Noise Sensitivity in Continuum Percolation

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In critical bond percolation on $\mathbb{Z}^2$ (i.e., with $p = 1/2$), consider the event that there is a horizontal crossing of the box $[n]^2$. Suppose that this event occurs for a particular (random) configuration $x \in \{0,1\}^E$, and let x^ε be obtained by re-randomizing each edge with probability $\varepsilon > 0$. What is the probability that there is a crossing in x^ε?

This question was first asked by Benjamini, Kalai and Schramm [1], who proved that the probability converges to $1/2$ as $n \to \infty$, and termed this phenomenon *noise sensitivity*. Their proof used techniques from discrete Fourier analysis, and built on ideas introduced in the famous paper of Kahn, Kalai and Linial [3]. In recent years much more precise results have been obtained about the Fourier spectrum of percolation, and about dynamical percolation on the triangular lattice (see [2, 5]).

In this talk we consider the corresponding question for Continuum Percolation, and in particular for the Poisson Boolean model (also known as the Gilbert disc model). Let η be a Poisson process of density λ in the plane, and connect two points of η by an edge if they are at distance at most 1. We prove that, at criticality, the event that there is a crossing of the box $[n]^2$ is noise sensitive. The proof is based on two extremely general tools: a version of the BKS Theorem for product measure, and a new extremal result on hypergraphs. The former result was first proved in [4]; we shall describe how it may be easily deduced from the uniform version.

This is joint work with Daniel Ahlberg, Erik Broman and Simon Griffiths.

References

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