New Space Shift Keying Modulation with Hamming Code-Aided Constellation Design

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***Abstract*—A modulation scheme that maps the information onto the antenna indices, such as space shift keying (SSK) and its generalized form (namely, generalized SSK or GSSK), presents an attractive option for the emerging large-scale MIMO system due to the reduced algorithm and hardware cost. In this letter, we present a new modulation scheme in this category, where we propose use of the Hamming code construction technique to systematically design the constellation. An illustrative example and experimental studies demonstrate that the proposed scheme introduces rich design flexibility and achieves better transmission rate, performance, and power tradeoffs with comparable hardware costs as compared with existing schemes.**

***Index Terms*—MIMO systems, spatial modulation, Hamming code.**

I. Introduction

Using antenna indices as a means of modulation has received increasing attention recently. In this type of modulation, the information is encoded partially or fully in the varying indices of the activated and idle antennas in space. First, the spatial modulation (SM) [1] was proposed to encode information in the combination of antenna indices and traditional phase and amplitude modulations. Later, the SSK [2] and its generalized form GSSK [3] suggested use of only the antenna indices to encode information. Space-time shift keying (STSK) [4] extended SSK to both space and time dimensions by combining SSK with space-time block codes. Using antenna indices presents an attractive means of modulation especially for large-scale multiple-input multiple-output (MIMO) systems, since: 1) The hardware expense is reduced, since by the nature of this modulation the required number of radio frequency (RF) chains is a subset of the total transmit antennas; 2) The detection complexity is lowered, as the information is contained *entirely* in the indexing of the antennas; 3) The transceiver requirement such as synchronization is reduced due to the absence of phase and amplitude modulations [2]. A disadvantage of this type of modulation, however, is a relatively small-sized modulation alphabet and therefore the reduced supportable transmission rates compared to conventional modulations. This problem can be alleviated by employing a large antenna array and GSSK.

In GSSK, a fixed number of activated antennas is employed at any given time. As a natural extension to SSK, GSSK however presents many limitations on its design flexibility and performance, such as the achievable transmission rates, utilization of the constellation space, and selection of an optimal constellation [3]. In this letter, we propose a new modulation scheme employing a possibly varying number of activated antennas. By observing the error probability expression, we further link the constellation design of the proposed modulation with the construction of Hamming codes [5]. The results show that the proposed scheme, termed SSK with Hamming code-aided constellation design, or HSSK for short, offers a rich selection of design options and achieves better transmission rate, performance, and power tradeoffs than GSSK. The hardware expenses for GSSK and the proposed scheme are comparable, as the transmitter requirement on the RF switches and RF chains [6] are almost identical. Furthermore, since the design criterion is based on the Hamming code construction technique, the proposed scheme does not suffer from the same computational burden in choosing a good constellation in GSSK.

This letter is organized as follows. Sec. II presents the system description. The proposed new modulation scheme is described in Sec. III. Performance results are demonstrated in Sec. IV and concluding remarks are given in Sec. V.

Notations: In this letter, $\mathbf{I}_M$ represents the $M \times M$ identity matrix, $\|\cdot\|$ the l_2-norm of a vector, $(\cdot)^T$ and $(\cdot)^H$ the matrix/vector transpose and conjugate matrix/vector transpose, respectively. $\mathrm{E}[\cdot]$ denotes expectation, $\Re(\cdot)$ the real part of its argument, and $Q(x)$ the Q-function defined as $\frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-(\alpha^2/2)}d\alpha$.

II. System Description

Consider an uncoded spatial multiplexing system with N_T transmit antennas and N_R receive antennas. The system employs the modulation scheme that uses only the antenna indices to carry information. The baseband signal model is given by

$$\mathbf{y} = \sqrt{E_b}\mathbf{H}\tilde{\mathbf{x}} + \mathbf{v}, \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{N_R\times 1}$ is the received signal, $\tilde{\mathbf{x}}$ is the $N_T\times 1$ transmitted symbol vector comprising one-elements (corresponding to activated antennas) and zero-elements (corresponding to idle antennas), $\mathbf{H} \in \mathbb{C}^{N_R\times N_T}$ is the flat-fading channel, $\mathbf{v} \in \mathbb{C}^{N_R\times 1}$ is the additive white Gaussian noise (AWGN), and E_b is the power amplification factor. The channel matrix $\mathbf{H}$ has independent and identically distributed (i.i.d.) complex Gaussian entries with zero mean and covariance matrix $\sigma_H^2\mathbf{I}_{N_R}$, where $\sigma_H^2 = 1$. The channel information is assumed perfectly known at the receiver. The noise $\mathbf{v}$ has i.i.d. complex elements with zero mean and covariance matrix $(N_0/2)\mathbf{I}_{N_R}$. The transmitted symbol vector $\tilde{\mathbf{x}}$ is drawn equally probably

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TABLE I
AN EXAMPLE OF GSSK SYMBOL MAPPING ($N_T = 5, n_t = 2$)

Information bits	GSSK symbol vector $\tilde{\mathbf{x}} \in \mathbb{A}^{(\text{GSSK})}$	Activated antenna indices $\tilde{\mathbf{i}} \in \mathbb{I}^{(\text{GSSK})}$
000	$[1,1,0,0,0]^T$	{1,2}
001	$[1,0,1,0,0]^T$	{1,3}
010	$[1,0,0,1,0]^T$	{1,4}
011	$[1,0,0,0,1]^T$	{1,5}
100	$[0,1,1,0,0]^T$	{2,3}
101	$[0,1,0,1,0]^T$	{2,4}
110	$[0,1,0,0,1]^T$	{2,5}
111	$[0,0,1,1,0]^T$	{3,4}

from the modulation alphabet (or the constellation set) $\mathbb{A}$, which is of size 2^b for b bits transmission. An example of $\mathbb{A}$ is shown in Table I for GSSK modulation employed in a 5×5 MIMO system to transmit 3 bits. The location of the $n_t = 2$ one-elements is varied to yield different symbol vectors (or constellation points). Since $\binom{N_T}{n_t} > 2^b$ in this case, $\mathbb{A}^{(\text{GSSK})}$ is not unique, and can be chosen lexicographically as shown in Table I or based on some optimization criterion [3] (see Sec. III).

A. Detection

Given the signal model in (1), the optimal maximum likelihood (ML) detection is to solve a constrained least-square problem, i.e.,

$$\tilde{\mathbf{x}}_{\text{ML}} = \arg\min_{\mathbf{x}\in\mathbb{A}} \left\| \mathbf{y} - \sqrt{E_b}\mathbf{H}\mathbf{x} \right\|^2. \tag{2}$$

The task in (2) is equivalent to searching column indices of $\mathbf{H}$ that match the one-elements of $\mathbf{x} \in \mathbb{A}$. Let $\mathbf{h}_k$ be the kth column of $\mathbf{H}$, then (2) can equivalently be expressed as

$$\tilde{\mathbf{i}}_{\text{ML}} = \arg\min_{\mathbf{i}\in\mathbb{I}} \left\| \mathbf{y} - \sqrt{E_b}\mathbf{h}_\mathbf{i} \right\|^2, \tag{3}$$

where $\mathbf{h}_\mathbf{i} = \sum_{k\in\mathbf{i}} \mathbf{h}_k$ and $\mathbb{I}$ is the set of activated antenna indices. An example of $\mathbb{I}$ is shown in Table I.

B. Pairwise Error Probability

Let $\mathbf{x}_i$ and $\mathbf{x}_j$ be two distinct symbol vectors in $\mathbb{A}$ with $\mathbf{i}$ and $\mathbf{j}$ being their activated antenna indices. The pairwise error probability (PEP) of deciding on $\mathbf{x}_j$ given that $\mathbf{x}_i = \tilde{\mathbf{x}}$ is transmitted conditioned on $\mathbf{H}$ is given by [3]

$$\begin{aligned} P(\mathbf{x}_i \to \mathbf{x}_j|\mathbf{H}) &= P\Big(\left\| \mathbf{y} - \sqrt{E_b}\mathbf{h}_\mathbf{i} \right\|^2 > \left\| \mathbf{y} - \sqrt{E_b}\mathbf{h}_\mathbf{j} \right\|^2 \Big) \\ &\overset{(a)}{=} P\Big(\Re\big(\mathbf{v}^H(\mathbf{h}_\mathbf{j} - \mathbf{h}_\mathbf{i})\big) > \frac{\sqrt{E_b}}{2} \left\| \mathbf{h}_\mathbf{i} - \mathbf{h}_\mathbf{j} \right\|^2 \Big) \\ &\overset{(b)}{=} Q(\sqrt{Z}), \end{aligned} \tag{4}$$

where $Z = (E_b/N_0) \left\| \mathbf{h}_\mathbf{i} - \mathbf{h}_\mathbf{j} \right\|^2$. (a) is derived by substituting $\mathbf{y} = \sqrt{E_b}\mathbf{h}_\mathbf{i} + \mathbf{v}$ and arranging the terms, and (b) is derived by assuming $\mathbf{h}_\mathbf{i}$ and $\mathbf{h}_\mathbf{j}$ are known and using the fact that entries of $\mathbf{v}$ are i.i.d. $\mathcal{CN}(0, N_0/2)$.

Since entries of $\mathbf{h}_\mathbf{i}$ or $\mathbf{h}_\mathbf{j}$ are i.i.d. $\mathcal{CN}(0, 1/N_T)$, entries of $\mathbf{h}_\mathbf{i} - \mathbf{h}_\mathbf{j}$ are i.i.d. $\mathcal{CN}(0, d(\mathbf{x}_i, \mathbf{x}_j)/N_T)$, where $d(\mathbf{x}_i, \mathbf{x}_j)$ is the number of distinct elements between $\{\mathbf{h}_k | k \in \mathbf{i}\}$ and $\{\mathbf{h}_k | k \in \mathbf{j}\}$, or equivalently, the *Hamming distance* between $\mathbf{x}_i$ and $\mathbf{x}_j$. As a result, Z is chi-square distributed with $2N_R$ degrees of freedom that can be expressed as $Z = \sum_{k=1}^{2N_R} z_k^2$, where z_k's are i.i.d. $\mathcal{N}(0, \sigma_z^2)$ with $\sigma_z^2 = E_b d(\mathbf{x}_i, \mathbf{x}_j)/(2N_T N_0)$. The PEP is then given by

$$\begin{aligned} P(\mathbf{x}_i \to \mathbf{x}_j) &= \mathrm{E}_Z\big[P(\mathbf{x}_i \to \mathbf{x}_j|\mathbf{H})\big] \\ &= \int_0^\infty Q(\sqrt{x}) f_Z(x) dx, \end{aligned} \tag{5}$$

where f_Z is the probability density function (pdf) of Z. Using Chernoff bound $Q(\sqrt{x}) \le (1/2)e^{-x/2}$ and substituting the known expression of f_Z in (5) we obtain

$$\begin{aligned} P(\mathbf{x}_i \to \mathbf{x}_j) &\le \frac{1}{2(\sigma_z^2 + 1)^{N_R}} \\ &\le a \cdot \left(\frac{E_b}{N_0}\right)^{-N_R} \cdot d(\mathbf{x}_i, \mathbf{x}_j)^{-N_R}, \end{aligned} \tag{6}$$

where $a = (2N_T)^{N_R}/2$. As suggested by (6), the performance of a modulation scheme that uses antenna indices to carry information depends on the Hamming distance between two possible symbol vectors.

C. System Error Probability

The system error probability can be derived by averaging the result in (6) over all pairwise combinations, i.e.,

$$P_s \le \frac{1}{2^{b-1}(2^b - 1)} \sum_{\mathbf{x}_i \ne \mathbf{x}_j \in \mathbb{A}} a \cdot \left(\frac{E_b}{N_0}\right)^{-N_R} \cdot d(\mathbf{x}_i, \mathbf{x}_j)^{-N_R}. \tag{7}$$

Note that the dominant terms in (7) correspond to pairs of symbol vectors with a small $d(\mathbf{x}_i, \mathbf{x}_j)$. In other words, the minimum Hamming distance between arbitrary two distinct symbol vectors, $d_{\min} = \min_{\mathbf{x}_i \ne \mathbf{x}_j \in \mathbb{A}} d(\mathbf{x}_i, \mathbf{x}_j)$, determines the system performance, as will be verified in Sec. IV.

III. PROPOSED NEW MODULATION

In GSSK, given a target transmission rate of b bits, the number of activated antennas n_t is chosen such that $\binom{N_T}{n_t} \ge 2^b$. This scheme presents several limitations:

1) A fixed n_t limits the amount of information that can be transmitted. For instance, a 4×4 system with GSSK can transmit at most two bits, with $n_t = 1$ or $n_t = 2$.
2) There is no simple and systematic method to design the constellation for GSSK that minimizes the error probability. The design method proposed in [3] chooses the specific $\mathbb{A}$ that maximizes $\sum_{\mathbf{x}_i \ne \mathbf{x}_j \in \mathbb{A}} d(\mathbf{x}_i, \mathbf{x}_j)$ as $\mathbb{A}^{(\text{GSSK})}$, which is a computationally intense search over the $\binom{\binom{N_T}{n_t}}{2^b}$ possibilities of $\mathbb{A}$.
3) Any constellation design, lexicographic as in Table I or optimal based on the above criterion, leads to $d_{\min} = 2$. This limits the system performance as suggested by (7).

The above limitations can be mitigated by relaxing the fixed n_t constraint and leveraging the design techniques for Hamming codes, as described as follows.

A. HSSK

The first key to the enhanced modulation design is to allow the number of activated antennas to be varied. Consider adopting a new modulation alphabet for a 4×4 system:

$$\mathbb{A} = \Big\{ [0,0,0,1]^T, [0,0,1,0]^T, [0,1,0,0]^T, [1,0,0,0]^T, [0,1,1,1]^T, [1,0,1,1]^T, [1,1,0,1]^T, [1,1,1,0]^T \Big\}.$$

As compared to GSSK with $n_t = 1$ (whose alphabet contains the first four constellation points above), this scheme can transmit three bits instead of two. The higher transmission rate is achieved at the cost of higher average power and more RF chains needed. This scheme and GSSK with $n_t = 1$ both have $d_{\min} = 2$. The elements of this alphabet are, in fact, identical to the eight codewords of the $(4, 3)$ Hamming code with the last bit of each codeword complemented.[1] Given the same length of codewords, a larger $d_{\min}$ can also be achieved at the compromise of the transmission rate, e.g., $\mathbb{A} = \{[0, 0, 0, 1]^T, [1, 1, 1, 0]^T\}$ with $d_{\min} = 4$. This motivates a systematic modulation design strategy based on Hamming codes, where the transmission rate, minimum Hamming distance, and power consumption can be customized.

In HSSK, given a target transmission rate of b bits, the constellation design procedure is summarized as follows:

1) Select $d_{\min}$ for which $n(N_T, d_{\min}) \geq 2^b$, where $n(N_T, d_{\min})$ is the number of codewords having fixed length N_T and satisfying $d_{\min}$ between any pair of codewords. Generate the codeword set $\mathbb{C}$ through readily available sources, e.g., [7].
2) Complement the last bit of all codewords in $\mathbb{C}$, i.e., $c_{N_T} = c_{N_T} \oplus 1, \forall \mathbf{c} = [c_1, c_2, \ldots, c_{N_T}] \in \mathbb{C}$.
3) Select the 2^b minimum Hamming-weight codewords from $\mathbb{C}$ (break ties arbitrarily), and transpose each of them to form $\mathbb{A}^{(\text{HSSK})}$.

B. An Example: 7×7 System

The capacity of flexible configuration in the proposed modulation scheme can be best seen by an example. In Fig. 1 we plot the interplay of transmission rate b, minimum Hamming distance $d_{\min}$, and average (constellation) power consumption $\mathrm{E}[\tilde{\mathbf{x}}^H \tilde{\mathbf{x}}]$ for different configurations. Each triangle represents a feasible modulation scheme. For GSSK in Fig. 1(a), only three configurations are possible, with a transmission rate of 3–5 bits, an average power equal to n_t, and a fixed $d_{\min} = 2$. In comparison, a rich array of configurations can be produced in HSSK, as shown in Fig. 1(b). Specifically, HSSK can transmit 3–6 bits, where 6 bits transmission is realized by using all 64 codewords in the $(7, 6)$ Hamming code with $d_{\min} = 2$, and 4 bits transmission is realized by either selecting 16 minimum Hamming-weight codewords from the $(7, 6)$ Hamming code or using all 16 codewords in the $(7, 4)$ Hamming code with $d_{\min} = 3$, etc. In addition, given a certain target transmission rate, HSSK allows several design choices that strike favorable tradeoffs between performance (related to $d_{\min}$) and power. Making specific comparisons side-by-side, we observe

- *Power consumption:* At the transmission rate $b = 3, 4,$ or 5 and fixed minimum Hamming distance $d_{\min} = 2$, HSSK consumes less power for $b = 3$ (1.25 vs. 2) and $b = 5$ (2.5625 vs. 3), and slightly more power for $b = 4$ (2.125 vs. 2).
- *Performance:* At a fixed transmission rate, e.g., $b = 3$, HSSK can be configured with $d_{\min} = 3$ (besides $d_{\min} = 2$) at a slight cost of power (2.375), and with $d_{\min} = 4$ at a higher cost of power (3.5). As will be seen in Sec. IV, the performance gain yielded by HSSK with $d_{\min} = 3$ cannot be duplicated by increasing the same amount of power for GSSK.
- *Number of RF chains:* The number of required RF chains at the transmitter, which concerns the hardware expense of the modulation scheme, can be interpreted as the maximum power consumption over all constellation points. For GSSK, this is equal to the average power consumption n_t. For HSSK, it is determined by the largest Hamming-weight codeword(s) among the 2^b selected codewords. The comparison is shown in Table II for different system settings. As can be seen, for the common configurations, HSSK may require an equal, greater, or smaller number of RF chains than GSSK depending on the scenario.

[1]Typically, Hamming codes are generated with an all-zero codeword. Complementing the last bit of all codewords guarantees no all-zero element in the modulation alphabet while retaining the minimum Hamming distance. Hereafter in this letter, a Hamming code refers to a set of codewords with the last bit complemented.

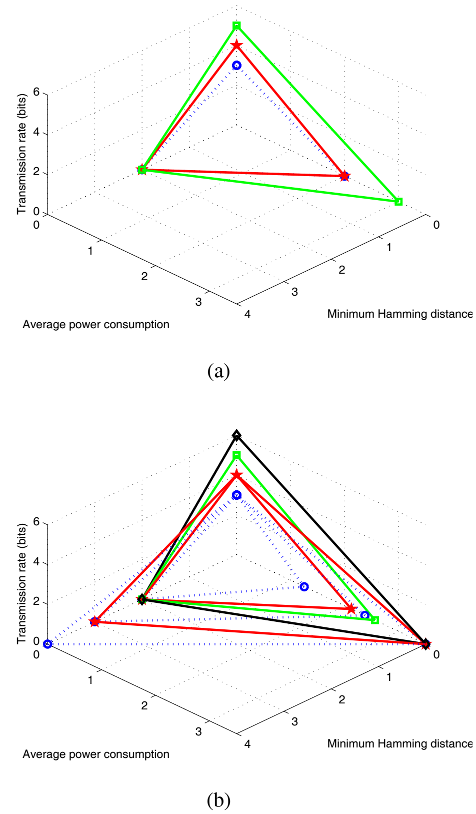

Fig. 1. The relation between transmission rate, minimum Hamming distance, and average power consumption for feasible configurations of (a) GSSK, (b) HSSK, for a 7×7 MIMO system with a target transmission rate of 3 bits or more.

IV. Simulation Results

Here we present the symbol error rate (SER) performance of the considered modulation schemes. In Fig. 2, we target 3 bits transmission in a 7×7 MIMO system. This requires $n_t = 2$ for GSSK and permits three different $d_{\min}$ for HSSK. Both the lexicographic and optimal [3] constellation designs are considered for GSSK. As can be seen, GSSK with optimal constellation design is about 0.6 dB better than the lexicographic design, which is comparable with HSSK ($d_{\min} = 2$). With the same $d_{\min} = 2$, GSSK with optimal

TABLE II
THE REQUIRED NUMBER OF RF CHAINS AT THE TRANSMITTER FOR DIFFERENT SYSTEM SETTINGS (⋆ DENOTES UNACHIEVABLE CONFIGURATIONS)

MIMO System	7 × 7							10 × 10								12 × 12							
Transmission rate (bits)	3			4		5	6	5			6		7	8	9	7			8		9	10	11
Min. Hamming distance	2	3	4	2	3	2	2	2	3	4	2	3	2	2	2	2	3	4	2	3	2	2	2
# RF chains (GSSK)	2	⋆	⋆	2	⋆	3	⋆	2	⋆	⋆	3	⋆	4	⋆	⋆	3	⋆	⋆	4	⋆	5	⋆	⋆
# RF chains (HSSK)	3	3	5	3	6	3	7	3	5	9	3	9	3	5	9	3	6	11	5	10	5	5	11

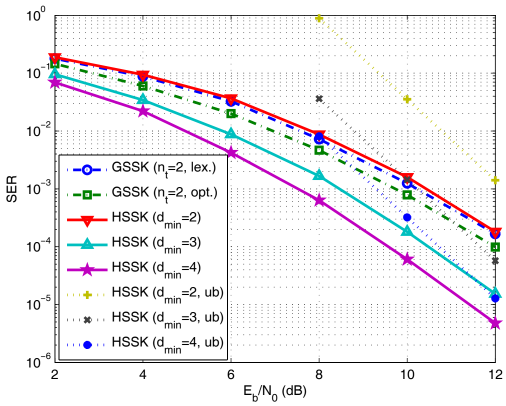

Fig. 2. SER performance for a 7×7 MIMO system.

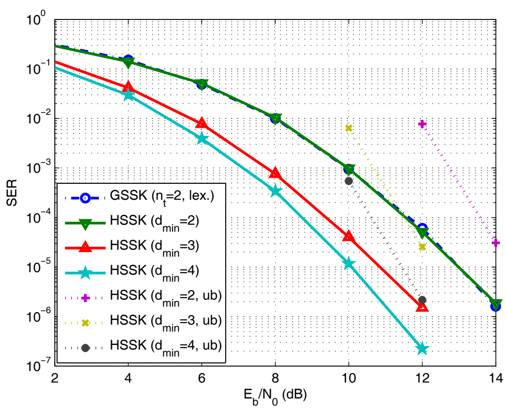

Fig. 3. SER performance for a 12×12 MIMO system.

constellation slightly outperforms HSSK, since GSSK finds constellation points with pairwise Hamming distances as large as possible whereas HSSK does not pursue an optimal combination (and spend such computational efforts) but rather selects constellation points with Hamming weights as small as possible.[2] HSSK however requires 37.5% less power than GSSK (1.25 vs. 2), as discussed in Sec. III-B. Note that there is a limit to what optimal constellation design can do for GSSK, since the same $d_{\min} = 2$ is yielded. By increasing $d_{\min}$ as enabled by HSSK, the performance can be further improved as predicted by (7); specifically, a 1.5 dB and 2.5 dB gain over optimal GSSK for HSSK ($d_{\min} = 3$) and HSSK ($d_{\min} = 4$). The analytical performance upper bound in (7) is plotted for HSSK, which is loose for low E_b/N_0 and tighter for high E_b/N_0.[3] It should be emphasized that the performance advantage of HSSK is yielded by a more efficient constellation design and not just power. In fact, increasing the power of GSSK to what is required for HSSK to achieve $d_{\min} = 3$, which amounts to an increase of $10\log(2.375/2) = 0.75$ dB, will not compensate the gap between HSSK ($d_{\min} = 3$) and GSSK which is at least 1.5 dB.

In Fig. 3, we target 5 bits transmission in a 12×12 MIMO system. In this scenario, HSSK ($d_{\min} = 2$) overlaps with GSSK while HSSK ($d_{\min} = 3$) and HSSK ($d_{\min} = 4$) outperform GSSK by about 2.3 dB and 3 dB, respectively. The error rate decays more quickly due to a larger N_R, as indicated by (7). Optimal constellation design for GSSK becomes computationally infeasible in this case (entailing a search over $\binom{\binom{12}{2}}{32} = 7 \times 10^{18}$ combinations), which shows a disadvantage of GSSK if optimal constellation is pursued in large systems. Since HSSK does not pursue such an optimization but adopts a systematic Hamming code design technique, it presents an inexpensive way for designing spatial modulation schemes.

V. CONCLUSION

A novel spatial modulation scheme has been proposed. In the proposed modulation scheme, the constellation is designed via the Hamming code construction technique, offering a rich selection of design options and carrying the potential of better transmission rate, performance, and power tradeoffs unachievable by previous schemes. The demonstrated performance as well as design simplicity and flexibility make the proposed scheme attractive for use in next-generation MIMO systems.

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[2]In accordance with the design rule of HSSK, HSSK constellation can also be optimized by breaking the ties of minimum Hamming-weight codewords in a more sophisticated way. That is, we can choose the combination that yields maximum pairwise Hamming distances. This usually entails a computationally feasible search, even for large systems. For example, in optimizing HSSK ($d_{\min} = 3$) for 5 bits transmission in a 12×12 system, the $2^5 = 32$ codewords would consist of 30 codewords with Hamming weight of 4 or less, and two codewords chosen from the 52 codewords with Hamming weight of 5.

[3]Clearly, a tighter performance upper bound can directly be achieved by substituting in (5) a tighter upper bound on the Q-function that makes the whole integrand integrable. The results in [8] are useful in this regard.