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New Results on Stability Analysis of Networked Control Systems

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Abstract— **This paper studies the problem of stability analysis for continuous-time networked control systems (NCSs). In NCSs, time-delay terms are piecewise differentiable, and their derivatives are equal to 1 except at countable interrupted points. By taking into account this feature of time-delay terms, new stability condition for NCSs is derived in terms of solutions to a set of linear matrix inequalities (LMIs). The new proposed stability criterion is less conservative than the existing ones without considering this feature, and the computational complexity is also reduced. Numerical examples are given to illustrate the effectiveness of the proposed methods.**

I. Introduction

Networked control systems (NCSs) are the feedback control loops closed through real time networks. The main advantages of NCSs are low cost, reduced weight, simple installation and maintenance, and high reliability. Despite of the great advantages and wide applications, communication networks in the control loops make the analysis and design of NCSs complicated. One main issue is the network-induced delays (sensor-to-controller and controller-to-actuator), it is known that the occurrence of delay degrades the stability and control performance of the NCSs.

Many researchers have paid attention to the study of the stability, which is the basic problem in NCSs. [1] modeled NCSs as ordinary linear systems with time-varying delay and studied the design of robust H_∞ controllers for uncertain NCSs with the effects of both the network-induced delay and data dropout taken into consideration. In [2], the feedback gain of a memoryless controller and the maximum allowable value of the network-induced delay were derived. [3] studied the problem of packet dropout and transmission delays induced by communication network of NCSs in both continuous time and discrete time cases. By using the Lyapunov-Razumikhin function techniques, [4] obtained the delay-dependent condition on the stabilization of NCSs in terms of linear matrix inequalities (LMIs). The admissible upper bounds of data packet loss and delays can be computed by using the quasi-convex optimization algorithm. [5] proposed a numerical procedure to design a linear output-feedback controller for a remote linear plant in which the loop is closed through a network. For other results dealing delay, see also [6]-[7].

Network-induced delay is the main reason of instability and poor performance of NCSs, so it is necessary to take the character of delay into consideration for stability analysis of NCSs. The control input of NCSs is piecewise constant, so the NCSs can be modeled as time delay systems, and the derivatives of time-delay terms are equal to 1 except at countable interrupted points. However, this feature is not taken into account for the stability analysis of NCSs in the above mentioned literature. By taking this feature into consideration and defining a novel Lyapunov function, this paper presents a less conservative stability criterion. Since fewer slack variables are involved in the stability criterion, the computational complexity of the presented result is also reduced.

The organization of this paper is as follows. Section 2 presents the model of an NCS with data packet dropout and transmission delays, where the presented model is able to capture the network-induced delay feature. In Section 3, a new method for stability analysis which takes the network feature into consideration is proposed. The obtained result is less conservative than the existing ones and includes fewer decision variables. Two numerical examples are given to show the effectiveness of the criteria in Section 4. Section 5 concludes this paper.

II. System description

Throughout this paper, we assume that the sensor is clock-driven, the controller and actuator are event-driven and hold the latest data, and the latest available control inputs will be used by the actuator. The sampling period is denoted as h. Single packet transmission is considered, where all the sensor data sampled at the same sampling instant are lumped together into one data packet and transmitted through the network. Data packet dropout and disordering in an NCS are unavoidable because the actuator and the sensor are connected through a communication network with finite bandwidth. An NCS with the possibility of dropping data packet and disordering can be described as in Figure 1.

The model presented here is the same as that in [1]:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

$$x(t) = \phi(t), \qquad t \in [t_1 - \eta,\ t_1], \tag{2}$$

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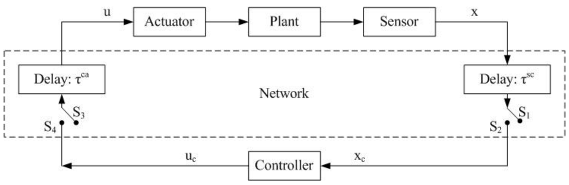

Fig. 1. An NCS with data packet dropout and transmission delays

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^p$ is the control input vector, and t_1 denotes the instant the actuator receives the 1st control signal, η is the upper bound of time delay. A, B are constant matrices of appropriate dimensions; x_c is the delayed version of x, u is the delayed version of u_c. Denote the instant the actuator receives the kth control signal as t_k, and this control signal is based on the state of plant at instant $i_k h$, thus $\{i_1, i_2, i_3, \cdots\} \subset Z^+$, and

$$i_k < i_{k+1}, \qquad \forall\ k \in \{1,\ 2,\ \cdots\} \tag{3}$$

due to the actuator will be updated until the new data comes. So, the control signal can be described as

$$u(t) = Kx(i_k h), \quad t \in [i_k h + \tau_k,\ i_{k+1}h + \tau_{k+1}) \quad \forall\ k = 1, 2, \cdots, \tag{4}$$

where K is the state feedback gain matrix, time-delay τ_k denotes the time from the instant $i_k h$ when the sensor node samples the plant states to the instant t_k when the actuator receives the control signal, i.e., $\tau_k = t_k - i_k h$, and $\tau_k = \tau_k^{sc} + \tau_k^{ca}$, where τ_k^{sc} is the time-delay of $x(i_k h)$ from sensor to controller, and τ_k^{ca} is the time-delay of $u_c(i_k h + \tau_k^{sc})$ from controller to actuator. Obviously, $\cup_{k=1}^{\infty}[i_k h + \tau_k,\ i_{k+1}h + \tau_{k+1}) = [t_1,\ \infty),\ t_1 \geq 0$.

As pointed out in [1], under assumption:

$$(i_{k+1} - i_k)h + \tau_{k+1} \leq \eta, \qquad k = 1, 2, \cdots, \tag{5}$$

$$\tau \leq \tau_k, \qquad k = 1, 2, \cdots, \tag{6}$$

where η and τ are constants satisfying $0 \leq \tau < \eta$, then the system (1)-(6) can be rewritten as follows:

$$\dot{x}(t) = Ax(t) + A_d x(t - d(t)), \tag{7}$$

$$x(t) = \phi(t), \qquad t \in [t_1 - \eta,\ t_1] \tag{8}$$

$$\tau \leq d(t) \leq \eta, \tag{9}$$

$$\dot{d}(t) = 1, \quad t \in [t_k,\ t_{k+1}), \tag{10}$$

where $A_d = BK$, $d(t) = t - i_k h\ \forall t \in [t_k,\ t_{k+1})$, which denotes the time-varying delay in the control signal. Similar to [6], Figure 2 shows the chart of $d(t)$.

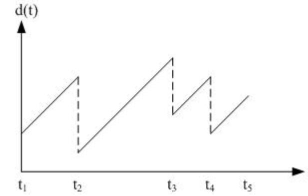

Fig. 2. The chart of $d(t)$ in an NCS

Obviously, $d(t)$ is always differentiable in the interval $[t_1,\ \infty)$ except at the points of $t_k\ (k = 1,\ 2,\ \cdots)$ and $\dot{d}(t) = 1$ *a.e.*, this is the most distinct character in NCSs. Unfortunately, this character has not ever been utilized in stability analysis of NCSs up to now.

In this paper, we will take this character into account for the stability analysis of NCSs.

Remark 1. The system described by (7)-(10) is a special case of the ordinary time-delay systems. Many results of time-delay systems can be applied to deal with stability analysis of this system, for example, the results in [8] (a simplified and equivalent form of the result in [8] was presented in [10]) and in [9]. For convenience of comparison, these results are listed as follows.

Lemma 1. [9] For given scalars $0 \leq \tau < \eta$, the system (7)-(10) is asymptotically stable there exist matrices $P = P^T > 0$, $Q_i = Q_i^T \geq 0\ (i = 1,\ 2)$, $Z_j = Z_j^T > 0\ (j = 1,\ 2)$ and $N_i,\ S_i,\ M_i\ (i = 1,\ 2)$, such that

$$\Phi = \begin{bmatrix} \Phi_1 + \Phi_2 + \Phi_2^T & \Phi_3 \\ * & \Phi_4 \end{bmatrix} < 0 \tag{11}$$

holds, where

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & PA_d & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & -Q_1 & 0 \\ * & * & * & -Q_2 \end{bmatrix} + \bar{A}^T U \bar{A},$$

$$\Phi_{11} = \sum_{i=1}^{2} Q_i + PA + (PA)^T,$$

$$\Phi_2 = \begin{bmatrix} N & S - N - M & M & -S \end{bmatrix},$$

$$\Phi_3 = \begin{bmatrix} \eta N & (\eta - \tau)S & (\eta - \tau)M \end{bmatrix},$$

$$\Phi_4 = diag\Big(-\eta Z_1,\ -(\eta - \tau)(Z_1 + Z_2),\ -(\eta - \tau)Z_2\Big),$$

$$U = \eta Z_1 + (\eta - \tau)Z_2,$$

$$\bar{A} = \begin{bmatrix} A & A_d & 0 & 0 & 0 \end{bmatrix},$$

$$N = \begin{bmatrix} N_1 \\ N_2 \\ 0 \\ 0 \end{bmatrix},\ S = \begin{bmatrix} S_1 \\ S_2 \\ 0 \\ 0 \end{bmatrix},\ M = \begin{bmatrix} M_1 \\ M_2 \\ 0 \\ 0 \end{bmatrix}.$$

Lemma 2. ([8], [10]) For given scalars $0 < \eta$, the NCS described by (7)-(10) with $\tau = 0$ is asymptotically stable if there exist matrices $P_1 = P_1^T > 0$, $R = R^T > 0$ and $P_2,\ P_3$, such that

$$\Psi = \begin{bmatrix} \Psi_1 & -\eta G^T \begin{bmatrix} 0 \\ A_d \end{bmatrix} \\ * & -\eta R \end{bmatrix} < 0 \tag{12}$$

holds, where

$$\Psi_1 = G^T \begin{bmatrix} 0 & I \\ A + A_d & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A + A_d & -I \end{bmatrix}^T G + \begin{bmatrix} 0 & 0 \\ 0 & \eta R \end{bmatrix},$$

$$G = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}.$$

III. Stability analysis

A. New Stability Criteria

In this subsection, a new type of Lyapunov functionals is proposed to derive new delay-dependent asymptotical stability criteria for the system (7)-(9), and the character of NCSs described by (10) is used.

In NCSs, the time-delay term $d(t)$ is piecewise differentiable, and except those countable interrupted points $\{t_k,\ k = 1,\ 2,\ \cdots\}$, its derivative is equal to 1, i.e., $\dot{d}(t) = 1$, *a.e.* According to the definition of $d(t) = t - i_k\ (t \in [t_k,\ t_{k+1}))$, and $i_k < i_{k+1}$ are always true for any $k \in \{1,\ 2,\ \cdots\}$, if choosing a positive scalar $0 < \alpha < 1$, then it yields that

$$t - \alpha d(t) \to (1-\alpha)t_{k+1} + \alpha i_k h, \qquad \text{if } t \to t_{k+1}^{-}. \tag{13}$$

Similarly, it gets that

$$t - \alpha d(t) \to (1-\alpha)t_{k+1} + \alpha i_{k+1} h, \qquad \text{if } t \to t_{k+1}^{+}. \tag{14}$$

Thus, it is known that $\int_{t-\alpha d(t)}^{t} x^T(s)Qx(s)ds$ decreases monotonously at the interrupted points $t_k\ (k = 1,\ 2,\ \cdots)$.

Based on this fact, we can derive the stability criteria of NCSs as follows.

Theorem 1. For given scalars $\tau, \eta\ (0 \le \tau < \eta)$, and $\alpha\ (0 < \alpha < 1)$, the NCS described by (7)-(10) is asymptotically stable if there exist matrices $P = P^T > 0$, $Q_i = Q_i^T \ge 0\ (i = 1,\ 2,\ 3)$, $Z_j = Z_j^T > 0\ (j = 1,\ 2)$, such that

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 & 0 & \frac{1}{\alpha\eta}Z_1 \\ * & \Omega_{22} & \frac{1}{\eta-\tau}Z_2 & \frac{1}{\eta-\tau}(Z_1+Z_2) & \frac{1}{(1-\alpha)\eta}Z_1 \\ * & * & \Omega_{33} & 0 & 0 \\ * & * & * & \Omega_{44} & 0 \\ * & * & * & * & \Omega_{55} \end{bmatrix} < 0 \tag{15}$$

holds, where

$$\begin{aligned}
\Omega_{11} &= PA + (PA)^T + Q_1 + Q_2 + Q_3 - \tfrac{1}{\alpha\eta}Z_1 + A^T UA, \\
\Omega_{12} &= PA_d + A^T UA_d, \\
\Omega_{22} &= -\tfrac{1}{(1-\alpha)\eta}Z_1 - \tfrac{1}{\eta-\tau}Z_2 - \tfrac{1}{\eta-\tau}(Z_1+Z_2) + A_d^T UA_d, \\
\Omega_{33} &= -Q_1 - \tfrac{1}{\eta-\tau}Z_2, \\
\Omega_{44} &= -Q_2 - \tfrac{1}{\eta-\tau}(Z_1+Z_2), \\
\Omega_{55} &= -(1-\alpha)Q_3 - \tfrac{1}{\alpha\eta}Z_1 - \tfrac{1}{(1-\alpha)\eta}Z_1, \\
U &= \eta Z_1 + (\eta-\tau)Z_2.
\end{aligned}$$

Proof: Construct a Lyapunov functional as

$$\begin{aligned}
V(t) = {} & x^T(t)Px(t) + \textstyle\int_{t-\tau}^{t} x^T(s)Q_1x(s)ds \\
& + \textstyle\int_{t-\eta}^{t} x^T(s)Q_2x(s)ds \\
& + \textstyle\int_{t-\alpha d(t)}^{t} x^T(s)Q_3x(s)ds \\
& + \textstyle\int_{-\eta}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)dsd\beta \\
& + \textstyle\int_{-\eta}^{-\tau}\int_{t+\beta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)dsd\beta,
\end{aligned} \tag{16}$$

where $P > 0,\ Q_i \ge 0\ (i = 1,\ 2,\ 3),\ Z_1 > 0,\ Z_2 > 0$.

Denoting

$$\zeta(t) = [\, x^T(t)\ x^T(t-d(t))\ x^T(t-\tau)\ x^T(t-\eta)\ x^T(t-\alpha d(t))]^T,$$

from the Jensen integral inequality ([13], [14]) with (9) that one can obtain

$$\begin{aligned}
& -\int_{t-\alpha d(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds \\
& \le -\frac{1}{\alpha\eta}\Big(\int_{t-\alpha d(t)}^{t} \dot{x}(s)ds\Big)^T Z_1 \int_{t-\alpha d(t)}^{t} \dot{x}(s)ds \\
& = -\frac{1}{\alpha\eta}[x(t) - x(t-\alpha d(t))]^T Z_1 [x(t) - x(t-\alpha d(t))],
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
& -\int_{t-d(t)}^{t-\alpha d(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds \\
& \le -\frac{1}{(1-\alpha)\eta}[x(t-\alpha d(t)) - x(t-d(t))]^T Z_1 \\
& \qquad \times [x(t-\alpha d(t)) - x(t-d(t))],
\end{aligned} \tag{18}$$

$$\begin{aligned}
& -\int_{t-d(t)}^{t-\tau} \dot{x}^T(s)Z_2\dot{x}(s)ds \\
& \le -\frac{1}{\eta-\tau}[x(t-\tau) - x(t-d(t))]^T Z_2 \\
& \qquad \times [x(t-\tau) - x(t-d(t))],
\end{aligned} \tag{19}$$

$$\begin{aligned}
& -\int_{t-\eta}^{t-d(t)} \dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)ds \\
& \le -\frac{1}{\eta-\tau}[x(t-d(t)) - x(t-\eta)]^T (Z_1+Z_2) \\
& \qquad \times [x(t-d(t)) - x(t-\eta)].
\end{aligned} \tag{20}$$

Taking the time derivative of $V(t)$ for $t \in [t_k,\ t_{k+1})$ along the trajectory of (7), and combining (17)-(20), it yields that

$$\begin{aligned}
\dot{V}(t) = {} & 2x^T(t)P\dot{x}(t) + \sum_{i=1}^{3} x^T(t)Q_ix(t) - x^T(t-\tau)Q_1x(t-\tau) \\
& - x^T(t-\eta)Q_2x(t-\eta) \\
& - (1-\alpha)x^T(t-\alpha d(t))Q_3x(t-\alpha d(t)) \\
& + \dot{x}^T(t)\Big(\eta Z_1 + (\eta-\tau)Z_2\Big)\dot{x}(t) \\
& - \textstyle\int_{t-\eta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-\eta}^{t-\tau} \dot{x}^T(s)Z_2\dot{x}(s)ds \\
= {} & 2x^T(t)P[Ax(t) + A_d(t-d(t))] + \sum_{i=1}^{3} x^T(t)Q_ix(t) \\
& - x^T(t-\tau)Q_1x(t-\tau) - x^T(t-\eta)Q_2x(t-\eta) \\
& - (1-\alpha)x^T(t-\alpha d(t))Q_3x(t-\alpha d(t)) \\
& + [Ax(t) + A_d(t-d(t))]^T U [Ax(t) + A_d(t-d(t))] \\
& - \textstyle\int_{t-\alpha d(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-d(t)}^{t-\alpha d(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds \\
& - \textstyle\int_{t-d(t)}^{t-\tau} \dot{x}^T(s)Z_2\dot{x}(s)ds \\
& - \textstyle\int_{t-\eta}^{t-d(t)} \dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)ds \\
\le {} & \zeta^T(t)\Omega\zeta(t),
\end{aligned} \tag{21}$$

where Ω, U are defined in (15).

So, if $\Omega < 0$, i.e., (15) holds, then $\dot{V}(t) < 0$ for any $t \in [t_k,\ t_{k+1})$.

Noting that $V(t_{k+1}^{-}) > V(t_{k+1}^{+})$ are true for any $k = 1,\ 2,\ \cdots$, so the proof is complete. ■

Remark 2. By using the Jensen integral inequality, Theorem 1 gives a new stability criterion for NCSs in terms of solutions to a set of LMIs. By introducing the term $\int_{t-\alpha d(t)}^{t} x^T(s)Qx(s)ds$ into the Lyapunov functional, the derivative character of time delay term $d(t)$ is taken into consideration. Compared with Lemma 1 ([9]), fewer slack variables are involved in the stability condition given by Theorem 1, hence the computational complexity is reduced.

B. Comparison with The Existing Results

In this subsection, we will show that Theorem 1 is less conservative than the existing results (Lemmas 1 and 2) in [9], [8] and [10]. For proving that Theorem 1 is less conservative than Lemma 1 ([9]), we give a lemma as follows.

Lemma 3. The following two statements are equivalent:
i) For given scalars $0 \le \tau < \eta$, and $0 < \alpha < 1$, there exist matrices $P = P^T > 0$, $Q_i = Q_i^T \ge 0$ $(i = 1,\ 2,\ 3)$, $Z_j = Z_j^T > 0$ $(j = 1,\ 2)$ and $Y_i,\ T_i,\ M_i,\ S_i$ $(i = 1,\ 2,\ \cdots,\ 5)$, such that

$$\Gamma = \begin{bmatrix} \Gamma_1 + \Gamma_2 + \Gamma_2^T & \Gamma_3 \\ * & \Gamma_4 \end{bmatrix} < 0 \tag{22}$$

holds, where

$$\Gamma_1 = \begin{bmatrix} \Gamma_{11} & PA_d & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & -Q_1 & 0 & 0 \\ * & * & * & -Q_2 & 0 \\ * & * & * & * & -(1-\alpha)Q_3 \end{bmatrix} + \bar{A}^T U \bar{A},$$

$$\Gamma_{11} = \sum_{i=1}^{3} Q_i + PA + (PA)^T,$$

$$\Gamma_2 = \begin{bmatrix} Y & -T - M + S & M & -S & -Y + T \end{bmatrix},$$

$$\Gamma_3 = \begin{bmatrix} \alpha\eta Y & (1-\alpha)\eta T & (\eta-\tau)M & (\eta-\tau)S \end{bmatrix},$$

$$\Gamma_4 = diag\Big(-\alpha\eta Z_1,\ -(1-\alpha)\eta Z_1,\ -(\eta-\tau)Z_2,\ -(\eta-\tau)(Z_1+Z_2)\Big),$$

$$U = \eta Z_1 + (\eta-\tau)Z_2,$$

$$\bar{A} = \begin{bmatrix} A & A_d & 0 & 0 & 0 \end{bmatrix},$$

$$Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_5 \end{bmatrix},\ T = \begin{bmatrix} T_1 \\ \vdots \\ T_5 \end{bmatrix},\ M = \begin{bmatrix} M_1 \\ \vdots \\ M_5 \end{bmatrix},\ S = \begin{bmatrix} S_1 \\ \vdots \\ S_5 \end{bmatrix}.$$

ii) For given scalars $0 \le \tau < \eta$, and $0 < \alpha < 1$, there exist matrices $P = P^T > 0$, $Q_i = Q_i^T \ge 0$ $(i = 1,\ 2,\ 3)$, $Z_j = Z_j^T > 0$ $(j = 1,\ 2)$, such that

$$\Omega < 0 \tag{23}$$

holds, where Ω is defined in Theorem 1.

Proof: i) $\Rightarrow$ ii). Pre- and post-multiplying both sides of Γ in (22) with

$$\Pi = \begin{bmatrix} I & 0 & 0 & 0 & 0 & -\frac{1}{\alpha\eta} & 0 & 0 & 0 \\ * & I & 0 & 0 & 0 & 0 & \frac{1}{(1-\alpha)\eta} & \frac{1}{\eta-\tau} & -\frac{1}{\eta-\tau} \\ * & * & I & 0 & 0 & 0 & 0 & -\frac{1}{\eta-\tau} & 0 \\ * & * & * & I & 0 & 0 & 0 & 0 & \frac{1}{\eta-\tau} \\ * & * & * & * & I & \frac{1}{\alpha\eta} & -\frac{1}{(1-\alpha)\eta} & 0 & 0 \\ * & * & * & * & * & I & 0 & 0 & 0 \\ * & * & * & * & * & * & I & 0 & 0 \\ * & * & * & * & * & * & * & I & 0 \\ * & * & * & * & * & * & * & * & I \end{bmatrix} \tag{24}$$

and its transpose, it yields that

$$\Delta = \Pi \Gamma \Pi^T = \begin{bmatrix} \Omega & \bar{\Gamma}_3 \\ * & \Gamma_4 \end{bmatrix} < 0, \tag{25}$$

where

$$\bar{\Gamma}_3 = \begin{bmatrix} \alpha\eta Y & (1-\alpha)\eta T & (\eta-\tau)M & (\eta-\tau)S \end{bmatrix} + \begin{bmatrix} Z_1 & 0 & 0 & 0 \\ 0 & -Z_1 & -Z_2 & Z_1 + Z_2 \\ 0 & 0 & Z_2 & 0 \\ 0 & 0 & 0 & -Z_1 - Z_2 \\ -Z_1 & Z_1 & 0 & 0 \end{bmatrix}$$

and Γ_4 is defined in (22).

Noting that Z_1, Z_2 are positive definite, $0 < \alpha < 1$, $0 \le \tau < \eta$, and Π is nonsingular, so Γ_4 is negative definite. Thus, it is easy to see that $\Delta < 0$ if $\Gamma < 0$, and by the Schur complement it follows that

$$\Omega \le \Omega - \bar{\Gamma}_3 \Gamma_4^{-1} \bar{\Gamma}_3^T < 0. \tag{26}$$

ii) $\Rightarrow$ i). If $\Omega < 0$ holds, then $\Omega - \bar{\Gamma}_3 \Gamma_4^{-1} \bar{\Gamma}_3^T = \Omega < 0$ is true by taking

$$Y_1 = -\tfrac{1}{\alpha\eta} Z_1,\ Y_2 = Y_3 = Y_4 = 0,\ Y_5 = \tfrac{1}{\alpha\eta} Z_1;$$
$$T_1 = T_3 = T_4 = 0,\ T_2 = \tfrac{1}{(1-\alpha)\eta} Z_1,\ T_5 = -\tfrac{1}{(1-\alpha)\eta} Z_1;$$
$$M_1 = M_4 = M_5 = 0,\ M_2 = \tfrac{1}{\eta-\tau} Z_2,\ M_3 = -\tfrac{1}{\eta-\tau} Z_2;$$
$$S_1 = S_3 = S_5 = 0,\ S_2 = -\tfrac{1}{\eta-\tau}(Z_1 + Z_2),\ S_4 = \tfrac{1}{\eta-\tau}(Z_1 + Z_2).$$

So, from (25), $\Delta < 0$ is immediately obtained. Meanwhile, because $\Delta < 0$ is equivalent to $\Gamma < 0$, then $\Gamma < 0$ also holds when $\Omega < 0$. This completes the proof. ■

The following theorem shows that Theorem 1 is less conservative than Lemma 1.

Theorem 2. If the condition of Lemma 1 is feasible, then the condition of Theorem 1 is also feasible.

Proof: When (11) in Lemma 1 is feasible, then $\Gamma < 0$ in Lemma 3 is also feasible, which can be seen by the Schur complement with taking $Y = T$, $Y_i = T_i = M_i = S_i = 0$ $(i = 3,\ 4,\ 5)$ and $Q_3 = \varepsilon I$ in Γ, where ε being a sufficient small positive scalar. So, from Lemma 3, the inequality (15) in Theorem 1 is also feasible. ■

Remark 3. From Theorem 2, one can see that Theorem 1 is less conservative than the result in [9] (Lemma 1).

The following theorem shows that Theorem 1 also is less conservative than Lemma 2.

Theorem 3. For $\tau = 0$, if the condition of Lemma 2 is feasible, then the condition of Theorem 1 is also feasible.

Proof: From Lemma 2, it gets that

$$\Psi = \begin{bmatrix} \Psi_{11} & P_1 - P_2^T + (A + A_d)^T P_3 & -\eta P_2^T A_d \\ * & -P_3 - P_3^T + \eta R & -\eta P_3^T A_d \\ * & * & -\eta R \end{bmatrix} < 0, \tag{27}$$

where

$$\Psi_{11} = P_2^T (A + A_d) + (A + A_d)^T P_2.$$

Pre- and post-multiplying both sides of Ψ in (27) with

$$\Delta = \begin{bmatrix} I & A^T & \eta^{-1} I \\ 0 & A_d^T & -\eta^{-1} I \end{bmatrix}, \tag{28}$$

and its transpose, then it follows that

$$\begin{bmatrix} \Delta_1 & P_1A_d+\eta^{-1}R+\eta A^TRA_d \\ * & -\eta^{-1}R+\eta A_d^TRA_d \end{bmatrix} < 0, \quad (29)$$

where

$$\Delta_1 = P_1A+A^TP_1-\eta^{-1}R+\eta A^TRA.$$

Similar to the proof of Lemma 3, (29) holds if and only if there exist matrices Y and T, such that

$$\begin{bmatrix} \Delta_2 & P_1A_d+\eta A^TRA_d+T^T-Y & \eta Y \\ * & -T-T^T+\eta A_d^TRA_d & \eta T \\ * & * & -\eta R \end{bmatrix} < 0, \quad (30)$$

where

$$\Delta_2 = P_1A+A^TP_1+\eta A^TRA+Y+Y^T.$$

So, if setting

$$P=P_1,\ Q_i=0\ (i=1,\ 2,\ 3),\ Z_1+Z_2=R,$$
$$N_1=Y,\ N_2=T,\ S_j=0,\ M_j=0\ (j=1,\ 2),$$

then it follows that $\Phi<0$ in Lemma 1 holds. Therefore, by Theorem 2, the inequality (15) in Theorem 1 is also feasible. ■

Remark 4. From the proof of Theorem 3, one can see that Lemma 1 is less conservative than Lemma 2.

About how to seek an appropriate α satisfying $0<\alpha<1$, such that the upper bound η of delay $d(t)$ satisfying (9) is maximal, we give an algorithm as follows.

Algorithm 1. (Maximizing $\eta>0$):

Step 1. Set appropriate step lengths, η_{step} and α_{step}, for η and α, respectively. Set $\alpha=\alpha_{step}$. For given μ and τ, choose an upper bound on η satisfying (15), and then select this upper bound as the initial value η_0 of η.

Step 2. Set k as a counter, and choose $k=1$. Meanwhile, let $\eta=\eta_0+\eta_{step}$ and the initial value α_0 of α equals to α_{step}.

Step 3. Let $\alpha=k\alpha_{step}$, if the inequality (15) is feasible, go to step 4; otherwise, go to step 5.

Step 4. Let $\eta_0=\eta$, $\alpha_0=\alpha$, $k=1$ and $\eta=\eta_0+\eta_{step}$, go to step 3.

Step 5. Let $k=k+1$. If $k\alpha_{step}<1$ and $k\alpha_{step}\mu<1$, then go to step 3; otherwise, stop.

Remark 5. In the above algorithm, the final η_0 is the desired maximum of the upper bound of delay $d(t)$ satisfying (15), and α_0 is the corresponding value of α.

IV. Numerical Examples

In this section, two examples are given to illustrate that the stability conditions with the feature of network-induced delay taken into consideration are less conservative than the corresponding ones without considering the feature. In Example 1, the lower-bound of network-induced delay is 0, and in Example 2, the lower-bound of network-induced delay is τ ($\tau \geq 0$).

TABLE I

Allowable upper bound of η with given τ

Methods	$\tau=0$	$\tau=0.1$	$\tau=0.2$	$\tau=0.3$	$\tau=0.4$
Yue et al. [1]	0.82	0.89	0.96	1.04	1.13
He et al. [9]	0.92	0.95	1.02	1.09	1.16
Theorem 1	1.09	1.10	1.12	1.14	1.17

Example 1. Consider the NCS described by

$$\dot{x}(t)=\begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}x(t)+\begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}x(t-d(t)), \quad (31)$$

and

$$0\leq d(t)\leq \eta, \quad (32)$$

where η is a constant. The maximum upper bound of η was 0.99 by the method in [8], while the maximum upper bound of η was 1.34 by the method in [9]. However, by taking the feature of network-induced delay ($\dot{d}(t)=1$ a.e.) into consideration, we can obtain the maximum upper bound of η is 1.39 by Theorem 1 with $\alpha=0.75$. It justifies that the method given in Theorem 1 is less conservative than one given in [8] and [9].

Example 2. Consider the following NCS described by

$$\dot{x}(t)=\begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix}x(t)+\begin{bmatrix} -1.4 & 0 \\ -0.8 & -1.5 \end{bmatrix}x(t-d(t)), \quad (33)$$

and

$$\tau\leq d(t)\leq \eta, \quad (34)$$

where $\tau\geq 0$ and $\eta\geq 0$ are constants.

For various τ, the computed upper bounds, η, which guarantee the stability of NCS (33), are listed in Table 1. It shows that the new proposed method can give better results than the ones in [1] and [9].

V. Conclusion

In this paper, the problem of stability analysis for continuous-time networked control systems (NCSs) has been investigated. Unlike the previous methods, the derivative character of time-delay terms are taken into full consideration, and new asymptotic stability condition for NCSs is proposed. It is proved that the obtained result for NCSs is less conservative than the corresponding ones in the existing literature. Since fewer slack variables are involved in the stability criterion, the computational complexity is reduced. The numerical examples have shown the effectiveness of the proposed method.

References

[1] D. Yue, Q. L. Han, and J. Lam, Network-based robust H_∞ control of systems with uncertainty, *Automatica*, Vol. 41, pp. 999-1007, 2005.

[2] D. Yue, Q. L. Han, and P. Chen, State feedback controller design of networked control systems, *IEEE Transactions on Circuits and Systems-II: Express Briefs*, Vol. 51, no. 11, pp. 640-644, 2004.

[3] M. Yu, L. Wang, T. G. Chu, and F. Hao, An LMI approach to networked control systems with data packet dropout and transmission delays, *Proceedings of the 43rd IEEE Conference on Decision and Control*, 2004, pp. 3545-3550.

[4] M. Yu, L. Wang, T. G. Chu, and F. Hao, Stabilization of networked control systems with data packet dropout and transmission delays: continuous-time case, *European Journal of Control*, Vol. 11, pp. 40-49, 2005.
[5] P. Naghshtabrizi and J. P. Hespanha, Designing an observer-based controller for a network control system, *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference*, 2005, pp. 848-853.
[6] W. Zhang, M. S. Branicky, and S. M. Phillips, Stability of networked control system, *IEEE Control Systems Magazine*, Vol. 21, No. 2, pp. 84-99, 2001.
[7] G. C. Walsh, H. Ye, and L.G. Bushnell, Stability analysis of networked control systems, *IEEE Transactions on Control Systems Technology*, Vol. 10, No. 3, pp. 438-446, 2002.
[8] E. Fridman and U. Shaked, An improved stabilization method for linear time-delay systems, *IEEE Transactions on Automatic Control*, Vol. 47, No. 11, pp. 1931-1937, 2002.
[9] Y. He, Q. G. Wang, C. Lin, and M. Wu, Delay-range-dependent stability for systems with time-varying delay, *Automatica*, Vol. 43, pp. 371-376, 2007.
[10] S. Xu, J. Lam, and Y. Zuo, Simplified descriptor system approach to delay-dependent stability and performance analyses for time-delay systems. *IEE Proceedings— Control Theorey and Applications*, Vol. 152, pp. 147-151, 2005.
[11] V. Suplin, E. Fridman, and U. Shaked, H_∞ control of linear uncertain time-delay systems – a projection approach, *IEEE Transactions on Automatic Control*, Vol. 51, No. 4, pp. 680-685, 2006.
[12] D. S. Kim, Y. S. Lee, W. H. Kwon, and H. S. Park, Maximum allowable delay bounds of networked control systems, *Control Engineering Practice*, Vol. 11, pp. 1301-1313, 2003.
[13] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of time-delay systems*, Birkhauser, 2003.
[14] M. N. A. Parlakci, Robust stability of uncertain time-varying state-delayed systems, *IEE Proceedings-Control Theory and Applications*, Vol. 153, No. 4, pp. 469-477, 2006.
[15] M. Yu, L. Wang, and T. Chu, Sampled-data stabilisation of networked control systems with nonlinearity, *IEE Proceedings-Control Theory and Applications*, Vol. 152, No. 6, pp. 609-614, 2005.
[16] M. Yu, L. Wang, T. Chu, and F. Hao, An LMI approach to networked control systems with data packet dropout and transmission delays, *International Journal of Hybrid Systems*, 3(2&3), pp. 291-303, 2003.