NEW WIDE-APERTURE CATA-DIOPTRIC SYSTEMS

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Summary

The inherent limitations of the Maksutov lens are examined, both as regards the higher orders of spherical aberration (and hence maximum relative aperture) and residual oblique aberrations. Two-meniscus spherical surfaced systems are described giving a good axial correction up to $f/0{\cdot}7$ with almost complete correction of all orders of oblique aberration, other than field curvature, for a 30° field. Very large glass masses may be avoided by " par-axial division " of the menisci, and considerable departure from symmetry leads to forms in which the small residual higher order spherical aberration terms may be balanced. The degree of correction attainable, and the influence of the probable glass inhomogeneity are discussed.

The systems described by Maksutov (1), in which the aberration of a spherical mirror is corrected by a single achromatic meniscus lens, give remarkably fine results for the astronomical work for which they were designed, where small angles of field and relative apertures are required. For purposes where larger angles of field are needed, it is clear from a consideration of the form of the third-order aberration coefficients that astigmatism and oblique colour cannot be corrected simultaneously with spherical aberration, coma and axial colour. Thus if A_1, A_2 and A_3 be the spherical aberration of the two meniscus surfaces and the mirror respectively, then for spherical aberration correction

$$A_1+A_2+A_3=0$$

and taking the stop at the centre of curvation of the mirror, the coma condition requires that

$$A_1B_1+A_2B_2=0,$$

where the factors B depend on the position of the stop relative to the surface; since A_3 is not zero, (A_1+A_2) must be finite, so that for coma correction, B_1 and B_2 must be unequal*; hence the Seidel coefficient for astigmatism, of the form $(A_1B_1^2+A_2B_2^2)$ must also be finite. Similarly, since the mirror has no chromatic aberration, axial colour correction requires that the first order aberrations of the two meniscus surfaces, $C_1+C_2=0$ and the oblique colour $(C_1B_1+C_2B_2)$ must be finite.

Moreover, the correction of first order coma requires equal and opposite coma terms on the two meniscus surfaces, with respect to a stop at the centre of curvature of the mirror; it necessarily follows from this that the meniscus must be so placed that the stop position is some way removed from the meniscus

* Except for the case $B_1=B_2=0$, where the two meniscus surfaces are concentric with the mirror; but this form does not satisfy the condition for axial achromatism.

centres of curvature, that the angles of incidence of the principal ray are appreciable, and that higher orders of coma than the first will be present.*

Field curvature is, of course, not corrected in simple Maksutov systems, nor in the modifications to be described later.

The size of the uncorrected residual oblique aberrations may be reduced by reducing the thickness of the meniscus lens, but this results in an increase of the higher-order spherical aberration, both since deeper curvatures and larger angles of incidence are then required for first order correction, and since a reduced thickness reduces the effect, characteristic of meniscus lenses, of a reduction of higher order over-correction arising from the effect of the aberration of the first surface on that of the second. In the systems described in detail by Maksutov, the thickness of the meniscus is maintained at one-tenth of the aperture, and for a relative aperture of $f/1$ (which is of course outside the range for which these systems are intended) the focal length must not exceed 1·37 mm. for the axial aberration to fall within the Rayleigh limit; the astigmatic focal distance is about 0·002 of the focal length at 15° from the axis and the ratio of image sizes for the C and F wave-lengths 1·0006. These uncorrected oblique aberrations are therefore comparatively small, but as the meniscus is thickened to give any appreciable improvement in the spherical zone, they are rapidly increased.

For purposes where systems of very large relative aperture, of the order of 1 : 1, are required, having excellent correction in fairly large focal lengths, the Maksutov system cannot therefore take the place of the Schmidt system. However, with present methods the difficulty of production of aspheric surfaces is so much greater than for spherical ones that it appears worth while to investigate the possibility of designing an effective substitute using spherical curves. The alternatives hitherto proposed either fail to give the required high degree of correction of spherical or chromatic aberration (**2**) or still require the use of figured surfaces of less asphericity than the Schmidt (**3**). Systems employing two spherical surfaced menisci have therefore been investigated, one concave and one convex to the mirror. This immediately results in a reduction of the zonal aberration of the Maksutov system to about one-tenth with the same thickness of meniscus, and in addition a number of quite new properties appear. In the first place there are two sets of solutions.

(*a*) There is a set of solutions having each meniscus substantially corrected for first-order coma, and standing in the same relationship to the mirror centre as in Maksutov's system; astigmatism may be corrected, but the oblique colour is of the same sign on each meniscus, and increases with meniscus thickness.

* In the Maksutov meniscus, A_1 and A_2 have opposite signs, so that B_1 and B_2 must have the same sign if $A_1B_1+A_2B_2=0$. Now B (which is invariant on refraction) may be written as

$$B=\frac{H}{nu^2}\cdot\frac{(x-r)}{(x-s)(s-r)},$$

where H is the Smith-Helmholtz constant, n the refractive index, u the angular semi-aperture, r the radius of the refracting surface, and x, s, the distances of the (intermediate) stop and image positions from the vertex of the surface, all to a paraxial approximation. From the achromatic condition, $Q_1h_1^2=Q_2h_2^2$ (Q being the Abbé invariant and h the incidence height of a marginal ray to paraxial accuracy), so that Q_1 and Q_2 have the same sign, and since for a meniscus r_1 and r_2 have the same sign, so have $(s_1'-r_1')$ and (s_2-r_2) ; in the glass space $(x_1'-s_1')=(x_2-s_2)$, so that considering B_1 and B_2 in that space, $(x_1'-r_1')$ and (x_2-r_2) have the same sign, i.e. the image of the stop in the glass space cannot lie between the centres of curvature ; the meniscus index being greater than that of air, and the stop on its concave side, the actual stop must be further removed from the centres of curvature than its image in the glass space.

(*b*) In a second set of solutions, small coma terms of like sign on one meniscus are corrected by equal and opposite terms on the other, the effective stop being situated between the centres of curvature of the two surfaces of each meniscus (Fig. 1). With such a construction, the principal ray through the stop centre makes extremely small angles of incidence with all the surfaces, and is substantially coincident with the auxiliary axis for each surface (Table I), so that the higher-order oblique aberrations become very small. In addition astigmatism, oblique colour and distortion may be corrected, and thickening of the menisci is possible to give further improvement of the higher-order spherical aberrations without affecting the oblique corrections. The overall length is much less than for the first type.

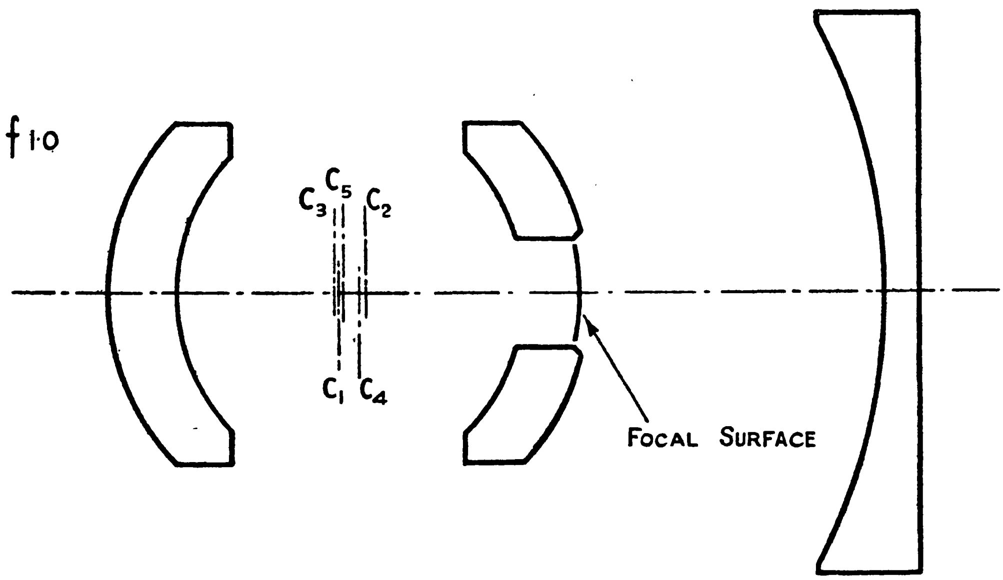

FIG. 1.

			n_d	v
R_1	+0·9704			
		0·291	1·613	59·3
R_2	+0·7820			
		1·456		
R_3	−0·7820			
		0·281	1·613	59·3
R_4	−0·9704			
		1·232		
R_5	−2·268			

TABLE I

Angles of incidence for principal ray at 12° *to axis*

Surface 1	2° 08′
,, 2	0° 27′
,, 3	1° 25′
,, 4	1° 06′
,, 5	0° 00′

It is this second set of solutions which is considered in this paper. In the lens shown in Fig. 1, for an angle of field of 30°, the oblique aberrations in the meridional plane are identical with the axial to six-figure accuracy, while skew-ray traces show that in the secondary plane there is higher-order aberration producing about a 20 per cent improvement over the axial spherical aberration.

The axial spherical aberration for a relative aperture of $f/1$ produces a departure of the emergent wave-front from the spherical of about a quarter of a wave-length per 25 mm. of focal length.

Fig. 2 shows a similar design, having considerably thicker lenses, and hence better axial correction; in this case the emergent wave-front departs from the spherical form by about 1/7 of a wave-length for 25 mm. of focal length at a relative aperture of $f/1{\cdot}0$, and about one wave-length at an aperture of $f/0{\cdot}7$. The oblique aberrations at $f/1{\cdot}0$, as in the previous system, are substantially identical with the axial.

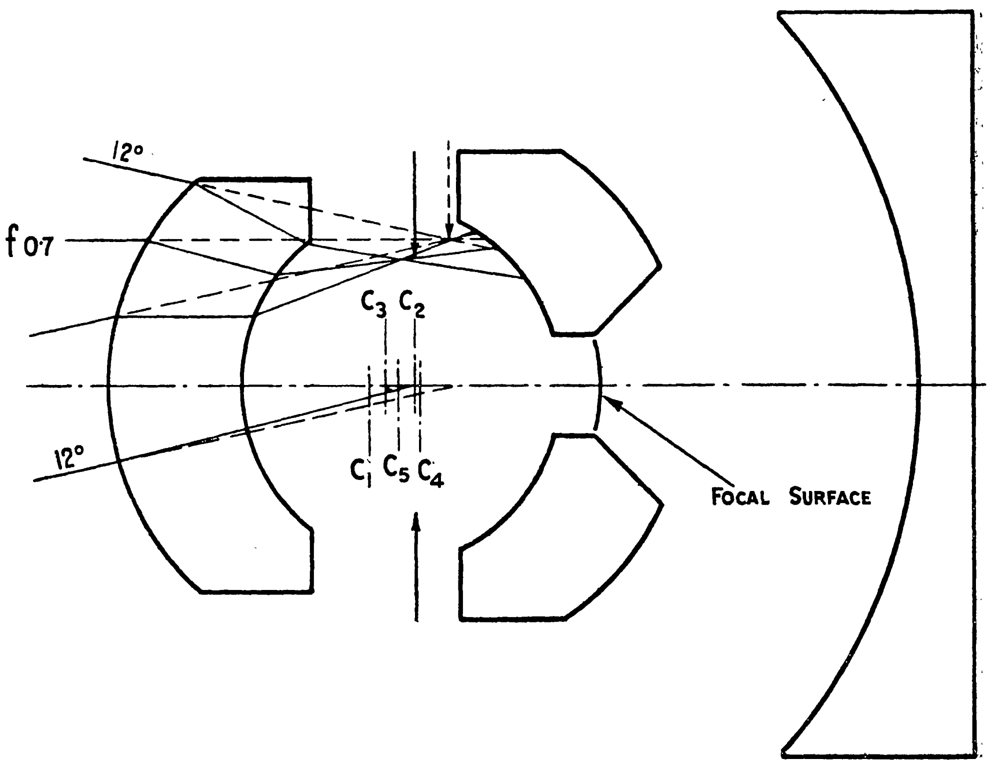

FIG. 2.

			n_d	ν
R_1	$+1{\cdot}237$			
		$0{\cdot}609$	$1{\cdot}613$	$59{\cdot}3$
R_2	$+0{\cdot}8378$			
		$1{\cdot}443$		
R_3	$-0{\cdot}8378$			
		$0{\cdot}609$	$1{\cdot}613$	$59{\cdot}3$
R_4	$-1{\cdot}274$			
		$1{\cdot}103$		
R_5	$-2{\cdot}466$			

At the aperture of $f/0{\cdot}7$ another effect becomes significant; the axial point of the stop is virtually free from aberration, but the edges of the stop suffer from a higher order aberration which, in the meridional plane, is of symmetrical form, as shown in Fig. 2; this corresponds to a displacement of the effective stop position, variable with incidence angle. This means that, although at any angle up to 30° of field the coma is completely corrected, to six-figure accuracy, with respect to the stop position appropriate to that angle, the necessary use of a fixed stop introduces a higher order coma in the final image; with the original stop position, this may be completely eliminated for a field angle of 30°, by introducing vignetting stops reducing the effective aperture by 6 per cent; or, by taking a new stop position which is a compromise between the positions required for different incidence angles, a much smaller vignetting factor is needed to give oblique correction equivalent to the axial.

At apertures much greater than $f/1{\cdot}0$ the chromatic difference of spherical aberration also becomes more significant, but by balancing this against a small first order chromatic term, the residual errors, from *d* to F, at a relative aperture of $f/0{\cdot}7$, are reduced to less than one (yellow) wave-length for 25 mm. of focal length, and less than half this from C to *d*.

It will be clear that in double meniscus lenses of these types the variables at the designer's disposal are more than are required to fulfil the conditions for the correction of the first-order aberrations. Regarding the ratio of the combined axial thicknesses of the menisci to the equivalent focal length as fixed by the degree of correction of zonal spherical aberration required, and disregarding the possibility of variation of refractive index (which has little effect on the performance of the systems *) the variables are five curvatures, one thickness and two separations, making eight in all; the conditions to be fulfilled are the determination of focal length, four Seidel conditions (neglecting field curvature) and two chromatic conditions, making seven.

Moreover, on account of the very small angles of incidence of the principal rays throughout the system, the distortion remains small for modifications of the system which maintain these small incidence angles. Neglecting distortion, therefore, the variables are sufficiently numerous to enable two other conditions to be fulfilled. This flexibility has been used in the example shown in Fig. 1 by making the curvatures of the concave and convex surfaces the same on each meniscus, the axial thicknesses being then slightly different for full correction; the uncorrected "pin-cushion" distortion amounts to 1·1 minutes of arc for the worst zone of a 30° field.

This possible adaptability of the system may also be employed in another way, to overcome the practical difficulties likely to be encountered in making lenses of great focal length and aperture having a high degree of correction. If such lenses were made, of the form shown in Fig. 2, for example, very large pieces of glass would be required, which would be disadvantageous on account of absorption, lack of homogeneity, and difficulty of supply. Now if, in a thick meniscus lens, a central air-lens be introduced, bounded by concentric surfaces centred on the paraxial focus in the glass space, then two thin lenses will result, having the same power, first-order spherical aberration, coma, axial and oblique chromatic aberrations as the original thick lens (Fig. 3). The higher-order spherical aberration introduced is of the correct sign to improve the correction of the system, but is quite small in amount; first-order astigmatism results, which may readily be corrected by small modifications in the separations of the

* In the lenses described in the earlier part of this paper, the calculated performance of lenses computed for different refractive indices is very nearly constant ; but lower indices lead to deeper curves and higher angles of incidence at large apertures, with consequent greater reflection losses ; moreover, the "blooming" of lens surfaces to reduce reflection is less efficient for low index glasses, due to the absence of film materials with very low indices. For small sized lenses, therefore, where difficulties of supply and the variation of probable inhomogeneity between glass types need not be considered, dense barium crowns have some slight advantage. For the lenses described later in the paper, with balanced high-order spherical aberration, lower indices lead to a rather more rapid loss of this correction as field angle is increased.

menisci and the mirror; the field curvature is very slightly improved; and distortion is introduced in an amount which though small is not wholly correctable without an undesirable departure from the near-concentricity of the original system. The higher-order coma introduced is small for an aperture of $f/0{\cdot}7$. This division of a single thick meniscus into two thin ones, with a great saving of size of glass could, of course, be applied to each meniscus in the systems already described, and this would provide the possibility of balancing the distortion, but it represents a considerable increase in the complexity of the system.

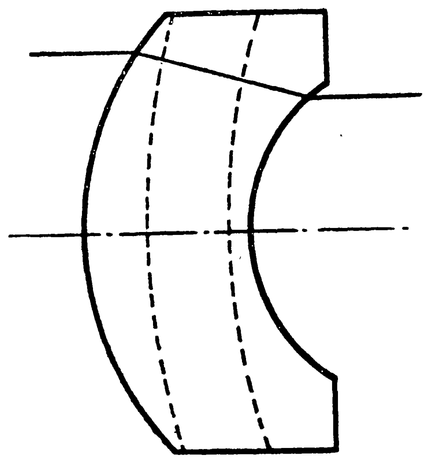

FIG. 3.

Alternatively, since the improvement of the spherical zone is achieved by an increase of thickness of either meniscus, a form may be chosen having one very thick meniscus, divided by an air-lens, and one thin meniscus (Fig. 4). This process of increasing the thickness of the front meniscus and decreasing that of the back necessarily requires, for full correction, a departure from separate achromatism of the menisci, and an increase of the relative contribution of the front to the correction of the spherical aberration of the mirror. Over a considerable range of change of relative thicknesses, the higher-order spherical aberration remains approximately constant for a given sum of axial thicknesses; this continues to be the case even after the total effect of the back meniscus on the spherical aberration of the system changes from over-correction to under-correction, but as the process is pushed further a point is reached at which the higher-order under-correction on the back surface of the back meniscus increases very rapidly, making possible a controlled balancing of the higher orders of aberration and hence a very great improvement in the axial correction. Thus in the lens shown in Fig. 4 with a relative aperture of $f/0{\cdot}75$ the emergent wave-front for *d* light on the axis departs from the spherical by less than $\pm 0{\cdot}09$ wave-length per inch of focal length.

This higher-order spherical aberration arises, of course, from the fact that to the Seidel degree of approximation the aberration on any surface is independent of the aberration of previous surfaces, and in this system light incident on the sixth surface is heavily over-corrected, this having the effect of increasing the higher-order aberration at that surface; and the angles of incidence for this surface being relatively and controllably large for large apertures, this increase is significant and controllable in magnitude. A similar effect arises in Gauss objectives, and is used in the design of wide-aperture double-Gauss photographic lenses.

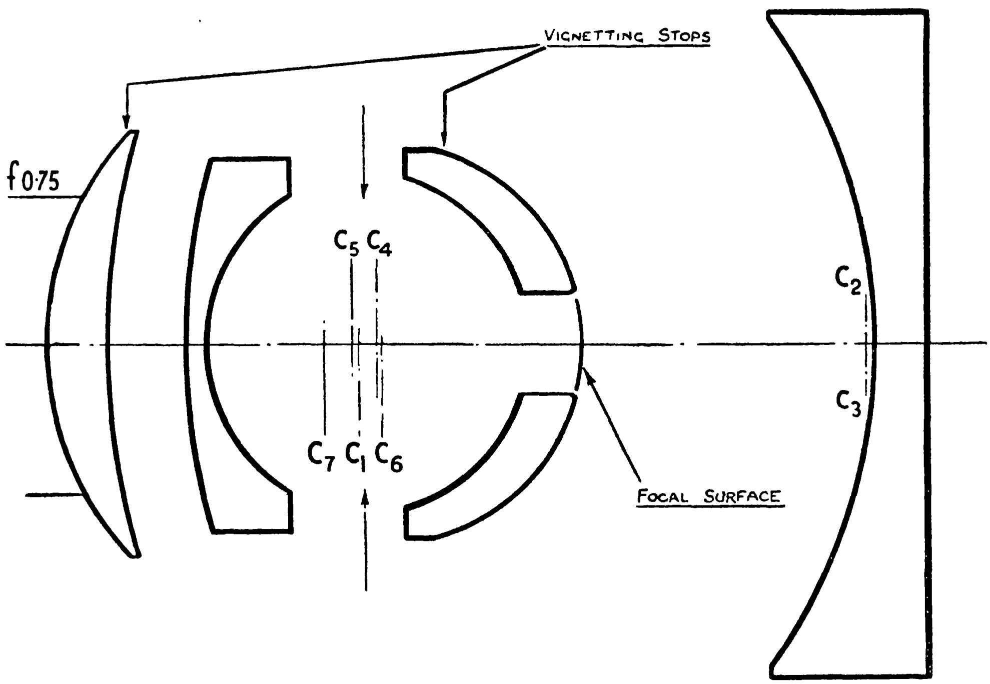

FIG. 4.—*Equivalent focal length* 1·00.

			n_d	ν
R_1	+1·4430			
		0·2748	1·613	59·3
R_2	+3·5222			
		0·3665		
R_3	+3·1557			
		0·0916	1·613	59·3
R_4	+0·8172			
		1·4468		
R_5	−0·7811			
		0·2639	1·613	59·3
R_6	−0·9143			
		1·4173		
R_7	−2·5280			

For points off the axis this higher-order spherical aberration affects the two sides of the aperture differently; and since the second meniscus is convergent, its thickness decreases away from the axis, with a consequent reduction in the under-correction with increasing angle of field, so that the advantage of this form decreases; but some advantage is retained over small field angles. Fig. 5 shows the departures of the wave-fronts from the spherical for different field angles, the spherical image surface having been chosen to give the best general result for a 31° field; for this image surface there is "barrel" distortion of 1·2 minutes of arc for the worst zone. If a smaller field were to be used, the best radius of image surface would be slightly different, giving rather less aberration for the smaller angles than that shown. For the larger angles of field, aberration increases very rapidly at the edge of the aperture, making it desirable to employ vignetting stops; these are shown in Fig. 4; they reduce the size of the nominal pupil area (neglecting the central obstruction) to 97·8 per cent of its axial value at $12\frac{1}{3}°$ from the axis.

This same method of higher spherical aberration control by the use of one thick and one thin meniscus does not of course depend on the thick one being

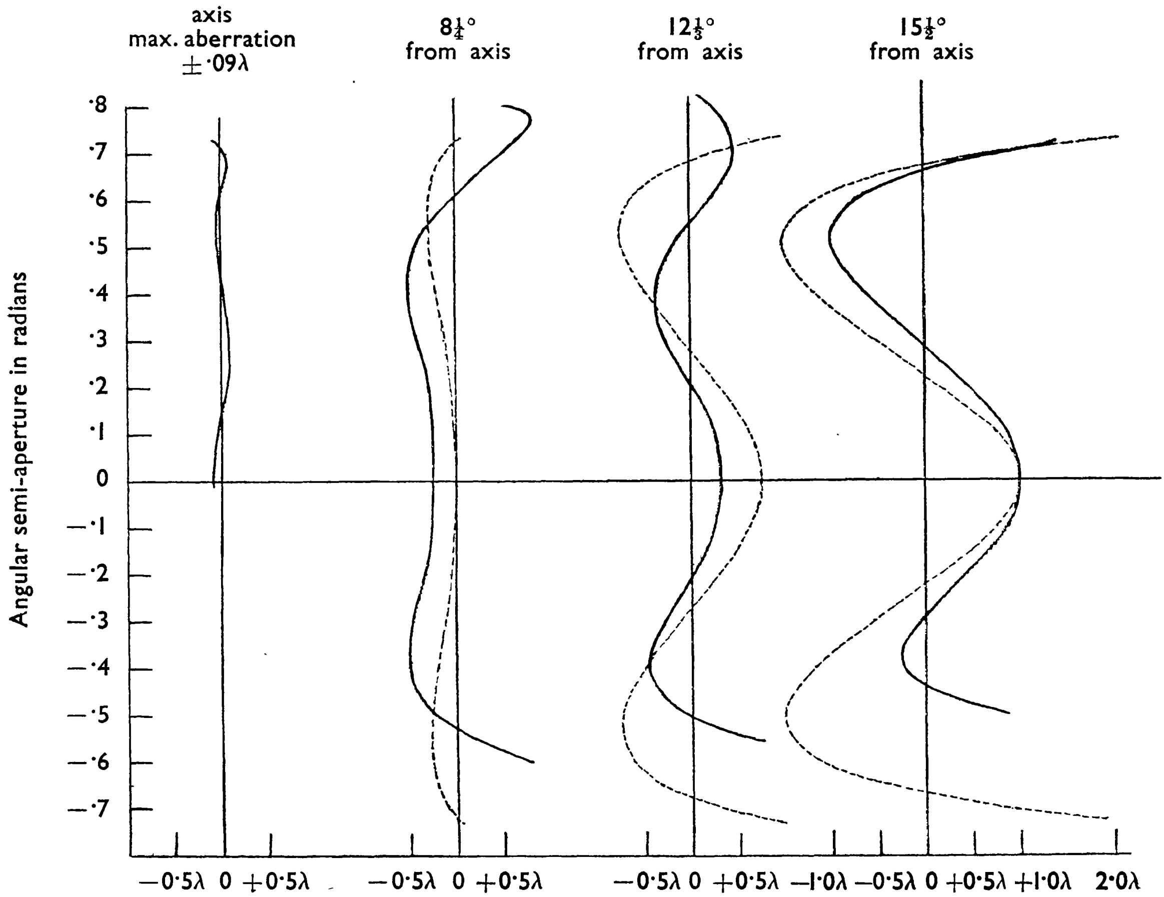

FIG. 5.—*f*/0·75 *system. 1 inch focal length.*

Departure of the emergent wave-front from a sphere centred on the spherical image surface.
Full line—Meridian section.
Dotted line—Sagittal section.

"paraxially divided" into two, and for smaller apertures the thicknesses involved may not require this. Fig. 6 shows a *f*/1·0 lens using two single menisci in which the axial aberration is less than ± 0·04 wave-length per inch of focal length, the aberration of the completely unvignetted emergent wave-fronts being shown in Fig. 7 for different field angles up to 30° (Fig. 7A shows, on the same scale, the aberrations of the meridional and sagittal sections for a semi-field of 12° of a *f*/1 Schmidt lens, having a corrector plate whose central and marginal thicknesses are equal). The effect of increase of angle is similar to that on the *f*/0·75 system. In all cases the wave-fronts are shown neglecting the necessary central obstruction, since the size of this depends on the field angle used. In both the *f*/1·0 and the *f*/0·75 systems, a small first-order axial chromatic aberration has been introduced to compromise the effects of chromatic difference of spherical aberration, and for the *f*/0·75 a small first-order oblique chromatic aberration is also required. The curves for the deviation of the C and F wave-fronts from that for *d* light for a meridian section is shown in Figs. 8 and 9. The virtual elimination of secondary spectrum characteristic of Maksutov systems is of course shared by all these meniscus lenses. It will be clear that, except when nearly monochromatic light is used, the residual chromatic aberrations are the limiting factor in the performance of both systems on the axis, and for the *f*/0·75 over most of the field. This is also true of the axial correction of Schmidt systems,

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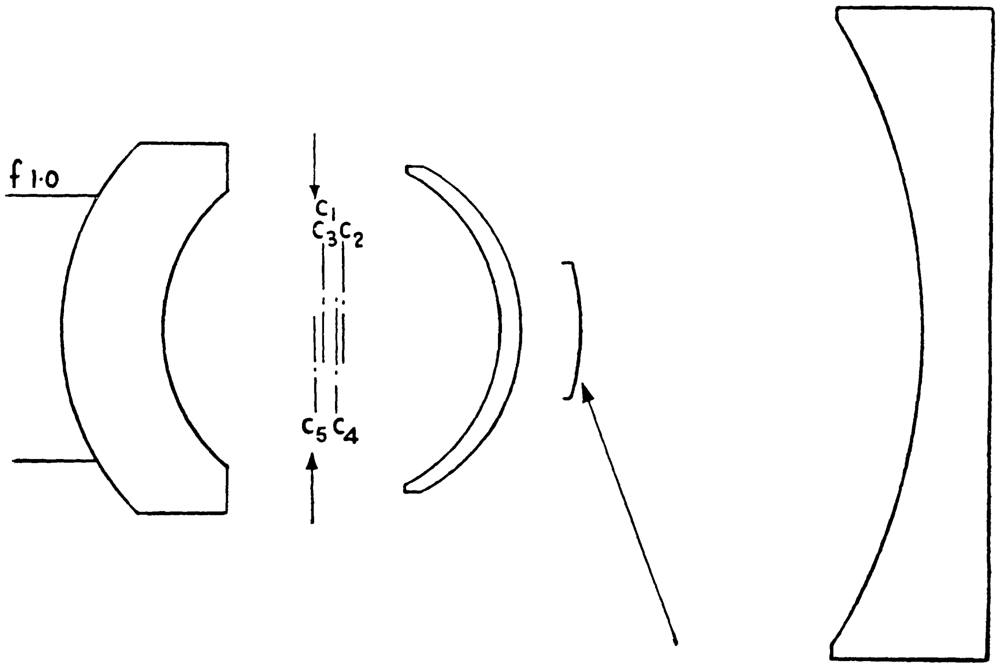

FIG. 6.—*Equivalent focal length* = 1·00.

			n_d	ν
R_1	+0·9676			
		0·3810	1·613	59·3
R_2	+0·6718			
		1·2529		
R_3	−0·6666			
		0·0762	1·613	59·3
R_4	−0·6837			
		1·4772		
R_5	−2·2455			

and the "Meniscus-Schmidt" systems described by Hawkins and Linfoot (3) *, and in all three systems the amount of the axial chromatic aberrations is similar. In the double meniscus and Schmidt systems the axial chromatic aberration curves are almost identical in form, but the errors are of opposite sign (Fig. 10). This may be expressed physically as the fact that the paraxially achromatic Schmidt plate, having the form $(n-1)x = ay^4 + by^6$ etc., has an effective glass thickness increasing for large values of the aperture y, whereas the paraxially achromatic meniscus lens decreases in effective thickness for large apertures in similar amount; these trends are unaffected by small departures from paraxial achromatism.

The similarity of form of the curves in the two cases is remarkable, and it is clear that the possibility exists of virtually complete corrections of the axial residual chromatic aberrations of the Schmidt by a combination of aspherized plate and meniscus lens correctors, in which the spherical aberration correction is divided appropriately between the two. The amount of figuring of the

* I am indebted to Dr Linfoot for the numerical data of the $f/1{\cdot}2$ Meniscus-Schmidt lens to which the curves of Fig. 10 relate; this was computed by him and Miss Hawkins after the publication of their paper (3) and departs slightly from the example given there. The plate glasses are changed to MBC 576594 and LF 575426, the front face of the plate is slightly convexed ($r = 39{,}000$ in.) and the interface radius $r_2 = -65{\cdot}83$ in. The effect of the changes is to improve the colour correction, which was incorrectly treated in (3) owing to a mistake in calculation. It is clear from the curves that the system was designed to give the optimum correction in the C–F range.

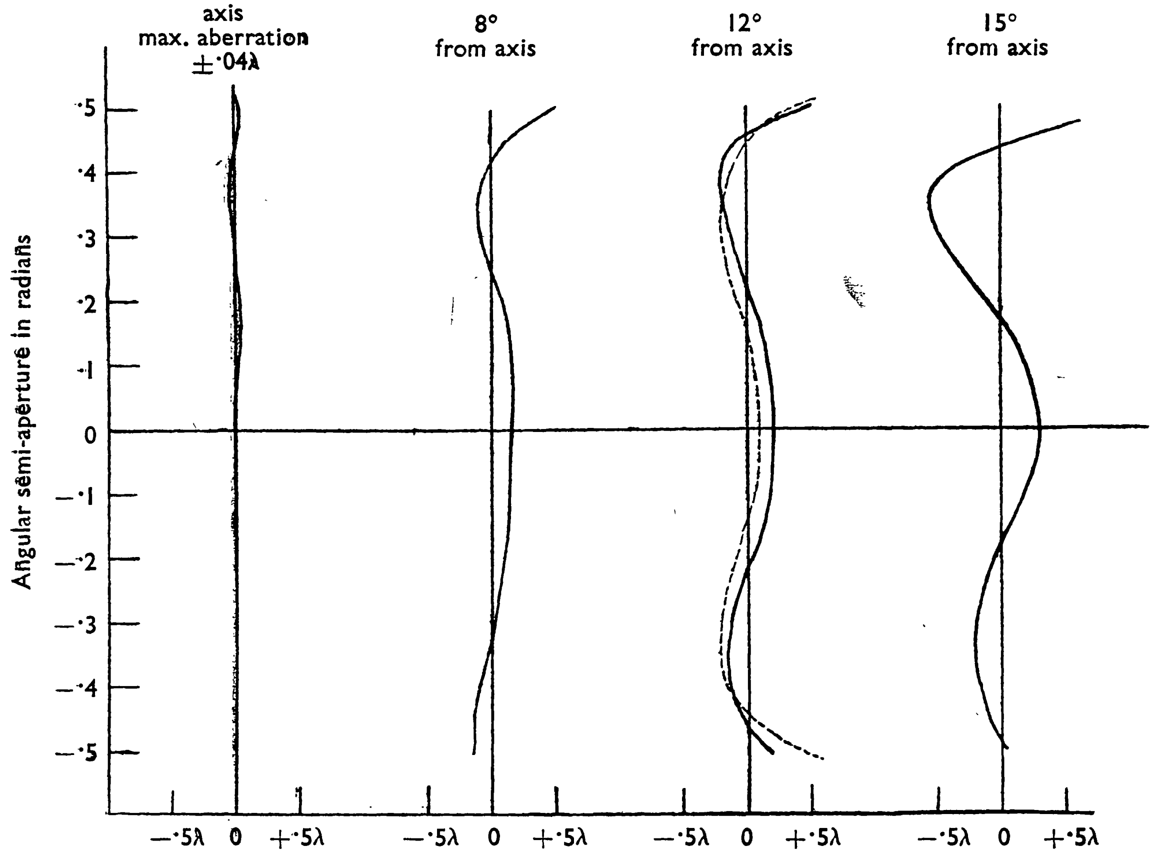

FIG. 7.—*Unvignetted f/1·0 system. 1 inch focal length.*
Departure of the emergent wave-front from a sphere centred on the spherical image surface for different angles of field.
Full line—Meridian section.
Dotted line—Sagittal section.

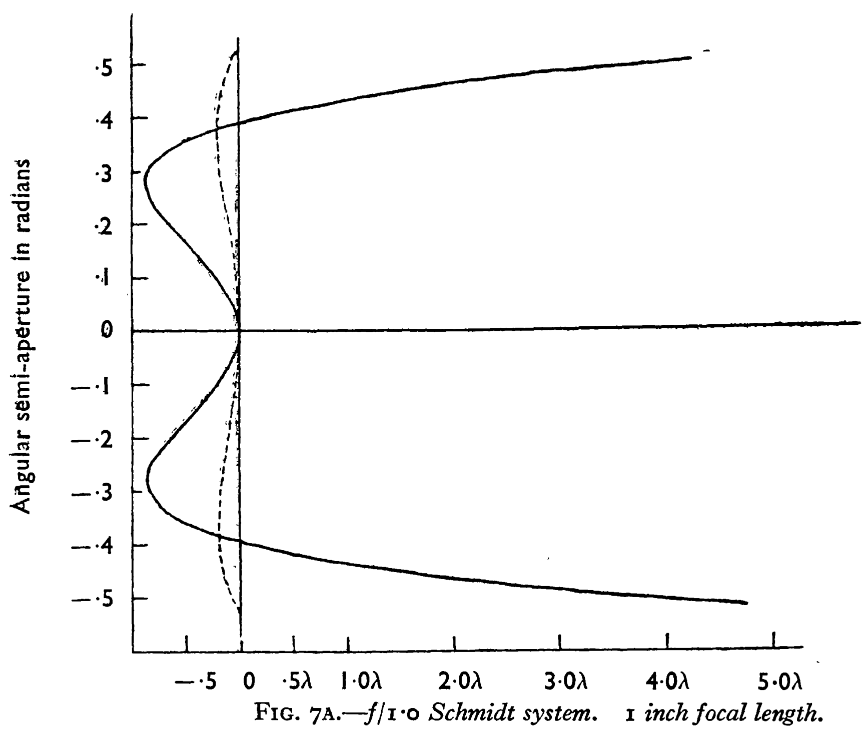

FIG. 7A.—*f/1·0 Schmidt system. 1 inch focal length.*
Departure of the emergent wave-front from a sphere for an object 12° from the axis.
Full line—Meridian section.
Dotted line—Sagittal section.

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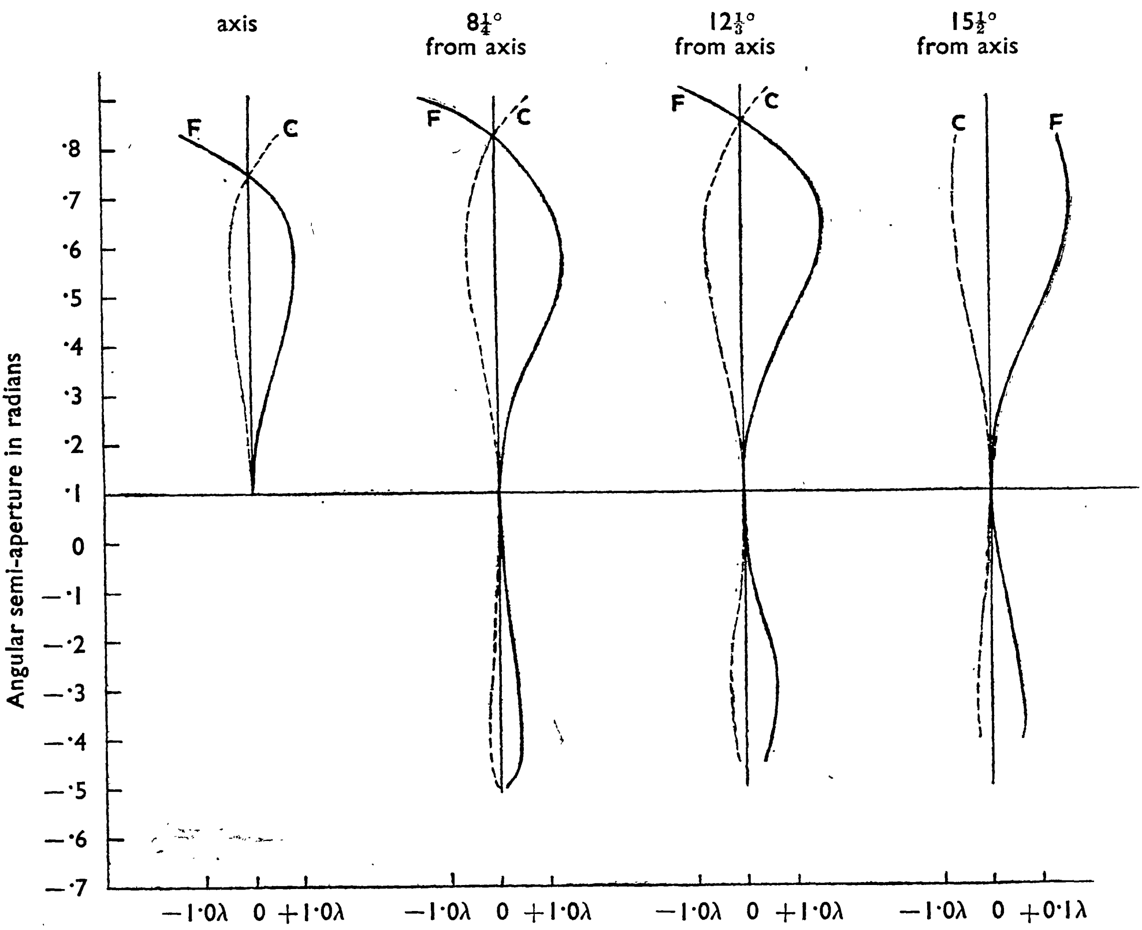

FIG. 8.—*f/0·75 system. 1 inch focal length.*
Aberration of the emergent wave-fronts for C and F with respect to the d wave-front.

Schmidt plate will of course be greater than in the "Meniscus-Schmidt" systems, and in consequence the oblique monochromatic corrections, arising from the inclination of the plate for oblique pencils, though superior to the Schmidt, will be inferior to those of either the "Meniscus-Schmidt" or double meniscus systems; but for small angles of field such systems offer the possibility of very wide aperture systems with perfection hitherto unapproached. Such combinations would seem to merit further investigation (4).

Although the slight figuring of the "Meniscus-Schmidt" lens is too small to produce this effect, it is interesting to observe that the chromatic difference of spherical aberration of the cemented doublet plate does, for C light, produce a considerable correction, but this is not maintained for all wave-lengths, due to the different relative partial dispersions of the crown and flint glasses used.

In comparing the performance of Schmidt and double meniscus systems, the axial corrections may be considered as effectively equal, since the chromatic aberration is almost identical, and in the presence of this the difference between the theoretically perfect monochromatic aberration of the Schmidt and that attained in the meniscus systems is clearly negligible; for oblique imagery the aberrations of the meniscus system are notably smaller.

Secondly, account must be taken of the probable vitiation of the calculated performance by the inhomogeneity of glass which must always be present to some extent. Unfortunately precise numerical information on this point does

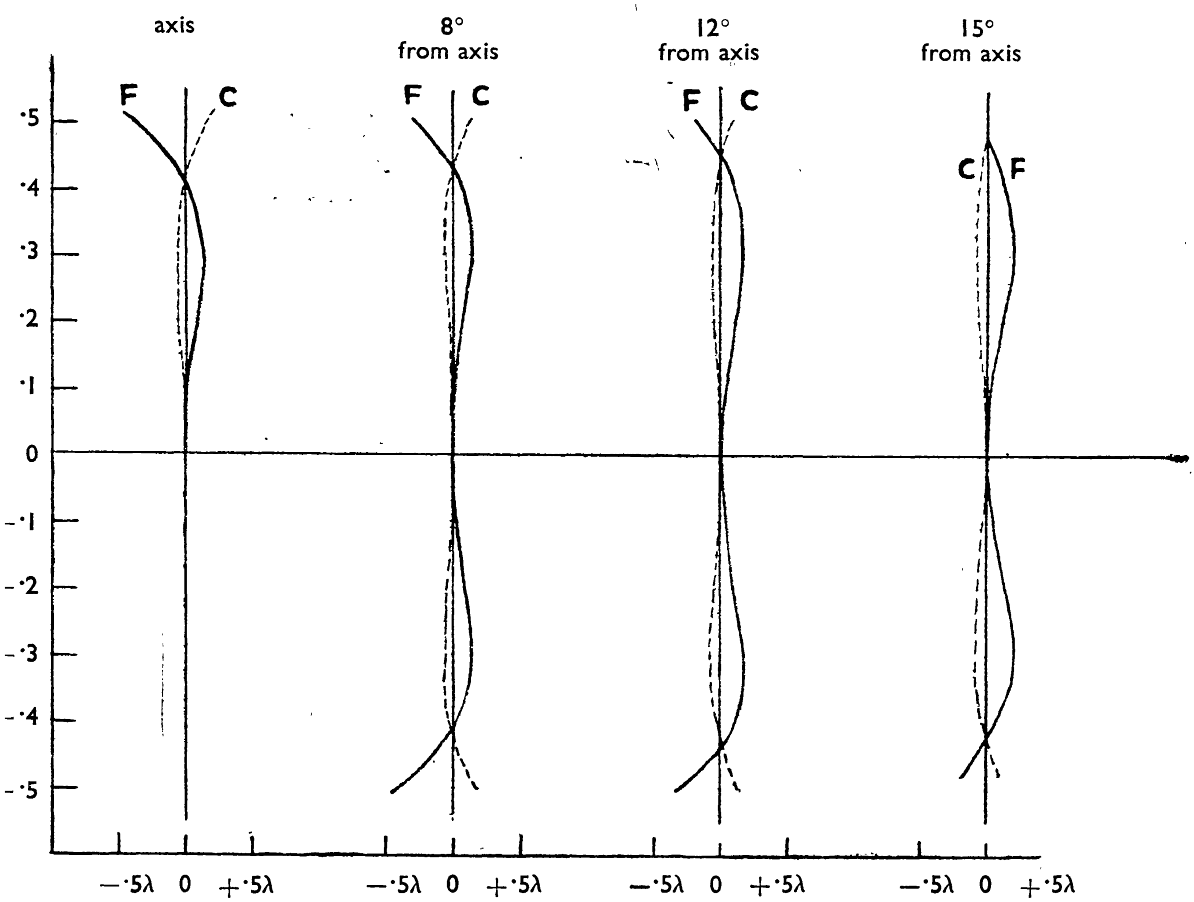

FIG. 9.—*f*/1·0 *system. Aberration per inch of focal length, of the C and F wave-fronts, with respect to the d wave-front.*

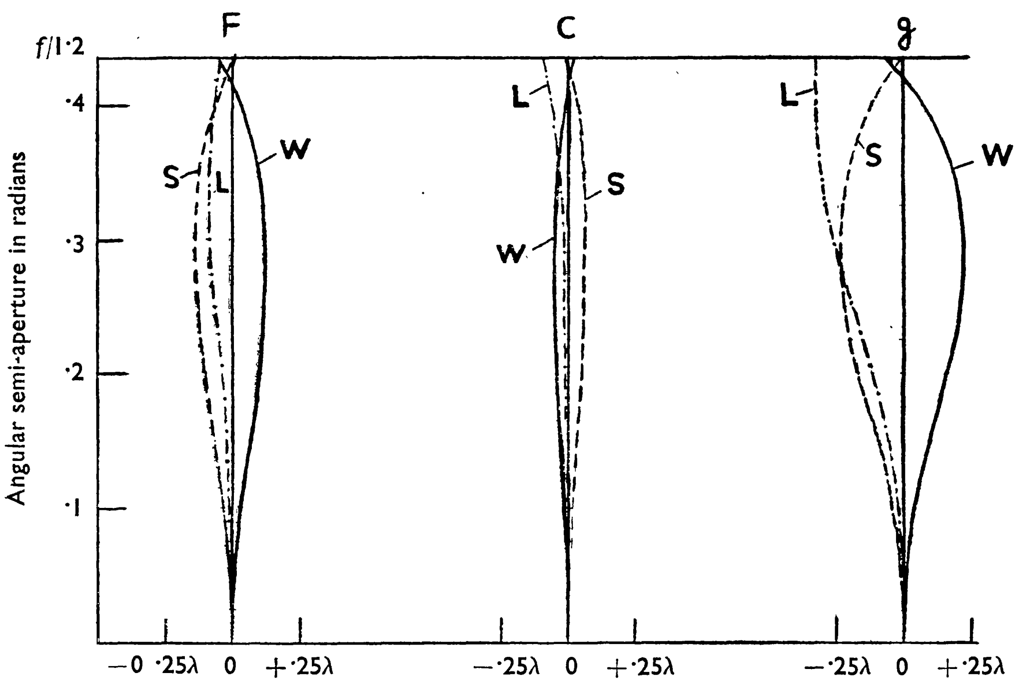

FIG. 10.—*Comparison, at f/1·2, of the axial chromatic aberrations for F, C and g with respect to d.*
S—Schmidt.
L—Linfoot's Meniscus-Schmidt.
W—Two Meniscus System.

not appear to have been published; but it is probable that the variation of refractive index increases with the size of glass block, and for lenses of considerable thickness it is obvious that quite small amounts of heterogeneity will introduce several wave-lengths of aberration. Inhomogeneity thus clearly sets a limit to the degree of correction that it is profitable to pursue in design. For very large systems, the Schmidt type will have a definite advantage from its smaller glass thickness despite the much greater difficulty of production.

Inhomogeneity errors may of course be corrected by the very laborious process of two-dimensional figuring, in any of these systems; but the diameter of the Airy-disk is 0·0014 mm. for an aperture of $f/1$ and 0·0010 mm. for $f/0{\cdot}7$ *, and in practice rather smaller than this, since an annular aperture is used, so that it appears probable that for many purposes other considerations, such as grain-size of photographic emulsions, would preclude the use of the calculated resolving power, even if this were attained. But for systems of fairly small focal length, say 100 mm., the optical inhomogeneity becomes of negligible importance, and it is possible that applications may be found for such lenses of relative aperture $f/1$ to $f/0{\cdot}7$, giving the full theoretical resolving power over a considerable angle of field.

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* Dr H. H. Hopkins has shown (5) that the Airy-disk formula is substantially correct for a relative aperture of $f/0{\cdot}7$.