Noise Characteristics of Single-Mode Semiconductor Lasers Under External Light Injection

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Abstract—This paper presents a theoretical investigation of the noise behavior of a semiconductor laser operating under relatively strong light injection. Equations have been presented to describe the noise effect through the calculation of the relative intensity noise as well as the carrier-to-noise ratio available at the receiving end. Illustrative examples are given, showing the impact of the master and slave laser bias currents. Also, the injection-locked and free-running operation regimes have been comparatively analyzed. The results show how the noise characteristics are affected by optical injection and, consequently, how the operating conditions must be chosen to reduce this effect. In particular, it is shown, in agreement with previous works, that the master laser emission noise will essentially take the lead. As a result, to improve the noise behavior by injection locking a solitary laser, the use of a low-noise master laser is required. To make it easy to apply the present results to any laser diode under the stable-locking condition, the necessary relations are explicitly given before specifying the parameters of simulation.

Index Terms—Injection locking, noise, optical communications, semiconductor lasers.

I. INTRODUCTION

INJECTION-LOCKING consists of suitably injecting the light from a CW operated laser (master) into the cavity of another laser (slave) which represents the transmitting source. The effectiveness of this technique has been confirmed previously in a large variety of experiments. External light injection into the cavity of a semiconductor laser affects the emission characteristics. The four-wave mixing regime is well known and may be developed into a technique for optical-frequency conversion [1] or else for an accurate characterization of many intrinsic laser parameters [2]. When a free-running laser is optically injected, locked or unstable regimes may also be observed under different operating conditions. Locking occurs when the optical frequency of the external light is chosen within the so-called locking range [3]. This is the range over which the frequency of the slave laser can be tuned while still locked to the master laser frequency. Once a perfect locking is reached, all power of the slave laser is emitted at the optical frequency of the master laser. If the injection conditions are fixed outside the locking range or if the injected light is not strong enough to effect locking, then the slave laser may become unstable or it may operate in a four-wave mixing regime.

The locking regime of an optically injected semiconductor laser has many advantages in the area of telecommunications.

Locking can be used, for example, to generate narrow spectral width microwave signals [4]. Besides, its feasibility has been proven as a method for eliminating the frequency chirping in semiconductor lasers subject to digital or analog modulation [3], [5]. This is so because the chirped frequency of the slave laser is likely to lock to the constant frequency of the master laser. This feature is advantageous and can be employed in many digital fiber-optic communications systems to combat, for example, the problem of pulse broadening due to chirp in combination with chromatic dispersion.

Additionally, we have recently shown that injection locking may be used to enhance the 3-dB bandwidth [5] and hence to increase significantly the direct-modulation speed of semiconductor lasers. In addition, by suitably choosing the injection parameters, it may be possible to reduce nonlinear distortion in laser diodes used in analog modulation systems [6]. For these systems, however, the laser linearity is not the only figure of merit, even though this characteristic is weighted heavily in deciding the system performance. In addition to performance criteria such as composite-triple-beat (CTB) and composite-second-order (CSO), noise from the laser and receiver may be a severe limiting factor and, hence, is an important parameter. The most serious noise terms that can impair the transmission quality include the receiver shot and thermal noise as well as the laser relative intensity noise (RIN). Since light injection into the laser cavity affects its emission characteristics, inherent noise will also be affected. In this paper, we have developed a model which, under small-signal approximation, allowed us to derive the RIN spectrum. The theoretical treatment is based on the rate equations in which Langevin noise forces are incorporated. Illustrative curves are reported bringing about the influence of the injected light. It is principally demonstrated that inherent RIN can be significantly reduced over a large frequency range, provided that a low-noise master laser is employed. Because the performance assessment of an analog-modulation system is a compromise between nonlinearity and noise, this work completes the distortion analysis carried out in [5] and [6]. Furthermore, this theoretical model is useful for predicting the behavior of optical communications systems which aim to apply the injection-locking technique.

II. THEORY

In a strict sense, the description of laser diodes with spontaneous emission noise inclusion requires a quantum-mechanical formulation of the rate equations, because of the quantum nature of the emission process. Here we use the simpler semiclassical

treatment and incorporate the spontaneous emission noise in the single-mode rate equations as Langevin noise functions, i.e.,

$$\frac{ds}{d\tau} = \gamma \left[n_{\text{th}} g(n)(1 - \varepsilon_n s)s - s + n_{\text{th}}\beta n \right.$$
$$\left. + 2\frac{\tau_p}{T}\left(\frac{s_i}{s}\right)^{1/2}\cos(\Phi_i - \Phi)s \right] + \ell_s \qquad (1)$$

$$\frac{dn}{d\tau} = j - g(n)(1 - \varepsilon_n s)s - n + \ell_n \qquad (2)$$

$$\frac{d\Phi}{d\tau} = \gamma \left\{ -2\pi\tau_p \Delta\nu_0 + \frac{\alpha}{2}\left[n_{th}g(n)(1 - k_n s) - 1 \right] \right.$$
$$\left. + \frac{\tau_p}{T}\left(\frac{s_i}{s}\right)^{1/2}\sin(\Phi_i - \Phi) \right\} + \ell_\Phi \qquad (3)$$

with

$$g(n) = \frac{1}{n_{\text{th}}}\left(n_{\text{th}}n + 1 - n_{\text{th}} \right) \qquad (4)$$

where

s, n	normalized photon and electron densities;
Φ	optical phase;
τ	time normalized to the electron lifetime;
j	ratio between the drive current and the threshold current;
$\Delta\nu_0$	frequency detuning between the slave and master lasers;
Φ_i	master laser optical phase;
s_i	fraction of the master laser emitted power injected in the cavity of the slave laser;
T	cavity round-trip time;
α	linewidth enhancement factor;
$\gamma = \tau_e/\tau_p$	ratio between the electron and photon lifetimes;
n_{th}	normalized carrier density at threshold;
ε_n	normalized gain compression factor;
β	fraction of spontaneous emission coupled to the lasing mode;
$\ell_s, \ell_n, \ell_\Phi$	Langevin noise forces.

The different quantities and variables in (1)–(4) are normalized in accordance with previous works [7].

Equation (3) slightly differs from the corresponding equation in [5] and [6] in the fact that the parameter ε_n has been replaced by the parameter k_n. The term involving k_n describes the refractive index nonlinearities. Since the refractive index is supposed to be affected in a different fashion than photon or electron densities, we believe this new parameter is more general for modeling the chirp. The index nonlinearity originates from the fact that the laser mode does not always coincide with the gain peak. This type of operation is often the case for structures containing a Bragg grating such as distributed feedback (DFB) lasers. More explanation can be found in [8] as well as a mathematical justification of the modified phase rate equation. Always based on the same reference, the parameter k_n can be derived for the situation where the laser operates well below saturation. This assumption is often justified in analog optical fiber communications since, in order to avoid static distortion,

the bias condition of the transmitting source must be chosen in the linear portion of the power–current curve. The calculation of k_n leads to $k_n = d\varepsilon_n/\alpha$ with $d \simeq -\delta\nu_0/\delta\nu_g$, where $\delta\nu_0$ stands for the difference between the lasing and gain-peak frequencies and $\delta\nu_g$ is the gain bandwidth. Referring to [8], it will be noted that, here, d is taken for β, not to be confused with the spontaneous emission factor. It is useful to remark that k_n can be negative or positive, depending on whether the laser operates in the red- or blue-shifted area. For devices emitting at the gain peak, k_n cancels out and can be taken out of the rate equation for the optical phase. In the general case, the damping effects should be included in theory. This was not the case in previous publications such as [9] and [10].

To obtain the noise characteristics, the fluctuations of the involved variables are assumed to remain small at all times in comparison with the respective steady-state average values (small-signal approximation). Under this assumption, if the random deviations are denoted, respectively, by δs, δn, and $\delta\Phi$, the rate equations can be easily linearized and subsequently solved in the frequency domain using Fourier analysis. For the following, the Fourier transforms of these variables will be denoted, respectively, by $\hat{s}_\omega$, $\hat{n}_\omega$, and $\hat{\Phi}_\omega$, while the Fourier transforms of the Langevin noise functions will be denoted by $\hat{\ell}_s$, $\hat{\ell}_n$, and $\hat{\ell}_\Phi$.

Inspection of the above relations shows that the rate equations for the photon density and optical phase explicitly involve the master laser emission parameters (emitted power and phase of the optical field). This implies that, in addition to the intrinsic spontaneous emission noise, the slave laser will be affected by the emission noise of the master laser. Thus, the determination of the spectral characteristics of the master laser must first be undertaken in order to derive those of the slave laser. For simplicity, the master laser is considered to be a one-section single-mode laser described by the same rate equations as the slave laser without optical injection, i.e., $s_i = 0$ in (1) and (3). Then, by following the calculation process explained above, we obtain

$$\begin{bmatrix} \hat{s}_{i\omega} \\ \hat{n}_{i\omega} \end{bmatrix} = \begin{bmatrix} j\omega + m_{11} & m_{12} \\ m_{21} & j\omega + m_{22} \end{bmatrix}^{-1} \begin{bmatrix} \hat{\ell}_{\text{is}} \\ \hat{\ell}_{\text{in}} \end{bmatrix} \qquad (5)$$

$$\hat{\Phi}_{i\omega} = \frac{1}{j\omega}\left(m_{31}\hat{s}_{i\omega} + m_{32}\hat{n}_{i\omega} + \hat{\ell}_{i\Phi} \right) \qquad (6)$$

where the m_{ij}'s are constants depending on the physical parameters and the bias point of the master laser. The m_{ij}'s are expressed in Appendix A.

Hence, the spectral characteristics of the master laser can be determined once the properties of the Langevin noise forces have been determined precisely. It is often admitted that the three noise terms have zero mean and are delta-correlated (Markovian approximation). This leads to the relationship

$$\langle \hat{\ell}_{iq}(\omega)\hat{\ell}_{ir}^*(\omega')\rangle = 2d_{qr}^i\delta(\omega - \omega'), \qquad q, r \in \{s, n, \Phi\} \qquad (7)$$

where the star symbol stands for the complex conjugate and the d_{qr}'s are the normalized diffusion coefficients associated with the noise forces. Because these coefficients are constant numbers (frequency-independent), (7) shows that the Langevin

noise sources have a white spectrum. The d_{qr}^i's can be expressed by the following formulas deduced from the results of [11] given in terms of number of photons and electrons:

$$d_{ss}^i = \tau_{ie}\beta_i\gamma_i n_{\text{th}}^i n_{i0} s_{i0},$$
$$d_{\Phi\Phi}^i = \frac{1}{4}\tau_{ie}\beta_i\gamma_i n_{\text{th}}^i \frac{n_{i0}}{s_{i0}}$$
$$d_{s\Phi}^i = 0 \tag{8}$$

$$d_{nn}^i = \Gamma_i\tau_{ip} n_{\text{th}}^i n_{i0}(\Gamma_i\beta_i s_{i0} + g_{i0}\tau_{ie}),$$
$$d_{sn}^i = -\tau_{ie}\Gamma_i\beta_i n_{\text{th}}^i n_{i0} s_{i0}$$
$$d_{n\Phi}^i = 0. \tag{9}$$

where Γ_i is the optical confinement factor and g_{i0} is the gain slope (in s^{-1}). The subscript "i" in (5)–(9) means these equations describe the master laser. However, although these relations are established for the master laser, they must be viewed as being general, also allowing one to describe the slave laser without optical injection. In the latter case, (8) and (9) are used further without the "i" symbol in order to differentiate the slave laser from the master one.

The Fourier transform of (1)–(3), after linearization, yields

$$\begin{bmatrix} \hat{s}_\omega \\ \hat{n}_\omega \\ \hat{\phi}_\omega \end{bmatrix} = \begin{bmatrix} j\omega + a_{11} & a_{12} & a_{13} \\ a_{21} & j\omega + a_{22} & a_{23} \\ a_{31} & a_{32} & j\omega + a_{33} \end{bmatrix}^{-1} \begin{bmatrix} \hat{h}_s(\omega) \\ \hat{h}_n(\omega) \\ \hat{h}_\Phi(\omega) \end{bmatrix} \tag{10}$$

where $\hat{\phi}_\omega = -2s_0\hat{\Phi}_\omega$. The a_{ij}'s are constant numbers depending on the physical parameters and the bias point of the laser and are expressed in Appendix B. The driving terms in (10) are given by the following expressions:

$$\hat{h}_s(\omega) = \hat{\ell}_s + s_0 a_{33}\frac{\hat{s}_{i\omega}}{s_{i0}} - 2s_0 a_{13}\hat{\Phi}_{i\omega} \tag{11}$$

$$\hat{h}_n(\omega) = \hat{\ell}_n \tag{12}$$

$$\hat{h}_\Phi(\omega) = -2s_0\left(\hat{\ell}_\Phi + \frac{a_{13}}{2}\frac{\hat{s}_{i\omega}}{s_{i0}} + a_{33}\hat{\Phi}_{i\omega}\right). \tag{13}$$

For simplicity, we will assume that the Langevin noise sources of the slave laser are independent from those of the master laser. In other words, the two sets of noise sources are assumed to be uncorrelated. It follows that relations can be established for the slave laser, similar to (7)–(9), as

$$\langle \hat{h}_q(\omega)\hat{h}_r^*(\omega)\rangle = 2e_{qr}(\omega) \tag{14}$$

where

$$e_{ss}(\omega) = d_{ss} + \frac{1}{2}\left(\frac{s_0 a_{33}}{s_{i0}}\right)^2\langle|\hat{s}_{i\omega}|^2\rangle + 2s_0^2 a_{13}^2\langle|\hat{\Phi}_{i\omega}|^2\rangle$$
$$-2\frac{s_0^2}{s_{i0}}a_{33}a_{13}\mathcal{R}_e\left[\langle\hat{s}_{i\omega}\hat{\Phi}_{i\omega}^*\rangle\right] \tag{15}$$

$$e_{nn}(\omega) = d_{nn} \tag{16}$$

$$e_{\Phi\Phi}(\omega) = 2s_0^2\left\{2d_{\Phi\Phi} + \frac{1}{4}\left(\frac{a_{13}}{s_{i0}}\right)^2\langle|\hat{s}_{i\omega}|^2\rangle\right.$$
$$\left. + a_{33}^2\langle|\hat{\Phi}_{i\omega}|^2\rangle + \frac{a_{13}a_{33}}{s_{i0}}\mathcal{R}_e\left[\langle\hat{s}_{i\omega}\hat{\Phi}_{i\omega}^*\rangle\right]\right\} \tag{17}$$

$$e_{sn}(\omega) = d_{sn} \tag{18}$$

$$e_{s\Phi}(\omega) = s_0^2\left(-\frac{a_{13}a_{33}}{2s_{i0}^2}\langle|\hat{s}_{i\omega}|^2\rangle + 2a_{13}a_{33}\langle|\hat{\Phi}_{i\omega}|^2\rangle\right.$$
$$\left. - \frac{a_{33}^2}{s_{i0}}\langle\hat{s}_{i\omega}\hat{\Phi}_{i\omega}^*\rangle + \frac{a_{13}^2}{s_{i0}}\langle\hat{s}_{i\omega}^*\hat{\Phi}_{i\omega}\rangle\right) \tag{19}$$

$$e_{n\Phi}(\omega) = 0 \tag{20}$$

in which $\mathcal{R}_e$ denotes the real part.

The RIN can be easily derived from (10)–(20) using the definition

$$\text{RIN}(\omega) = \frac{\langle|\hat{s}_\omega|^2\rangle}{s_0^2}. \tag{21}$$

By using (21), the RIN of the free-running laser ($s_i/s_0 = 0$) can be analytically derived and written as

$$\text{RIN}(\omega) = \frac{2}{\gamma^2 s_0^4 g^2(\omega)}\left[\left(m_{22}^2 + \omega^2\right)d_{ss}\right.$$
$$\left. + m_{12}^2 d_{nn} - 2m_{12}m_{22}d_{sn}\right] \tag{22}$$

with

$$g(\omega) = \left[\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + 4\eta^2\frac{\omega^2}{\omega_0^2}\right]^{1/2} \tag{23}$$

where $\omega_0 = (\gamma s_0)^{1/2}$ and $\eta = (1 + s_0 + \gamma\varepsilon_n s_0 + \beta\gamma n_{\text{th}}n_0/s_0)/(2\omega_0)$. It should be mentioned that $g(\omega)$ is the same function as expressed in [7]. One must, however, notice the η parameter that can be deduced from [7] does not include the term involving the spontaneous emission factor. Very often, β can be neglected in η, but here we keep it in case its effect becomes important. Hence, the inverse function of $g(\omega)$ corresponds to the transfer function of the free-running laser to within a multiplication constant. Therefore, observation of relation (22) shows that the RIN will exhibit a resonance phenomenon at the resonant frequency of the laser diode. This will be more easily seen by reference to illustrative curves.

It should be noted that if the RIN is the major inherent noise term generally considered, in the literature, nonlinear distortion has been sometimes treated as another intrinsic noise term, as suggested in [12] and [13]. We believe that such an approach is a surplus of requirements and is, therefore, inappropriate. In multichannel systems indeed, the CSO and CTB are widely accepted measures of nonlinear distortion effect on a given channel, and these parameters are treated separately from noise.

The meaningful quantity commonly used to characterize the total noise effect in an actual system is the carrier-to-noise ratio (CNR). The CNR is defined as the power of the carrier signal to the total noise power. The receiver contribution on CNR largely

depends on the particular receiver used. For a direct-detection system employing a PIN-AMP receiver, it is often admitted that the thermal noise of the preamplifier and the shot noise of the photodiode are the main noise terms that limit the CNR. The thermal noise can, however, be alleviated by using APD's in place of PIN-AMP's, taking advantage of their internal gain. But, on the premise that the optical receiver must remain cost-effective in order to minimize the subscriber's equipment cost, the PIN-based receiver must be considered. Therefore, the CNR at the receiver site for a single channel occupying bandwidth B can be expressed (in decibels) by the standard equation

$$\mathrm{CNR} = 10 \log \left\{ \frac{\mathrm{OMD}^2 (r P_r)^2}{2\mathrm{B} \left[\mathrm{RIN}(f_c)(r P_r)^2 + 2 e r P_r + \langle I_a^2 \rangle \right]} \right\} \quad (24)$$

where

OMD   optical modulation depth;
r quantum efficiency of the photodiode;
P_r average optical power received;
e absolute value of the electron charge;
I_a electronic preamplifier thermal noise current (typically 15 pA).

Formula (24) implies that the RIN value at the channel central frequency f_c is taken for the entire band. This is a realistic assumption since the signal bandwidth is generally narrow.

III. RESULTS AND DISCUSSION

Illustrative examples are given in this section. The computer simulation is carried out using $\tau_e = 1$ ns, $\tau_p = 2$ ps, $g_0 = 2.222 \times 10^{-5}$ GHz, $\Gamma = 0.3$, $T = 7$ ps, $\beta = 10^{-4}$, $\varepsilon_n = 0.01$, $n_{\mathrm{th}} = 1.9$, and $\alpha = 6$. For simplicity, the physical parameters of the master and slave lasers are supposed to be identical. The injection-locked regime is illustrated for three injection levels of -26, -20, and -14 dB, corresponding, respectively, to dynamically stable locking ranges of $-30.9/-12.5$, $-40.7/-12.4$, and $-61.8/-11.1$ GHz. These numbers result from computation using a slave laser bias current equal to two times the threshold current. The exact values of the injection parameters for simulation are chosen within the above ranges and are indicated in the captions.

Fig. 1 plots the free-running RIN as decibels in function of frequency, in log-coordinate. The parameter of the curves is the bias current of the laser. At a given bias current, the noise exhibits a constant behavior and is relatively low in the low frequency side. For lasers biased at two times or more above threshold, the RIN will fall below -150 dB/Hz for frequencies less than 0.3 GHz. It further rises gradually above 0.1 GHz and peaks at the resonant frequency of the transfer function. When the bias current is increased, the RIN decreases at a given frequency. These conclusions have been qualitatively shown in some previous work in which the optical power was taken as a parameter [11].

The first results concerning the injection-locked regime are given in Fig. 2 showing the impact of the master laser bias cur-
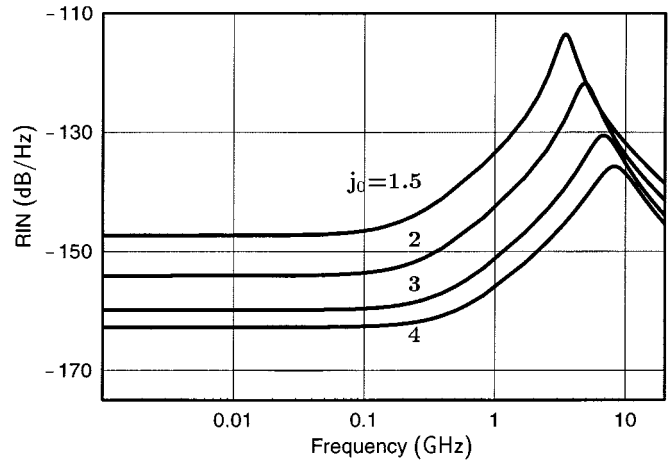

Fig. 1. RIN of the free-running laser as a function of frequency for several bias conditions (j_0 stands for the ratio between the slave laser bias and threshold currents).
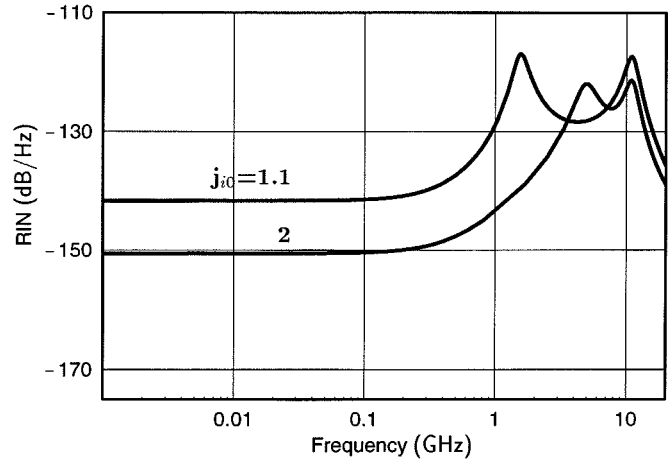

Fig. 2. Influence of the master laser bias current on the RIN with the injection parameters $s_i/s_0 = -20$ dB and $\Delta\nu_0 = -20$ GHz. The slave laser is considered to be biased at two times above threshold, and j_{i0} stands for the ratio between the master laser bias and threshold currents.

rent upon RIN. These examples are obtained using the bias condition $j_0 = 2$ for the slave laser and the injection parameters $s_i/s_0 = -20$ dB, $\Delta\nu_0 = -20$ GHz. The characteristics exhibit qualitatively similar shapes as in the case of a free-running operation regime. The difference is that, in addition to the maximum appearing at the resonant frequency of the injection-locked slave laser, another peak appears at the resonant frequency of the master laser. This result has also been reached in [10] and can be understood in connection with the fact that the noise characteristics of the master laser are included in the determination of the slave laser noise in the stable-locking condition. An interesting feature observed in Fig. 2 is the important decrease of RIN with an increase of the master laser bias current. This result indicates that, practically, the master laser will provide the main contribution to the slave laser noise. This view is supported by Fig. 3 which shows the modeled RIN, comparing the general result [Fig. 3(a)] with two cases where the Langevin noise forces of the slave laser are neglected [Fig. 3(b)] and the
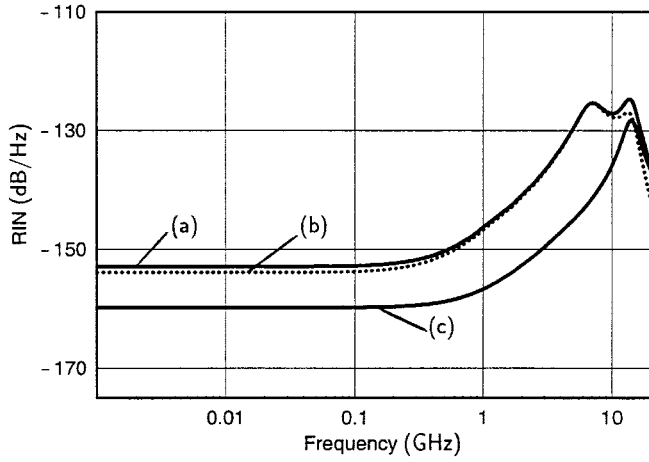
Fig. 3.   RIN as a function of frequency with the bias conditions $j_0 = 2$ and $j_{i0} = 3$ and the injection parameters $s_i/s_0 = -14$ dB and $\Delta\nu_0 - 20$ GHz. The curves correspond to three different situations: (a) general case simulation, (b) the Langevin noise forces of the slave laser are neglected, and (c) the master laser emission fluctuations are neglected.

Fig. 5.   Influence of the master laser spontaneous emission factor on the RIN with the bias and injection-locking conditions $j_0 = 2$, $j_{i0} = 5$, $s_i/s_0 = -26$ dB, and $\Delta\nu_0 = -20$ GHz. Solid line: injection-locked regime; dotted line: solitary laser.
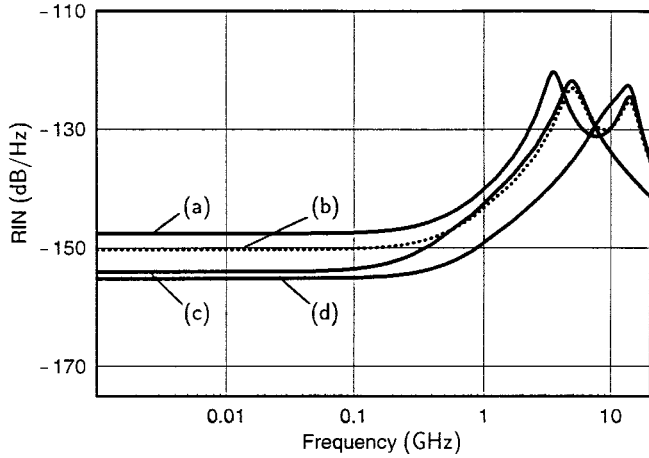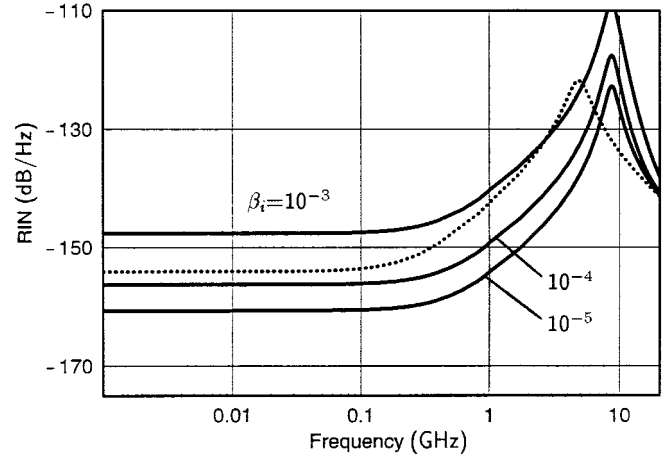

Fig. 4.   Influence of the master laser bias current on the RIN under the bias condition $j_0 = 2$ and the same injection parameters as for Fig. 3. The curves correspond to four different cases: (a) $j_{i0} = 1.5$, (b) $j_{i0} = 2$, (c) free-running laser, and (d) $j_{i0} = 5$.

master laser fluctuations are ignored [Fig. 3(c)]. The curves were obtained for the injection parameters $s_i/s_0 = -14$ dB and $\Delta\nu_0 = -20$ GHz and bias conditions $j_{i0} = 2$ and $j_{i0} = 3$. It is clearly seen that the influence of the slave laser emission noise on RIN becomes negligible. In other words, the terms d_{qr} [$q, r \in \{s, n, \Phi\}$] are negligible in (15)–(19). This result, in a way, is consistent with the prior works of Gallion *et al.* who have shown that the injection-locked slave laser linewidth turns to be locked on the master laser linewidth for any output power [14], [15]. As a consequence, to reduce the noise effect using the injection-locking technique, a low-noise master laser is required.

Fig. 4 shows the spectral characteristics of the RIN for three bias conditions of the master laser, keeping the same injection parameters as in Fig. 3. The free-running laser RIN is also given in the same figure for comparison. For $j_{i0} = 1.5$, the injection-locked RIN shows higher levels. When $j_{i0} = 2$ (the same

bias condition as the slave laser), the two results nearly coincide over frequencies extending from 0.5 GHz to the resonant frequency of the free-running laser. The injection-locked RIN remains, nevertheless, slightly higher at low frequencies. On the other hand, the RIN is reduced all along the plot range in the case $j_{i0} = 5$. At low frequencies, the reduction is small, but it is seen to increase at frequencies ranging from 1 GHz to the resonant frequency of the solitary operation. A comparison between Fig. 4(c) and (d) gives a reduction in RIN of more than 5 dB/Hz within the 1–5-GHz band. This once more illustrates the important contribution of the master laser to RIN. Since noise originates mostly from the spontaneous emission, it is of interest to know how the master laser spontaneous emission factor impacts the characteristics. This feature is presented in Fig. 5 for three values of β_i. Inspection of the curves corresponding to $\beta_i = 10^{-3}$ and $\beta_i = 10^{-4}$ shows a reduced RIN by about 10 dB/Hz all along the plot range. When β_i is decreased from 10^{-4} to 10^{-5}, the reduction in RIN is only about 5 dB/Hz. This behavior could have been predicted from the set of relations (8) and (9). For relatively high values of β_i, the second term of the d_{nn}^i expression is negligible. Likewise, (8) and (9) show that RIN is related proportionally to β_i. On the other hand, for small values of the spontaneous emission factor, the second term in the d_{nn}^i expression takes the lead and the RIN–β_i relation is no longer proportional. More quantitatively, the comparison of the injection-locked RIN in the case of $\beta_i = 10^{-5}$ with the free-running RIN shows a reduction of more than 10 dB/Hz over 1–5 GHz. The reduction in RIN attains about 15 dB/Hz near resonance.

The impact of injection locking on RIN is of great interest. This feature is displayed in Figs. 6 and 7. These simulations were carried out for the slave laser biased at two times above its threshold for oscillation ($j_0 = 2$). On the other hand, the value of the spontaneous emission factor of the master laser is deliberately shifted from 10^{-4} to 10^{-5} in order to obtain a sufficient difference between its noise behavior and that of the slave laser (in the free-running state). Curves in Fig. 6 are plotted for a constant detuning of -20 GHz and a varying injection level
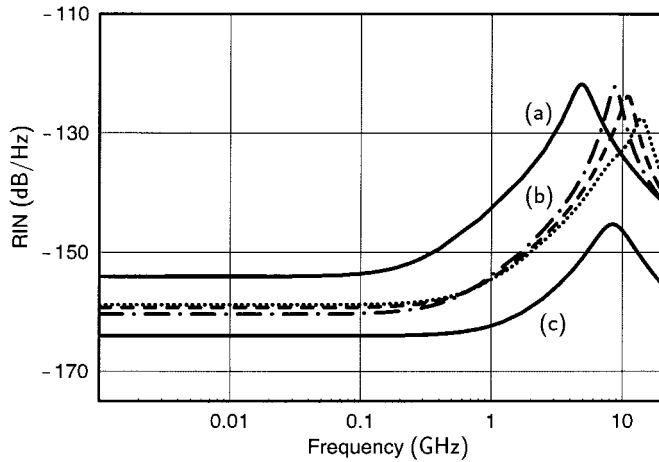
Fig. 6.   Influence of the injection level on the RIN for $j_0 = 2$, $j_{i0} = 4$, and $\beta_i = 10^{-5}$. (a) Slave laser in the free-running regime. (b) Slave laser in the locking condition $\Delta\nu_0 = -20$ GHz: $s_i/s_0 = -26$ dB, $s_i/s_0 = -20$ dB, and $s_i/s_0 = -14$ dB. (c) Master laser (in a free-running state).

Fig. 8.   CNR as a function of the averaged received optical power for a channel centered at 4 GHz. A comparison between the free-running and the injection-locking regimes. The bias and injection-locking conditions are the same as in Fig. 5. Injection-locked regime for $\beta_i = 10^{-5}$; injection-locked regime for $\beta_i = 10^{-4}$; free-running regime.
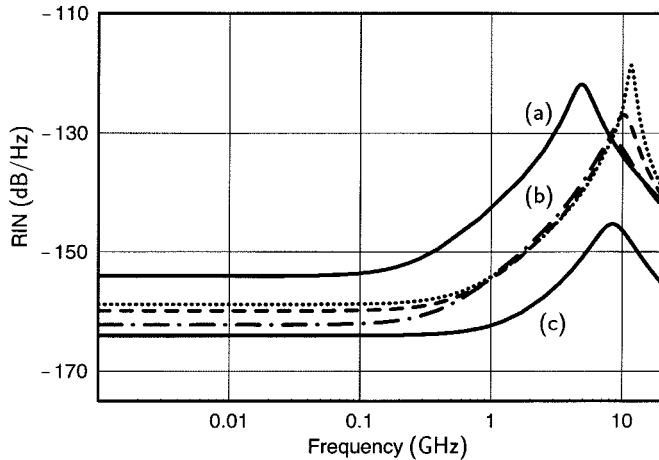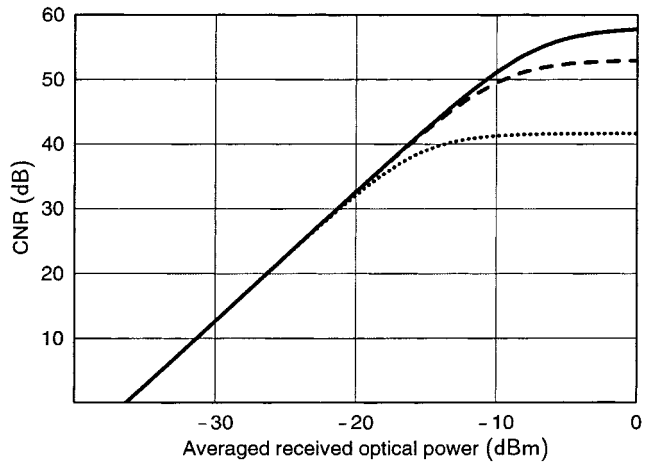

Fig. 7.   (a) Influence of frequency detuning on RIN for $j_0 = 2$, $j_{i0} = 4$, and $\beta_i = 10^{-5}$. (b) Slave laser in the locking condition $s_i/s_0 = -20$ dB. Dot–dashed line: $\Delta\nu_0 = -35$ GHz; dashed line: $\Delta\nu_0 = -25$ GHz; dotted line: $\Delta\nu_0 = -15$ GHz. (a) and (c) have the same significance as in Fig. 6.

of −26, −20, and −14 dB while Fig. 7 corresponds to a fixed injection level of −20 dB and a varying detuning of −35, −25, and −15 GHz. It is observed that, in the locking condition, the peak height of the RIN decreases with higher injection and increases with higher detuning. This is the same effect as on the frequency response [6]. It can be seen that, in the low-frequency side, the injection-locked RIN exhibits approximately a constant level, whatever the injection parameters (injection level and frequency detuning). But, in contrast to the general view of [10], the constant value does not necessarily correspond to the master laser RIN. In the high-frequency side, Figs. 6 and 7 show that a certain reduction in RIN with higher injection levels and less negative detunings can be expected. The amount of reduction that can be obtained with a varied injection level is found to be much less than theoretically predicted in [16]. Our results rather tend to confirm the general discussion of [16] which suggests that the noninclusion of the noise term from the master laser

in the theoretical model may mostly explain the large discrepancy between the computation results and the measurements. It should be mentioned, however, that the noise of the slave laser in the locking state may happen to be accurately described without including the noise from the master laser. Owing to the large number of parameters involved, these types of situations are not easy to determine. We believe that the present work reports a more complete description of the slave laser in the locking condition. The importance of the master laser noise can be best understood by returning back to Fig. 2: if account was not taken of this noise term, a single peak would have been observed instead of two. It can be noticed that the linewidth enhancement factor greatly affects the phase noise and therefore may play a critical role in enhancing the effect of the master laser noise. We have indeed verified that the phase noise of the master laser is the most dominant contributor to the noise level of the slave laser in the locking state. This noise component even seems to continue to have a significant influence under weak injection. This was previously confirmed by measurements [15]. Also, let us notice that the phase noise from the master laser does produce a singularity at $\omega = 0$ in the noise power of the slave laser, a singularity which is not offset by a compensating ω in the numerator. We are still investigating these aspects for a better understanding.

The comparison is of practical interest between individual RIN's and that obtained when the free-running laser is shifted to injection-locking. Inspection of Figs. 6 and 7 shows that, even though the master laser noise takes the lead, the level of RIN in the locking condition is higher than that of the master laser itself, but lower than that of the free-running slave laser over the frequencies of interest. As indicated above, however, these results correspond to the situation where the master laser presents less noise than the slave one, as will most likely be the case in practice.

We have reported the CNR in Fig. 8 for a channel centered at 4 GHz as a function of the averaged received optical power. The figure provides comparison between the free-running laser and the same laser operating under injection-locking with

$s_i/s_0 = -20$ dB and $\Delta\nu_0 = -20$ GHz. The curves correspond to the same optical modulation depth of 10%. The results demonstrate first that the CNR initially increases linearly with the input optical power before settling at a saturation value from a certain level of power. Equation (24) shows that the linear portion of the characteristics corresponds to thermal noise domination whereas the high-power portion mostly gives the RIN contribution. Accordingly, Fig. 8 shows that the CNR can be increased by means of optical injection only at high received powers. At 0 dBm, for example, the CNR is observed to increase by about 10 dB for $\beta_i = 10^{-4}$ and more than 15 dB for $\beta_i = 10^{-5}$.

It is worth mentioning that, since many users are often served by the same transmitter, the received power at a given subscriber's facility is generally weak. This means that the receiver noise will contribute the most to CNR. This is the case with FM-based systems because of the low values of CNR (less than 17 dB) required. The present analysis shows that no benefit can be expected regarding the noise performance using optical injection. On the other hand, for AM-based systems such as video, a CNR of 50 dB or more has to be virtually provided to achieve an ideal picture quality. Because in AM-based SCM systems no improvement factor is obtained during the demodulation process, the received power must be several orders of magnitude greater than that of FM systems to reach the above requirement. This problem can be solved by reducing the optical fiber length or by inserting optical amplifiers if care is taken to remain within a safe power level. In this case, the implementation of injection-locking techniques on the transmitter side can be advantageous to boost the CNR.

Finally, let us notice that the noise performance of a system can significantly degrade in the presence of back-reflected signals into the master and slave lasers. Potential sources of reflections include all device facets, fiber ends, and Rayleigh backscattering within the fiber. In actual applications, optical isolators should be equipped to protect the lasers against large feedback effects. Optical reflections may also induce interferometers within the optical fiber that interact with the laser chirp to convert FM into IM, hence causing an extra intensity noise. This has been investigated previously for digital [17] as well as for analog modulation systems [18], [19]. Even though overcoming reflections due to Rayleigh backscattering is not easy, feedback from fiber discontinuities (connectors, splices) can be minimized if tilts and antireflection coatings are realized.

IV. CONCLUSIONS

We have theoretically investigated the transmitted intensity fluctuation noise of a semiconductor under strong optical injection. This analysis is based on the rate equations in which Langevin noise forces are incorporated. Although a laser subject to optical injection may exhibit a variety of instable regimes [20], we have chosen all the parameter values within the dynamically stable locking range in which the envisaged applications are intended to operate. The noise effect has been analyzed through the study of both the RIN and the CNR. The RIN is seen to exhibit higher levels at the resonanc and is reduced with increased bias current. A comparison is given between the

free-running and injection-locked regimes. The most important results can be summarized as follows.

1) In the low-frequency side, the injection-locked RIN exhibits approximately a constant level, regardless of the injection level and frequency detuning.
2) In the relaxation resonance region, a certain amount of reduction in RIN can be expected within the locking range for higher injection levels and less negative detunings.
3) On the assumption that the master laser presents less noise than the slave laser in the free-running condition, the injection-locked RIN is higher than that of the master laser but much less than that of the free-running slave laser over the frequency range of interest.
4) The master laser, to a large extent, determines the noise characteristics in the locking regime. This result, in some way, is in line with the phase noise analysis presented in [21]. Obviously, the noise of the master laser cannot systematically be ignored in the noise model of the slave laser subject to optical injection. The noise theory of the slave laser may sometimes be fairly accurate without including the noise from the master laser, but, owing to the large number of parameters, these cases are difficult to identify. Accordingly, the exclusion of the master laser noise from the analysis presented in [16] is not necessarily the main reason behind the discrepancy between computation results and measured data. However, as we saw here, it would be unsafe to ignore the contribution of the master laser. Thus, we believe that the present work constitutes an improvement to work previously carried out in [16] and [22].

The present analysis predicts the injection-locking technique to improve the noise performance of SCM systems operating near resonance, provided that a low-noise master laser is employed. Since narrow linewidth means low noise, a large variety of laser diodes meet this condition and can be foreseen to be suitable as master lasers. These devices include DFB or DBR lasers, external cavity lasers, or quantum-well structures. Furthermore, the wavelength tunability of these devices is of interest if the study of the frequency detuning influence on the slave laser characteristics has to be realized.

Finally, it is worth mentioning that, because the master laser to a large extent determines the noise, external feedback from the slave laser or from an inserted optical amplifier (if there is any) into its cavity, even minor, can have a nonnegligible influence. Even if this phenomenon can be minimized using an optical isolation, a detailed study is required to know how perfect the isolator must be. This aspect of the investigation is in progress.

Remark: The above analysis has been performed using negative frequency detunings for which the conditions for locking are fulfilled. However, a semiconductor laser subject to light injection may show different types of behaviors, as indicated earlier. A four-wave mixing regime [1], [2] occurs when the frequency detuning between the interacting lasers is chosen far enough from the locking range. On the other hand, unstable operation can be observed for positive detunings chosen sufficiently close to the locking range. This behavior is still not fully

understood. In practice, the slave laser can lock to the reference laser, but the locking state does generally switch off from time to time, hence the term "unstable." Recent works have shown that, if the level of injection is strong enough, locking will not take place at all, but the slave laser will exhibit self-sustained oscillations with a modulation index close to unity [23]. This operation was additionally found to be accompanied by an extremely low phase noise and should therefore be expected to present a low RIN. However, the potential areas of applications of this finding will differ from those considered here. Because the slave laser is no longer continuously operating, it can hardly be used for transmitting direct modulation signals. A possible and interesting application of the sinusoidal output oscillations is certainly the generation of spectrally pure microwave signals for wireless communication systems [24].

APPENDIX A

The elements m_{ij} in (5) and (6) are given by

$$m_{11} = -\gamma_i \left[(n_{th}^i n_{i0} + 1 - n_{\text{th}}^i)(1 - 2\varepsilon_{in} s_{i0}) - 1 \right]$$

$$m_{12} = -\gamma_i n_{\text{th}}^i \left[s_{i0}(1 - \varepsilon_{in} s_{i0}) + \beta_i \right]$$

$$m_{21} = \frac{1}{n_{\text{th}}^i}(n_{\text{th}}^i n_{i0} + 1 - n_{\text{th}}^i)(1 - 2\varepsilon_{in} s_{i0})$$

$$m_{22} = 1 + s_{i0}(1 - \varepsilon_{in} s_{i0})$$

$$m_{31} = -\frac{\alpha_i}{2}\varepsilon_{in}\gamma_i (n_{th}^i n_{i0} + 1 - n_{\text{th}}^i)$$

$$m_{32} = \frac{\alpha_i}{2} n_{\text{th}}^i \gamma_i (1 - \varepsilon_{in} s_{i0})$$

where s_{i0} and n_{i0} denote the steady-state average values of the normalized photon and electron densities inside the master laser cavity.

APPENDIX B

The elements a_{ij} in (10) are given by

$$a_{11} = -\gamma \left[(n_{\text{th}} n_0 + 1 - n_{\text{th}})(1 - 2\varepsilon_n s_0) - 1 + \rho_0 \tau_p \cos \theta_0 \right]$$

$$a_{12} = -\gamma n_{\text{th}} \left[s_0(1 - \varepsilon_n s_0) + \beta \right]$$

$$a_{13} = \gamma \rho_0 \tau_p \sin \theta_0$$

$$a_{21} = \frac{1}{n_{\text{th}}}(n_{\text{th}} n_0 + 1 - n_{\text{th}})(1 - 2\varepsilon_n s_0)$$

$$a_{22} = 1 + s_0(1 - \varepsilon_n s_0)$$

$$a_{23} = 0$$

$$a_{31} = -\gamma \left[\varepsilon_n \alpha s_0(n_{\text{th}} n_0 + 1 - n_{\text{th}}) + \rho_0 \tau_p \sin \theta_0 \right]$$

$$a_{32} = \gamma \alpha n_{\text{th}} s_0 (1 - \varepsilon_n s_0)$$

$$a_{33} = \gamma \rho_0 \tau_p \cos \theta_0$$

where s_0 and n_0 denote the steady-state average values of the normalized photon and electron densities inside the slave laser cavity, respectively. The other parameters are given by $\theta_0 = \Phi_{i0} - \Phi_0$ and $\rho_0 = T^{-1}(s_i/s_0)^{1/2}$, the ratio s_i/s_0 being defined as the injection level.

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