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Reference

LEVKIVSKYI, Ivan, SUKHORUKOV, Eugene. Noise-Induced Phase Transition in the Electronic Mach-Zehnder Interferometer. *Physical review letters*, 2009, vol. 103, no. 3

DOI : 10.1103/PhysRevLett.103.036801

Noise-Induced Phase Transition in the Electronic Mach-Zehnder Interferometer

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(Received 19 March 2009; published 13 July 2009)

We consider dephasing in the electronic Mach-Zehnder interferometer strongly coupled to current noise created by a voltage biased quantum point contact (QPC). We find the visibility of Aharonov-Bohm oscillations as a function of voltage bias and express it via the cumulant generating function of noise. In the large-bias regime, high-order cumulants of current add up to cancel the dilution effect of a QPC. This leads to an abrupt change in the dependence of the visibility on voltage bias which occurs at the QPC's transparency $T = 1/2$. Quantum fluctuations in the vicinity of this point smear out the sharp transition.

DOI: 10.1103/PhysRevLett.103.036801 PACS numbers: 73.23.−b, 03.65.Yz, 85.35.Ds

The effective theory of quantum Hall (QH) edge states [1] suggests that at integer filling factors the low-energy edge excitations are free chiral electrons. If this were the case, it would imply that edge excitations remain coherent at long distances, and would call for various quantum information applications. Results of tunneling spectroscopy experiments [2] reasonably agree with the free-electron description of edge states. In contrast, the first experiment on Aharonov-Bohm (AB) oscillations of a charge current in the electronic Mach-Zehnder (MZ) interferometer [3] has shown that the phase coherence is strongly suppressed at energies, which are inverse proportional to the interferometer's size. Moreover, subsequent experiments [4–7] have found that the visibility of AB oscillations as a function of voltage bias applied to the interferometer shows unusual lobe-type behavior, suggesting that a strong Coulomb interaction might be responsible for dephasing of edge electrons.

Early attempts to explain the unusual AB effect in MZ interferometers have focused on the filling factor $\nu = 1$ state, and suggested different mechanisms of dephasing, including the resonant interaction with a counterpropagating edge state [8], the dispersion of the Coulomb interaction potential [9], and non-Gaussian noise effects [10,11]. To date, however, all the experiments, reporting multiple side lobes in the visibility function of voltage bias, have been done at filling factor $\nu = 2$. In one of our previous works [12], we have shown that in this case the long-range Coulomb interaction splits the spectrum of collective charge excitations at the QH edge (plasmons) in two modes: a fast charge mode and a slow dipole mode. At low energies, only slow mode is excited at the first quantum point contact (QPC). It carries away the electron phase information, but may be absorbed at the second QPC. This process partially restores the phase coherence at specific values of voltage bias, and generates multiple lobes in the visibility. At the same time, thanks to the chirality of edge states, the electron transport through a single QPC is not affected by interaction.

Importantly, the experiments [4–7] can be roughly grouped into two categories according to whether dephasing in MZ interferometers is caused by spontaneous emission of plasmons, addressed earlier in Refs. [8,9,12], or it is induced by external noise sources. In the present Letter, we consider the second group of experiments, where electrons are injected into a MZ interferometer via an additional QPC, as shown in Fig. 1. Apart from diluting the incoming electron channel, this additional QPC generates a partition noise [13]. The MZ interferometer turns out to be strongly coupled to this noise, so that non-Gaussian effects, characterized by irreducible moments (cumulants) of the current noise, become important. We express the visibility of AB oscillations in the differential conductance in terms of the cumulant generating function, and find that in the limit of large voltage bias, all the current cumulants add up to cancel the dilution effect of an additional QPC. We predict that this leads to a phase transition at the QPC's transparency $T = 1/2$, where the visibility function of voltage bias abruptly changes its behavior.

Electronic Mach-Zehnder interferometer.—The model of a MZ interferometer, introduced earlier in

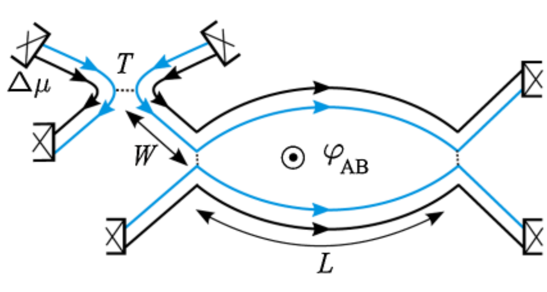

FIG. 1 (color online). Schematic of the electronic MZ interferometer. Two chiral channels are formed at the edge of a quantum Hall liquid at filling factor $\nu = 2$. Outer channels (shown by blue or gray lines) are mixed at two QPCs and form an Aharonov-Bohm loop. Electrons are injected into the interferometer through an additional voltage biased QPC, which is placed at the distance W from the interferometer and has transparency T.

Refs. [8,12], is discussed here only briefly. We note that experimentally relevant energy scales are very small [4–7]. Therefore, it is appropriate to use an effective theory [14] describing edge states at filling factor $\nu = 2$ as collective fluctuations of the charge density $\rho_{s\alpha}(x)$, where $\alpha = 1, 2$ enumerates channels at the QH edge, and $s = U, D$ enumerates arms of the interferometer. The charge density fields are expressed in terms of chiral boson fields, $\phi_{s\alpha}(x)$, satisfying the commutation relations

$$[\phi_{s\alpha}(x), \phi_{k\beta}(y)] = i\pi\delta_{sk}\delta_{\alpha\beta}\mathrm{sgn}(x-y), \tag{1}$$

namely, $\rho_{s\alpha}(x) = (1/2\pi)\partial_x\phi_{s\alpha}(x)$. The total Hamiltonian of a MZ interferometer, $\mathcal{H} = \mathcal{H}_0 + \sum_\ell(A_\ell + A_\ell^\dagger)$, contains a term describing edge states

$$\mathcal{H}_0 = \frac{1}{8\pi^2}\sum_{s,\alpha,\beta}\int dxdy V_{\alpha\beta}(x-y)\partial_x\phi_{s\alpha}(x)\partial_y\phi_{s\beta}(y), \tag{2}$$

where the kernel, $V_{\alpha\beta}(x-y) = 2\pi v_F\delta_{\alpha\beta}\delta(x-y) + U_{\alpha\beta}(x-y)$, includes a free fermion contribution with the Fermi velocity v_F, and the Coulomb interaction potential $U_{\alpha\beta}$. Vertex operators $A_\ell = t_\ell\exp[i\phi_{D1}(x_\ell) - i\phi_{U1}(x_\ell)]$, $\ell = L, R$, describing electron tunneling between outer edge channels of the interferometer at the left and right QPC, are treated perturbatively. The AB phase $\varphi_{\rm AB}$ is taken into account via the relation for tunneling amplitudes, $t_R^* t_L = |t_R t_L|e^{i\varphi_{\rm AB}}$.

The electron current is defined as a rate of change of the electron number N_D in the lower arm: $I = i[\mathcal{H}, N_D]$. To leading order in tunneling amplitudes, its average value is given by $\langle I\rangle = \int_{-\infty}^{\infty} dt \sum_{\ell\ell'}\langle[A_\ell^\dagger(t), A_{\ell'}(0)]\rangle$. The AB oscillations in the differential conductance $G \equiv d\langle I\rangle/d\Delta\mu$ are characterized by the visibility $\mathcal{V}_{\rm AB}(\Delta\mu) = (G_{\max} - G_{\min})/(G_{\max} + G_{\min})$. Using the expression for the average current, one easily finds that both the visibility and the phase shift of AB oscillations are expressed in terms of the same complex function [12], namely

$$\mathcal{V}_{\rm AB} = \mathcal{V}_0|\mathcal{I}(\Delta\mu)|, \qquad \Delta\varphi_{\rm AB} = \arg\mathcal{I}(\Delta\mu), \tag{3a}$$

$$\mathcal{I}(\Delta\mu) = \partial_{\Delta\mu}\int_{-\infty}^{\infty}\frac{dt}{2\pi}K_U(L,t)K_D^*(L,t), \tag{3b}$$

where $\mathcal{V}_0 \propto 2|t_L t_R|/(|t_L|^2 + |t_R|^2)$, and

$$K_s(x,t) \propto \langle\exp[-i\phi_{s1}(x,t)]\exp[i\phi_{s1}(0,0)]\rangle \tag{4}$$

are the electron correlation functions [15] at the outer channels of the interferometer.

Correlation functions and FCS.—The Hamiltonian (2), together with the commutation relations (1), generates equations of motion for the fields $\phi_{s\alpha}$, which have to be accompanied with a boundary condition:

$$\partial_t\phi_{s\alpha}(x,t) = -\frac{1}{2\pi}\sum_\beta\int_{-\infty}^{\infty}dy V_{\alpha\beta}(x-y)\partial_y\phi_{s\beta}(y,t), \tag{5a}$$

$$\partial_t\phi_{s\alpha}(-W,t) = 2\pi j_{s\alpha}(t), \tag{5b}$$

where $j_{s\alpha}$ is the charge current flowing out of the QPC at the point $x = -W$ [16]. In general, the fields $\phi_{s\alpha}$ influence fluctuations of the currents $j_{s\alpha}$ at a QPC, leading to the dynamical Coulomb blockade in the quantum, low-energy regime [17], and to cascade corrections to noise in the classical limit [18]. An important simplification in the present case arises from the fact that such backaction effects are absent for chiral edge states [8,12]. As a consequence, in the case of $\nu = 2$ the electron transport through a single QPC is not affected by interactions, which has been recently confirmed in the experiment [5]. Therefore, by solving Eqs. (5), one may express the correlation functions of the fields $\phi_{s\alpha}$ in terms of irreducible moments (cumulants) of the currents, $\langle\langle j_{s\alpha}^n\rangle\rangle$, and equivalently, via the generator of full counting statistics (FCS) defined as [19],

$$\chi_{s\alpha}(\lambda,t) = \langle e^{i\lambda Q_{s\alpha}(t)}e^{-i\lambda Q_{s\alpha}(0)}\rangle, \tag{6}$$

where $\partial_{i\lambda}^n\log(\chi_{s\alpha})/t = \langle\langle j_{s\alpha}^n\rangle\rangle$ in the long-time limit. Here, averaging is defined over free electrons, and $Q_{s\alpha}(t) = \int_{-\infty}^{t}dt' j_{s\alpha}(t')$.

All the interaction effects are encoded in a solution of Eq. (5a). We assume that the Coulomb potential is screened at distances d, with $L \gg d \gg a$, where a is the distance between edge channels. The screening may occur due to the presence of either a back gate, or a massive air bridge [12]. Therefore, at low energies one can neglect the logarithmic dispersion of the Coulomb potential and simply write $U_{\alpha\beta}(x-y) = U_{\alpha\beta}\delta(x-y)$. Nevertheless, the long-range character of the interaction, i.e., the fact that $d \gg a$, allows one to approximate $U_{\alpha\beta} = \pi u$, where $u/v_F \sim \log(d/a) \gg 1$. As a result, the spectrum of collective charge excitations splits in two modes: a fast charged mode with the speed u, and a slow dipole mode with the speed $v \simeq v_F$. At relevant energies, v/L, the charged mode is not excited, which leads to a universality in the electron transport predicted in Ref. [12] and observed in experiments [4–7]. Here, taking the limit $u \to \infty$ simplifies the solution of Eq. (5a), and we obtain the result $\phi_{s1}(x,t) = -\pi[Q_{s1}(t) + Q_{s2}(t) + Q_{s1}(t_W) - Q_{s2}(t_W)]$, where $t_W = t - (x+W)/v$.

Finally, we further assume that the noise source is located far away from the interferometer, $W \gg L$, which reasonably agrees with the experimental situation [4–7]. This assumption implies that the charges $Q_{s\alpha}(t_W)$ and $Q_{s\alpha}(t)$ in the solution for the field $\phi_{s1}(x,t)$ are well separated in time, and therefore contribute independently to the correlation function (4). Therefore, the correlator $K_s(x,t)$ splits in the product of four terms

$$K_s(L,t) \propto \chi_{s1}(\pi,t)\chi_{s1}(\pi,t-L/v)\chi_{s2}(\pi,t) \times \chi_{s2}(-\pi,t-L/v), \tag{7}$$

where we used the definition (6) for the generator of FCS.

Gaussian noise approximation.—We note that the variable λ in the expression (7) plays a role of a coupling constant in the context of the noise detection physics [19].

It is typically small, so the contribution of high-order cumulants of noise to the detector signal is negligible [20]. Here, in contrast, $\lambda = \pm\pi$, implying that a MZ interferometer is strongly coupled to noise. Nevertheless, it is instructive, for comparison purpose, to consider Gaussian fluctuations first. Expanding the generator (6) up to second order in charge operators, we obtain

$$\log[\chi_{s\alpha}(\lambda, t)] = i\lambda\langle j_{s\alpha}\rangle t - \lambda^2 J_{s\alpha}(t), \tag{8}$$

where a Gaussian noise contribution is given by the integral

$$J_{s\alpha}(t) \equiv \frac{1}{2\pi}\int \frac{d\omega S_{s\alpha}(\omega)}{\omega^2 + \eta^2}(1 - e^{-i\omega t}), \qquad \eta \to 0, \tag{9}$$

and $S_{s\alpha}(\omega) = \int dt e^{i\omega t}\langle \delta j_{s\alpha}(t)\delta j_{s\alpha}(0)\rangle$ is the noise power.

The expression (9) for the correlation function $J_{s\alpha}(t)$ is typical in the context of the noise detection physics (see, e.g., Ref. [20]). In the long-time (classical) limit, a dominant contribution to this function is linear in time: $J_{s\alpha}(t) = (1/2)\langle\langle j_{s\alpha}^2\rangle\rangle|t|$, where $\langle\langle j_{s\alpha}^2\rangle\rangle \equiv S_{s\alpha}(0)$, in agreement with definition (6) of the FCS generator. For a QPC at zero temperature, the scattering theory [13] gives

$$S_{s\alpha}(\omega) = S_q(\omega) + R_{s\alpha}T_{s\alpha}S_n(\omega), \tag{10}$$

where $S_q(\omega) = (1/2\pi)\omega\theta(\omega)$ is the quantum, ground-state spectral function, and $S_n(\omega) = \sum_{\pm} S_q(\omega \pm \Delta\mu) - 2S_q(\omega)$, is the nonequilibrium contribution (see Fig. 2). Note that the noise power (10) differs from the one for a nonchiral case [20].

We now focus on the specific situation shown in Fig. 1, namely, we set $T_{D1} = T_{D2} = T_{U2} = 1$ and $T_{U1} = T = 1 - R$. We evaluate the electron correlation function (7) in the upper arm of the MZ interferometer, using Eqs. (8)–(10), and arrive at the result

$$K_U(L, t) \propto \frac{\exp\{i\Delta\mu T(t - L/2v)\}}{\sqrt{t(t - L/v)}} \exp\{-\pi^2 RT[J_n(\Delta\mu t) + J_n(\Delta\mu t - \Delta\mu L/v)]\}, \tag{11}$$

where the function J_n is given by the integral (9) with $S_{s\alpha}(\omega)$ replaced by $S_n(\omega)$. In expression (11), the numerator in the first term originates from the average current $T\Delta\mu/2\pi$ in (8), the denominator is the contribution of the quantum noise $S_q(\omega)$, and the last term comes from the nonequilibrium noise $S_n(\omega)$ and describes dephasing. The correlation function in the lower arm of the interferometer

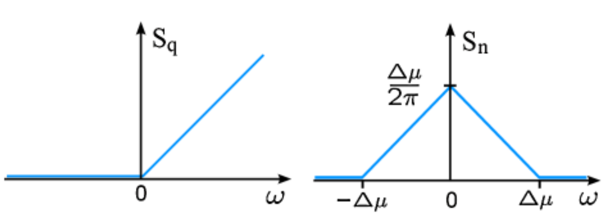

FIG. 2 (color online). Two spectral functions that contribute to the noise power (10).

can be obtained from Eq. (11) by setting $\Delta\mu = R = 0$ with the result $K_D(L, t) \propto 1/\sqrt{t(t - L/v)}$. Thus for a ballistic channel, and for $L = 0$, the electron correlation function coincides with the one for free electrons. This explains the fact that in the $\nu = 2$ case, the Coulomb interaction does not affect an electron transport through a single QPC [5], and justifies our approach.

Next, we use the results for correlation functions K_s to evaluate the integral (3b). For a large voltage bias $L\Delta\mu/v \gg 1$, we obtain

$$\mathcal{I}(\Delta\mu) \propto E_{\text{lb}}\partial_{\Delta\mu}\sin\left(\frac{\pi\Delta\mu}{E_{\text{lb}}}\right)e^{-\Delta\mu/E_{\text{df}}}, \tag{12a}$$

$$E_{\text{lb}} = \frac{2\pi v}{TL}, \qquad E_{\text{df}} = \frac{4v}{\pi RTL}. \tag{12b}$$

Thus the visibility $\mathcal{V}_{\text{AB}}$, given by Eq. (3a), shows a lobe-type behavior: It oscillates as a function of voltage bias $\Delta\mu$, vanishes at certain values of bias, and decays. Since the function $\mathcal{I}(\Delta\mu)$ is real, the AB phase shift $\Delta\varphi_{\text{AB}}$ jumps by π at zeros of the visibility and remains constant between zeros, thus showing the phase rigidity [4]. The distance between zeros of the visibility, E_{lb}, is determined by the average current of transmitted electrons, and can be viewed as a "mean-field" contribution to the correlator (11). The dephasing rate E_{df} is determined by the current noise power. The ratio $2E_{\text{lb}}/(\pi E_{\text{df}}) = R$ is given, in general, by the Fano factor of Gaussian noise.

Noise induced phase transition.—In what follows, we consider non-Gaussian noise, and show that the contribution of high-order cumulants of current is indeed not small. Note that the ground-state contribution of the current noise, S_q, that dominates at short times, is pure Gaussian. Therefore, the denominator in expression (11) remains unchanged. In the long-time limit, the dominant contribution to the FCS generator comes from the nonequilibrium part of noise, S_n. For a QPC, it is given by the well known expression [19] for a binomial process: $\chi_{U1}(\lambda, t) = (R + Te^{i\lambda})^N$, where $N = \Delta\mu t/2\pi$ is the number of electrons that contribute to noise. Applying the analytical continuation $\lambda \to \pi$, we obtain

$$\log[\chi_{U1}(\pi, t)] = \frac{\Delta\mu t}{2\pi}[\log|T - R| + i\pi\theta(T - R)], \tag{13}$$

where the imaginary part contributes to the effective voltage bias in the first term of the correlator (11), while the real part is responsible for dephasing.

A remarkable property of the expression (13) is that high-order cumulants of current add up to cancel the dilution effect of a QPC. Therefore, the continuous variation of the mean-field contribution in the correlator (11) is replaced with the jump in the voltage bias across a MZ interferometer at the point $T = 1/2$. We evaluate the integral (3b) in the limit $L\Delta\mu/v \gg 1$ and arrive at the result (12a), as in the Gaussian case, but with new energy scales:

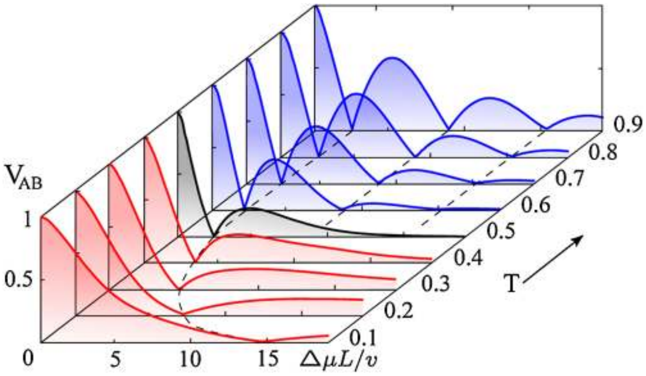

FIG. 3 (color online). The visibility of AB oscillations is shown as a function of the normalized voltage bias for different transparencies of the QPC that injects electrons. It is evaluated numerically using the Gaussian approximation at low bias, and Markovian FCS at large bias. The visibility shows several lobes for $T > 1/2$, while it has only one side lobe for $T < 1/2$. The black curve shows the visibility at critical point of the phase transition. Dashed lines indicate the position of zeros.

$$E_{\rm lb} = \frac{2\pi v}{L}, \qquad E_{\rm df} = \frac{2\pi v}{L|\log(T-R)|}, \qquad T > 1/2. \quad (14)$$

The rigidity of zeros of the visibility for $T > 1/2$ is clearly seen in Fig. 3. For $T < 1/2$, the visibility may be found by taking the limit $E_{\rm lb} \to \infty$ in the expression (12a) with the result $\mathcal{I}(\Delta\mu) \propto (1 - \Delta\mu/E_{\rm df})e^{-\Delta\mu/E_{\rm df}}$. Thus, the only zero of the visibility scales as $\Delta\mu = E_{\rm df}$, given by the expression in (14).

The behavior of the visibility of AB oscillations, shown in Fig. 3, may be considered a phase transition, because strictly speaking, it arises in the classical regime, where the number of electrons that contribute to this effect is large, $N \gg 1$. The transition occurs at the critical point, $\lambda = \pi$, $T = 1/2$, where the moment generator $\chi_{U1}(\lambda, t)$ of a binomial process vanishes, and can be viewed as a result of entanglement between electrons of the noise source and those that contribute to AB oscillations. However, quantum fluctuations of N at critical point smear out the sharp transition.

Quantum correction at critical point.—Finding quantum corrections to the long-time asymptotic of the FCS of noninteracting electrons requires the evaluation of Fredholm determinants, which is best formulated in the wave-packet basis [19]. In the present situation a simplification arises from the fact that in the long-time limit the dominant contribution to the generator (6) comes from nonequilibrium electrons in the energy interval $\Delta\mu$. Such electrons can be viewed as a "train" of incoming wave packets $W(s_n) = \sqrt{\Delta\mu/2\pi v_F}\sin(s_n)/s_n$, where $s_n = (\Delta\mu/2)(x/v_F - t) + \pi n$, which are normalized as $\int dx|W(s_n)|^2 = 1$. If electrons were transmitted through the QPC (placed at $x = 0$ for the convenience) with the probability T and reflected with the probability $R = 1 - T$, this would lead to a binomial process. However, the fact that wave packets have a finite width leads to the small probability $P_n = \int_{-\infty}^0 dx W^2(s_n)$ for electrons not to reach the QPC, which can be well approximated with $P_n = [\pi(\Delta\mu t - 2\pi n)]^{-1}$. Thus, taking into account all three possibilities, we write the moment generating function as $\chi_{U1}(\lambda, t) = \prod_n[(1 - P_n)(R + Te^{i\lambda}) + P_n]$. At critical point, $\lambda = \pi$, $T = 1/2$, this gives the following result:

$$\log[\chi_{U1}] = \sum_n \log(P_n) = -\frac{\Delta\mu t}{2\pi}[\log(\pi\Delta\mu t) - 1]. \quad (15)$$

The imaginary part of $\log[\chi_{U1}]$ comes from a branch cut of the logarithm and grows gradually in the interval $T - R \approx 1/(2\pi^2 N)$, smearing out the discontinuity in (13). Using Eq. (15) we find that at critical point the visibility scales as $\mathcal{V}_{\rm AB} \propto \partial_\varepsilon \exp\{-\varepsilon[\log(\pi^2\varepsilon) - 1]\}/\sqrt{\varepsilon}$, $\varepsilon = \Delta\mu L/2\pi v \gg 1$. The result of a numerical evaluation, shown by the black line in Fig. 3, demonstrates the residual phase coherence at critical point due to quantum fluctuations of the number N.

We acknowledge support from the Swiss NSF.

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