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# New Structures of Proximity Spaces

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**Abstract:** The purpose of this paper is to construct a new approach of proximity and generalized proximity based on the soft set. Some results on these spaces have obtained and three of the important results are: every soft $T_4-$ space is compatible with a proximity relation on $P(X)^E$. In addition, every soft space is compatible with a Pervin proximity relation on $P(X)^E$. It is also shown that every soft $R_o-$space is compatible with a Lodato proximity relation on $P(X)^E$.

**Keywords:** Proximity of soft set, Generalized proximity of soft set, compatibility.

## 1 Introduction

The fundamental concept of Efremovič proximity space has been introduced by Efremovič [1]. In addition to, Leader [2,3] and Lodato [4,5] have worked with weaker axioms than those of Efremovič proximity space enabling them to introduce an arbitrary topology on the underlying set. Furthermore, proximity relations are useful in solving problems based on human perception [6].

Several theories, such as the theory of fuzzy sets [7], theory of intuitionists fuzzy sets [8], theory of vague sets, theory of interval mathematics [9], theory of rough sets and theory of probability [10] can be considered as mathematical tools for dealing with uncertainties. These theories have inherent difficulties due to the inadequacy of the parameterization tool of the theories as pointed out by Molodtsov.

In 1999 Molodtsov [11] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. In [12], Molodtsov applied successfully in directions such as, smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability and theory of measurement. Maji et al. [13,14] gave the first practical application of soft sets in decision making problems. In 2003, Maji et al. [14] defined and studied several basic notions of soft set theory. In 2005, Pei et al.[15] and Chen [16] improved the work of Maji et al. [13,14]. Recently, A. Kandil et.al. [17,18] introduced a new approach of proximity structure [19] based on the ideal notion. In this paper, we introduce the notions of proximity and generalized proximities by using the soft sets. The main theorems in our work is to exhibit the relation between the topology generated via these proximities and the soft topological space $(X,\tau,E)$.

## 2 Preliminaries

Now we recall some definitions and results defined and discussed in [20,21,11,22,14].

**Definition 2.1.** Let $X$ be a nonempty set, $E$ be a set of parameters, and $P(X)$ denotes the power set of $X$. A pair $(F,E)$ is called a soft set over $X$ , where $F$ is a mapping given by $F : E \to P(X)$. In other words, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For a particular $e \in E$ , $F(e)$ may be considered the set of $e$-approximate elements of the soft set $(F,E)$, i.e. $F = \{F(e) : e \in E,\ F : E \to P(X)\}$. The family of all these soft sets on $(X,E)$ denoted by $P(X)^E$.

**Definition 2.2.** Let $F,\ G \in P(X)^E$. Then $F$ is soft subset of $G$, denoted by $F\tilde{\subseteq}G$, if $F(e) \subseteq G(e)$, $\forall e \in E$.

**Definition 2.3.** Two soft subset $F$ and $G$ over a common universe set $X$ are said to be soft equal if $F$ is a soft subset of $G$ and $G$ is a soft subset of $F$.

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**Definition 2.4.** The complement of a soft set $(F,E)$, denoted by $(F,E)^c$, is defined by $(F,E)^c = (F^c,E)$, $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$, $\forall e \in E$ and $F^c$ is called the soft complement function of $F$.

**Definition 2.5.** The difference of two soft sets $(F,E)$ and $(G,E)$ over the common universe $X$, denoted by $(F,E) - (G,E)$ is the soft set $(H,E)$ where for all $e \in E$, $H(e) = F(e) - G(e)$.

**Definition 2.6.** Let $(F,E)$ be a soft set over $X$ and $x \in X$. We say that $x \in (F,E)$ read as $x$ belongs to the soft set $(F,E)$ whenever $x \in F(e)$ for all $e \in E$.

**Definition 2.7.** A soft set $(F,E)$ over $X$ is said to be a null soft set denoted by $\tilde{\phi}$ if for all $e \in E$, $F(e) = \phi$ (null set).

**Definition 2.8.** A soft set $(F,E)$ over $X$ is said to be an absolute soft set denoted by $\tilde{X}$ if for all $e \in E$, $F(e) = X$.

**Definition 2.9.** The intersection of two soft sets $(F,E)$ and $(G,E)$ over the common universe $X$ is the soft set $(H,E)$, where for all $e \in E, H(e) = F(e) \cap G(e)$.

**Definition 2.10.** Let $I$ be an arbitrary indexed set and $L = \{(F_i,E), i \in I\}$ be a subfamily of $P(X)^E$.

(1)The union of $L$ is the soft set $(H,E)$, where $H(e) = \cup_{i\in I} F_i(e)$ for each $e \in E$ . We write $\tilde{\cup}_{i\in I}(F_i,E) = (H,E)$.
(2)The intersection of $L$ is the soft set $(M,E)$, where $M(e) = \cap_{i\in I} F_i(e)$ for each $e \in E$ . We write $\tilde{\cap}_{i\in I}(F_i,E) = (M,E)$.

**Definition 2.11.** Let $\tau$ be a collection of soft sets over a universe $X$ with a fixed set of parameters $E$, then $\tau \subseteq P(X)^E$ is called a soft topology on $X$ if

(1)$\tilde{X}, \tilde{\phi} \in \tau$,
(2)the union of any number of soft sets in $\tau$ belongs to $\tau$,
(3)the intersection of any two soft sets in $\tau$ belongs to $\tau$.

The triplet $(X,\tau,E)$ is called a soft topological space over $X$. The members of $\tau$ are called open soft sets in X. Also, a soft set $(F,E)$ is called closed soft if the complement $(F,A)^c$ belongs to $\tau$ . The family of closed soft sets is denoted by $\tau^c$.

**Definition 2.12.** Let $(X,\tau,E)$ be a soft topological space. A soft set $(F,E)$ over $X$ is said to be closed soft set in $X$, if its complement $(F,A)^c$ is an open soft set.

**Definition 2.13.** Let $(X,\tau,E)$ be a soft topological space and $F \in P(X)^E$. The soft closure of $F$, denoted by $C(F)$ is the intersection of all closed soft super sets of $F$ i.e $\overline{F} = \tilde{\cap}\{H \in P(X)^E : (H,E) \text{ is closed soft set and } F\tilde{\subseteq}H\}$.

**Definition 2.14.** The soft set $F \in P(X)^E$ is called a soft point if there exists $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point $(F,E)$ is denoted by $x_e$.

**Definition 2.15.** The soft point $x_e$ is said to be belonging to the soft set $G \in P(X)^E$, denoted by $x_e \tilde{\in} G$, if for $e \in E$, $F(e) \subseteq G(e)$.

## 3 Compatibility of Proximity Soft Spaces

**Definition 3.1.** A binary relation $\delta$ on $P(X)^E$ is called a proximity of soft sets on $(X,E)$ if for all $F,G,H \in P(X)^E$, $\delta$ satisfies the following conditions:-

$(S_1) F\delta G \Rightarrow G\delta F$,
$(S_2) F\delta(G\tilde{\cup}H) \Leftrightarrow F\delta G$ or $F\delta H$,
$(S_3) F\delta G \Rightarrow F \neq \tilde{\phi}$, and $G \neq \tilde{\phi}$,
$(S_4) F\tilde{\cap}G \neq \tilde{\phi} \Rightarrow F\delta G$.
$(S_5) F \not\delta G \Rightarrow \exists H,K \in P(X)^E$ such that $F \not\delta H^c, K^c \not\delta G$ and $H\tilde{\cap}K = \tilde{\phi}$,

A soft proximity space is a triple $(X,E,\delta)$ consisting of a set $X$, a set of Parameters $E$, and a proximity relation on $P(X)^E$. We shall write $F\delta G$ if the soft sets $F,G \in P(X)^E$ are $\delta-$related, otherwise we shall write $F \not\delta G$.

**Lemma 3.2.** Let $(X,\delta,E)$ be a soft proximity space, $F\delta G$, $F\tilde{\subseteq}H$, and $G\tilde{\subseteq}K$, then $H\delta K$.

**Proof.** The result follows immediately from $S_1$ and $S_2$.

**Theorem 3.3.** Let $(X,E,\delta)$ be a soft proximity space. Then the $C^\delta$- operator

$$C^\delta : P(X)^E \to P(X)^E$$

defined by:

$$C^\delta(F) = \tilde{\cup}\{x_e \in P(X)^E : x_e\delta F\} \tag{1}$$

satisfies the following:-

1.$C^\delta(\tilde{\phi}) = \tilde{\phi}$,
2.$F\tilde{\subseteq}C^\delta(F)$,
3.$F\tilde{\subseteq}G \Rightarrow C^\delta(F)\tilde{\subseteq}C^\delta(G)$,
4.$C^\delta(F\tilde{\cup}G) = C^\delta(F)\tilde{\cup}C^\delta(G)$,
5.$C^\delta(C^\delta(F)) = C^\delta(F)$,
6.$C^\delta(F\tilde{\cap}G)\tilde{\subseteq}C^\delta(F)\tilde{\cap}C^\delta(G)$,
7.$C^\delta(F) - C^\delta(G)\tilde{\subseteq}C^\delta(F - G)$

**Proof.** The results follow immediately from Definition 3.1, formula (1), and Lemma 3.2

**Corollary 3.4.** Let $(X,E,\delta)$ be a soft proximity space. Then the operator

$$C^\delta : P(X)^E \to P(X)^E$$

defined by the formula (1) satisfies Kuratwski's axioms and induces a topology on $(X,E)$ called $\tau_\delta$ given by:

$$\tau_\delta = \{F \in P(X)^E : C^\delta(F^c) = F^c\} \tag{2}$$

**Definition 3.5.** A soft topological space $(X,\tau,E)$ is compatible with the Proximity relation of soft sets $\delta$ , denoted $\tau \sim \delta$, if $\tau_\delta = \tau$.

**Definition 3.6.** A soft topological space $(X,\tau,E)$ is called a soft normal space if for every $F_1,F_2 \in \tau^c$ such that $F_1\tilde{\cap}F_2 = \tilde{\phi}$ then there exists $H_1,H_2 \in \tau$ such that $F_1\tilde{\subseteq}H_1$, $F_2\tilde{\subseteq}H_2$, and $H_1\tilde{\cap}H_2 = \tilde{\phi}$.

**Definition 3.7.** A soft topological space $(X,\tau,E)$ is called a soft $T_1$ space if it satisfies $\overline{x_e} = x_e$ for every $x_e \in P(X)^E$.

**Definition 3.8.** A soft topological space $(X,\tau,E)$ is called a soft $T_4$ space if it is soft normal and soft $T_1$ space.

**Example 3.9.** Let $(X,\tau,E)$ be a soft normal space, and $\delta$ be a binary relation on $P(X)^E$ defined as:

$$F\delta G \Leftrightarrow \overline{F}\tilde{\cap}\overline{G} \neq \tilde{\phi}. \quad (3)$$

Then $\delta$ is a Proximity relation on $P(X)^E$. Indeed, one easily sees that $\delta$ satisfies conditions $(S_1)$-$(S_4)$. So, to check that $\delta$ also satisfies condition $(S_5)$, let $F \not\delta G$. It follows that $\overline{F}\tilde{\cap}\overline{G} = \tilde{\phi}$. Since $(X,\tau,E)$ is a soft normal space, then there exists tow open soft set $H_1,H_2$ such that $\overline{F}\tilde{\subseteq}H_1$, $\overline{G}\tilde{\subseteq}H_2$, and $H_1\tilde{\cap}H_2 = \tilde{\phi}$. Let $K_1 = H_1$, and $K_2 = H_2$. It follows that $F \not\delta K_1^c$, $K_2^c \not\delta G$, and $K_1\tilde{\cap}K_2 = \tilde{\phi}$. Then the result.

**Theorem 3.10.** Let $(X,\tau,E)$ be a soft $T_4$ space and $\delta$ be the formula (3). Then $\tau \sim \delta$.

**Proof.** To prove the theorem, it suffices to show that $\forall F \in P(X)^E$, $\overline{F} = C^\delta(F)$. Let $x_e \in C^\delta(F)$, then $x_e\delta F$, and hence $\overline{x_e}\tilde{\cap}\overline{F} \neq \tilde{\phi}$. Since $(X,\tau,E)$ is a soft $T_1$ space then $x_e\tilde{\cap}\overline{F} \neq \tilde{\phi}$. Consequently, $x_e\tilde{\in}\overline{F}$. Hence

$$C^\delta(F)\tilde{\subseteq}\overline{F}. \quad (4)$$

Now, we want to prove that $\overline{F}\tilde{\subseteq}C^\delta(F)$ or equivalently, if $x_e\tilde{\notin}C^\delta(F)$, then $x_e\tilde{\notin}\overline{F}$. Let $x_e\tilde{\notin}C^\delta(F)$, then $x_e \not\delta F$ and hence formula (3) implies that $\overline{\{x_e\}}\tilde{\cap}\overline{F} = \tilde{\phi}$. Since $(X,\tau,E)$ is a soft $T_1$ space, then $x_e\tilde{\cap}\overline{F} = \tilde{\phi}$. Hence $x_e\tilde{\notin}\overline{F}$. It follows that

$$\overline{F}\tilde{\subseteq}C^\delta(F).$$

This result, combined with formula (4), completes the proof of the theorem.

## 4 Generalized Soft Proximity Spaces

**Definition 4.1.** Let $\delta$ be a binary relation on $P(X)^E$. For any $F,G,H \in P(X)^E$, consider the following axioms:-

$(L_1)$ $F\delta G \Rightarrow G\delta F$,
$(L_2)$ $F\delta(G\tilde{\cup}H) \Leftrightarrow F\delta G$ or $F\delta H$, and $(G\tilde{\cup}H)\delta F \Leftrightarrow G\delta F$ or $H\delta F$,
$(L_3)$ $F\delta G \Rightarrow F \neq \tilde{\phi}$, and $G \neq \tilde{\phi}$,
$(L_4)$ $F\delta G$ and $x_e\delta H$ $\forall x_e\tilde{\in}G \Rightarrow F\delta H$,
$(L_5)$ $F\tilde{\cap}G \neq \tilde{\phi} \Rightarrow F\delta G$.
$(L_6)$ $F \not\delta G \Rightarrow \exists H,K \in P(X)^E$ such that $F \not\delta H^c, K^c \not\delta G$ and $H\tilde{\cap}K = \tilde{\phi}$,

Then $\delta$ is said to be:-

(*a*) A Leader proximity of soft sets on $(X,E)$, if it satisfies $(L_2)$, $(L_3)$, $(L_4)$ and $(L_5)$.
(*b*) A Lodato proximity of soft sets on $(X,E)$, if it is satisfies $(L_1)$-$(L_5)$.
(*c*) A Pervin proximity of soft sets on $(X,E)$, if it satisfies $(L_2)$, $(L_3)$, $(L_5)$ and $(L_6)$.

If $\delta$ is a Leader (respectively Lodato and Pervin) proximity relation on $P(X)^E$, then the triple $(X,\delta,E)$ is called a Leader (respectively Lodato and Pervin) soft proximity space.

**Definition 4.2.** A binary relation $\delta$ on $P(X)^E$ is called a generalized Proximity relation on $P(X)^E$ if it is a Leader or a Pervin or a Lodato Proximity relation on $P(X)^E$. Moreover, if $\delta$ is a generalized proximity relation of soft sets on $P(X)^E$, then the triple $(X,\delta,E)$ is called a generalized soft proximity space.

**Lemma 4.3.** Let $(X,\delta,E)$ be a Generalized soft proximity space, $F\delta G$ and $G\tilde{\subseteq}H$, then $F\delta H$.

**Proof.** The result follows immediately from $(L_2)$.

**Theorem 4.4.** Every Pervin proximity of soft sets on $(X,E)$ is also Leader proximity of soft sets on $(X,E)$.

**Proof.** Let $\delta$ be a Pervin proximity of soft sets on $(X,E)$. It is sufficient to show that $\delta$ satisfies $(L_4)$. Let $F\delta G$ and $\forall x_e\tilde{\in}G$, $x_e\delta H$. If $F \not\delta H$, then by $(L_6)$ there exists $\exists K_1,K_2\tilde{\in}P(X)^E$ such that $F \not\delta K_1^c, K_2^c \not\delta H$, and $K_1\tilde{\cap}K_2 = \tilde{\phi}$. This result, combined with $F\delta G$, implies $G\tilde{\nsubseteq}K_1^c$, i.e. $G\tilde{\cap}K_1 \neq \tilde{\phi}$. It follows that $\exists x_e \in p(X)^E$ such that $x_e\delta H$ and $x_e\tilde{\in}K_1\tilde{\subseteq}K_2^c$. This result, combined with Lemma 4.3, shows that $K_2^c\delta H$, which is contradiction. So, $F\delta H$.

**Theorem 4.5.** Let $(X,E,\delta)$ be a generalized soft proximity space. Then the $C^\delta$- operator

$$C^\delta : P(X)^E \to P(X)^E$$

defined by:

$$C^\delta(F) = \tilde{\cup}\{x_e \in P(X)^E : x_e\delta F\} \quad (5)$$

satisfies the following:-

1. $C^\delta(\tilde{\phi}) = \tilde{\phi}$,
2. $F\tilde{\subseteq}C^\delta(F)$,
3. $F\tilde{\subseteq}G \Rightarrow C^\delta(F)\tilde{\subseteq}C^\delta(G)$,
4. $C^\delta(F\tilde{\cup}G) = C^\delta(F)\tilde{\cup}C^\delta(G)$,
5. $C^\delta(C^\delta(F)) = C^\delta(F)$,
6. $C^\delta(F\tilde{\cap}G)\tilde{\subseteq}C^\delta(F)\tilde{\cap}C^\delta(G)$,
7. $C^\delta(F) - C^\delta(G)\tilde{\subseteq}C^\delta(F-G)$.

**Proof.** The results follow immediately from Definition 4.1, formula (5), and Lemma 4.3.

**Corollary 4.6.** Let $(X,E,\delta)$ be a generalized soft proximity space. Then the operator

$$C^\delta : P(X)^E \to P(X)^E$$

defined by the formula (5) satisfies Kuratwski's axioms and induces a topology on $(X,E)$ called $\tau_\delta$ given by:

$$\tau_\delta = \{F \in P(X)^E : C^\delta(F^c) = F^c\} \quad (6)$$

**Proof.**

**Theorem 4.7.** Let $(X,\delta,E)$ be a generalized soft proximity space. Then

$$F\delta G \Leftrightarrow F\delta C^{\delta}(G). \tag{7}$$

**Proof.** $F\delta C^{\delta}(G)$ and $\forall\, x_e\tilde{\in}C^{\delta}(G)$, we have $x_e\delta G$. Hence $(L_4)$ implies $F\delta G$. The other inclusion follows directly from Theorem 4.5 (2) and Lemma 4.3.

**Example 4.8.** Let $(X,\tau,E)$ be a soft topological space, and $\delta$ be a binary relation on $P(X)^E$ defined as:

$$F\delta G \Leftrightarrow F\tilde{\cap}\overline{G} \neq \tilde{\phi}. \tag{8}$$

Then $\delta$ is a Pervin Proximity relation on $P(X)^E$. Indeed, one easily sees that $\delta$ satisfies conditions $(L_2)$, $(L_3)$ and $(L_5)$. So, to check that $\delta$ also satisfies condition $(L_6)$, let $F\,\not{\delta}_I G$. It follows that $F\tilde{\cap}\overline{G} = \tilde{\phi}$ and by taking $H = (\overline{G})^c$ and $K = \overline{G}$ have the required properties.

**Theorem 4.9.** Let $(X,\tau,E)$ be a soft topological space and $\delta$ be the formula (8). Then $\tau \sim \delta$ and $\delta$ is the smallest compatible Leader or Pervin proximity relation on $P(X)^E$.

**Proof.** Indeed, $\tau = \tau_{\delta}$ follows from the fact that $x_e\tilde{\in}C^{\delta}(F) \Leftrightarrow x_e\tilde{\cap}\overline{F} \neq \tilde{\phi} \Leftrightarrow x_e\tilde{\in}\overline{F}$. To prove that $\delta$ is the smallest compatible Leader or Pervin proximity relation on $P(X)^E$. Let $\alpha$ be a Leader or a Pervin proximity relation on $P(X)^E$ and $F\,\not{\alpha}H$. Theorem 4.7 implies $F\,\not{\alpha}C^{\delta}(H)$. Therefore, $F\tilde{\cap}C^{\delta}(H) = \tilde{\phi}$. Hence, $F\,\not{\delta}H$.

**Definition 4.10.** A soft topological space $(X,\tau,E)$ is called a soft $R_o$ space if for all $x_e, y_e \in P(X)^E$ such that $x_e \neq y_e$ then $x_e\tilde{\in}\overline{y_e} \Rightarrow y_e\tilde{\in}\overline{x_e}$.

**Example 4.11.** Let $(X,\tau,E)$ be a soft topological $R_o$ space, and $\delta$ be a binary relation on $P(X)^E$ defined as:

$$F\delta G \Leftrightarrow \overline{F}\tilde{\cap}\overline{G} \neq \tilde{\phi}. \tag{9}$$

Then $\delta$ is a Lodato Proximity relation on $P(X)^E$. It follows directly from formula (9) that $\delta$ satisfies conditions $(L_1)$-$(L_3)$ and $(L_5)$. So, to check that $\delta$ also satisfies condition $(L_4)$, let $F\delta G$ and $x_e\delta H\ \forall\, x_e\tilde{\in}G$. It follows that $\overline{F}\tilde{\cap}\overline{G} \neq \tilde{\phi}$ and $\overline{x_e}\tilde{\cap}\overline{H} \neq \tilde{\phi}$. Hence there exists $y_e\tilde{\in}\overline{H}$ such that $y_e\tilde{\in}\overline{x_e}$. Since $(X,\tau,E)$ is a soft $R_o$ space, then $x_e\tilde{\in}\overline{y_e}\tilde{\subseteq}\overline{H}$, showing that $G\tilde{\subseteq}\overline{H}$. As a consequence, $\overline{F}\tilde{\cap}\overline{H} \neq \tilde{\phi}$, i.e. $F\delta H$. Then the result.

**Theorem 4.12.** Let $(X,\tau,E)$ be a soft $R_o$ space, and $\delta$ be the formula (9). Then $\tau \sim \delta$.

**Proof.** To prove the theorem, it suffices to show that $\tau_{\delta} = \tau$. In other words, we show that $\forall F \in P(X)^E$, $\overline{F} = C^{\delta}(F)$. Indeed, the result follows from the fact that $x_e\tilde{\in}C^{\delta}(F) \Leftrightarrow \overline{x_e}\tilde{\cap}\overline{F} \neq \tilde{\phi} \Leftrightarrow x_e\tilde{\in}\overline{F}$.

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