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Noise-Sustained Structure, Intermittency, and the Ginzburg-Landau Equation

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The time-dependent generalized Ginzburg-Landau equation is a partial differential equation that is related to many physical systems. In the stationary (ie. laboratory) frame of reference the equation is:

$$\frac{\partial \psi}{\partial t} = a\,\psi - v\frac{\partial \psi}{\partial x} + b\frac{\partial^2 \psi}{\partial x^2} - c\,|\psi|^2\psi \qquad (1)$$

where the dependent variable ψ is in general complex; a , b , and c are constants which are in general complex; and v is the group velocity.

Consider a small initial localized perturbation about the equilibrium state $\psi = 0$. A linear stability analysis reveals that there are three types of behavior which this perturbation can undergo. 1) The perturbation will be damped in any frame of reference. This behavior corresponds to the system being *abolutely stable* . 2) The perturbation will grow and spread such that the edges of the perturbation move in opposite directions. This behavior corresponds to the system being *absolutely unstable* . 3) The perturbation will be damped at any given stationary point but a frame of reference may be found in which the perturbation is growing. In other words, even though the perturbation is growing and spreading, it is moving at a sufficiently large velocity such that both edges of the perturbation are moving in the same direction. Thus the system behind the perturbation returns to its undisturbed state. This behavior corresponds to the system being *spatially unstable* (ie convectively unstable).

Under conditions when the system is absolutely unstable, an initial small (microscopic) perturbation will grow to macroscopic size, saturate (assuming $c_r > 0$), and produce a structure for all time. In contrast, if the system is spatially unstable, the perturbation and resulting structure will move spatially such that the structure eventually leaves the boundaries of the system. Thus the system returns to the equilibrium state

A *single* perturbation therefore produces only a *temporary* structure. However if the the system is continuously perturbed by microscopic external noise it will be unable to return to the equilibrium state and a new state will be established which is sustained by the presence of the noise (ie. a *noise-sustained state*).

Under conditions in which the system is spatially unstable (ie. $a_r - \frac{v^2 b_r}{4|b|^2} < 0$ and $a_r > 0$), eq. (1) is numerically solved in the presence of low-level external noise. At the conference the time evolution of ψ plotted as a function of x was shown via a movie. Two striking features were the coherent structure forming as a result of the selective amplification of the external noise and the interspersion of this coherent structure with turbulent behavior.

Fig. 1 shows one frame of the movie -- ψ plotted as a function of x for a particular t after the system has reached a statistically steady state. The microscopic noise near the left boundary grows spatially to macroscopic proportion resulting in the observed structure. This is a *noise-sustained structure*. If the external noise is removed the structure moves out through the right boundary and the system returns to the state $\psi = 0$ everywhere except for some slight ($\sim 10^{-100}$) fluctuations do to computer roundoff.

The intermittency is seen in fig. 2 which shows ψ plotted as a function of t at $x = 150$. This intermittent behavior may be qualitatively understood as follows. The microscopic noise near the left boundary is spatially and selectively amplified resulting in the formation of spatially growing waves. These waves are sufficiently regular to produce the coherent structure seen in fig. 1. However the coherent structure itself is *spatially unstable* (ie. a secondary spatial instability). This spatial instability causes the coherent structure to break up at some spatial point. The point at which it breaks up depends on the degree of irregularity in the spatially growing waves. Since the irregularities in the spatially growing waves change with time (the source of the waves being random noise), the point at which the coherent structure breaks up changes with time resulting in intermittency.

A few other points are: 1) The external noise (or other external perturbation) was necessary for the formation of the structure -- no noise → no structure. 2) The *microscopic* noise played an important role in the *macroscopic* dynamics of the system (eg. the intermittency). 3) The chaotic behavior is not associated with a strange attractor (ie. not deterministic chaos). 4) The system exhibits a laminar region followed spatially by a turbulent region. This type of behavior occurs in many fluid systems such as pipe flow, fluid flow over a flat plate, and smoke rising from a cigarette. 5) A few fluid systems from which eq. (1) (with nonzero group velocity v) has been derived are plane Poiseulle flow and wind-induced water waves. A nonzero group velocity is necessary in order for the equation to exhibit a spatial instability.

For references and a more detailed account the reader is referred to ref. [1] and the references contained therein.

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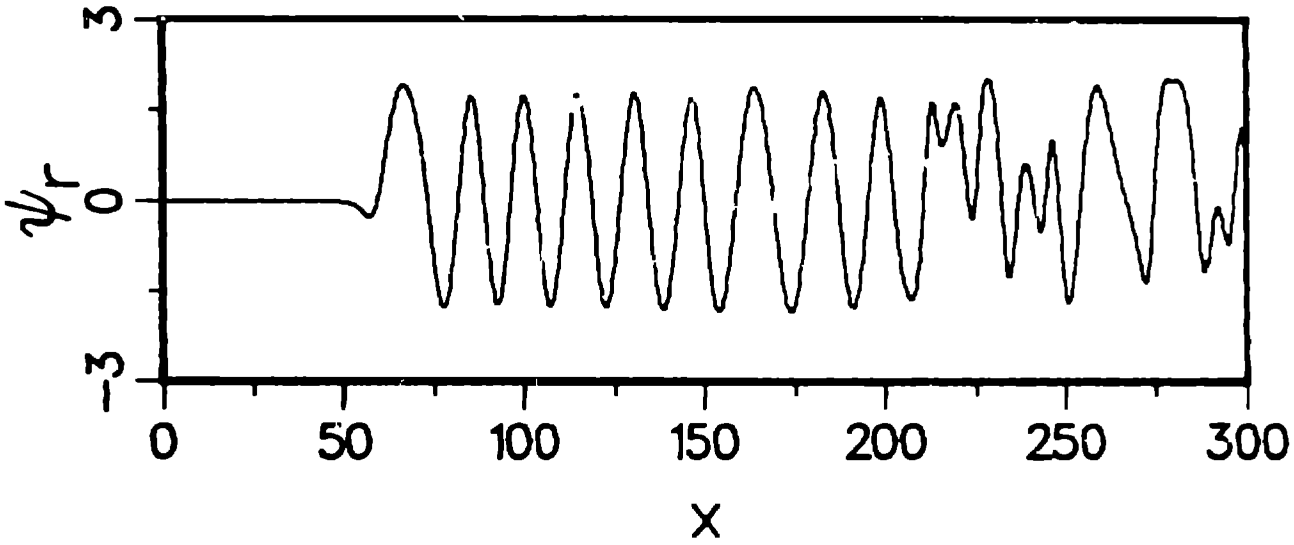
References

1) R. J. Deissler, "Noise-Sustained Structure, Intermittency, and the Ginzburg-Landau Equation", Los Alamos preprint #LA-UR-84-3629 and submitted to J. Stat. Phys.

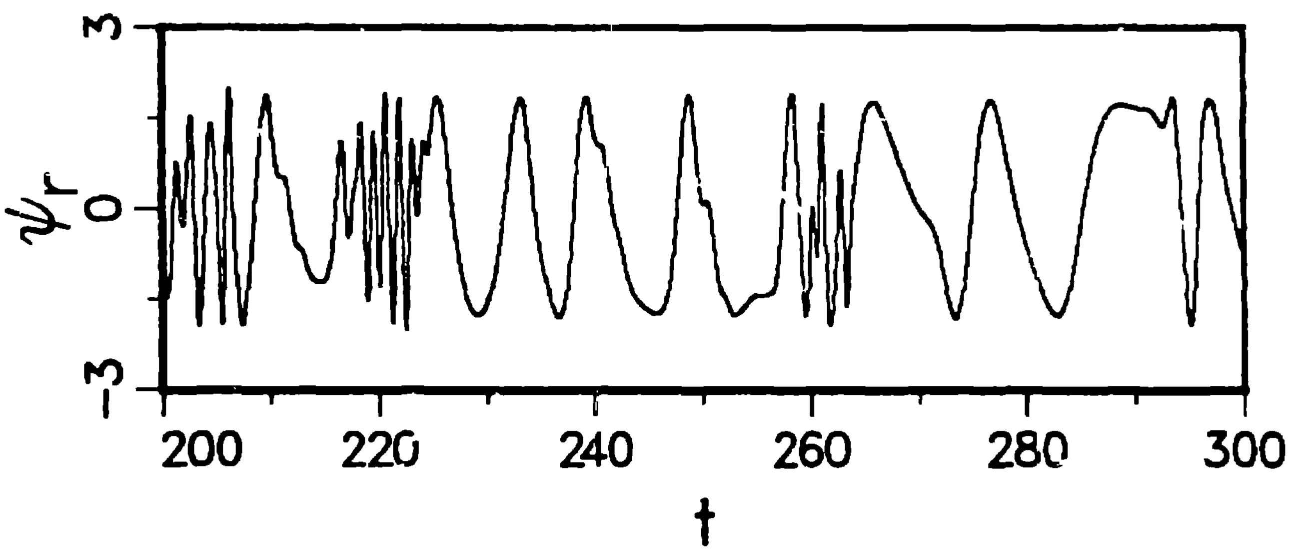
Figures

1) Plot of ψ_r as a function of x for a given t ($t=300$) after transients have settled down. $a=2$, $v=6$, $b_r=2.8$, $b_i=-1$, $c_r=.5$, $c_i=1$. Noise level $= r = 10^{-8}$. The noise, a random number uniformly distributed between $-r$ and r, is added to the grid point adjacent to the left boundary. The microscopic noise at this grid point grows spatially to macroscopic proportion resulting in the observed structure.

2) Plots of ψ_r as a function of t. $a=2$, $v=6$, $b_r=2.8$, $b_i=-1$, $c_r=.5$, $c_i=1.$, $x=150$. High and low frequencies are seen to be interspersed.



3
ψ_r
0
-3
0
50
100
150
200
250
300
X



3
0
-3
ψ_r
200
220
240
260
280
300
t