

NEW UPPER BOUNDS ON THE PROBABILITY OF EVENTS BASED ON GRAPH STRUCTURES

BÉLA VIZVÁRI

Abstract. We present upper bounds for the probability of the union of events based on the individual probabilities and joint probabilities of pairs. The bounds generalize Hunter's upper bound and can be interpreted as objective function values corresponding to feasible solutions of the dual of the Boolean probability bounding LP.

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