \forall i, \ \forall j, \ \forall k.
\end{aligned}
\tag{10}
$$

parametrized over $\Gamma'_L \in \{0, \ldots \lfloor 0.5\Gamma_L \rfloor\}$ and selecting as optimal Γ'_L the one that maximizes the optimal objective in the series of Problems (10).

(ii) Problem (9) is equivalent to solving the series of problems:

$$
\begin{aligned}
\min_{v} \quad & \sum_{i=1}^{N}\sum_{\tau=1}^{S}\frac{x_{i\tau}}{(1+r)^{\tau-1}}\left[\sum_{s=1}^{K_i}\left(\frac{m_i}{(1+r)^s}\left[\mu_i\left(f_i^0(s)+\sum_{k=1}^{8}v_{is}^k[f_i^k(s)-f_i^0(s)]\right)-f_i^+(s)\Theta_i^*(\alpha,\Gamma_L)\right]\right)\right]\\
s.t. \quad & \sum_{k=1}^{8}v_{is}^k \le 1, \forall i,\ \forall s \le K_i,\\
& \sum_{i=1}^{N}\sum_{s=1}^{K_i}\sum_{k=1}^{4}v_{is}^k \le \Gamma_B - 2\Gamma'_B,\\
& \sum_{i=1}^{N}\sum_{s=1}^{K_i}\sum_{k=5}^{8}v_{is}^k \le \Gamma'_B,\\
& v_{is}^k \in \{0,1\},\ \forall i,\ \forall s,\ \forall k.
\end{aligned}
\tag{11}
$$

parametrized over $\Gamma'_B \in \{0,\dots\lfloor 0.5\Gamma_B\rfloor\}$ *and selecting as optimal* Γ'_B *the one that minimizes the optimal objective in the series of Problems (11).*

Proof. The result is immediate if the constraints $\sum_{i=1}^{N}\sum_{j\in A^i} w_{ij}^3 \le \Gamma'_L$ and $\sum_{i=1}^{N}\sum_{s=1}^{K_i}\sum_{k=5}^{8} v_{is}^k \le \Gamma'_B$ in Problems (10) and (11) are equality constraints; however, to obtain a key result below, we need to relax the equal sign without losing optimality. We now show how to do so. The minimum payment under an uncertainty scenario is always greater than or equal to the minimum payment under the nominal case, because we always have an ordering on the probability functions (the probability of yes in the nominal case is no less than the probability of yes with uncertainty) since the coefficients of the logit function are smaller. This matches intuition since the partner must be compensated for the uncertainty; otherwise he will not accept the offer. Similarly, for (ii), the adoption rate of product i t periods after the start of the diffusion process is no greater in the case with high-uncertainty than in the nominal case. Thus, the coefficients in front of the decision variables w_{ij}^k, resp. v_{ij}^k, are always non-negative, resp. non-positive. □

Through the parametrization above, the structure of the constraint matrix becomes one where (i) all the decision variables are present in the first group of constraints, (ii) only the decision variables corresponding to scenarios using only 1 unit of uncertainty are present in the second constraint group (which is in fact a single constraint, corresponding to the inequality with $\Gamma_L - 2\Gamma'_L$ or $\Gamma_B - 2\Gamma'_B$ in the right-hand side), and (iii) only the decision variables corresponding to scenarios using 2 units of uncertainty are present in the third constraint group. We use the following lemma to conclude.

Lemma 3 (Nemhauser and Wolsey (1999)) *Let A be a $(0, -1, 1)$ matrix with no more than two nonzero elements in each column. Then, A is totally unimodular if and only if the rows of A can be partitioned into two subsets Q_1 and Q_2 such that if a column contains two nonzero elements, the following statements are true:*

- *If both nonzero elements have the same sign, then one is in a row contained in Q_1 and the other is in a row contained in Q_2.*
- *If the two nonzero elements have opposite sign, then both are in rows contained in the same subset.*

This result is important because the extreme points of a polyhedron with a totally unimodular constraint matrix and an integer right-hand side are integer, so that we are able to relax the integrality constraints in the inner problems and obtain linear problems. This in turn allows us to invoke strong duality in linear programming to derive a tractable reformulation.

Lemma 4 *The constraint matrices of Problem (10) and Problem (11), namely $\mathbf{P}'^1$ and $\mathbf{P}'^2$, are totally unimodular.*

Proof. A totally unimodular matrix stays totally unimodular after multiplying a row by -1 (Nemhauser and Wolsey, 1999). Therefore, we multiply the second group of constraint coefficients by -1 and obtain the structure required to conclude, putting the rows in the first and third groups of constraints in Q_1 and those in the second group in Q_2, for both problems.

In mathematical terms, let $h_1 = \sum_{i=1}^{N} |A^i|$ and $h_2 = \sum_{i=1}^{N} K_i$. The constraint matrices of Problem (10) and Problem (11) have the following structures, respectively, where the decision matrices have been redefined as decision vectors. $\mathbf{I}_k$ refers to the identity $k \times k$ matrix. Similarly, $\mathbf{0}_k$ and $\mathbf{1}_k$ refer to the $1 \times k$ row vectors of all 0 and all 1.

$$\mathbf{P}'^{1} = \begin{pmatrix} \mathbf{I}_{\mathbf{h_1}} & \mathbf{I}_{h_1} & \mathbf{I}_{h_1} \\ \mathbf{1}_{h_1} & \mathbf{1}_{h_1} & \mathbf{0}_{h_1} \\ \mathbf{0}_{h_1} & \mathbf{0}_{h_1} & \mathbf{1}_{h_1} \end{pmatrix}$$

$$\mathbf{P}'^{2} = \begin{pmatrix} \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} \\ \mathbf{1}_{h_2} & \mathbf{1}_{h_2} & \mathbf{1}_{h_2} & \mathbf{1}_{h_2} & \mathbf{0}_{h_2} & \mathbf{0}_{h_2} & \mathbf{0}_{h_2} & \mathbf{0}_{h_2} \\ \mathbf{0}_{h_2} & \mathbf{0}_{h_2} & \mathbf{0}_{h_2} & \mathbf{0}_{h_2} & \mathbf{1}_{h_2} & \mathbf{1}_{h_2} & \mathbf{1}_{h_2} & \mathbf{1}_{h_2} \end{pmatrix}$$

Note that the 3 column-blocks for $\mathbf{P}'^{1}$ and 8 column-blocks for $\mathbf{P}'^{2}$ reflect the 3 and 8 scenarios (without the baseline scenario of all-nominal values), respectively, for the logit choice model and the Bass diffusion model. Multiplying the second row-block in the matrices by -1 yields:

$$\mathbf{P}''^{1} = \begin{pmatrix} \mathbf{I}_{h_1} & \mathbf{I}_{h_1} & \mathbf{I}_{h_1} \\ -\mathbf{1}_{h_1} & -\mathbf{1}_{h_1} & \mathbf{0}_{h_1} \\ \mathbf{0}_{h_1} & \mathbf{0}_{h_1} & \mathbf{1}_{h_1} \end{pmatrix}$$

$$\mathbf{P}''^{2} = \begin{pmatrix} \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} & \mathbf{I}_{h_2} \\ -\mathbf{1}_{h_2} & -1_{h_2} & -\mathbf{1}_{h_2} & -\mathbf{1}_{h_2} & \mathbf{0}_{h_2} & \mathbf{0}_{h_2} & \mathbf{0}_{h_2} & \mathbf{0}_{h_2} \\ \mathbf{0}_{h_2} & \mathbf{0}_{h_2} & \mathbf{0}_{h_2} & \mathbf{0}_{h_2} & \mathbf{1}_{h_2} & \mathbf{1}_{h_2} & \mathbf{1}_{h_2} & \mathbf{1}_{h_2} \end{pmatrix}$$

The matrices $\mathbf{P}''^{1}$ and $\mathbf{P}''^{2}$ satisfy the desired property. □

We now present the main result in this section.

Theorem 5 *The robust optimization problem (7) is equivalent to solving the mixed integer pro-*

gramming problem:

$$
\begin{aligned}
\max_{x,\sigma,\gamma,\delta,\epsilon,Z} \quad & \sum_{i=1}^{N}\sum_{\tau=1}^{S}\frac{x_{i\tau}}{(1+r)^{\tau-1}}\left[\sum_{s=1}^{K_i}\left(\frac{m_i}{(1+r)^s}\left(\mu_i f_i^0(s)-f_i^+(s)\Theta_i^*(\alpha,\Gamma_L)\right)\right)\right]-Z \\
s.t. \quad & Z \geq \left(\sum_{i=1}^{N}\sum_{s=1}^{K_i}\epsilon_{is}+\sigma(\Gamma_B-2\Gamma'_B)+\gamma\Gamma'_B+\sum_{i=1}^{N}\sum_{s=1}^{K_i}\sum_{k=1}^{8}\delta_{is}^k\right),\ \forall\Gamma'_B\in\{0,...,\lfloor 0.5\Gamma_B\rfloor\}, \\
& \epsilon_{is}+\sigma+\delta_{is}^k \geq \sum_{\tau=1}^{S}\frac{x_{i\tau}\mu_i m_i[f_{is}^0-f_{is}^k]}{(1+r)^{(s+\tau-1)}},\ \forall i,\ \forall s\in\{1,...,K_i\},\ \forall k\in\{1,..,4\}, \\
& \epsilon_{is}+\gamma+\delta_{is}^k \geq \sum_{\tau=1}^{S}\frac{x_{i\tau}\mu_i m_i[f_{is}^0-f_{is}^k]}{(1+r)^{(s+\tau-1)}},\ \forall i,\ \forall s\in\{1,...,K_i\},\ \forall k\in\{5,..,8\}, \\
& \sum_{\tau=1}^{S}x_{i\tau}\leq 1,\ \forall i, \\
& \sum_{i=1}^{N}x_{i\tau}D_i \leq \frac{B_\tau}{(1+r)^{\tau-1}},\ \forall\tau, \\
& x_{i,\tau}\in\{0,1\},\ \forall i,\ \forall\tau \\
& \epsilon,\gamma,\delta,\sigma\geq 0,
\end{aligned}
\tag{12}
$$

where $\Theta_i^*(\alpha,\Gamma_L)$ *is the part of the company's payment to partners for product* i*, with the worst-*

case total payment being the optimal objective of:

$$
\begin{aligned}
\min_{\theta,\beta,\chi,z,y,Y} \quad & \sum_{i=1}^{N}\sum_{j\in A^i} y_{ij}O_{ij}^0 + Y \\
s.t. \quad & Y \geq \sum_{i=1}^{N}\sum_{j\in A^i}\beta_{ij} + \chi\Gamma_L + (\theta-2\chi)\Gamma'_L + \sum_{i=1}^{N}\sum_{j\in A^i}\sum_{k=1}^{3} z_{ij}^k,\ \forall \Gamma'_L \in \{0,...,\lfloor 0.5\Gamma_L\rfloor\}, \\
& \beta_{ij} + \chi + z_{ij}^k - y_{ij}[O_{ij}^k - O_{ij}^0] \geq 0, \forall i,\ \forall j\in A^i,\ \forall k \in \{1,2\}, \\
& \beta_{ij} + \theta + z_{ij}^k - y_{ij}[O_{ij}^3 - O_{ij}^0] \geq 0, \forall i,\ \forall j\in A^i, \\
& \sum_{j\in A^i} y_{ij} = 1,\ \forall i, \\
& y_{ij} \in \{0,1\},\ \forall i,j, \\
& \theta, \beta_{ij}, \chi, z_{ij}^k \geq 0.
\end{aligned} \tag{13}
$$

Proof. Consider the inner problem (11). Because the left-hand side constraint matrix is totally unimodular and the right-hand side coefficients are integer, the feasible set of the linear relaxation has integer extreme points, therefore its optimal objective is equal to the optimal objective of the original problem with binary variables and we can use strong duality for the linear relaxation of the problem. This technique was first used in Duzgun and Thiele (2010). Another example of use of robust optimization combined with limited probabilistic knowledge can be found in Mak et al (2014). This leads to the reformulation:

$$
\begin{aligned}
\max_{\sigma,\gamma,\delta,\epsilon} \quad & -\left(\sum_{i=1}^{N}\sum_{s=1}^{K_i}\epsilon_{is} + \sigma(\Gamma_B - 2\Gamma'_B) + \gamma\Gamma'_B + \sum_{i=1}^{N}\sum_{s=1}^{K_i}\sum_{k=1}^{8}\delta_{is}^k\right) \\
s.t. \quad & \epsilon_{is} + \sigma + \delta_{is}^k \geq \sum_{\tau=1}^{S}\frac{x_{i\tau}\mu_i m_i[f_{is}^0 - f_{is}^k]}{(1+r)^{(s+\tau-1)}},\ \forall i,\ \forall s\in\{1,...,K_i\},\ \forall k\in\{1,..,4\}, \\
& \epsilon_{is} + \gamma + \delta_{is}^k \geq \sum_{\tau=1}^{S}\frac{x_{i\tau}\mu_i m_i[f_{is}^0 - f_{is}^k]}{(1+r)^{(s+\tau-1)}},\ \forall i,\ \forall s\in\{1,...,K_i\},\ \forall k\in\{5,..,8\}, \\
& \epsilon,\gamma,\delta,\sigma \geq 0.
\end{aligned} \tag{14}
$$

However, while the decision maker seeks to maximize the worst-case NPV over the product launch sequences x, we must first minimize the objective of Problem (14) over Γ'_B taking integer values between 0 and $\lfloor 0.5\Gamma_B \rfloor$ before we can maximize over x. In other words, our problem involves, from left to right, a maximization over x, a minimization over Γ'_B and a maximization over the auxiliary dual variables $\epsilon, \gamma, \delta, \sigma$ of a function that is bilinear in Γ'_B and γ, σ, and linear in all other variables. But because the minimization is over a finite set $\Gamma'_B \in \{0, ..., \lfloor 0.5\Gamma_B \rfloor\}$, we can linearize this part of the problem by introducing an auxiliary variable Z' that will replace $\min_{\Gamma'_B} \max_{\sigma,\gamma,\delta,\epsilon} - \left(\sum_{i=1}^{N} \sum_{s=1}^{K_i} \epsilon_{is} + \sigma(\Gamma_B - 2\Gamma'_B) + \gamma\Gamma'_B + \sum_{i=1}^{N} \sum_{s=1}^{K_i} \sum_{k=1}^{8} \delta_{is}^k \right)$ and be constrained so that $Z' \leq \max_{\sigma,\gamma,\delta,\epsilon} - \left(\sum_{i=1}^{N} \sum_{s=1}^{K_i} \epsilon_{is} + \sigma(\Gamma_B - 2\Gamma'_B) + \gamma\Gamma'_B + \sum_{i=1}^{N} \sum_{s=1}^{K_i} \sum_{k=1}^{8} \delta_{is}^k \right)$ for all $\Gamma'_B \in \{0, ..., \lfloor 0.5\Gamma_B \rfloor\}$. Then, because of the sign of the inequality, the maximum can be dropped and it is necessary and sufficient to find a feasible solution $(\sigma, \gamma, \delta, \epsilon)$ that satisfies the inequality. We then perform the change of variables $Z = -Z'$ to conclude. The proof for Problem (13) is very similar and therefore is left to the reader. □

2.4 Robust Product Launch with a Real Option: Option to Update the Contract Size

In this section, we address the difficulty in estimating accurately the parameter m_i, the ultimate number of adopters of product i. We build upon the setup provided in Section 1.3.

We assume that the partners are rational decision makers, and they use the NPV of the net profit they obtain as the metric to compare the two alternatives: the business contract with the innovative company in the absence of the real option and in the presence of the real option. However, the innovative company does not know the unit cost that the partner company encounters for the product i. Therefore, we use the industry profit margin average ψ_i as an estimation for the profit margin of each of the potential partners for product i. (We use the data on S&P 500 sectors and industries profit margins in Yardeni and Abbott (2014) as a reference.) The potential partner is expected to be paid an amount of money equal to $m_i f_i^+(s)$ at the s^{th} period of their partnership where s ranges from 1 to K_i. Therefore, the NPV of the total payments foreseen to be made to partner j for product i, which is introduced to the market at time period τ when the real option is not available, is formulated as (with τ the launch time of product i):

$$NPV(\text{Total Payment}) = \frac{1}{(1+r)^{\tau-1}} \sum_{s=1}^{K_i} \frac{f_i^+(s) m_i \Theta_i^*(\alpha, \Gamma_L)}{(1+r)^s}.$$

Then, using the industry profit margin for the partner for product i, the NPV of the profit that the partner obtains from this business is formulated as:

$$NPV(\text{Total Profit}) = \frac{\psi_i}{(1+r)^{\tau-1}} \sum_{s=1}^{K_i} \frac{f_i^+(s) m_i \Theta_i^*(\alpha, \Gamma_L)}{(1+r)^s}.$$

If the innovative company has purchased the real option and exercises it, the partner will be paid:

$$NPV(\text{Total Payment})' = \frac{1}{(1+r)^{\tau-1}} \left(\Omega_i + \sum_{s=1}^{\eta_i} \frac{f_i^+(s) m_i \Theta_i^*(\alpha, \Gamma_L)}{(1+r)^s} + \sum_{s=\eta_i+1}^{K_i} \frac{f_i^+(s) \kappa_i m_i \Theta_i^*(\alpha, \Gamma_L)}{(1+r)^s} \right)$$

and the total net profit of the partner will be:

$$NPV(\text{Total Profit})' = \frac{1}{(1+r)^{\tau-1}} \left(\Omega_i + \psi_i \sum_{s=1}^{\eta_i} \frac{f_i^+(s) m_i \Theta_i^*(\alpha, \Gamma_L)}{(1+r)^s} + \psi_i \sum_{s=\eta_i+1}^{K_i} \frac{f_i^+(s) \kappa_i m_i \Theta_i^*(\alpha, \Gamma_L)}{(1+r)^s} \right)$$

The partner company needs to have at least the same profit when the innovative company exercises the option in order to accept the contract with the real option. Therefore, the option premium should be at least equal to the NPV of the profit that could have been obtained by the decreased portion of the contract. In other words, we should have $NPV(\text{Total Profit}) <= NPV(\text{Total Profit})'$ so that the partner company accepts the requirements of the real option. This results in the following option premium formulation:

$$\Omega_i^* = \psi_i \sum_{s=\eta_i+1}^{K_i} \frac{f_i^+(s)(1-\kappa_i)m_i\Theta_i^*(\alpha,\Gamma_L)}{(1+r)^s}.$$

In the presence of the real option, the innovative company exercises the real option if the new estimate $\tilde{m}_i$ is less than the amount $\kappa_i m_i$ specified in the option. It is assumed that the innovative company has a set of scenarios for the possible future estimates for the ultimate number of adopters ($\tilde{m}_i^l$) and their corresponding probabilities (π_i^l). Let us define a set N^i such that $N^i = \{l : \tilde{m}_i^l < \kappa_i m_i\}$. Then, the probability that the innovative firm exercises the option (ρ_i) for the product i is calculated as:

$$\rho_i = \sum_{l\in N^i} \pi_i^l.$$

The expected value of the NPV of the payments foreseen to be made by the innovative company to the prospective partner is equal to (distinguishing between the time periods 1 to η_i before the option can be exercised and the time periods $\eta_i + 1$ onwards, with the possible size adjustment):

$$\frac{1}{(1+r)^{\tau-1}}\Bigg(\Omega_i^* + \sum_{s=1}^{\eta_i} \frac{f_i^+(s)m_i\Theta_i^*(\alpha,\Gamma_L)}{(1+r)^s} + \rho_i \sum_{s=\eta_i+1}^{K_i} \frac{f_i^+(s)\kappa_i m_i\Theta_i^*(\alpha,\Gamma_L)}{(1+r)^s} + (1-\rho_i)\sum_{s=\eta_i+1}^{K_i} \frac{f_i^+(s)m_i\Theta_i^*(\alpha,\Gamma_L)}{(1+r)^s}\Bigg).$$

Therefore, in the presence of real options, Problem (12) becomes:

$$
\begin{aligned}
\max_{x,\sigma,\gamma,\delta,\epsilon,\nu} \quad & \sum_{i=1}^{N}\sum_{\tau=1}^{S}\left(\frac{x_{i\tau}}{(1+r)^{\tau-1}}\left[\sum_{s=1}^{\eta_i}\left(\frac{m_i}{(1+r)^s}\left(\mu_i f_i^0(s)-f_i^+(s)\Theta_i^*(\alpha,\Gamma_L)\right)\right)\right]\right) \\
& \sum_{i=1}^{N}\sum_{l\in N^i}\pi_i^l\sum_{\tau=1}^{S}\left(\frac{x_{i\tau}}{(1+r)^{\tau-1}}\left[\sum_{s=\eta_i+1}^{K_i}\left(\frac{1}{(1+r)^s}\left(m_i^l\mu_i f_i^0(s)-\kappa_i m_i f_i^+(s)\Theta_i^*(\alpha,\Gamma_L)\right)\right)\right]\right) \\
& \sum_{i=1}^{N}\sum_{l\in N^{i'}}\pi_i^l\sum_{\tau=1}^{S}\left(\frac{x_{i\tau}}{(1+r)^{\tau-1}}\left[\sum_{s=\eta_i+1}^{K_i}\left(\frac{1}{(1+r)^s}\left(\min(m_i^l,m_i)\mu_i f_i^0(s)-m_i f_i^+(s)\Theta_i^*(\alpha,\Gamma_L)\right)\right)\right]\right)-Z \\
s.t. \quad & Z\geq\left(\sum_{i=1}^{N}\sum_{s=1}^{K_i}\epsilon_{is}+\sigma(\Gamma_B-2\Gamma_B')+\gamma\Gamma_B'+\sum_{i=1}^{N}\sum_{s=1}^{K_i}\sum_{k=1}^{8}\delta_{is}^k\right),\ \forall\Gamma_B'\in\{0,...,\lfloor 0.5\Gamma_B\rfloor\}, \\
& \epsilon_{is}+\sigma+\delta_{is}^k\geq\sum_{\tau=1}^{S}\frac{x_{i\tau}\mu_i\left(\sum_{l\in N^i}m_i^l\pi_i^l+\sum_{l\in N^{i'}}\pi_i^l\min(m_i,m_i^l)\right)[f_{is}^0-f_{is}^k]}{(1+r)^{(s+\tau-1)}},\ \forall i,\ \forall s\geq\eta_i+1,\ \forall k=\{1,..,4\}, \\
& \epsilon_{is}+\sigma+\delta_{is}^k\geq\sum_{\tau=1}^{S}\frac{x_{i\tau}\mu_i m_i[f_{is}^0-f_{is}^k]}{(1+r)^{(s+\tau-1)}},\ \forall i,\ \forall s\in\{1,...,\eta_i\},\ \forall k=\{1,..,4\}, \\
& \epsilon_{is}+\gamma+\delta_{is}^k\geq\sum_{\tau=1}^{S}\frac{x_{i\tau}\mu_i m_i[f_{is}^0-f_{is}^k]}{(1+r)^{(s+\tau-1)}},\ \forall i,\ \forall s\in\{1,...,\eta_i\},\ \forall k=\{5,..,8\}, \\
& \epsilon_{is}+\gamma+\delta_{is}^k\geq\sum_{\tau=1}^{S}\frac{x_{i\tau}\mu_i\left(\sum_{l\in N^i}m_i^l\pi_i^l+\sum_{l\in N^{i'}}\pi_i^l\min(m_i,m_i^l)\right)[f_{is}^0-f_{is}^k]}{(1+r)^{(s+\tau-1)}},\ \forall i,\ \forall s\geq\eta_i+1,\ \forall k=\{5,..,8\}, \\
& \sum_{\tau=1}^{S}x_{i\tau}\leq 1,\ \forall i, \\
& \sum_{i=1}^{N}x_{i\tau}(D_i(1+r)^{(\tau-1)}+\Omega_i^*(\alpha,\Gamma_L))\leq B_\tau,\ \forall\tau, \\
& x_{i,\tau}\in\{0,1\},\ \forall i,\ \forall\tau, \\
& \epsilon,\gamma,\delta,\sigma\geq 0.
\end{aligned}
\tag{15}
$$

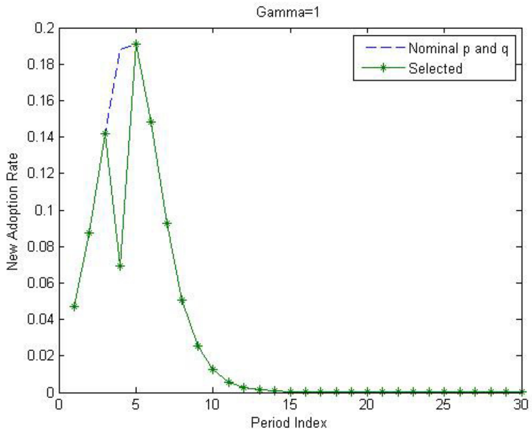
2.5 Numerical Experiments

We consider 10 new products and 5 potential partners for each product. All parameters are randomly selected using uniform distributions. The Bass model parameters are selected so that the value of the parameter K_i (the longest possible time period from the beginning of the diffusion process until it terminates) varies between 12 and 15 periods. Market size is between 1200 and 1450 customers for each product. The unit price of each product is between \$200 and \$300. The logit choice parameters are chosen so that the nominal unit cost of outsourcing is between \$15 and \$30 for each product. The parameters η_i, κ_i, ρ_i, are taken to be 5, 0.9, and 0.6 for all the products, for simplicity. The random parameters can deviate from their nominal values by $\pm 20\%$.

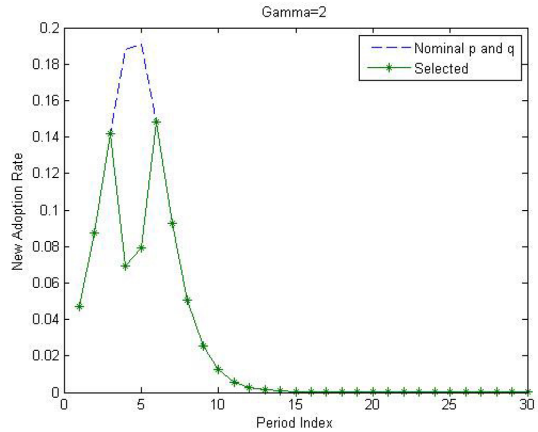
We observe in Figures 3-4, showing the marginal adoption rates in the robust model, that the computer first selects a marginal rate lower than expected around the time of peak marginal adoption and, if the budget of uncertainty in the Bass model increases so that more parameters can take their worst-case value, the time periods for which the parameters deviate from their nominal values are those around that peak-adoption time, where the impact of unrealized sales is most acute. (Additional figures about marginal adoption rates for higher values of Γ_B, and about all cumulative adoption rates, are omitted here due to space constraints but are available in Cetinkaya (2014).) This translates into significant drops in the cumulative adoption rates and the ultimate market size. The impact of increasing the budget of uncertainty Γ_B decreases with time when the marginal adoption rate becomes closer to zero, and deviations from nominal values thus have little impact. In our example, there are 30 time periods but the adoption rate process reaches steady state after about 15 periods and has about 6-7 time periods of high marginal adoption rates. We observe that increasing Γ_B beyond 7 does not noticeably affect the adoption curves, but that increasing Γ_B from 0 to 7 significantly decrease the overall number of adopters. In fact, adoption rates reach a steady state about 60% below their nominal values in this example for $\Gamma_B = 7$.

We are particularly interested in the impact of the Bass-model budget of uncertainty Γ_B on the solution, since of the two budgets, that one is the less problem-specific and is the more likely to be applied to other robust optimization problems in new product launch and innovation management. Therefore, we solve the problem both without and with options for $\Gamma_L = 30$ and vary Γ_B from 1 to 75 in increments of 1. The problems were solved at the COR@L lab at Lehigh University using AMPL/CPLEX 12.5 on a 64-bit computer with AMD Opteron 2.0 GHz (x16) processor and 32 GB memory. All the problems were solved within milliseconds (about 300 MIP simplex iterations for each product launch optimization problem at given uncertainty budgets); therefore, we do not report computational times here.

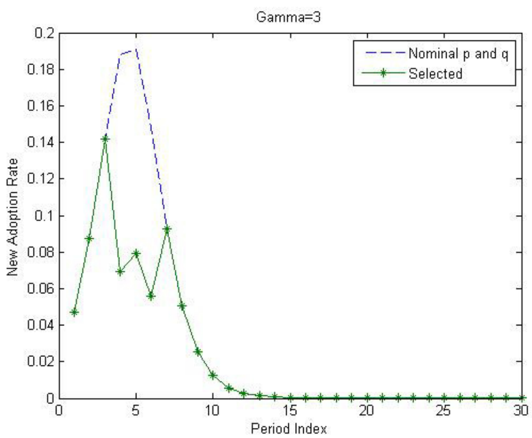
Without real options, there are 14 breakpoints in Γ_B, at which which the optimal launch sequence x changes. All the strategies have the following in common:



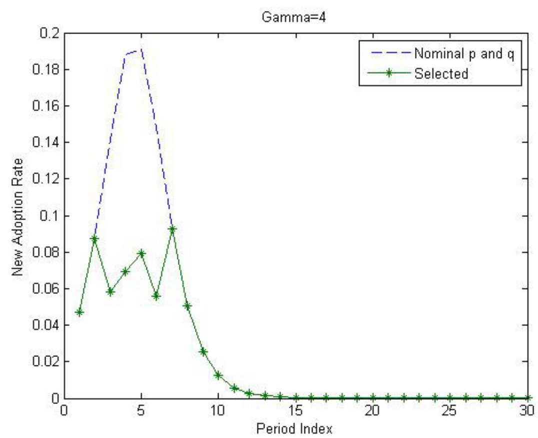
(a) $\Gamma_B = 1$



(b) $\Gamma_B = 2$



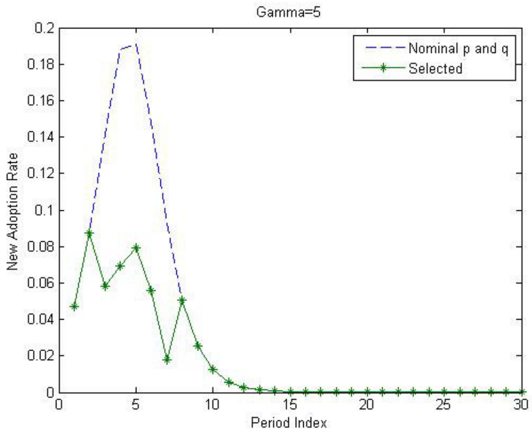
(c) $\Gamma_B = 3$



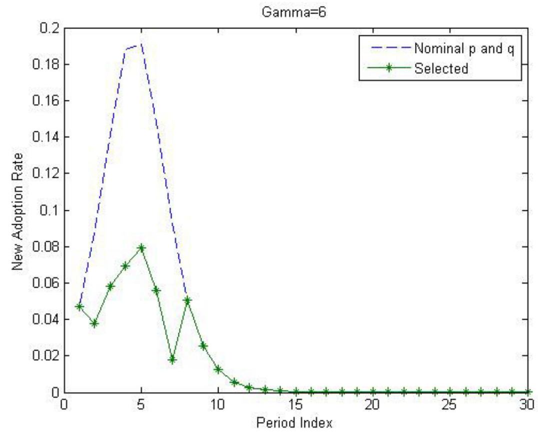
(d) $\Gamma_B = 4$

Figure 3: The Impact of the Uncertainty Budget Parameter on the Robust Marginal Adoption Rates ($1 \leq \Gamma_B \leq 4$)

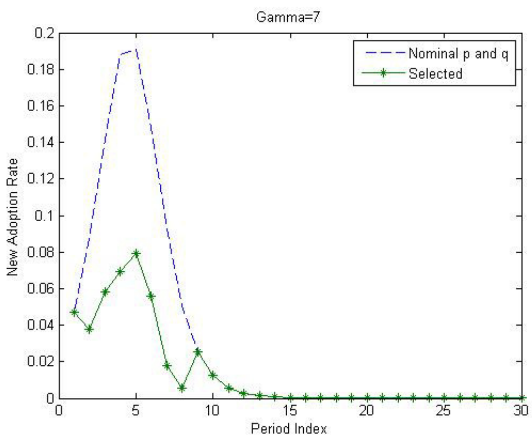
- 1 and 8 are launched first, followed by 6.
- 9 is always launched at the 6^{th} time period, and 4 at the 9^{th} time period.
- All the products are always launched prior to the 10^{th} time period, which is never used for a launch.



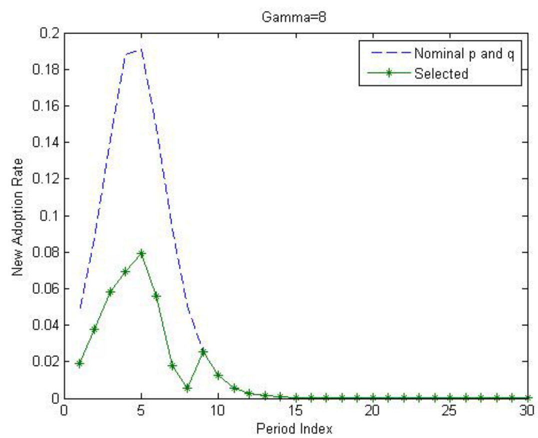
(a) $\Gamma_B = 5$



(b) $\Gamma_B = 6$



(c) $\Gamma_B = 7$



(d) $\Gamma_B = 8$

Figure 4: The Impact of the Uncertainty Budget Parameter on the Robust Marginal Adoption Rates ($5 \leq \Gamma_B \leq 8$)

- 2, 7 and 10 will be launched in periods 3, 4 and 5 (one in each time period), and 3 and 5 will be launched in periods 7 and 8 (one in each time period).
    - · 7 is usually launched at time 3, 2 at time 4 and 10 at time 5.
    - · 5 is usually launched at time 7 and 3 at time 8.
    - · The specific sequence depends on Γ_B.

With real options, there are 17 breakpoints in Γ_B, at which which the optimal launch sequence x changes. We observe in this numerical experiment that the i^{th} smallest breakpoint without real options is at most the i^{th} breakpoint with real options, for $i \leq 12$, but the breakpoints without real options for $i = 13$ and $i = 14$ are higher than their counterparts with real options. The i^{th} launch sequence without real options is only identical to the i^{th} launch sequence with real options for $i \leq 7$. In other words, without real options, for small values of Γ_B, the decision-maker simply changes his strategy sooner to protect against uncertainty. As the difference between the i^{th} Γ_B breakpoints grows, the decision-maker skips strategies that had been optimal without real options, and in some cases implements new ones. For instance, the 8^{th} strategy with real options is the 10^{th} strategy without real options, but strategies 11 to 15 in the case with real options launch product 5 at time 6, which never happened in the case without real options. All the strategies have the following in common:

- 1 and 8 are launched first, followed by 6.
- 4 is always launched at the 9^{th} time period.
- All the products are always launched prior to the 10^{th} time period, which is never used for a launch.
- 2, 7 and 10 will be launched in periods 3, 4 and 5 (one in each time period), and 3, 5 and 9 will be launched in periods 6 to 8 (one in each time period).
    - · 7 is usually launched at time 3, 2 at time 4 and 10 at time 5.
    - · 9 is usually launched at time 6 (otherwise 5 is launched at time 6) and 3 at time 8 (otherwise 5 is launched at time 8). The product launched at time 7 can be 3, 5 or 9.
    - · The specific sequence depends on Γ_B.

A key change due to the presence of real options here is to introduce more variability on the launch time of items 5 and 9.

The robust optimization approach thus presents the added benefit of identifying a set of candidate strategies that the ambiguity-averse decision-maker can then study in more depth in the light of the specific business environment he is in.

3 Conclusions

We have presented an approach to new product introduction under parameter uncertainty that incorporates robust optimization to the Bass diffusion model in order to determine the optimal product launch sequence for the decision maker. Future work includes (i) extending our results to include correlation between products' success with customers through a robust dynamic optimization approach, (ii) providing an adaptive decision tool that would have the choice of the next product launched depend on, for instance, the market size reached to date or more specific estimates of the innovation parameters and (iii) further analyzing the impact of real options in the robust launch sequence.

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Table 2: Sensitivity of the optimal strategy to the parameter q_i

Coefficient of q_i	only q_1 changes	only q_2 changes	only q_3 changes	only q_4 changes	only q_5 changes
0.6	87	9	3	7	88
0.64	87	9	3	7	88
0.68	87	9	3	7	88
0.72	87	9	3	7	88
0.76	85	7	3	9	90
0.8	85	7	3	9	90
0.84	85	7	1	9	90
0.88	85	1	7	9	90
0.92	85	1	7	9	90
0.96	87	7	7	9	90
1	7	7	7	7	7
1.04	112	33	7	111	7
1.08	112	113	3	111	7
1.12	112	113	107	111	7
1.16	112	113	107	111	7
1.2	112	113	107	111	7
1.24	112	113	107	111	7
1.28	112	113	107	111	7
1.32	112	113	107	111	7
1.36	112	113	107	111	7
1.4	112	113	107	111	7

Table 3: Sensitivity of the optimal strategy to the parameter p_i

Coefficient of p_i	only p_1 changes	only p_2 changes	only p_3 changes	only p_4 changes	only p_5 changes
0.6	112	7	1	9	7
0.64	112	7	1	9	7
0.68	112	7	1	9	7
0.72	112	7	1	9	7
0.76	112	7	1	9	7
0.8	112	7	1	9	7
0.84	112	7	1	9	7
0.88	112	7	7	9	7
0.92	112	7	7	9	7
0.96	112	7	7	7	7
1	7	7	7	7	7
1.04	7	7	7	7	112
1.08	7	9	7	7	112
1.12	7	9	7	7	112
1.16	7	9	7	7	112
1.2	7	9	7	87	112
1.24	7	9	7	87	112
1.28	7	9	7	87	112
1.32	7	33	7	87	112
1.36	7	33	7	87	112
1.4	7	33	7	87	112