New Training Sequence Structure for Zero-Padded SC-FDE System in Presence of Carrier Frequency Offset

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Abstract— **Frequency Domain Equalization (FDE) is an attractive solution for wireless broadband transmission because of its strong capability in handling multipath environment. However, frequency domain channel estimation suffers from the inter-carrier interference(ICI) caused by carrier frequency offset (CFO). In this paper, we proposed new training sequence structure for channel estimation to reduce the estimation errors caused by residual CFO. The proposed new training sequence structure can be easily obtained from any existing channel training sequences without introducing significant changes to their original property. Simulation results show that, this scheme can efficiently reduce the channel estimation errors in FDE system, and improve the system performance.**

I. Introduction

Frequency Domain Equalization (FDE) is a low complexity solution for broadband wireless systems, where severe time-dispersive channels are faced. Typical FDE systems include Orthogonal Frequency Division Multiplexing (OFDM) and Single Carrier with Frequency Domain Equalized (SC-FDE) systems. OFDM has been proposed as the physical layer transmission technique for many standards. SC-FDE technique attracts more and more attention from both research and practical fields due to its low complexity equalization and the signals' low peak to average power ratio (PAPR) [1]. However, because of the frequency domain equalization, FDE systems are sensitive to some frequency domain interferences caused by the nonlinearity of the analogue front end.

Carrier Frequency Offset (CFO) is caused by the impairment of the transmitter and receiver oscillators. OFDM systems are known to be sensitive to CFO [2], which causes the destruction of orthogonality among subcarriers and introduces inter-carrier interference (ICI). The frequency domain channel estimation procedure for both SC-FDE and OFDM are the same. In training-based channel estimation, ICI introduced by CFO causes extra channel estimation error. In the literature, some special signal structures, called ICI-self cancellation, are proposed to reduce the ICI effect in data symbols [3] [4]. However they are not suitable for the channel estimation purpose as training sequence in the preamble needs to have some general properties, e.g., good auto-and cross- correlation.

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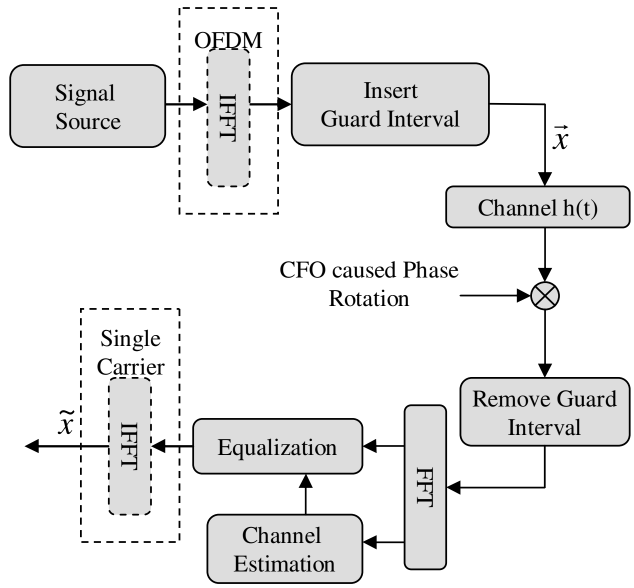

Fig. 1. Frequency Domain Equalization System in the presence of Carrier Frequency Offset

In this paper, we first analyze the increasing of channel estimation error caused by residual CFO in Zero Padded (ZP) SC-FDE system. Based on the Cramer-Rao Low Bound (CRLB) of channel estimation error, we then proposed a novel training sequence to reduce the impact of residual CFO. The scheme is also suitable for OFDM systems. For simplity, we focus on SC-FDE which suffers more performance degradation from channel estimation error. Section II formulates the ZP-SC-FDE system in the presence of CFO. Section III shows the new training sequence structure. Simulation results are given in Section IV. Section V concludes the paper.

In the following analysis, $\vec{\cdot}$ denotes vector, bold capital symbol denotes matrix, $^{-1}$ denotes matrix inverse and $diag[\vec{x}]$ denotes a diagonal matrix with $\vec{x}$ as the elements on diagonal line.

II. Zero Padded SC-FDE System

The system structure of an SC-FDE system is shown in Fig. 1. In the system, symbols to be transmitted are organized in blocks, and a guarding interval is appended to each block.

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The guarding interval could be padded by zero or cyclic prefix. In this paper, without loss of generality, we focus on ZP systems. In the receiver, a frequency domain equalizer is applied to remove the channel effect, and an inverse fast fourier transform (IFFT) is applied to recover the data symbols. For detailed information of SC-FDE systems, the readers are referred to [1]. An SC-FDE system can be regarded as obtained from an OFDM system by shifting the IFFT module from the transmitter to the receiver. Thus they can be easily formulated in a common mathematical model, as done in this paper.

Denote a block of M symbols to be transmitted in the time domain as $\vec{x} = [x_1, x_2, \ldots, x_M]^T$, and their frequency domain dual as $\vec{X} = [X_1, X_2, \ldots, X_M]^T$. Let the channel impulse response be $\vec{h} = [h_1, h_2, \ldots, h_L]^T$. Without loss of generality, we assume that L equals to the length of the guarding interval (channel could be extended with zeros if it is shorter).

Consider a CFO term ω_d, normalized with respect to the signal bandwidth. Since accumulated phase shift only causes a fixed phase shift for all samples within a block, we will ignore its effect in the following analysis. In a ZP-FDE system, overlapping and Sum (O&S) [5] is implemented in the receiver to convert a linear convolution to a circular convolution between the transmitted signal and the channel. For every block of length $M+L$, the O&S operation adds the last L samples of the received signal to the first L samples. Denote the $M+L$ received samples as $\vec{y}_0$, which is given by

$$\vec{y}_0 = \begin{pmatrix} p_1 & 0 & \ldots & 0 \\ 0 & p_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & p_{M+L} \end{pmatrix} \begin{pmatrix} h_1 & 0 & \ldots & 0 \\ h_2 & h_1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & h_1 \end{pmatrix} \begin{pmatrix} \vec{x} \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \vec{w}_0, \tag{1}$$

where the elements $p_k = e^{-jk\omega_d}$ in the diagonal matrix are phase shifting terms due to the CFO, $\vec{w}_0$ denotes the AWGN samples, and L zeros are appended to $\vec{x}$. The M samples obtained after O&S operation can be written as

$$\begin{aligned} \vec{y} &= \mathbf{HP}\vec{x} + \vec{w} \\ &= \begin{pmatrix} h_1 & 0 & \ldots & p_1 h_2 \\ p_1 h_2 & h_1 & \ldots & p_2 h_3 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & h_1 \end{pmatrix} \begin{pmatrix} p_1 & 0 & \ldots & 0 \\ 0 & p_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & p_M \end{pmatrix} \vec{x} + \vec{w}, \end{aligned} \tag{2}$$

where $\vec{w}$ is the noise vector with the first L samples including the noise components added from the last L rows with L denoting the channel length, $\mathbf{P} = diag[p_1, p_2, \ldots, p_M]$ with $p_k = e^{-jk\omega_d}$ representing the phase shift caused by normalized CFO ω_d. $\mathbf{H}$ is the equivalent circulant channel matrix after O&S operation, with first column $[h_1, h_2 p_1, \ldots, h_L p_{L-1}]^T$ where h_i denotes the original channel taps without CFO.

Representing the FFT and IFFT matrices as $\mathbf{F}$ and $\mathbf{F}^H$ respectively, the frequency domain signal is given by

$$\vec{Y} = \mathbf{DC}\vec{X} + \vec{W} \tag{3}$$

where $\vec{Y}$, $\vec{X}$ and $\vec{W}$ denote the frequency domain received signal, transmitted signal and Gaussian noise respectively, $\mathbf{D} = diag[d_1, d_2, ..., d_M]$ is the diagnose frequency domain channel matrix, and $\mathbf{C} = \mathbf{FPF}^H$ denotes the frequency domain phase shift matrix.

The frequency domain channel estimation in the SC-FDE system is the same to that in an corresponding OFDM system. Assuming the training sequence for channel estimation has unit energy in frequency domain, i.e. $X_i X_i^* = 1$, according to (3), when a zero-forcing channel estimation is used, the estimated i^{th} channel response d_i is given by

$$\begin{aligned} \hat{d}_i &= C_0 d_i + \sum_{k=1, k\neq i}^{M} X_k X_i^* I_{i,k}^d + w_i X_i^* \\ &= C_0 d_i + \eta_i \end{aligned} \tag{4}$$

where

$$\begin{aligned} I_{i,k}^d &= \frac{e^{j(\pi\omega_d(M-1)/M)} \sin(\pi\omega_d)}{M \sin(\pi(k-i+\omega_d)/M)} \\ C_0 &= \frac{\sin \pi\omega_d}{M \sin(\pi\omega_d/M)} e^{j(\pi\omega_d(M-1)/M)}. \end{aligned} \tag{5}$$

From (4), we can see that the channel estimation error are from scale factor C_0, ICI interference I_i^d and the noise $w_i X_i^*$. Since the scale factor will be absorbed by the equalization, η_i is the real estimation error. In [6], it is shown that the channel estimation error η_i could affect the system performance significantly. There are some methods proposed to reduce η_i. Generally, two or more training symbols are used for channel estimation to achieve better results. Assuming Q training symbols are used for channel estimation, using CRLB [7], the error variance of the i^{th} channel estimation can be written as

$$\sigma_\eta^2(i) = \frac{\sigma_w^2}{Q\|X_i\|^2\|C_0\|^2} + \sum_{k=1, k\neq i}^{M} \frac{\|X_k\|^2\|I_{i,k}^d\|^2}{\|C_0\|^2} \tag{6}$$

Equation (6) reveals that the noise effect on channel estimation errors can be reduced by increasing the number of identical training symbols, however, the impact of ICI could not be alleviated in the same way. In the next section, we propose new structure of training sequence to reduce the ICI $\|I_{i,k}^d\|^2$.

III. Proposed Training Sequence

Lets first consider the case when two training sequences are available. From (5), the CFO impacts are included in $I_{i,k}^d$, which depends on $\sin(\pi(k-i+\omega_d)/M)$. We can see that the smaller $\|k-i\|$, the larger $\|I_{i,k}^d\|^2$. So ICI from adjacent subcarriers, $(i+1)^{th}$ and $(i-1)^{th}$, are the largest ones to the i^{th} subcarrier. Based on this observation, we propose to use two frequency symbol $[\vec{Z^a}; \vec{Z^b}]$, where $\vec{Z^a} = [Z_1^a, Z_2^a, \ldots, Z_M^a]^T$ can be any training sequence and $\vec{Z^b} = [Z_1^b, Z_2^b, \ldots, Z_M^b]^T$ is defined as

$$Z_k^b = \begin{cases} Z_k^a, & k = 2v-1 \\ -Z_k^a, & k = 2v \end{cases}. \tag{7}$$

where v is an integer and $v = 1, 2, ..., M/2$. So the frequency domain training sequence is given by

$$\begin{bmatrix} Z_1^a, & Z_2^a, & Z_3^a, & \ldots, & Z_{M-1}^a, & Z_M^a \\ Z_1^a, & -Z_2^a, & Z_3^a, & \ldots, & Z_{M-1}^a, & -Z_M^a \end{bmatrix}^T. \tag{8}$$

Denote the frequency domain received signal corresponding to the training symbols as $\vec{Y}^a = [Y_1^a, Y_2^a, ..., Y_M^a]$ and $\vec{Y}^b = [Y_1^b, Y_2^b, ..., Y_M^b]$. The estimated channel matrix $\hat{\mathbf{D}} = diag[\hat{d}_1, \hat{d}_2, \cdot, \hat{d}_M]$ is given by

$$\begin{aligned}\hat{d}_i &= \begin{cases} (Y_i^a + Y_i^b)(Z_i^a)^*/2, & i = 2v-1 \\ (Y_i^a - Y_i^b)(Z_i^a)^*/2, & i = 2v \end{cases} \\ &= \begin{cases} (\hat{d}_i^a + Y_i^b(Z_i^a)^*)/2, & i = 2v-1 \\ (\hat{d}_i^a - Y_i^b(Z_i^a)^*)/2, & i = 2v \end{cases}, \end{aligned} \tag{9}$$

where v is integer and $v \in [1, \cdot, M/2]$.

The channel estimation based on $\vec{Z}^a$ is given by

$$\hat{d}_i^a = C_0 d_i + \sum_{k=1,k\neq i}^{M} Z_k^a (Z_i^a)^* I_{i,k}^d + w_i^a (Z_i^a)^*. \tag{10}$$

Similarly, the $Y_i^b(Z_i^a)^*$ is given by

$$\begin{aligned} Y_i^b(Z_i^a)^* &= C_0 d_i Z_b (Z_i^a)^* + \sum_{k=1,k\neq i}^{M} Z_k^b (Z_i^a)^* I_{i,k}^d + w_i^b (Z_i^a)^* \\ &= C_0 d_i + w_i^b (Z_i^a)^* + \sum_{v=1,2v\neq i}^{M/2} Z_{2v}^b (Z_i^a)^* I_{i,k}^d \\ &+ \sum_{v=1,2v-1\neq i}^{M/2} Z_{2v-1}^b (Z_i^a)^* I_{i,k}^d \end{aligned} \tag{11}$$

So when i is odd, from (7), $(Z_i^a)^* = (Z_i^b)^*$, and $Y_i^b(Z_i^a)^*$ is given by

$$\begin{aligned} Y_i^b(Z_i^a)^* &= C_0 d_i + w_i^b (Z_i^a)^* - \sum_{v=1,2v\neq i}^{M/2} Z_{2v}^a (Z_i^a)^* I_{i,k}^d \\ &+ \sum_{v=1,2v-1\neq i}^{M/2} Z_{2v-1}^a (Z_i^a)^* I_{i,k}^d. \end{aligned} \tag{12}$$

When i is even, from (7), $(Z_i^a)^* = -(Z_i^b)^*$, and $Y_i^b(Z_i^a)^*$ is given by

$$\begin{aligned} Y_i^b(Z_i^a)^* &= -C_0 d_i + w_i^b (Z_i^a)^* - \sum_{v=1,2v\neq i}^{M/2} Z_{2v}^a (Z_i^a)^* I_{i,k}^d \\ &+ \sum_{v=1,2v-1\neq i}^{M/2} Z_{2v-1}^a (Z_i^a)^* I_{i,k}^d. \end{aligned} \tag{13}$$

So the final channel estimation $\hat{d}_i$ is given by

$$\hat{d}_i = \begin{cases} C_0 d_i + \frac{(w_i^b + w_i^a)(Z_i^a)^*}{2} + I_i^{odd} & i = 2v-1 \\ C_0 d_i + \frac{(w_i^a - w_i^b)(Z_i^a)^*}{2} + I_i^{even} & i = 2v \end{cases} \tag{14}$$

where

$$\begin{aligned} I_i^{odd} &= \sum_{v=1,2v-1\neq i}^{M/2} Z_{2v-1}^a (Z_i^a)^* I_{i,k}^d \\ I_i^{even} &= \sum_{v=1,2v\neq i}^{M/2} Z_{2v}^a (Z_i^a)^* I_{i,k}^d. \end{aligned} \tag{15}$$

So for any odd subcarrier, ICI is only from other odd subcarriers, and ICI from $(i+1)^{th}$ and $(i-1)^{th}$ subcarriers are removed. Similarly, for even subcarrier, ICI is only from other even subcarriers, so ICI from adjacent subcarrier are removed. In addition, there is still averaging effect of AWGN noise. So SNR in the channel training sequence is still 3dB higher than that in the data symbols. Therefore, significant ICI rejection can be achieved simply by changing sign of the alternative subcarriers in the second training symbol. The proposed scheme is simple and can be integrated into any system without effecting the properties of the original training sequence.

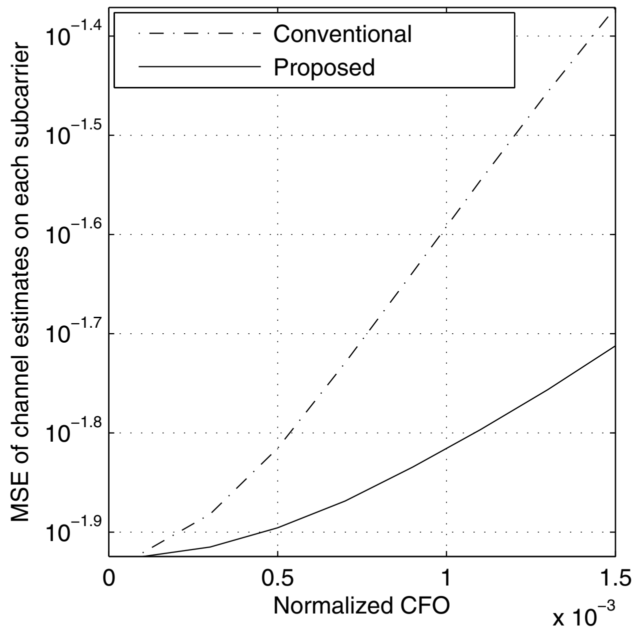

Fig. 2. MSE of channel estimates at each subcarrrier for conventional training sequence and the proposed training sequence, SNR = 16dB.

IV. Simulation Results

Although the preceding analysis was focused on SC-FDE system, this scheme can also be applied to ZP OFDM systems. Both Zero-padded SC-FDE and Zero-Padded OFDM systems are simulated. The proposed scheme, labelled as 'Proposed', is compared with the conventional training sequence scheme, labelled as 'conventional'. To validate the generality of the proposed scheme, random $\vec{Z}^a$ with $\|Z_i^a\|^2 = 1$ are used in the simulation. For both SC and OFDM systems, $M = 64$ subcarriers and uncoded QPSK modulation are adopted. Accumulated CFO per block, is assumed to be known and compensated in the simulation. The ETSI Multipath A [8], an indoor channel model, is used. Since CFO is generally estimated and compensated before channel estimation, only residual CFO is entered into channel estimation, and the simulated residual range is $[0.0001, \quad 0.0015]$.

Fig. 2 shows the mean square error (MSE) of the channel estimates at each subcarrrier versus normalized CFO. It is shown that, small residual CFO causes notable MSE variation in channel estimation, when either conventional or proposed training sequences is used. However, MSE of channel estimates with the proposed training sequence increases slower

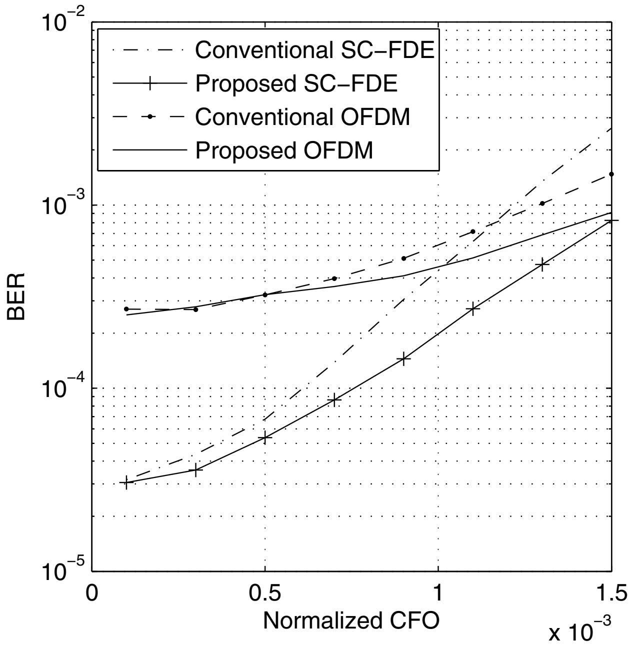

Fig. 3. BER vs normalized CFO for ZP SC-FDE and ZP OFDM systems with conventional and the proposed training sequences, SNR=16dB.

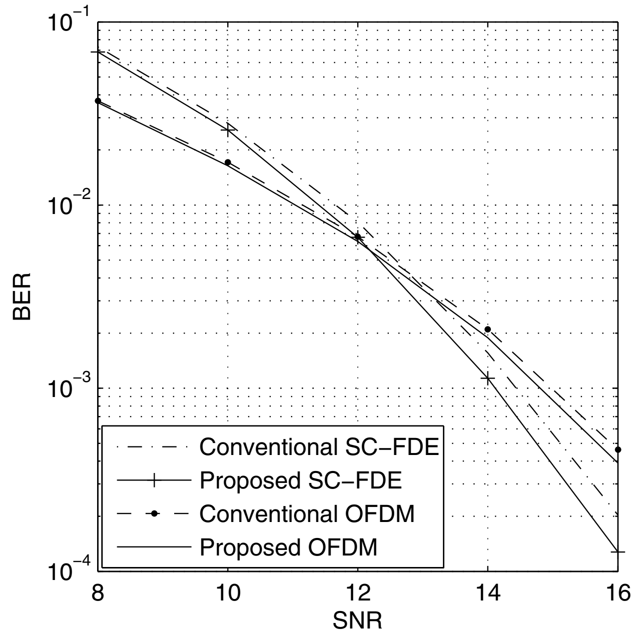

Fig. 4. BER vs SNR for ZP SC-FDE and ZP OFDM systems with conventional and the proposed training sequences, and Normalized CFO is 8×10^{-4}.

than that in the system with the conventional one. So the proposed training sequence is more robust to residual CFO.

Fig. 3 shows the Bit Error Rate (BER) versus normalized CFO for both ZP-SC-FDE and ZP-OFDM systems. The SNR is 16dB. The figure shows that in all cases, the system performance degrades with the increasing of CFO. The SC-FDE system experiences larger performance degradation than the OFDM system. In both OFDM and SC-FDE systems, by using the proposed training sequence, the system performance can be improved.

Fig. 4 shows BER versus SNR for both ZP-SC-FDE and ZP-OFDM systems. The normalized residual CFO is assumed to be 8×10^{-4}. The figure shows that when SNR is small, the system with the conventional and proposed training sequence have similar performance, because the noise dominates the signal to noise and interference ratio (SINR) and the MSE of channel estimates as shown in (6). With SNR increasing, systems with proposed training sequence structure outperform the system with conventional one, because the ICI now dominates system performance.

V. Conclusion

In this paper, we show that Carrier Frequency Offset could cause channel estimation error in frequency domain equalized systems. To improve the accuracy of channel estimation, new training sequence structure is proposed which can remove major ICI from adjacent subcarriers without making significant changes to the property of original training sequence. Simulation result shows that the proposed training sequence structure can improve system performance significantly.

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