No-Scale Supergravity Realization of the Starobinsky Model of Inflation

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We present a model for cosmological inflation based on a no-scale supergravity sector with an $SU(2,1)/SU(2) \times U(1)$ Kähler potential, a single modulus T, and an inflaton superfield Φ described by a Wess-Zumino model with superpotential parameters (μ, λ). When T is fixed, this model yields a scalar spectral index n_s and a tensor-to-scalar ratio r that are compatible with the Planck measurements for values of $\lambda \simeq \mu/3M_P$. For the specific choice $\lambda = \mu/3M_P$, the model is a no-scale supergravity realization of the $R + R^2$ Starobinsky model.

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The initial release of cosmic microwave background data from the Planck satellite [1] confronts theorists of cosmological inflation [2,3] with a challenge. On the one hand, the data have many important features that are predicted qualitatively by the inflationary paradigm. For example, there are no significant signs of non-Gaussian fluctuations or hints of nontrivial topological features such as cosmic strings, and the spectrum of scalar density perturbations exhibits a significant tilt: $n_s \simeq 0.960 \pm 0.007$, as would be expected if the effective scalar energy density decreased gradually during inflation. On the other hand, many previously popular field-theoretical models of inflation are ruled out by a combination of the constraint on n_s and the tensor-to-scalar ratio $r < 0.08$ as now imposed by Planck *et al*: see, e.g., Fig. 1 of [1]. The only model with truly successful predictions displayed in Fig. 1 of [1] is the R^2 inflation model of Starobinsky [4], though similar predictions are made in Higgs inflation [5] and related models [6].

In the following paragraphs we motivate the approach to inflation taken in this Letter, which casts a new light on the Starobinsky model [4] and embeds it in a more general theoretical context that connects with other ideas in particle physics. Specifically, the upper limit on r implies that the energy scale during inflation must be much smaller than the Planck energy, $\sim 10^{19}$ GeV. Such a hierarchy of energy scales can be maintained naturally, without fine-tuning, in a theory with supersymmetry [7]. As is well known, (approximate) supersymmetry has many attractive features, such as providing a natural candidate for dark matter and facilitating grand unification, as well as alleviating the fine-tuning of the electroweak scale. In the context of early-universe cosmology, one must combine supersymmetry with gravity via a suitable supergravity theory [8], which should accommodate an effective inflationary potential that varies slowly over a large range of inflaton field values. This occurs naturally in a particular class of supergravity models [9], which are called "no scale" because the scale at which supersymmetry is broken is undetermined in a first approximation, and the energy scale of the effective potential can be naturally much smaller than $\sim 10^{19}$ GeV, as required by the cosmic microwave background data. No-scale models have the additional attractive feature that they arise in generic four-dimensional reductions of string theory [10], though this does not play an essential role in our analysis. The attractive features of this no-scale supergravity framework for inflation do not depend sensitively on the supersymmetry-breaking scale, which could be anywhere between the experimental lower limit ~ 1 TeV from the LHC [11] and $\sim 10^{10}$ TeV from the tensor-to-scalar ratio.

We now discuss these motivations at greater length before entering into the details of our inflationary model.

Since the energy scale during the inflationary epoch is typically $\ll M_P$, it is natural to study renormalizable models, i.e., some combination of ϕ^2, ϕ^3, and ϕ^4 in the single-field case. In this spirit, it was shown in [12,13] that a single-field model with a potential of the form

$$V = A\phi^2(v - \phi)^2 \qquad (1)$$

could easily produce Planck-compatible values of (n_s, r) for a suitable number of e-folds before the end of inflation $N \sim 50$–60. This simple symmetry-breaking potential has a long pedigree, having been proposed initially in [14] (for a review, see [2]), where it was argued that successful inflation would require a small value of A and $v > M_P$.

As we pointed out in [7], in addition to all the well-known reasons for postulating low-scale supersymmetry, the small values of the quartic and quadratic couplings that would be required in a successful inflationary model, e.g., A in the above example, become technically natural in the presence of low-scale supersymmetry. In particular, small values of $\delta\rho/\rho$ become technically natural if approximate supersymmetry is invoked [7], and if the grand unified theory Higgs boson is distinguished from the singlet field that produces inflation, that later became known as the inflaton [15].

The simplest globally supersymmetric model is the Wess-Zumino model with a single chiral superfield Φ [16], which is characterized by a mass term $\hat{\mu}$ and a trilinear coupling λ, with the superpotential

$$W = \frac{\hat{\mu}}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3. \tag{2}$$

As was discussed in [13], the effective potential of the Wess-Zumino model reduces to (1) when the imaginary part of the scalar component of Φ vanishes, in which case this model yields Planck-compatible inflation for a suitable small value of λ.

However, global symmetry is not enough. As discussed above, in the context of early-universe cosmology one should certainly include gravity and hence construct a locally supersymmetric model, i.e., upgrade to supergravity [8]. The first attempt at constructing an inflationary model in $N = 1$ supergravity proposed a generic form for the superpotential for a single inflaton [17], the simplest form being $W = m^2(1 - a\Phi)^2$ [18]. As discussed in [19], while this relatively simple model is capable of sufficient inflation, it is an example of accidental inflation in the sense that the coefficient of the linear term in the superpotential a must be extremely close to unity. This model has also become one of Planck's casualties. The scalar-to-tensor ratio in this model is very small, but the value of n_s predicted in this model is $n_s \simeq 1–4/N = 0.933$ for $N = 60$ [20], since the effective potential varies insufficiently slowly.

In a supergravity model with a generic Kähler potential for the chiral supermultiplets there are quadratic $|\phi|^2$ terms, which cause variations in the effective potential that destroy its suitability for inflation, an obstacle known as the η problem [3]. As was pointed out in [21], a natural solution to this problem is offered by no-scale supergravity [9], whose motivations were summarized earlier. In such a model, quadratic terms are suppressed, and the effective scalar potential resembles that in a globally supersymmetric model, thanks to an underlying noncompact $SU(N,1)/SU(N) \times U(1)$ symmetry.

Other no-scale supergravity approaches have also been proposed [22], as well as models based on a noncompact Heisenberg symmetry [23], a shift symmetry [24–26], or string theory [27]. The $SU(N,1)$ model [21] was based on the superpotential $W = m^2(\phi - \phi^4/4)$ and gives similar predictions for the inflationary parameters as the minimal $N = 1$ model discussed above. This too is an example of accidental inflation [19], and a small change in the coefficient of the quartic term would lead to parameters consistent with Planck data [1].

In this Letter we show how one can elevate the simplest globally supersymmetric Wess-Zumino inflationary model of [13] to a no-scale supergravity version (NSWZ). Concretely, we study a model in which the inflaton superfield is embedded in an $SU(2,1)/SU(2) \times U(1)$ no-scale supergravity sector together with a modulus field T (which we assume to be fixed by other dynamics [28]) and find a range of the parameters where it is compatible with the Planck data [1]. Quite remarkably, as we show, the NSWZ model is the conformal equivalent of an $R + R^2$ model of gravity for one specific value of $\hat{\mu}/\lambda$, so that in this case our realization of inflation in the NSWZ model is equivalent to the Starobinsky model of inflation [4]. Thus, we embed this model in a broader and attractive theoretical framework.

We first recall the basic relevant formulas governing the kinetic term and the effective potential of scalar fields ϕ in $\mathcal{N} = 1$ supergravity, specializing to the no-scale case with noncompact $SU(N,1)/SU(N) \times U(1)$ symmetry. The scalar sector may be characterized in general by a Hermitian Kähler function K and a holomorphic superpotential W via the combination $G \equiv K + \ln W + \ln W^*$. The kinetic term is then given by $K_i^{j^*}\partial_\mu\phi^i\partial\phi^*_j$, where the Kähler metric $K_i^{j^*} \equiv \partial^2 K/\partial\phi^i\partial\phi^*_j$, and the effective potential is

$$V = e^G\left[\frac{\partial G}{\partial\phi^i}K^i_{j^*}\frac{\partial G}{\partial\phi^*_j} - 3\right], \tag{3}$$

where $K^i_{j^*}$ is the inverse of the Kähler metric $K_i^{j^*}$.

In the minimal no-scale $SU(2,1)/SU(2) \times U(1)$ case, there are two complex scalar fields: T, a modulus field, and ϕ, which we identify as the inflaton field, with the Kähler function $K = -3\ln(T + T^* - |\phi|^2/3)$. In this case, the kinetic terms for the scalar fields T and ϕ become

$$\mathcal{L}_{KE} = (\partial_\mu\phi^*, \partial_\mu T^*)\left(\frac{3}{(T + T^* - |\phi|^2/3)^2}\right) \times \begin{pmatrix} (T + T^*)/3 & -\phi/3 \\ -\phi^*/3 & 1 \end{pmatrix}\begin{pmatrix} \partial^\mu\phi \\ \partial^\mu T \end{pmatrix}, \tag{4}$$

and the effective potential becomes

$$V = \frac{\hat{V}}{(T + T^* - |\phi|^2/3)^2}: \hat{V} \equiv \left|\frac{\partial W}{\partial\phi}\right|^2. \tag{5}$$

In early no-scale models [21,23] it was assumed that K was fixed so that the potential up to a rescaling was simply $\hat{V}$. Here we assume that the T field has a vacuum expectation value (VEV) $2\langle \mathrm{Re}T\rangle = c$ and $\langle \mathrm{Im}T\rangle = 0$ that is determined by nonperturbative high-scale dynamics [28], as in the Kähler correction provided in [29]. In this case, we may neglect the kinetic mixing between the T and ϕ fields in (4), and are left with the following effective Lagrangian for the inflaton field ϕ:

$$\mathcal{L}_{\text{eff}} = \frac{c}{(c - |\phi|^2/3)^2}|\partial_\mu \phi|^2 - \frac{\hat{V}}{(c - |\phi|^2/3)^2}. \quad (6)$$

We assume as in [13] the minimal Wess-Zumino superpotential (2) for the inflaton field.

To better study the potential for the inflaton, we first transform ϕ to the field χ:

$$\phi = \sqrt{3c}\tanh\left(\frac{\chi}{\sqrt{3}}\right). \quad (7)$$

With this field redefinition, the Lagrangian becomes

$$\mathcal{L}_{\text{eff}} = \text{sech}^2[(\chi - \chi^*)/\sqrt{3}]\left[|\partial_\mu \chi|^2 - \left(\frac{3}{c}\right)|\sinh(\chi/\sqrt{3}) \times [\hat{\mu}\cosh(\chi/\sqrt{3}) - \sqrt{3c}\lambda\sinh(\chi/\sqrt{3})]|^2\right]. \quad (8)$$

Clearly the VEV of the T field can be absorbed into the definition of the mass and, writing $\hat{\mu} = \mu\sqrt{c/3}$, the potential becomes

$$V = \mu^2\left|\sinh(\chi/\sqrt{3})\left(\cosh(\chi/\sqrt{3}) - \frac{3\lambda}{\mu}\sinh(\chi/\sqrt{3})\right)\right|^2. \quad (9)$$

Writing χ in terms of its real and imaginary parts: $\chi = (x + iy)/\sqrt{2}$, and, for reasons which will become clear, considering the specific case where the quartic coupling $\lambda = \mu/3$ (in Planck units), we have

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\sec^2(\sqrt{2/3}y)[(\partial_\mu x)^2 + (\partial_\mu y)^2] - \mu^2\frac{e^{-\sqrt{2/3}x}}{2}\sec^2(\sqrt{2/3}y)(\cosh\sqrt{2/3}x - \cos\sqrt{2/3}y). \quad (10)$$

The imaginary part of the inflaton is fixed to $y = 0$ by the potential, having a mass $m_y = \mu/\sqrt{3}$ during inflation when x is large and $m_y = \mu/\sqrt{6}$ at the end of inflation when $x = 0$. Thus we expand the Lagrangian about $y = 0$, in which case we have minimal kinetic terms for x and y, accompanied by derivative interaction terms. The potential for the real part of the inflaton now takes the form

$$V = \mu^2 e^{-\sqrt{2/3}x}\sinh^2(x/\sqrt{6}). \quad (11)$$

This potential is depicted in Fig. 1, where we also display the potential for values of λ slightly perturbed from the nominal value of $\mu/3$.

We use the standard slow-roll expressions for the tensor-to-scalar ratio r and the spectral index n_s for the scalar perturbations in terms of the slow-roll inflation parameters ϵ, η [3], which we evaluate in terms of the canonically normalized field x. In the NSWZ model described above, the VEV of T is absorbed in the definition of the mass parameter μ, which is determined by the normalization of the quadrupole. For the special case $\lambda = \mu/3$, we have

$$A_s = \frac{V}{24\pi^2\epsilon} = \frac{\mu^2}{8\pi^2}\sinh^4(x/\sqrt{6}), \quad (12)$$

implying a value $\mu = 2.2 \times 10^{-5}$ in Planck units for $N = 55$: μ varies between $(1.8\text{–}3.4) \times 10^{-5}$ over the range of NSWZ models considered here. Setting the remaining NSWZ parameter $\lambda = \mu/3$, we have

$$\epsilon = \frac{1}{3}\text{csch}^2(x/\sqrt{6})e^{-\sqrt{2/3}x}, \quad (13)$$

$$\eta = \frac{1}{3}\text{csch}^2(x/\sqrt{6})(2e^{-\sqrt{2/3}x} - 1), \quad (14)$$

which allows us to determine the quantities (n_s, r), once the value of the field x is fixed by requiring $N = 50$–60 e-folds. The nominal choice of $N = 55$ yields $x = 5.35$, $n_s = 0.965$, and $r = 0.0035$.

Figure 2 displays the predictions for (n_s, r) of the NSWZ model for five choices of the coupling λ that yield $n_s \in [0.93, 1.00]$ and $N \in [50, 60]$. The last 50–60 e-folds of inflation arise as x rolls to zero from $\sim$5.1–5.8, the exact value depending on λ and N. As one can see, the values of λ are constrained to be close to the critical value $\mu/3$, for which we find extremely good agreement with the Planck determination of n_s. The values of r are rather small for $\lambda = \mu/3$, varying over the range 0.0012–0.0084, in the models considered.

At first sight, this success might appear to be another example of accidental inflation [19], but, as we now show, this choice of λ has a more profound geometric interpretation. The alert reader may have noticed resemblances of both the potential shown in Fig. 1 and the values of (n_s, r) found for the $\lambda = \mu/3$ model with results for inflation in the $R + R^2$ model proposed by Starobinsky [4]. To further probe this resemblance, we examine the generalization of the Einstein-Hilbert action to contain an R^2 contribution, where R is the scalar curvature,

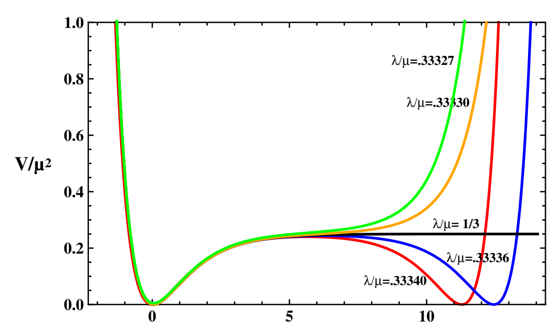

FIG. 1 (color online). The potential V in the NSWZ model for choices of $\lambda \sim \mu/3$ in Planck units, as indicated.

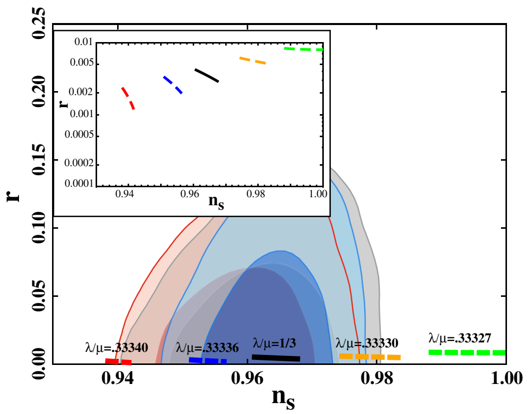

FIG. 2 (color online). Predictions from the NSWZ model for the tilt n_s in the spectral index of scalar perturbations and for the tensor-to-scalar ratio r, compared with the 68% and 95% C.L. regions found in analyses of Planck and other data [1]. In the main panel the lines are labeled by the values of λ/μ (in Planck units) assumed in each case. In the inset, the same cases are shown on a log scale to better display the values of r.

$$S = \frac{1}{2}\int d^4x\sqrt{-g}(R + R^2/6M^2), \qquad (15)$$

where $M \ll M_P$ is some mass scale. This theory is conformally equivalent to canonical gravity plus a scalar field φ [30]. Making the transformation $\tilde{g}_{\mu\nu} = (1 + \varphi/3M^2)g_{\mu\nu}$ and the field redefinition $\varphi' = \sqrt{3/2}\ln[1 + (\varphi/3M^2)]$, we obtain the action

$$S = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\left[\tilde{R} + (\partial_\mu\varphi')^2 - \frac{3}{2}M^2\left(1 - e^{-\sqrt{2/3}\varphi'}\right)^2\right], \qquad (16)$$

corresponding to a potential

$$V = \frac{3}{4}M^2(1 - e^{-\sqrt{2/3}\varphi'})^2. \qquad (17)$$

The potential (17) is identical with the potential (11) along the real direction of the NSWZ model. Moreover, we have the identification $M^2 = \mu^2/3$, which equals $\hat{\mu}^2$ for $c = \langle(T + T^*)\rangle = 1$. Thus the Starobinsky mass M is directly related to the NSWZ mass $\hat{\mu}$ in the superpotential (2). We note that similar potentials are also obtained in Higgs inflation and related models [5].

We have shown in this Letter that the simplest $SU(2,1)/SU(2)\times U(1)$ no-scale supergravity model with a single modulus field T and a single matter field ϕ with the simplest renormalizable Wess-Zumino superpotential, identified with the inflaton, is capable of yielding cosmological inflation with values of the scalar spectral tilt n_s and the tensor-to-scalar ratio r within the region favored by Planck and other data at the 68% C.L. Successful inflation is obtained for $\lambda \simeq \mu/3$ in Planck units. This NSWZ model is a proof of the existence of acceptable models of inflation based on no-scale supergravity, and normally we would not advocate that its details should necessarily be taken literally. For example, a realistic no-scale model derived from a generic compactification of string theory would have more moduli fields, with many matter fields that could be the inflaton, with a superpotential more complicated than assumed here.

However, it is truly striking that the NSWZ model is conformally equivalent to the Starobinsky R^2 model [4] for the specific choice $\lambda = \mu/3$ in Planck units. This correspondence suggests that there is a profound geometric interpretation of this model that remains to be understood.

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