spectral width of about $2\Delta\nu \approx 25$ GHz is required, the electrical power spectrum density is −136 dBm/Hz which is still 40 dB above the thermal noise limit.

As an example Fig. 2 shows the noise spectrum for unity P_{el} and $\Delta\nu = 1$ GHz. The solid line corresponds to eqn. 4 and the dashed line is the exact power spectrum accounting for the spectral width of the laser diode (presumed here to be $\Delta f = 50$ MHz). It can be seen in Fig. 2 that the spectrum of eqn. 4 is in good agreement with the actual power spectrum for frequencies $< 2\Delta\nu$. The evaluation of the power spectrum should therefore be limited to this frequency range.

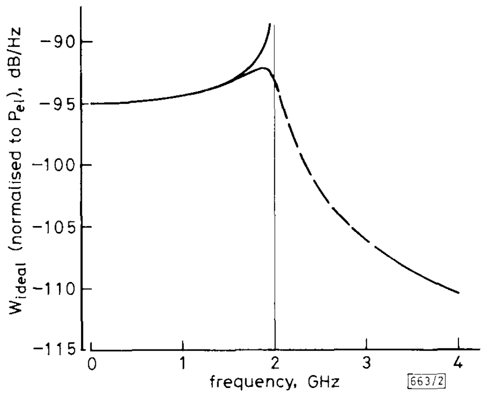

Fig. 2 *Power density spectrum for ideal photodetector with unity total power and $\Delta\nu = 1$ GHz*

—— laser diode with $\Delta f = 0$
- - - - laser diode with $\Delta f = 50$ MHz

Finally the power transfer function $H(f)$ of the photoreceiver can be expressed as

$$|H(f)|^2 = \frac{W_m(f)}{W_{ideal}(f)}$$

where $W_m(f)$ is the measured power spectrum.

Experiment: The proposed method was verified by a preliminary experiment. Owing to the equipment used in the experiment, only the frequency range up to 1·8 GHz was analysed. A DFB laser diode has been used with a current frequency transfer factor of about 0·25 GHz/mA at 100 kHz. For a 2 GHz-wide spectrum ($\Delta\nu = 1$ GHz) a current modulation amplitude of about 4 mA is thus required. Fig. 3 shows a set of results for the frequency response of the photoreceiver under test for different modulation indices m. For large m, large frequency modulations $\Delta\nu$ and wide measurement ranges are obtained, as shown in Fig. 3. The results show very good agreement for all modulation indices m, which demonstrates the good measurement reproducibility of this method.

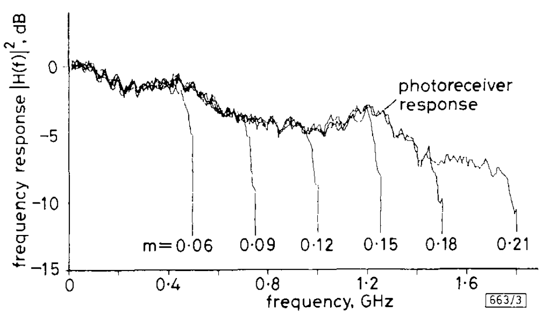

Fig. 3 *Frequency response of photoreceiver tested*

Conclusions: We have demonstrated a new technique for measuring the frequency response of a photoreceiver using a delayed self-homodyne set-up. This measurement technique has the advantage of a simple implementation. Even for measuring bandwidths of tens of GHz, the power spectral density at the photoreceiver is about 40 dB above the thermal noise limit yielding reliable results. The measurement range is ultimately limited by the maximum frequency modulation of the laser diode. A wide frequency range of $\Delta\nu = 10$–100 GHz would be possible, if tunable lasers[10] are used.

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NEW RADIX-2-BASED ALGORITHM FOR FAST MEDIAN FILTERING

Indexing terms: Signal processing, Algorithms, Filters, Image processing

A fast radix-2-based median filtering algorithm is proposed. The median is determined bit-by-bit successively by eliminating the samples whose previous bits are different to that of the median. The intermediate computations of the algorithm do not involve any array computation, nor any memory. The worst-case computational complexity of the algorithm is O(w) for w samples.

Introduction: Median filtering is a nonlinear smoothing technique used in signal and image processing to filter out the impulsive noises while preserving the edge-information. In the applications of median filtering, a window of size w moves over the sampled values of the signal or image, and then the median of the samples within the window is computed and written as the output pixel at the location of the centre of the window.[1]

Various median filtering algorithms for software and for hardware implementations have been published.[2–7] A comparison of the computational requirements of various software-based median filtering algorithms is given by Ataman *et al.*,[4] and it is claimed that their algorithm is more efficient than the others. More recently, V. Rao and S. Rao[6] have presented a software-based median filtering algorithm which requires less processing than Ataman's algorithm.

However, in both Ataman's[4] and Rao's[6] algorithms, an intermediate array is computed and updated. This requires too much computation. Since some of the elements of these arrays are not used in the median computation, there is some redundancy. Furthermore, the arrays of Ataman's and Rao's algorithms require 2^r bytes of memory.

In this letter, an alternative fast median computation procedure is proposed. The computation of full-word median requires r full-word comparisons and a number of bit-level operations proportional to $w \times r$ where w is the window size and r is the word-length. The proposed algorithm does not require any intermediate array computation, any memory except the storage of the window samples and a few registers, any histogram computation, nor any data sorting. Furthermore, the algorithm can be easily mapped to hardware either as a single chip or as a small-sized printed circuit board.

Algorithm: Let $\{x_1, x_2, x_3, \ldots, x_w\}$, w is odd, be the set of samples in the window. Let $\{u_{i1}, u_{i2}, u_{i3}, \ldots, u_{ir}\}$ and $\{m_1, m_2, m_3, \ldots, m_r\}$ be the radix-2 representations of the x_i and median, respectively. In other words, u_{ij} is the jth most significant bit of ith input sample, and m_i is the ith most significant bit of the median.

Let $t = (\omega + 1)/2$. Then the median is the tth smallest sample in the window. Thus the median search task is to find the tth smallest sample. The algorithm finds this sample successively bit-by-bit starting from the most significant bit. To do that, first we start counting the 0s in the most significant bits of the samples, and call that number Z_1. If $Z_1 \geq t$ then the first bit of the median is set to 0 ($m_1 = 0$), and t stays as it is. If $Z_1 < t$ then $m_1 = 1$ and t is updated as $t \leftarrow t - Z_1$. In the second step, Z_2 is found by counting 0s in the second most significant bits of the samples whose most significant bits are the same as m_1, and repeating the procedure of the first step. Consequently, in the kth step, Z_k is obtained by counting 0s in the kth bits of the samples whose first $(k - 1)$ bits are the same as that of the median.

To specify the samples whose first k bits are the same as that of the median, a flag is assigned to each sample. This flag can be checked and updated during the computations as follows: let c_i be the flag for the sample x_i, if $c_i = 1$ at kth step, then the sample x_i is enabled in the computation of Z_k; otherwise it is disabled (in the computation of Z_k, only enabled samples are considered). At the kth step, each c_i is updated in such a way that if $c_j = 1$ and $u_{jk} \neq m_k$ then $c_j = 0$, else it remains as it is, $c_j = 1$. Note that if a flag takes the value 0 at any time, then it never changes.

Let the number of enabled samples be denoted by S_c which is equal to the sum of c_is. Then, during the computation of the median when $S_c = t = 1$ the algorithm terminates. In this case, since there is only one enabled sample, the undetermined bits of the median are the same as that of this sample.

The complete procedure of the algorithm is as follows:

Step 1: Set $k = 1$, $t = (w + 1)/2$, and $c_i = 1$ for $1 \leq i \leq w$

Step 2: Compute Z_k

Step 3: If $Z_k < t$, then $m_k = 1$ and $t \leftarrow t - Z_k$, else $m_k = 0$

Step 4: Update c_is and find $S_c = \Sigma_{i=1}^{w} c_i$

Step 5: If $S_c = t = 1$, then set $m_j = u_{ij}$ whose $c_i = 1$ and $k \leq j \leq r$, and go to Step 8

Step 6: $k \leftarrow k + 1$

Step 7: If $k = r + 1$ go to Step 8, else go to Step 2

Step 8: Stop.

As an example, consider the set of samples $\{5, 4, 11, 1, 14, 7, 9, 5, 10\}$. Presume that each of these samples is a 4-bit number (i.e. $r = 4$). The procedure to find the median of this set is as follows:

x_i	5	4	11	1	14	7	9	5	10
u_{i1}	0	0	1	0	1	0	1	0	1
u_{i2}	1	1	0	0	1	1	0	1	0
u_{i3}	0	0	1	0	1	1	0	0	1
u_{i4}	1	0	1	1	0	1	1	1	0

$k = 1$	$k = 2$	$k = 3$
$\{c_i\}$: 111111111	$\{c_i\}$: 110101010	$\{c_i\}$: 110001010
$t = 5$	$t = 5$	$t = 4$
$Z_1 = 5$	$Z_2 = 1$	$Z_3 = 3$
$Z_1 \geq t$	$Z_2 < t$	$Z_3 < t$
$m_1 = 0$	$m_2 = 1$	$m_3 = 1$
$(t = t)$	$t = 5 - 1 = 4$	$t = 4 - 3 = 1$
$\{c_i\}$: 110101010	$\{c_i\}$: 110001010	$\{c_i\}$: 000001000
$S_c = 5$	$S_c = 4$	$S_c = 1$
$S_c \neq t \neq 1$	$S_c \neq t \neq 1$	$S_c = t = 1$
$k = 1 + 1 = 2$	$k = 2 + 1 = 3$	$m_4 = 1$

Thus, the median represented by $\{m_1, m_2, m_3, m_4\}$ is found to be $\{0, 1, 1, 1\}$. This is the radix-2 representation of 7 which can also be found by sorting the samples and taking the middle sample.

Concluding remarks: A fast radix-2-based median filtering algorithm with worst-case computational complexity $O(w \times r)$ is presented. All operations of the algorithm are performed recursively as the bits arrive, therefore there is no redundant computation. Since most of the operations are in bit-level, the algorithm is very efficient if it is implemented in a low-level programming language. For the real-time applications, since one has to consider the worst-case computational complexity, the algorithm proposed here is more efficient than recently proposed algorithms.[4,6] In addition, the algorithm does not require any memory except storage of the window samples and a few registers, whereas the other two algorithms require 2^r bytes of memory. Furthermore, for hardware implementation, the proposed algorithm requires considerably less circuitry than the others. A fast median filter unit can be built using VLSI techniques, or even using conventional components on a small printed circuit board. In any case, the cost of the hardware implementation grows linearly with both the window size w and the word-length r.

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