

**Nicephorus Blemmydes on the Holy Trinity and the Paraconsistent
Notion of Numbers:
A Logical Analysis of a Byzantine Approach to the *Filioque***

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Abstract:

The paper deals with the most controversial – in the modern scholarly discussion – episode within the Byzantine polemics on the *Filioque*, Nicephorus Blemmydes' acknowledgement of proceeding of the Spirit *through* the Son providing that the Son be considered as generated *through* the Spirit. The logical intuition behind this theological idea is explicated in the terms of paraconsistent logic and especially of a kind of paraconsistent numbers called by the author “pseudo-natural numbers”. Such numbers could not be interpreted *via* the notion of ordered pair. Instead, they imply a known (first described by Emil Post in 1941) but still little studied logical connective ternary exclusive OR.

Keywords: Nicephorus Blemmydes, *Filioque*, Spirituque, Triadology, paraconsistent logic, paraconsistent numbers.

...cette intuition du nombre pur, la seule qui ne puisse nous tromper.

Henri Poincaré [42, p. 122]

1. Introduction: Symmetric Formula “Through Each Other”

The modern view that the dogma of the Trinity must be – in some way, at least, – logically consistent is not that of Byzantine Patristics. For Byzantine Fathers, it was normal to insist that the Trinity is beyond the human mind, because God is beyond any rational consistency. Thus, an “implicit inconsistency” of the Trinity in Plantinga’s sense¹ was taken as granted. Nevertheless, the explicit logical frames of the dogma were often presented as consistent as possible.

As a rule, the Fathers have avoided usage of openly inconsistent logical categories and preferred to follow a kind of Niels Bohr’s Correspondence Principle, that is, to use classical logical notions in a non-classical way [45], [3]. Such a standard has been established by Cappadocians in their anti-Eunomian polemics and was normally kept by Byzantine participants of anti-Latin polemics on the *Filioque*. The cases of an explicit discussion of inconsistency of the Cappadocian Triadology were, in Byzantine Patristics, relatively rare, even though sometimes quite important (especially those by Dionysius the Areopagite) [33].

The case of Nicephorus Blemmydes was somewhat different. In his *Filioque* polemics, he eventually reopened discussion on the very notions of “Son”, “Spirit”, and “Father”, without limiting himself to discuss their mutual relations.

Blemmydes recalled the definitions of hypostasis through the notion of energy instead of the notion of essence (nature). Indeed, providing that the energy is “the movement of the essence”², such definitions must be mutually equivalent. The resulted “non-static” definitions of hypostasis (applied in particular to the hypostasis of the Spirit) – far from being invented by Blemmydes but taken verbatim from Athanasius of Alexandria³ – were much more useful for the further discussion of the proceedings of the two hypostases (those of the Son and the Spirit) from the one (that of the Father).

In this way, Blemmydes elaborated a compromise understanding of the *Filioque*, which has, however, never been explicitly discussed in full (not fragmentary) by anybody, without exception of Blemmydes’ followers, Gregory of Cyprus and the Palamites. The present article is aimed at such an explicit discussion of Blemmydes’ central triadological concept, namely, its symmetrical formula of proceeding of the Son and the Spirit “through each other”, or, more literally, “(any?)one (of the two) through another” – διὰ θατέρου θάτερον: “...si le Verbe et l’Esprit <existent> à partir du Père comme du principe sans que l’un des deux <existe> par l’autre (μὴ διὰ θατέρου θάτερον), une division s’introduit dans la Divinité” (*Epistle to Theodore Laskaris*, 10 [48, v.1, pp. 346/347 txt/tr.]). The symmetry of this formula could be understood as either complete or partial, depending on our understanding of θάτερος as either “anyone of the two” (the normal meaning unless otherwise is specified) or “a specific one of the two” (the meaning that could be specified with the context).

I have previously noticed that this formula is a unique one [57]. It could be interpreted in different ways, depending on the context. On the one hand, it could be put into a Latin context, as John Bekkos really did, but even the modern critics of Blemmydes agree that this was a too much forced interpretation of Blemmydes’ original thought [25]⁴. But, on the other hand, what about the Palamite reading of Blemmydes?⁵ Was the Byzantine Orthodoxy read into Blemmydes’ works by the Palamite readers – as now Jean-Claude Larchet claims – or did it actually exist in Blemmydes’ theological thinking?

The most of considerations put forward so far *pro* and *contra* Blemmydes’ orthodoxy are not more than a balance of probabilities. Anyway, Blemmydes normally does not decline from the already established Patristic language: even its “worst” (from, so-to-say, Larchet’s viewpoint) formulations allegedly confusing the notions of hypostasis and energy are based on Athanasius. No Blemmydes’ critic was able to take him at his words.

The value of Blemmydes’ unique “symmetric formula” is therefore exceptional. It was both original and never quoted by either Gregory of Cyprus or later Blemmydes’ followers. It is therefore the only “difficult” theological statement by Blemmydes.

There could be no doubt that it would have been not useful in any polemical context, where only familiar formulations were acceptable. This would explain why this formula has been put aside by Gregory of Cyprus and Gregory Palamas. Indeed, Blemmydes himself did not use it in polemics.

Be this as it may, Blemmydes' formula needs to be checked against its patristic background. If it turns out to be impossible with the usual methods of patrology – that is, tracing its prehistory in earlier theological texts – we have to explore the underlying logical model implied by Blemmydes.

Such a study will be inevitably limited in respect of Blemmydes' own "secret thoughts" (in what extent he realised himself the logic he followed, and so on). The latter, however, is not a matter of any scholarly interest – at least, in patrology. The *histoire des idées* has a logic of its own, and it does not matter in what extent any personality who contributed to it was psychologically fitting him – or herself with the intellectual flow of history. We have to study logic without a psychological commitment.

Fortunately, a recent publication of the previously unknown Blemmydes' texts by Michel Stavrou provides a decisive witness that the completely symmetrical understanding of the Blemmydes' formula ("through each other") was not that of Blemmydes himself. His original meaning was "a specific one of the two" (proceeding of the Spirit though the Son but not *vice versa*)⁶. Therefore, his own opinion on the mediatory role of the Son in the proceeding of the Spirit was either somewhat "subordinationist" (I would use this term as derived from the term "order", but Michel Stavrou prefers to say of a "réciprocité asymétrique" between the Son and the Spirit⁷) or still not completely clarified even to himself. Be this as it may, we know, from the further development of the Byzantine theology, that it was Blemmydes who gave an impetus to symmetrical explanations of the mutual relations between the Son and the Spirit.

2. The Basic Problem of Triadology: With or Without Pairs?

In Triadology, one counts to three but has a very big problem how to pass two. Can we speak about three without previously speaking about two? The "symmetric formula" would imply an answer like "Yes, but, in this case, the second will be also the third, and the third will be also the second".

The problem of order within the Trinity (especially who is there the second and who is the third) has been dealt with at length in *ca* 1335 by Gregory Palamas. He insisted (and argued with the testimonies taken from the Cappadocian Fathers and Ps.-Chrysostom = Severian of Gabala) that the Latin *Filioque* implies an order within the divine nature, whereas no order between the three hypostases is allowable in this sense (that is, in the sense of ἐκ τῆς φυσικῆς ἀκολουθίας / "from the natural consequence"). He avoided here, nevertheless, the explicitly paradoxical symmetric formula with its "quantum superposition" of the two proceedings⁸. In general, Palamas argues here "from the Fathers" but not from any logical system. This could be sufficient for demonstrating that, in the Cappadocian Triadology, any "natural" order between the Son and the Spirit is forbidden, but not for explaining why.

In *Filioque* doctrines, however, the intermediary step of counting to two has never been skipped, neither in the old Carolingian two-principle *Filioque*, nor in the *tanquam ab uno principio* of the 1274 Council of Lyon. Both kinds of *Filioque* were perfectly consistent, from a logical point of view. There were, however, some equally consistent alternatives to the *Filioque* Triadology. Let us briefly consider them all starting from the Triadologies implying pairs.

Both early Latin *Filioque* with its two "principles/beginnings" within the Holy Trinity (the Father and the Son are two different "principles" of the Holy Spirit: thus in the *Libri Carolini* (s. on them [18]) and the mainstream Latin doctrine before the 1274 Lyon Council, including the lifetime of Blemmydes) and Council of Lyon's 1274 doctrine of *tanquam ab uno principio* (the Father and the Son form a unique "principle" of the Spirit) imply some pairing. In the first case, these are the pairs of the Father and the Spirit and the Son and the Spirit = (F + Sp) + (S + Sp) = (1 + 1) + (1 + 1). In the latter case, this is the pair of the Father and the Son followed by the pair of them both and the Spirit = (F + S) + Sp = (1 + 1) + 1.

Given that the $\mu\upsilon\nu\alpha\rho\chi\iota\alpha$ of the Father is out of question (that is, the Father must remain on the first place everywhere), one can easily see that only the two following variants of pairing are left: $(F + Sp) + S = (1 + 1) + 1$, — and $(F + S) + (F + Sp) = (1 + 1) + (1 + 1)$.

In sum, there are logically possible only four “pairing” Triadologies which correspond to two schemes of pairing (Table 1). We have just discussed two Triadological doctrines where the schemes of pairing were different.

Table 1.

Doctrine	Scheme of Pairing	Historical Realisation
$(F + Sp) + (S + Sp)$	$(1 + 1) + (1 + 1)$	Earliest <i>Filioque</i> doctrine
$(F + S) + (F + Sp)$		Opposition to Gregory of Cyprus; Meletius Pegas <i>et al.</i>
$(F + S) + Sp$	$(1 + 1) + 1$	Lyon’s <i>Filioque</i> doctrine (<i>tanquam ab uno principio</i>)
$(F + Sp) + S$		Ethiopian doctrine <i>Qəbat</i>

The two remaining Triadological doctrines implying pairing are the following.

The doctrine $(F + Sp) + S$, that is, “the Son is born from the Father through the Spirit”, is another version of *tanquam ab uno principio*, where this “principle” is, however, the Spirit and not the Son. It has been realised in the Ethiopian seventeenth-nineteenth-century theological doctrine *Qəbat* (“Unction”: the Son is born through the function of the Spirit)⁹. It has never been in consideration in the middle and late Byzantium or among the Latins. In the eyes of the Ethiopian adherents of this doctrine, it was not without support in the works of Cyril of Alexandria and Cappadocian Fathers, and we will see that such claims were not completely unfounded. Nevertheless, such an attitude was hardly possible anywhere in Byzantium during the whole period of the *Filioque* polemics.

The doctrine $(F + S) + (F + Sp)$, that is, the generation of the Son and the procession of the Spirit are absolutely independent from each other, appears shortly after Blemmydes, within the Byzantine opposition to the Triadology of Gregory of Cyprus and his 1285 Blachernae Synod¹⁰. The imminent victory of Palamism led to a temporary suppression of this doctrine, but it will reappear near 1600 as a result of the Triadological quarrel between Maximus Margounios and Gabriel Severus supported by Meletius Pegas¹¹. The latter will become the main responsible for its *de facto* canonisation in the nominally Orthodox textbooks until the “rediscovery” of Gregory of Cyprus in the twentieth century. In Pegas and textbooks that followed him, the topic of uncreated energies became completely absent (whereas already Maxim Margounios made the first step in this direction limiting the divine energies to the revelation to the creation without allowing them to be *in divinis*).

This doctrine becomes vulnerable to the same arguments that were used against the Latin Carolingian *Filioque* doctrine, namely, that a distinction between the Father and the Son as the two “principles” would imply a division within the Trinity. Here, in a similar way, a division between the Son and the Spirit as the two separate “products” of the Father would imply an analogous division.

In fact, now, we have exhausted the list of the consistent and paradox-free treatments of the mutual relations between the hypostases within the Holy Trinity. Any other paradox-free approach would lead to either explicit Arianism or explicit Sabellianism. The traditional Byzantine approach was, however, still different.

3. Why Cappadocian Triadology Became Incomprehensible

The Byzantines reached the discussions on the *Filioque* when the logical and philosophical language of their theology was not in its best shape. We have to mention this, even though there is

no room here to go into details, not to say that the most of these details are still waiting for being studied.

The key to the original explanation of the unity in the trinity given by Cappadocian Fathers was definitively lost already in the sixth century, when – especially under the high pressure of John Philoponus’ influence [58], [21] – it became habitual to treat the unique essence/nature of the godhead as a common (“second”) nature in the Aristotelean sense. There was, then, a range of interpretations of what this “Aristotelean sense” means, within the span between, so-to-say, “nominalism” and “realism”, – but all of them were reducing the natural unity between the divine hypostases to the same level as the natural unity between three men, whatever explanation of the latter kind of unity would have been provided [53]. This was certainly not enough for explaining the unity of the Holy Trinity (unless one would have wished to adopt Philoponus’ own doctrine of the “Tritheism”).

Therefore, since the seventh century as the latest, Chalcedonian theologians were trying to elaborate other conceptions, as it was first witnessed by John of Damascus ([6], [49], [10], but s. criticisms of [7] in [23]). We have now to skip this part of the story but need to notice that the very intervention of Nicetas Stethatos into the 1054 discussion with Cardinal Humbert turned out to be an exacerbation of these Byzantine problems with the logical inconsistency of their Triadology¹². Probably, Dirk Krausmüller is not always right claiming the “nominalist” trend in the understanding of the divine nature “heterodoxy” (because there is no bijective correspondence between the philosophical/logical and theological concepts) but, at least, usage of such categories has made the logical construct of Triadology to be more and more far from Cappadocian Fathers.

Nicephorus Blemmydes avoided reopening the whole issue, but in his controversial formula, he puts his finger on its main logical knot, namely, the problem of logical consistency.

4. The Two *Spirituque*

In the realities of the twentieth century, the slogan of symmetric Triadologies became *Spirituque* “and from the Spirit” by Paul Evdokimov¹³ – patterned, of course, after *Filioque*. Such a label is somewhat misleading, because it does literally mention a diametric opposition to the *Filioque*, which is the doctrine *Qəbat*. The *Qəbat* and *Filioque* doctrines mirror each other, whereas the *Spirituque* doctrines are always symmetric, presuming *both* proceeding of the Spirit though the Son and begetting of the Son through the Spirit.

It would be difficult and irrelevant to our purpose trying to figure out Evdokimov’s original meaning of the *Spirituque*. Anyway, his uncritical relying on Bolotov makes *ipso facto* Larchet’s criticisms¹⁴, at least, partially justified. Bolotov was not only “anti-paraconsistent” but rather “positivistic”. In a consistent framework, any symmetric approach to two “second” hypostases would have been acceptable with a price of reducing their self-standing reality, that is, with a concession to the Sabellianism. It is worth noting that the *Filioque* doctrine *tanquam ab uno principio* has sounded Sabellianic to Byzantine Fathers [34]. No wonder that the *Spirituque* approach acquired a support from some Catholic theologians including such famous figures as Yves Congar (after some hesitations: s. [16, p. 79]) and especially Leonardo Boff who produced a detailed triadological doctrine in the *Spirituque* line supported with both patristic and modern witnesses [2, pp. 13, 106-108, 180-182, 224-227, 249-253, 286].

Nevertheless, the question is not as simple as that. Alexander Golitzin published, to my opinion, the most important contribution shedding light on the real dimension of the problem. Golitzin was commenting on a unique passage of Gregory of Nazianzus where the Holy Trinity is compared to the first human family in the way that Eve becomes an image of the Spirit who gives birth to the Son¹⁵. This passage, according to Golitzin, is to be read within a large mystical Semitic/Syriac tradition where the Holy Spirit is “Mother”¹⁶ and also within the mainstream Christian liturgical tradition (referring itself to the Annunciation narrative in Luke), where the flesh of Christ appears after an invocation of the Holy Spirit [15].

If we imagine Alexander Golitzin being an Ethiopian theologian belonging to the *Qəbat* faction, he could provide an ample patristic dossier favouring the generation of the Son through the Spirit. The real Golitzin puts forward, instead, a more balanced view endorsing Leonardo Boff's *Spirituque*. What is especially helpful, he recalled Dumitru Stăniloae's already elaborated exposition of the "non-causal reciprocity" between the Son and the Spirit¹⁷, based, in turn, on Joseph Bryennios' (ca 1350–1431) Trinitarian theology¹⁸.

Stăniloae, following Joseph Bryennios, distinguishes between two kinds of interpersonal relation within the Trinity, causal (between the Father and the Son and between the Father and the Spirit) and non-causal (between the Son and the Spirit). It is obvious that the reciprocity between the Son and the Spirit meant in the symmetric formula is non-causal, either: it implies that the Father is the only "cause" in the Trinity.

Stăniloae quotes from a long Bryennios' exposition: "The Son, because he is the one who is the Son, alone possesses the name of Son *vis-à-vis* the Father, for he is the Son of the Father only, not of two; but the name of Word which belongs to the Son alone within the Holy Trinity has reference not only to the Father as the one who is Mind, but also to the Spirit in another way..."¹⁹. And, in the same manner, the Spirit is He Who Proceeds only *vis-à-vis* the Father who caused him to proceed, whereas being the Spirit of both Father and Son²⁰.

The elaborated Triadology by Joseph Bryennios – not only in the *Hortatory Sermon* but also in some other works, including the *Twenty One Sermons on the Holy Trinity* – provides, to my opinion, an appropriate context for a deeper understanding of post-Blemmydian symmetric Triadology. Bryennios discusses at length the non-causal relations and the (in)existence of the physical order within the Holy Trinity.

Stăniloae and, after him and following him, Golitzin, provided us with the optics having a sufficient resolution for perceiving the inner traditionalism behind Blemmydes' apparent innovation, that is, his symmetric formula.

Nevertheless, the question remains: what is the logic implied in such a symmetric Triadology? It is already clear from the above that it is not any logic avoiding the contradictions, but now we need to define the kind of paraconsistent logic we are dealing with.

5. From a Logical Point of View

5.1. "Set-theoretical" Reformulation of the Problem

The symmetric formula implies a severe logical difficulty — at least, from a "classical" logical point of view. It does not allow counting to three and, therefore, makes the number three in "Trinity" unintelligible. These are not bad news, judging from a Cappadocian perspective, but certainly not easily digestible for Blemmydes' contemporaries.

The problem is the following. To be able counting to three, we have, normally, to count to two and, then, to repeat the same procedure when reaching three. In the modern set-theoretical language, we can say that, to be able to count, we need oriented pairs, that is, elementary sets having two elements, where one element is chosen to be the first and the remaining element is, thus, the second. Without this, no natural row of numbers is possible, neither any row of numbers known to our modern mathematics.

In the "symmetric" counting, no oriented pair and no pair at all are possible. The elementary set contains here three elements, not two. One of these three elements is chosen to be the first (the Father)²¹, whereas no element is chosen to be the second or the third. Alternatively, one can say that two elements simultaneously are the second ones.

Evidently, we have left the ground of classical logic in general and any usual mathematical logic in particular. We have, nevertheless, to make explicit the logic of the symmetric formula as it is.

Our explanation will go through the following steps:

1. Explanation of the logical connective implied: what means “to choose exactly one (Father = “cause”) from three”, that is, what is the difference between this logical procedure and the choice of exactly one from two, in a three-element set, repeated two times.
2. What kind of sets and, therefore, what kind of numbers we are dealing with.
3. What kind of non-classical logic is implied.

5.2. *The Principle of “Sabellianism”: Any Pairing Scheme in Triadology Implies an Implicit “Sabellianism”*

There are three different kinds of logical connective OR (disjunction) but only two of them are widely known: the inclusive disjunction (“at least, one of the two”) and the ordinary (that is, binary) exclusive disjunction (“exactly one of the two”). The third kind of disjunction has been first described – in the modern logic – by Emil Post in 1941 did not become widely known – at least, among the logicians – until recently [39]. This is the ternary exclusive disjunction (“exactly one of the three”) and its generalisation up to n -arity (“exactly one of the n ”).

The two kinds of exclusive disjunction show drastically different behaviour (truth-functions) starting from the arity 3 and, then, at the each odd value of the arity. It is almost self-evident that the mutual relations of the hypostases of the Trinity are to be described with the ternary exclusive OR and not with any other disjunction: a given hypostasis is exactly one of the three. Nevertheless, let us demonstrate it in a more detailed way. Let us begin with a demonstration of a principle concerning the possible triadological meaning of the ordinary (binary) exclusive disjunction.

At the arity 2, the two kinds of exclusive disjunction are not distinguishable and have the same truth-function that is described in Table 2. Let us consider, as an example, two arbitrary hypostases of the Trinity, say, the Father and the Son. Let the proposition φ_1 will be “the hypostasis X is the Father” and the proposition φ_2 will be “the hypostasis X is the Son”. These two propositions cannot be true simultaneously. Thus, the truth-function is the following (T means “true”, F means “false”, and \oplus is the symbol of the ordinary exclusive disjunction):

Table 2.

φ_1	φ_2	$\varphi_1 \oplus \varphi_2$
T	T	F
T	F	T
F	T	T
F	F	F

In our example, we have had a choice between the two possibilities only: the hypostasis X could be either Father or Son or, alternatively, neither of them, but there was no third value for the notion “hypostasis”. The exclusive binary disjunction allows only two situations to be true: when either φ_1 or φ_2 is true, whereas the remaining proposition is false. The situation where both φ_1 and φ_2 are true is false, that is, not allowed.

So far so good. Let us consider a more complex situation that corresponds to that of the Trinity. Thus, let us include the proposition φ_3 “the hypostasis X is the Spirit”. The ordinary exclusive disjunction would presuppose the choice between the pairs. The order of these pairs does not matter. Let us consider the sequence $(\varphi_1 \oplus \varphi_2) \oplus \varphi_3$; s. Table 3.

Table 3.

φ_1	φ_2	φ_3	$(\varphi_1 \oplus \varphi_2) \oplus \varphi_3$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

As in the previous example, the hypostasis X could not take more than one value: it is the Father or the Son or the Spirit, but never any two of them nor all the three. Our connective, however, whereas not allowing any two-value situation for X , does allow the three-value one: the first row of Table 3 contains the value of the truth-value function that renders the ordinary exclusive disjunction inapplicable to the Trinity.

The ordinary (binary) exclusive disjunction makes choice between the pairs. If there are more than two objects to choose between them, it repeats the choice from a pair. Thus, at an arity more than 2, this kind of exclusive disjunction is nothing but a reiteration of the binary disjunction. This is why the choice from three is performed in two binary steps, where the second step is a choice between the result of the former step and the remaining third object. This is why the first row of Table 3 contains such a value of the truth-function. At the first step, when there were two propositions claimed to be true, our connective results in the truth-value “false”, but then, at the second step, it has to deal with this resulting from the former choice false proposition and the apparently true third proposition, that results in the decisive truth-value “true”. This result is to be repeated at all odd numbers of options. If the number of options is even, the choice of all options simultaneously is impossible.

If in the Trinity there existed the pairs, our hypostasis X could be simultaneously the Father, the Son, and the Spirit. Let us notice that we have just demonstrated logically (using the *reductio ad absurdum* method) the following theological principle:

The Principle of “Sabellianism”

- Any triadological doctrine on the “triune God” (that is, any non-Arian Triadology) implying any kind of pairing between the hypostases would be intrinsically “Sabellianic” (to the Byzantine taste²²).

5.3. A Logical Connective Disallowing the Pairs: Ternary Exclusive OR

If we correct Table 3 making a unique change, namely, changing the truth-value in the first row from T to F, we obtain another truth-function that corresponds to another kind of disjunction, the ternary exclusive OR, the connective that never allows to choose all the three from three.

The ternary exclusive OR forbids the choice of all options simultaneously even at the odd numbers of options, which would be impossible with reiteration of the choice between pairs.

The three hypostases of the Trinity are connected with the ternary exclusive OR and not with the ordinary (binary) exclusive disjunction. This means that there are no pairs in the Holy Trinity.

Instead of ordered pairs, the ternary (n -ary) exclusive OR creates the groups “the chosen one + all others”, where these “all others” are different from each other but not distinguishable, except

It is clear from the above that, for the row of natural numbers, the existence of ordered pairs is a *conditio sine qua non*.

In our “pseudo-natural” numbers, we have, instead, “pseudo-ordered” pairs, which are the pairs where only the first element is defined, whereas all other elements of the set are the second. This is a paraconsistent conjunction based on the contrary (not contradictory) opposition: from a classical point of view, it is impossible that there are more than one “second” elements, whereas there is no problem, if none of given elements is the second one.

Thus, the pseudo-ordered pair is to be defined as following. In a set of n elements, there is one element chosen to be the first, a ; the remaining elements (designed with the letter b with an appropriate index) are in amount of $n-1$. Thus, the pseudo-ordered pair is

$$\left(a, \bigwedge_{n-1} b_{n-1} \right) = \bigwedge_{n-1} \{ \{a\}, \{a, b_{n-1}\} \}$$

The above formula is paraconsistent: it does not design $n-1$ pairs, but only a unique pair with $n-1$ “second” elements.

For the case of the Trinity, $n = 3$.

It is clear that the pseudo-natural numbers do not form any row. Instead, their set has only one “ordered” component: the element chosen to be the first. All other elements of their set are equally the “second”.

Thus, as Evagrius pointed out, there could be neither “two” nor “four” flanking the “three” in the Holy Trinity.

Let us notice that these paraconsistent relations in the Holy Trinity are not causal. In their respective causal relations, both Son and Spirit are completely distinct without forming any paraconsistent relations. However, this consistent and “classical” reasoning in Triadology is placed within a non-classical concept (our pseudo-natural numbers), exactly according to Niels Bohr’s Correspondence Principle.

However, in non-causal relations, the Father is not necessarily the first in the Holy Trinity. Many Byzantine authors, whereas not Blemmydes, dedicated detailed explanations to why there is no “physical order” among the hypostases of the Holy Trinity, that is, why any hypostasis could be counted as the first one²⁶. Thus, theoretically, there is not only one choice of the first element (discussed by Blemmydes) but all the three, and the resulting number of the pseudo-ordered pairs in the paraconsistent conjunction is equal to the number of permutations (ordered combinations) of two elements from n ,

$$P_n^2 = \frac{n!}{(n-2)!}$$

In the Holy Trinity, where $n = 3$, this results in 6. If one element from three is already chosen, we have to replace n in the above formula with $n-1$, which results in 2: the two paraconsistent non-causal conjunctions covered by the symmetric formula.

6. Conclusion

The Byzantine patristic tradition is certainly rich enough to provide us with an elaborated theory of pseudo-natural numbers. Blemmydes, however, was not a theoretician of it. His merit consists in making some first steps in its direction when it became semi-forgotten by his contemporaries.

There is no room here to go deeper into analysis of both theological, set-theoretic, and logical problems related to the pseudo-natural numbers²⁷. The purpose of the above study was to make visible a powerful flow of patristic logical thought to whom Blemmydes demonstrated an imperfect but unusual, for his epoch, sensitivity.

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Notes

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1. Cf. Plantinga's distinction between three kinds of contradiction in theology (explicit, formal, and implicit) in [40, p. 12-24] (first published in 1974).
2. The earliest witness of this definition is the textbook *Doctrina Patrum de incarnatione Verbi* (ca 700) where it is ascribed to Gregory of Nyssa (now unknown in his preserved works): Ἐνέργειά ἐστι ποιὰ τις κίνησις τῆς οὐσίας. <...> Ἐνέργειά ἐστι κίνησις δραστικὴ. κίνησις δὲ ἐστι παράλλαξις τοῦ προτέρου ["The energy is as if some movement of the essence. <...> The energy is an active movement, whereas the movement is an alteration of the former" (sc., state)] [9, p. 258, lines 4, 6-7].

3. See the most detailed analysis of the relevant passages of Blemmydes in [35]. The key Athanasius' passage (*Epistula I ad Serapionem*, 20–21) was here referred to by Makarov according to PG 26, 577C-580A, still without an access to the recent critical edition, which made clear that a striking difference between Athanasius original texts and its quotation in both Blemmydes and Gregory of Cyprus is not an “alteration” made by Blemmydes [35, pp. 206-207] but a different variant reading in manuscripts of Athanasius (where the Holy Spirit either “is the living energy” or “...being the life, that is the energy...”): [πνεῦμα] ἐνὸς γὰρ ὄντος τοῦ ζῶντος λόγου μίαν εἶναι δεῖ τελείαν καὶ πλήρη τὴν ἁγιαστικὴν καὶ φωτιστικὴν ζῶσαν [variant reading ζῶν οὖσαν] ἐνέργειαν αὐτοῦ καὶ δωρεάν, ἥτις ἐκ πατρὸς λέγεται ἐκπορεύεσθαι... (ch. 20:4; [46, pp. 29-31]).
4. Cf. my answer to this and other criticisms by Larchet [30, pp. 502-506], as well as my review of tome I of [48]: [31, esp. p. 426], where I endorsed Stavrou's claim that Blemmydes' theological work was not an “...evolution mais approfondissement ou *involution* de la pneumatologie byzantine” (quoted [48, v. 1, p. 117]).
5. Gregory Palamas' explicit quotation from Nicephorus Blemmydes has been misinterpreted by John Meyendorff and other scholars but is eventually rightly identified by Ioannis D. Polemis [43].
6. A series of syllogisms without title entitled by the editor “Autres syllogismes sur la procession du saint Esprit” and dated to the same period as Blemmydes' two major treatises on the procession of the Holy Spirit (1255–1256). Here, in the syllogism 4, Blemmydes explicitly denies the symmetry: Εἰ μὴ διὰ τοῦ Υἱοῦ τὸ Πνεῦμα τὸ ἅγιον, ἔσται διὰ τοῦ Πνεύματος ὁ Υἱός· ἀλλὰ μὴν τοῦτο οὐκ ἔστι, τὸ ἕτερον ἄρα (“Si l'Esprit saint n'existe pas par le Fils, le Fils existera par l'Esprit ; mais il n'est pas ainsi, donc l'autre hypothèse [est vraie]”) [48, v.2, p. 224/225 txt/tr.]. Moreover, in the syllogism 11, he uses almost the same formula as διὰ θατέρου θάτερον — ἐτέρας διὰ τῆς ἐτέρας, — but in a perfectly transparent context: Εἰ μὴ διὰ τοῦ Υἱοῦ τὸ Πνεῦμα τὸ ἅγιον, τόπω περιγραφίσηται ἡ Τριάς· αἱ γὰρ ὑποστάσεις τοῦ Υἱοῦ καὶ τοῦ Πνεύματος, μὴ ἐτέρας διὰ τῆς ἐτέρας προΐουσης, τόπον τινὰ οὐ περιλήψονται ἐν ᾧ θατέρα ἐσεῖται, ἀλλὰ παρὰ μέρος ἀντιλήψονται τούτου (“Si l'Esprit saint n'existe pas par le Fils, la Trinité sera localement circonscrite. En effet, les hypostases du Fils et de l'Esprit — aucune des deux ne venant [à l'être] par l'autre — n'embrasseront pas un lieu dans lequel existera l'une des deux, mais c'est en partie qu'elles s'attacheront à ce lieu”) [48, v. 2, pp. 226/227 txt/tr.]. I am very grateful to Michel Stavrou for pointing me out these formulations by Blemmydes.
7. As he suggested in an e-mail to me on 21 June 2016.
8. Gregory Palamas, *First Apodictic Sermon on the Procession of the Holy Spirit*, 32-33; ed. by B. Bobrinsky in [54, pp. 60-64] (quotation pp. 61, line 24).
9. See, as a short introduction, [50]. As the best introduction to the theological doctrine, s. [13].
10. For historical overview, [38], [26]; cf. [30]. In [30] I put forward a guess that the leading theologian of this group was Georges Moschampar. Unfortunately, there is no, so far, any detailed study of the theology of the adversaries of Gregory of Cyprus and its possible relation to the Triadology of Akindynos (a direct adversary of Gregory Palamas, whose pseudo-Photian quotations show a similar or the same theology).
11. No detailed study so far. Cf. some brief observations in [29, pp. 183-184]. Recently, Marcus Plested recalled this discussion but without any analysis [41, pp. 146-147].
12. Cf. now a series of Dirk Krausmüller's articles on Nicetas Stethatos (the third one is still in preparation).
13. “Le Père engendre le Fils avec la participation de l'Esprit Saint et il spire l'Esprit Saint avec la participation du Fils” [11, p. 72]; “...le Fils est la condition trinitaire de la spiration du Saint-Esprit par le Père, l'Esprit Saint est la condition trinitaire de l'engendrement du Fils par le Père. L'innascibilité, la génération et la procession sont sans confusion ni séparation un seul acte tri-un de Révélation, avec la participation simultanée et réciproque des Trois” [11, p. 75]. Evdokimov refers, here and elsewhere, to Vassily Vassiljevich Bolotov as his principal predecessor in this approach. See: [51], important translations and commentaries: [4], [55], [56].
14. “Il est inutile de dire que cette théorie, en décalage complet par rapport à la Tradition orthodoxe, est une innovation irrecevable qui ne fait qu'ajouter une erreur à une autre et des nouvelles confusions aux confusions précédentes” [27, p. 25-26] (quotation p. 26, n. 54). Larchet, however, equates this Evdokimov's and Bolotov's attitude with that of Blemmydes and Stavrou.
15. Gregory of Nazianzus, *Oratio* 31, *On the Holy Spirit (Theological 5)*, 11: “What was Adam? A creature of God. What then was Eve? A fragment (τμήμα) of the creature. And what was Seth? The begotten of both (Ἀμφοτέρων γέννημα). Does it then seem to you that Creature (πλάσμα) and Fragment (τμήμα) and Begotten (γέννημα) are the same thing? Of course, it does not. But were not these persons consubstantial (ὁμοούσια)? Of course they were. Well then, here it is an acknowledged fact that those that are differently hypostasised may have the same substance (τὰ διαφόρως ὑποστάντα τῆς αὐτῆς εἶναι οὐσίας ἐνδέχασθαι). I say this, not that I would attribute creation or fraction or any property of body to the Godhead (let none of your contenders for a word be down upon me again), but that I may contemplate in these, as on a stage, things which are objects of thought alone. For it is not possible to trace out any image exactly to the whole extent of the truth. But, they say, what is the meaning of all this? For is not the one an offspring, and the other a something else of the One (Οὐ γὰρ τοῦ ἐνὸς τὸ μὲν γέννημα, τὸ δὲ ἄλλο τι. Τί οὖν ;)? Did not both Eve and Seth come from the one Adam (οὐχὶ τοῦ αὐτοῦ Ἀδάμ)? Indeed, from whom else? And were they both begotten by him? No; but the one was a fragment (τμήμα) of him, and the other was begotten (γέννημα) by him. And yet the two were one and the same thing; both were human beings; no one will deny that. Will you then give up your contention against the Spirit, that He must be either altogether begotten, or else cannot

be consubstantial, or be God; and admit from human examples the possibility of our position? I think it will be well for you, unless you are determined to be very quarrelsome, and to fight against what is proved to demonstration.” Text: [12, pp. 294, 296], tr. [5].

16. Interesting, Boris Bobrinskoy whom Golitzin quotes among the like-minded theologians, was afraid of giving a pretext for speculations on the Spirit as “Mother”: “Comme l’écrit Paul Evokimoff, l’Esprit n’est pas étranger au mystère de la relation Père-Fils, car celui-ci est une relation triadique et non dyadique [11, p. 77]. Il est essentiel de poser cela ainsi au début même de toute réflexion sur le mystère trinitaire, tout en veillant à ne pas tomber dans la spéculation gnostique d’une « maternité » divine et éternelle de l’Esprit” [1, pp. 271-272].
17. In a paper published in Romanian in 1970 and reprinted as ch. III “The Holy Trinity: Structure of Supreme Love” of [47, pp. 73-108, 231-234, esp. 105-106]. Stăniloae refused to see, in Blemmydes, the germ of the same reasoning as in Bryennios (as Stăniloae wrote in a personal letter to me *ca* 1989).
18. Stăniloae referred to Bryennios in a rather confusing manner, only to the Romanian translation by Metropolitan Grigorie Dascalul, where the work referred to is added as the 22th sermon to Bryennios’ 21 sermons *On the Holy Trinity* [8, p. 346]. In fact, the quoted Bryennios’ work is *Λόγος συμβουλευτικός περι τῆς ἐνώσεως τῶν ἐκκλησιῶν (Hortatory Sermon on the Union of the Churches, 1422)* [52, pp. 469-500]; dated by Ph. Meyer, *pace* N. Tomadakes; cf. [28, p. 30, n. 1].
19. ...ὁ Υἱὸς, τὸ μὲν Υἱὸς ὄνομα τοῦτο, καθὼ Υἱὸς, πρὸς μόνον κέκτηται τὸν Πατέρα· Πατὴρ γὰρ ἐστὶν ἐνὸς Υἱοῦ μόνου, καὶ οὐ δυοῖν· τὸ δὲ Λόγος ὄνομα, ὃ μόνος ἐν τῇ ὑπερθέῳ Τριάδι πλουτεῖ, οὐ μόνον ἐστὶ τοῦ Πατρὸς καθὼ νοῦ, ἀλλὰ δὲ καὶ τοῦ Πνεύματος, καθ’ἕτερον τρόπον...
20. Joseph Bryennios, *Hortatory Sermon...*, [52, esp. 487-499 and chart 3 (σχῆμα Γ´) at the end of the volume], quoted p. 487; tr. from [47, p. 105].
21. No Byzantine patristic author would agree even with the claim that the Father is the first, because, in the Holy Trinity, there is no “the first”, “the second”, and “the third” in the natural order. However, in our present purely logical — or rather “set-theoretical” — language we are daring to say that the Father is the first (in a “non-Byzantine” sense of the word), where “the first” means “the cause (αἰτία)”.
22. That is, “Sabellianism” would be considered, by the Byzantine polemicists, as an appropriate heresiological label for the corresponding doctrine. Of course, the charges of implicit “Sabellianism” were compatible with those of “Arianism”, because any kind of pairing within the Trinity would lead to deepening of the opposition between this pair and the remaining hypostasis. However, such an “Arianism” has appeared as a secondary phenomenon.
23. Quoted according to the original recension S₂, but the text of S₁ does not contain any important differences: [17, pp. 221, 223]; s., for S₁, [17, pp. 220, 222].
24. Whether it is a definition or an interpretation depends on one’s attitude towards the foundations of mathematics. To von Neumann himself as well as other set-theoreticians of the early 20th cent., it was a definition, to Poincaré it would seem an interpretation. For such objects as our pseudo-natural numbers Poincaré’s argumentation against Couturat, Zermelo, and Russell is valid. Cf. [14].
25. I quote the standard modern formulation of von Neumann’s definition. For the original one: [37].
26. See above references to Gregory Palamas (who quotes classical passages of Severian of Gabala on the topic) and Joseph Bryennios. For a more general introduction to the pseudo-natural numbers, see now: [32].
27. I would like only to enumerate here the most striking among them. In theology: provenance of the “caused” hypostases (the Son and the Spirit) in relation to the divine energies. In set-theory and logic: foundation axiom, countability, axiom of choice.