The Be Phenomenon in Early-Type Stars, IAU Colloquium 175
ASP Conference Series, Vol. 214, 2000
M. A. Smith, H. F. Henrichs, and J. Fabregat, eds.

New Radiation Driven Wind Solutions Including Stellar Rotation: Applicability to Be stars

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Abstract. The theory of radiative-line driven wind including stellar rotation is re-examined. After a suitable change of variables, new analytical equations for the mass loss rate and position of the critical point are derived. A non-linear equation for the position of the critical point is obtained, in order to solve it, a β-field is used, and in addition to the standard CAK critical point two other critical points as function of star's rotational speed (V_ϕ) are founded. When the stellar rotation velocity is approximate or greater than $0.8 \times V_{break-up}$, the CAK critical point is located far away in the wind ($r > 4R_*$) and there we have again only one critical point. Numerical solutions from this *new* critical point are obtained, they have very low terminal velocities (approx. 400 km/sec). The applicability of this new solution in the frame of Be stars are discussed.

1. Hydrodynamic formulation

The standard model for radiation driven stellar winds consider one component fluid in a stationary regime with spherical symmetry, neglecting the effect of viscosity, heat conduction and magnetic field. The continuity equation reads $4\pi r^2 \rho v = \dot{M}$ and the momentum equation is given by

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM(1-\Gamma)}{r^2} + \frac{v_\phi^2}{r} + g^{line}(\rho, v, dv/dr, n_E) \qquad (1)$$

here v is the fluid velocity, dv/dr the velocity gradient, ρ the mass density, $\dot{M}$ is the star's mass loss rate, p the fluid pressure, v_ϕ the star's rotational speed, Γ the radiative acceleration caused by Thomson scattering in terms of gravitational acceleration and $g^{line}(\rho, v, dv/dr, n_E)$ is the acceleration due to the lines. The standard form of the line force is: $g^{line} = \frac{C}{r^2}\, CF(r, v, \frac{dv}{dr})\, \left(r^2 v \frac{dv}{dr}\right)^\alpha \left(\frac{n_E}{W(r)}\right)^\delta$

The coefficient C is given by $C = \Gamma G M k \left(\frac{4\pi}{\sigma_E\, v_{th}\, \dot{M}}\right)^\alpha \left(\frac{D\dot{M}}{2\pi}\right)^\delta$, here v_{th} is the thermal velocity of the protons, σ_E is the Thompson scattering absorption coefficient per density and n_E is the electron number density in units of $10^{-11} cm^{-3}$, $W(r)$ is the dilution factor, CF is the correction factor and all the other quantities have their usual meaning (see, e.g., Kudritzki et al. 1989).

Introducing now the following change of variables, $u = -R_*/r, w = v/a$ and $w = dw/du$, where R_* is the star's radius and a is the isothermal sound speed, i.e., $p = a^2\rho$. The momentum equation with the line force now reads:

$$F(u,w,\frac{dw}{du}) = (1-\frac{1}{w^2})w\frac{dw}{du} + A + \frac{2}{u} + a_\phi^2 u - C'\ CF\ g(u)(w)^{-\delta}\left(w\frac{dw}{du}\right)^\alpha \quad (2)$$

here the constant C transform to $C' = C\ (\frac{\dot{M}}{2\pi\mu}\frac{10^{-11}}{aR_*^2})^\delta\ (a^2R_*)^{(\alpha-1)}$, $g(u) = \left(\frac{u^2}{1-\sqrt{1-u^2}}\right)^\delta$ is the dilution factor and the constant $A = GM(1-\Gamma)/a^2R_* = v_{esc}^2/2a^2$ is related to the escape velocity v_{esc} and $a_\phi = v_\phi/a$ is the rotational velocity of the star in terms of the thermal velocity of the wind plasma.

The standard method for solve this non-linear momentum differential equation together with the constant C' (the eigenvalue of the problem) is imposing that the solution goes true the singular (critical) point. The critical point is located when the singularity condition is satisfied, $\partial F(u,w,w')/\partial w' = 0$ and at this point the regularity condition is imposed, namely, $dF(u,w,w')/du = 0$ Besides these equations a constrain (boundary condition) has to be imposed, usually the total optical depth is set to 2/3.

1.1. The new coordinate transformation

In order to solve the equations at the critical point, the next coordinate transformation are used, $Y = w\ w'$ and $Z = w/w'$ with this new coordinates, the equations transform to:

$$(1-\frac{1}{YZ})Y + A + \frac{2}{u} + a_\phi^2 u - C'f_1(u,Z)\ g(u)Z^{-\delta/2}Y^{\alpha-\delta/2} = 0 \quad (3)$$

$$(1-\frac{1}{YZ})Y \qquad - C'f_2(u,Z)\ g(u)Z^{-\delta/2}Y^{\alpha-\delta/2} = 0 \quad (4)$$

$$(1+\frac{1}{YZ})Y - \frac{2Z}{u^2} + a_\phi^2 Z \quad - C'f_3(u,Z)\ g(u)Z^{-\delta/2}Y^{\alpha-\delta/2} = 0 \quad (5)$$

functions f_1, f_2,f_3 and f_{123} (see Curé 1999) are combinations of the correction factor and its derivatives. Solving for C', u and Y , we get, $Y = \frac{1}{Z} + \left(\frac{f_2}{f_1-f_2}\right)\left(A + \frac{2}{u} + a_\phi^2 u\right)$ and $C' = \frac{1}{g\,f_2}\left(1 - \frac{1}{YZ}\right)\ Z^{\delta/2}Y^{1-\alpha+\delta/2}$ and the roots of the function R , defined by:

$$R(u,Z) \equiv \frac{2}{Z} - \frac{2Z}{u^2} + a_\phi^2 Z - f_{123}\left(A + \frac{2}{u} + a_\phi^2 u\right) \quad (6)$$

gives the location of the critical point(s). We want to notice that any approximation has been done until this point!

Introducing now a β-field approximation, not for the velocity field, but now for the quotient $Z = w/w' = (1+u)/\beta$, the critical point function becomes:

$$R(u) = \frac{2\beta}{(1+u)} - \frac{2(1+u)}{u^2\beta} + a_\phi^2\frac{(1+u)}{\beta} - f_{123}(u)\left(A + \frac{2}{u} + a_\phi^2 u\right) \quad (7)$$

From the features of $R(u)$ in the interval $-1 \le u \le 0$ (or $R_* \le r \le \infty$), the main conclusion of this study are achieved. Once the root(s) of $R(u) = 0$ is(are)

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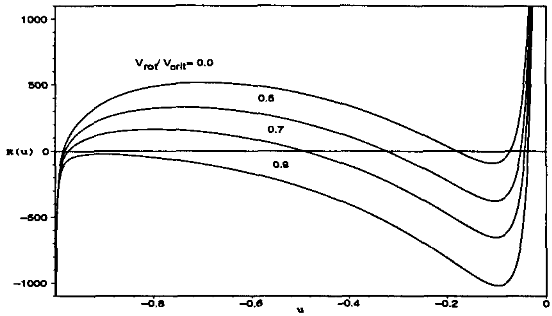

Figure 1. $R(u)$ versus u for different values of star's rotational speed for a typical B1 V Star

obtained, the values of Y_c and $C'(\dot{M})$ are straightforward to obtained from the equation in the above solutions.

In Figure 1 the behavior of $R(u)$ is show as function of the star's rotational speed ($V_{rot}/V_{break-up} = 0.0; 0.5; 0.7; 0.9$ respectively) for a typical B1 V Star for the m-CAK (CAK with the finite cone angle correction) model (see Kudritzki et al. 1989). Stellar parameters are $T_{\rm eff}$=25000 K, $M = 11$ M$_\odot$ and $R = 5.3$ R$_\odot$. The line-force parameters are $\alpha = 0.5$, $k = 0.3$ ($\delta = 0.0$). From this figure, in addition to the standard critical point, two other critical points (roots of $R(u)$) appear as the rotational speed increase. The location of the m-CAK singular point is shifted downstream as a function of a_ϕ. For rotational speeds of about $0.6 - 0.7 \times V_{break-up}$ no numerical solution starting at stellar surface, passing true *one* singular point and reaching infinity can be obtained. A more detailed study of the topology of the wind momentum equation is currently under way. It is not clear yet, that the solution should pass only for one critical point or have to pass true more critical points as in the magnetic model to reach infinity (Poe and Friend 1986 and references therein)

The standard m-CAK critical point now disappears when $a_\phi \sim 0.8 \times V_{break-up}$ (see $V_{rot}/V_{break-up} = 0.9$ in Figure 1). This is the reason why many authors have reported numerical difficulties in finding the location of the singular point: it does not exist. In this case, the CAK critical point is shifted far downstream in the wind ($r \sim 5R_*$).

2. New wind solutions

The function $R(u)$ has *again* only one root when the rotational speed is $V_\phi \sim 0.8 \times V_{break-up}$ as Figure 1 shows. This critical point correspond to the CAK critical point, and therefore a numerical solution can be achieved.

Figure 2 shows the polar and equatorial velocity profiles for a B1 V Start (see above stellar parameters) with $V_\phi = 0.9 \times V_{break-up}$. The critical point is at $r = 1.014R_*$ for the pole and $r = 23.63R_*$ for the equator. Terminal velocity and mass loss rates are: $V_\infty = 2025$ km/s, $\dot{M} = 3.178 \times 10^{-9}M_\odot$/year and $V_\infty = 443$ km/s, $\dot{M} = 6.854 \times 10^{-9}M_\odot$/year respectively. The quotient

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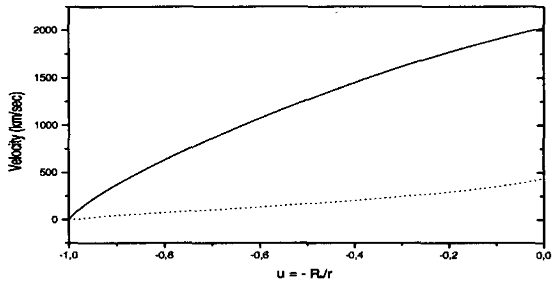

Figure 2. Velocity profile for the pole (continuous line) and the equator (dashed line), see text for details.

between equatorial and polar densities is about 100 near the stellar surface, but mainly about a factor 10 for the whole wind.

3. Conclusions

After a suitable change of coordinates, we were able to show that there are more than one critical point when rotational speed is taken into account for a radiation driven wind. In this work only the case of extreme high rotational rate (applied to Be stars) has been studied so far. Here only one critical point and numerical wind solution can be achieved. These *new* solutions gives very slow terminal velocities ($\sim$ 450 km/s) and ratio of about 10 for the quotient between equatorial and polar densities. The model presented here must be improved for Be stars. In order to do this, the following effects have to take into account: the equatorial radius distortion is a function of rotational rate, equatorial and polar dependence on the temperature as a function of rotational rate, and a self-consistent treatment of the line force parameters with the inclusion of the δ-parameter, and also a detailed study of the influence of the boundary conditions, re-analysis of angular momentum conservation (viscose forces), and the here ignored influence of the magnetic fields. Currently a detailed study of the topology of the singular points is under way. There are probably more solutions which involve more than one singular point as in the case of magnetic fields (see Curé 1999).

Acknowledgments. This work has been partially supported by Universidad de Valparaíso, internal project DIUV 16/97.

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