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Nominal Rigidities and the Term Structures of Equity and Bond Returns

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Abstract

A downward-sloping term structure of equity and upward-sloping term structures of interest rates arise endogenously in a general-equilibrium model with nominal rigidities and nonlinear habits in consumption. Countercyclical marginal costs exacerbate the procyclicality of dividends after a technology shock, and hence their riskiness, and generate countercyclical inflation. Marginal costs gradually fall after a negative technology shock as the price level increases sluggishly, so the payoffs of short-duration dividend claims (bonds) are more (less) procyclical than the payoffs of long-duration claims (bonds). The simultaneous presence of market and home consumption habits allows for uniting nonlinear habits and a production economy without compromising the ability of the model to fit macroeconomic variables.

JEL classification: E43; E44; G12.

Keywords: Structural term structure modeling, Equity and bond yields, Habit formation, Nominal rigidities, Macro-finance modeling.

Introduction

Recent evidence shows that the average term structure of equity risk premia is downward-sloping and starts from a high level. Also, it is well known that the term structure of nominal bonds slopes upwards on average. These facts are of interest to financial economists as the maturity structure of equity and bond risk premia reveals how investors form expectations about future macroeconomic variables and their marginal utilities at different horizons. A joint general-equilibrium explanation for this evidence has remained as yet elusive (e.g., Binsbergen and Koijen, 2015).

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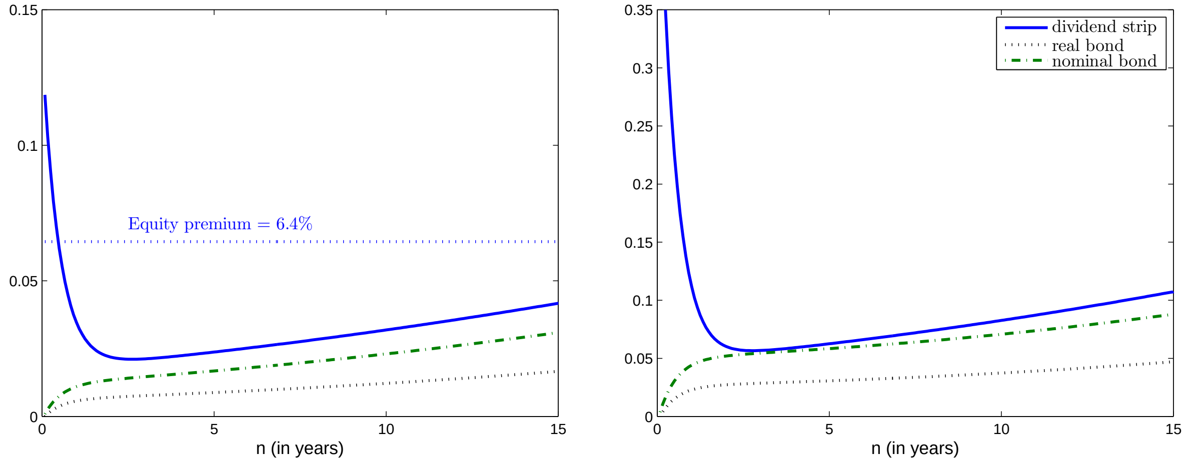


(a) *Risk premia,* $\ln E(R^{e,(n)}_{t+1})$. (b) *Volatilities,* $var(\ln R^{e,(n)}_{t+1})$.

Figure 1: Average term structures of annualized excess returns and volatilities for holding for one month the nth zero-coupon cashflow claim in our benchmark model specification. Different lines associate with the term structures of different cashflow claims: market equity (solid), real bonds (dotted) and nominal bonds (dash-dotted line). The dotted line is the average annualized holding-period equity premium.

We propose a simple framework with large and time-varying risk premia that endogenizes the payoff of nominal bonds and that rationalizes dividends as a levered version of consumption. Our model links macroeconomic fluctuations with asset pricing facts in full general equilibrium and offers a joint explanation of the documented term structure properties. In particular our model is able to capture simultaneously the negative slope of the average term structure of dividend strip returns and volatilities as well as upward-sloping average term structures of interest rates and a positive inflation risk premium at all horizons (Lettau and Wachter, 2011; Binsbergen, Brandt and Koijen, 2012a; Binsbergen, Hueskes, Koijen and Vrugt, 2013; Lopez, 2013). Figure 1 plots our model-implied term structures of returns and volatilities.

We unite a textbook New Keynesian model economy (Galí, 2008) and Campbell and Cochrane (1999) habits in consumption. Nominal rigidities induce a time-varying labor share of output, driven by countercyclical marginal costs. The countercyclical labor share implies a procyclical dividend share as well as countercyclical inflation. It follows that dividend claims and nominal bonds pay off badly in a technological recession and are therefore risky investments. Since marginal costs are stationary, the payoffs of long-duration dividends (bonds) are less (more) procyclical; the dividend share and the price level increase as more and more firms are able to adjust their prices to regain their markups.[2]

[2]A recent and rapidly growing literature is focusing on the asset pricing implications of nominal rigidities (e.g., Rudebusch and Swanson, 2008, 2012; Bekaert, Cho and Moreno, 2010; Palomino, 2012; Li and Palomino, 2014; Andreasen, 2013; Campbell, Pflueger and Viceira, 2013; Kung, 2015; Gorodnichenko and Weber, 2013; Weber, 2014).

Reconciling habit formation with business cycle facts

We avoid the well-known difficulties in reconciling business cycle facts with habit formation models in production economies (e.g., Jermann, 1998; Lettau and Uhlig, 2000; Uhlig, 2007; Rudebusch and Swanson, 2008, 2012; Swanson, 2012) by introducing nonlinear habits in two consumption goods, one purchased in the market and one produced at home.[3]

In a production economy habits affect equilibrium quantities by their effect on the intertemporal rate of substitution, which drives consumption-saving and investment decisions, and by their effect on the intratemporal rate of substitution, which controls the link between consumption and labor supply. The consequence is that equilibrium quantities (consumption, output, labor, investment and the capital stock) depend on the additional state variables that drive habit dynamics and countercyclical risk premia. The time-variation in the new state variables has quantity implications that are associated either with counterfactually large business cycle fluctuations in some real variables (such as labor, the capital stock, the real wage rate, or the real risk-free rate), or with small risk premia as households absorb aggregate shocks by varying labor or investment.

Campbell and Cochrane (1999) engineered a consumption habit sensitivity function that controls the intertemporal consumption-saving decisions and induces large, volatile, and time-varying risk premia with a low and stable risk-free rate. In the same spirit, we also engineer restrictions on the home-consumption habit sensitivity function to control the intratemporal effect of habits on consumption-labor decisions and thereby avoid risk-premia spillovers on macroeconomic quantities—and hence the quantity puzzles first documented by Lettau and Uhlig (2000). Finally, the effect of habits on the consumption-investment tradeoff is controlled by the curvature of capital adjustment costs, which determines the dependence of investment on Q; in the limit when capital adjustment is infinitely costly habits have no effect on investment.

To provide intuition about the model's implication in as simple a setting as possible, we calibrate the model to the polar case of an exact separation between risk premia and quantity dynamics that reconciles habit formation with business cycle facts, independently of the presence of other intratemporal distortions such as wage rigidities or other labor market frictions (as considered for example by Uhlig, 2007 and Rudebusch and Swanson, 2008).[4] The result that such a polar case exists extends to the habit formation setting a macro-finance separation result analogous to the one that Tallarini (2000) described in a setting with Epstein-Zin preferences (see also Cochrane, 2008).

[3]We choose to focus on the habit formation framework rather than on a 'long-run risk' framework building on Epstein-Zin-Weil preferences for two reasons. First, Croce, Lettau and Ludvigson (2015) show the difficulties of simultaneously producing in an Epstein-Zin-Weil context a downward-sloping term structure of equity and a sizeable equity premium for a level of risk aversion not exceeding the commonly accepted upper bound of 10 (Epstein, Farhi and Strzalecki, 2014). Second, the Campbell-Cochrane habit specification naturally generates time-variation in risk premia without relying on counterfactual heteroskedasticity in macroeconomic fundamentals (Campbell, Pflueger and Viceira, 2013).

[4]The extant literature offers examples of habit formation in small-scale production economies but they feature habits that are either linear (e.g., Jermann, 1998; Boldrin, Christiano and Fisher, 2001; De Paoli, Scott and Weeken, 2010; Challe and Giannitsarou, 2014) or that depart from the Campbell-Cochrane specification in order to grant exact exponential-affine term structures (e.g., Gallmeyer, Hollifield and Zin, 2005; Bekaert, Engstrom and Xing, 2009; Bekaert, Cho and Moreno, 2010; Palomino, 2012; Dew-Becker, 2013), so the financial spillovers onto the intratemporal or on the intertemporal rates of substitution are left unrestrained.

Term structures of equity strips and bond returns

Production economies provide endogenous restrictions on cashflows based on economic theory, in contrast with a reduced-form approach that may be difficult to reconcile with standard macroeconomic models. Additionally, jointly modeling macroeconomic quantities and asset prices with a structural approach allows for studying policy interventions, structural shifts and potential feedbacks between the real and the financial sides of the economy. Since the macro-finance separation ensures that discount rate variation does not compromise the ability of the model to fit macroeconomic variables, we can focus on the asset pricing implications of the restrictions placed on cashflows by the DSGE model.

Our framework preserves all the main achievements of Campbell and Cochrane (1999), including a solution to the average equity premium puzzle, the risk-free rate and the excess volatility puzzles, and the countercyclicality of stock market returns and volatility. Additionally, our nonlinear-habit model is able to explain the entire observed maturity structure of equity and bond returns and volatilities, disentangling the intertemporal risk-return tradeoffs at different horizons.[5] The key drivers of our results are nominal rigidities and some degree of mean reversion in the growth rate of technology.

Sticky prices endogenize the payoff of nominal bonds and rationalize dividends as a levered version of consumption. The payoff of short-term equity is positively correlated with consumption news but is much more volatile, and hence more risky, as long as positive short-run shocks to technology growth associate with negative long-run shocks to technology growth. In fact, short-run shocks increase consumption and dividends alike but negative long-run shocks push demand below potential, which lowers consumption while creating downward pressure on inflation and real wages that raises corporate profits.

Mean reversion in technology reduces the riskiness of dividend claims with longer duration, as a positive exposure to long-run technology growth risk provides consumption insurance. Thus, the model is able to generate an initially negative slope in the term structure of market equity for a sufficiently large degree of price stickiness, capturing the evidence by Binsbergen et al. (2012a). Moreover, the role of bonds (real and nominal) as a hedge for transitory shocks to the growth rate of technology does not produce a bond premium puzzle because the exposure to the state variable that drives the risk-free rate (the conditional mean of the growth rate of technology) commands a negative price.[6]

Finally, in our model the price of risk is a state variable that has a low unconditional correlation with technology growth and its conditional mean, so investors still fear long-duration equities because they do poorly in recessions unrelated on average with technology risk.

[5]While the search for a structural explanation of the positive slope of the term structure of interest rates has a rather long history (see, for example, Gürkaynak and Wright, 2012; Duffee, 2013), the search for a structural explanation for the negative slope of the term structure of equity has only recently received a lot of attention (Croce, Lettau and Ludvigson, 2015; Belo, Collin-Dufresne and Goldstein, 2015; Lynch and Randall, 2011; Ai, Croce, Diercks and Li, 2013; Marfè, 2013; Nakamura, Steinsson, Barro and Ursúa, 2013; Wachter, 2013); see also Binsbergen and Koijen (2015).

[6]This property motivates from first principles the descriptive structure assumed by Lettau and Wachter (2011) that lies behind their ability to capture the initial slopes of the term structures of equity and real interest rates.

1. Incorporating Campbell-Cochrane habit formation in a production economy

This section describes a textbook DSGE model with nominal rigidities that we augment with nonlinear habits in market and home consumption.

1.1. Households

As in Greenwood and Hercowitz (1991) our households obtain utility over consumption of two types of goods, nondurable goods and services purchased in the market, and goods and services produced at home. Households get used to an accustomed standard of living as represented by some particular levels of consumption of the market-purchased good and of the home-produced good.

Identical consumers indexed by $j \in [0, 1]$ have preferences captured by the function

$$U_0(j) = E_0 \sum_{t=0}^{\infty} \beta^t \Big(\frac{[C_t(j) - X_t^c]^{1-\gamma} - 1}{1-\gamma} + \chi \frac{[H_t(j) - X_t^h]^{1-\gamma} - 1}{1-\gamma} \Big) \tag{1}$$

where C_t is real consumption purchased in the market and H_t denotes the consumption produced at home, with production function $H_t = A_t(1 - N_t)$, with N_t the labor choice and A_t aggregate productivity. X_t^c and X_t^h represent habit levels that are a nonlinear function of contemporaneous and past consumption. Parameter β is the subjective discount rate and parameter χ controls the steady-state effect of habits, while the curvature of the utility function in market and home consumption is the same to ensure balanced growth (see Campbell and Ludvigson, 2001). We assume a calibration for χ to achieve the same steady state as under a power-utility specification ($X_t^c = X_t^h = 0$).

Habits are endogenous state variables that induce a departure of the equilibrium dynamics from the power-utility specification. As is customary in the extant literature, we assume that the law of motion of habits is specified indirectly through the processes for surplus market consumption $s_t \equiv \ln[(C_t - X_t^c)/C_t]$ and surplus home consumption $z_t \equiv \ln[(H_t - X_t^h)/H_t]$ in order to ensure consumption levels that never fall below their respective habit levels, and hence well-behaved marginal utilities. The law of motion of the surplus levels is driven by aggregate market and home consumption, $c_t \equiv \ln \int_0^1 C_t(j)dj$ and $h_t \equiv \ln \int_0^1 H_t(j)dj$; since each individual agent has zero mass, she takes the habit levels thus specified as external to her consumption decisions. This structure implies the following marginal utilities of market and home consumption

$$\frac{\partial U_t}{\partial C_t} = C_t^{-\gamma} S_t^{-\gamma}$$
$$\frac{\partial U_t}{\partial H_t} = \chi H_t^{-\gamma} Z_t^{-\gamma}$$

The inclusion of the two types of consumption allows us to maintain the separability between consumption and hours, while also remaining consistent with balanced growth and keeping the elasticity of intertemporal substitution as a free parameter. In addition to being able to reconcile our model with the available evidence of an intertemporal elasticity lower than one, this property preserves the original implications of Campbell and Cochrane (1999) for the stochastic discount factor.

1.1.1. Habit structure

We specify the following dynamics for the logarithms of aggregate surplus levels:[7]

$$\begin{aligned} \hat{s}_{t+1} &= \rho_s \hat{s}_t + \Lambda_c[\hat{s}_t](E_{t+1} - E_t) f_c[C_{t+1}] \\ \hat{z}_{t+1} &= \rho_s \hat{z}_t + \Lambda_h[\hat{z}_t](E_{t+1} - E_t) f_h[H_{t+1}] \end{aligned} \tag{2}$$

where $f_c[C_t] = \ln[C_t]$, $f_h[H_t] = \ln[A_t^{\alpha/(1-\alpha)}(A_t - H_t)]$, and the sensitivity functions (Λ_c and Λ_h) and steady state levels of the surplus variables are:

$$\Lambda_c[\hat{s}_t] = \begin{cases} S^{-1}\sqrt{1-2\hat{s}_t} - 1, & \hat{s}_t \leq \frac{1}{2}(1-S^2) \\ 0 & \hat{s}_t > \frac{1}{2}(1-S^2) \end{cases} \qquad S = \sqrt{\frac{\gamma\, var(\varepsilon_t^c)}{1-\rho_s-\xi_1/\gamma}}$$

$$\Lambda_h[\hat{z}_t] = (1-\alpha)(1+\xi_2)\Lambda_c[\hat{z}_t/(1+\xi_2)] \qquad Z = S\left(S + (1-S)\frac{var(\varepsilon_t^c)}{cov(\varepsilon_t^c, \varepsilon_t^h)}\right)^{-1}$$

The sensitivity functions and the steady state levels depend on the parameter $\xi = [\xi_1; \xi_2] \in \mathbb{R}^2$ that controls the spillover of habits dynamics onto the equilibrium quantities. Propositions 1 and 2 specifies restrictions on the spillover parameter that grant a macro-finance separation.

As in Campbell and Cochrane (1999) and Wachter (2006), ξ_1 controls the effect of time-varying risk aversion on the intertemporal rate of substitution.[8] Additionally ξ_2 controls the effect of time-varying risk aversion on the intratemporal rate of substitution. Note that $\xi_2 = -1$ describes the case with constant surplus home consumption, which is equivalent to a model without home consumption habits.

The market (home) consumption habit indirectly specified by the surplus process is a complex nonlinear function of current and past market (home) consumption; however, it is approximately a linear habit that adjusts slowly to unanticipated movements in market (home) consumption.

Like Campbell and Cochrane (1999), we choose the market consumption sensitivity function to satisfy the following conditions: (i) the market consumption habit does not produce a risk-free rate puzzle; (ii) the habit coincides with the consumptions level in the long run; (iii) the habit is locally predetermined; and (iv) the habit moves nonnegatively with consumption near the steady state. The first condition shows how habits can be engineered in such a way that the spillover on consumption-saving decisions can be kept under control (via parameter ξ_1); the remaining conditions can be interpreted as (local) microfoundations that add to the well-behaved marginal utilities and the local slow-moving representation of habits. Appendix A proves these properties.[9] In this context, habits pull the real risk-free rate in offsetting directions via an intertemporal substitution motive and a

[7]The notation $\varepsilon_t^x \equiv (E_t - E_{t-1})x_t$ stands for the one-period ahead forecast error in variable x, and 'hats' denote deviations from steady state.

[8]Unlike Wachter (2006), we do not impose exogenously a countercyclicality in real rates; rather, we let the production economy introduce a small departure from random-walk consumption (which inherits a near-zero, negative autocorrelation by the mean reversion in technology), and hence generate real rate movements.

[9]Moreover, we achieve two additional improvements in the microeconomic properties of the habits relative to Campbell and Cochrane (1999) because we operate in the context of a production economy. First, both habits move nonnegatively with market and home consumption, respectively, in *and around* the steady state. This property owes entirely to the endogeneization of equilibrium consumption choices; in fact, in the endowment economy of Campbell

precautionary savings motive; the spillover parameter ξ_1 controls whether an increase in surplus consumption drives rates up or down.

By analogous logic, we choose the home consumption sensitivity function as follows: (i) the home consumption habit does not produce a quantity puzzle; (ii) the habit coincides with the home consumption level in the long run; (iii) the habit is locally predetermined; and (iv) the habit moves nonnegatively with home consumption near the steady state. As in the case of market consumption habits, the last three conditions can be interpreted as local microfoundations, and the first condition shows how the habits can be engineered in such a way that the spillover on consumption-labor decisions is controlled (via parameter ξ_2). Our home consumption habit is locally predetermined in a weaker sense than the market consumption habit, as its first-order predeterminedness must deteriorate if we move sufficiently far from the benchmark perfect correlation in market and home consumption innovations. Appendix A discusses these properties.

1.1.2. Consumers' problem

Consumers maximize the intertemporal objective (1) subject to the sequence of budget constraints

$$P_tC_t(j) + \frac{1}{1+i_t}B_t(j) \leq W_tN_t(j) + B_{t-1}(j) + P_tD_t - T_t(j)$$

and the appropriate transversality condition, where B_t denotes their time-t holdings of one-period bonds discounted at the nominal rate i_t, D_t is the dividend they receive from owning the aggregate firm, and T_t are lump-sum taxes that the government levies on consumers to finance the corrective subsidy.

1.2. Firms

Our benchmark model considers the polar case of a macro-financially separate economy, which implies a deterministic capital stock. In appendix B we allow for nontrivial capital accumulation and describe one last spillover, controlled by parameter $\xi_3 \in \mathbb{R}_+$, which affects the consumption-investment tradeoff (proposition 3). To preserve macro-finance separation we need a deterministic capital accumulation because investment is determined by Tobin's Q, which is an asset price and therefore a channel that must break separation.

The production side of the economy is characterized by a unit mass of identical firms indexed by $i \in [0, 1]$ that maximize intertemporal profits and operate with production technology

$$Y_t(i) = [e^{\mu t}\widetilde{A}_tN_t(i)]^{1-\alpha}K_t(i)^\alpha$$

where Y_t is real output; N_t is the labor input, which they acquire at a unit cost equal to the nominal wage rate W_t; $K_t = e^{\mu t}$ is the deterministic capital stock, which grows at rate μ on a balanced-growth

and Cochrane there is only a zero relationship between habits and consumption in the steady state and a potentially strictly negative relationship near the steady state (Ljungqvist and Uhlig, 2015; see appendix A for more details).

Second, Campbell and Cochrane (1999) need to assume a *high* average relative risk aversion coefficient in their endowment economy and defend this choice against the objection that the assumed coefficient is implausibly large. In a production economy, households can absorb macroeconomic shocks along both the consumption and the labor margin, which dramatically reduces their risk aversion (Swanson, 2012). The online appendix shows how the steady-state risk aversion coefficient in our calibrated model is about the upper bound of 10.

path; and $e^{\mu t}\widetilde{A}_t$ denotes the exogenous labor-augmenting technology level. The ith good sells for the nominal price $P_t(i)$ and $P_t \equiv [\int_0^1 P_t(i)^{1-\varepsilon} di]^{1/(1-\varepsilon)}$ is the price index. The relationship between productivity in market- and home-produced goods is $A_t = e^{\mu t}\widetilde{A}_t^{1-\alpha}$ (Campbell and Ludvigson, 2001).

We also allow for Calvo-type nominal price rigidities and monopolistic competition in the market for goods. Each firm i can reset prices at any given time only with probability $1 - \eta$ and faces the demand curve for the good it produces $C_t(i) = [P_t(i)/P_t]^{-\varepsilon} C_t$, which arises as the cost-minimizing plan of individual consumers, $j \in [0, 1]$, who bundle the continuum of goods, $i \in [0, 1]$, via a Dixit-Stiglitz aggregator with constant elasticity of substitution between goods, ε. The government levies lump-sum taxes on each firm to finance an employment subsidy, $\tau = 1/\varepsilon$, which reduces the unit nominal cost of labor and is in place to offset any steady-state distortions caused by the monopolistic competition.[10]

Finally, market equity is the value of the aggregate firm, which pays out per-period equilibrium profits as dividends.

1.3. Equilibrium

Joint intertemporal and static optimality of market and home consumption decisions are described by the equations

$$\ln(1 + i_t) = -\ln E_t \beta e^{-\gamma\Delta c_{t+1} - \gamma\Delta s_{t+1} - \pi_{t+1}} \tag{3}$$

$$w_t - p_t = -\ln(\chi) + \gamma c_t - \gamma h_t + a_t + \gamma(\hat{s}_t - \hat{z}_t) \tag{4}$$

with the loglinearized home-production relation,

$$h_t = a_t - \frac{N}{1 - N} n_t \tag{5}$$

Market clearing for each good i implies market clearing at the aggregate level, $y_t = c_t$, and therefore market consumption relates with the loglinearized production function as

$$c_t = \ln(1 - \alpha) + a_t + (1 - \alpha) n_t \tag{6}$$

A standard New Keynesian Phillips curve describes the loglinearized optimal price-setting behavior of firms as the forward-looking optimality condition linking inflation, $\pi_t \equiv \ln(P_t/P_{t-1})$, and marginal costs, $mc_t \equiv (w_t - p_t) - \ln(\partial Y_t/\partial N_t)$,

$$\pi_t = \widetilde{\beta} E_t \pi_{t+1} + \lambda\, \widehat{mc}_t \tag{7}$$

where $\lambda \equiv (1 - \eta)(1 - \widetilde{\beta}\eta)(1 - \alpha)/\eta(1 - \alpha + \alpha\varepsilon)$ controls the slope of the curve, with $\widetilde{\beta} \equiv \beta e^{(1-\gamma)\mu}$. Inflation is high when firms expect long-run marginal costs above the flexible-price level, in which case resetting firms choose a price above the index to realign their marginal costs to the desired level.

[10]This assumption can be easily relaxed but simplifies notation.

Loglinearized equilibrium dividends can be written as

$$d_t = c_t - \frac{1-\alpha}{\alpha}\widehat{mc}_t \tag{8}$$

Corporate profits, and hence dividends, are low when marginal costs are high, holding the level of output fixed. Since marginal costs fluctuate when prices are sticky, the presence of nominal rigidities is entirely responsible for breaking down the equality between dividend growth and consumption growth.

Finally, firms' optimal labor demand schedule restricts aggregate real wages and marginal costs,

$$w_t - p_t = \widehat{mc}_t + c_t - n_t$$

and hence, matching labor demand and supply as the labor market clears,

$$\widehat{mc}_t = \frac{\gamma(1-\alpha)+\alpha+\varphi}{1-\alpha}(c_t - c_t^n) - \gamma(\hat{s}_t - \hat{z}_t) \tag{9}$$

where $\varphi \equiv \gamma N/(1-N)$ is the inverse steady-state quasi-Frisch's labor supply elasticity,[11] and where $c_t^n = a_t$ denotes optimal consumption under flexible prices. Deviations of aggregate marginal costs from the desired level are associated with a gap in aggregate activity relative to the flexible-price equilibrium.

1.3.1. Technology

The logarithm of the growth rate of technology evolves according to the process

$$\begin{aligned} \Delta a_{t+1} &= \mu + u_t + \sigma e_{t+1}^a \\ u_{t+1} &= \rho_u u_t + \phi\sigma e_{t+1}^u \end{aligned} \qquad \begin{bmatrix} e_t^a \\ e_t^u \end{bmatrix} \sim Niid\left(0, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right) \tag{10}$$

with $[\phi;\sigma] \in \mathbb{R}_+^2$, $\rho \in [-1,1]$ and $\rho_u \in [0,1)$. The stochastic component of the conditional mean of technology growth is a mean reverting process driven by shocks e_t^u, while shocks e_t^a have only a contemporaneous effect on technology growth. Accordingly, we refer to e_t^a as a short-run shock and to e_t^u as a long-run shock to technology growth. This structure allows for the unit-root dynamics in cashflows routinely used in the consumption-based asset pricing literature.

1.3.2. Monetary policy

Since in a sticky-price environment the level of inflation influences the equilibrium allocation, we must specify monetary policy, which we describe by a Taylor rule that reacts to inflation and the output gap,

$$i_t = \phi_\pi \pi_t + \phi_y(c_t - c_t^n) \tag{11}$$

which grants determinacy if $\kappa(\phi_\pi - 1) + (1-\widetilde{\beta})\phi_y > 0$, for $[\phi_\pi;\phi_y] \in \mathbb{R}_+^2$, with $\kappa \equiv \lambda[\gamma(1-\alpha)+\alpha+\varphi]/(1-\alpha)$ (Galí, 2008).

[11]The online appendix derives and discusses Frisch's elasticity in our setting.

1.3.3. Competitive equilibrium

For any specified policy process $\{i_t\}_{t=0}^{\infty}$ and exogenous state vector $\{a_t, u_t, s_t, z_t\}$, the loglinearized competitive equilibrium is an allocation $\{c_t, h_t, d_t, n_t\}_{t=0}^{\infty}$ and a price system $\{\pi_t, w_t, mc_t\}_{t=0}^{\infty}$ satisfying equations (2) to (11), and the initial condition for $[a_0; u_0; s_0; z_0]$.

2. Macro-finance separation by force of habit

Definition (Macro-finance separation)**.** An equilibrium (a feasible allocation and a price system that solve each household's and each firm's problem and clear markets) is macro-financially separate if the equilibrium allocation and equilibrium inflation are the same as in the model without habits (i.e., such that $X_t^c = X_t^h = 0$ at all dates).

The ability to preserve the quantity implications of the underlying real business cycle model is a crucial diagnostic to evaluate a macro-finance model. In making this claim we are taking to the logical extreme the critique made by Lettau and Uhlig (2000), and revived by Uhlig (2007), Rudebusch and Swanson (2008) and Swanson (2012), and applying it to DSGE models with habit formation in the spirit of Campbell and Cochrane (1999).[12]

In our context, risk premia are driven to first order by the price of risk, whose dynamics are fully determined by surplus consumption. Therefore, the notion of macro-finance separation boils down to the separation between the dynamics of risk premia and the dynamics of quantities (including inflation).[13] Time-varying risk aversion in turn spills over onto the flexible-price equilibrium allocation if and only if it does so onto the intratemporal rate of substitution that determines the optimal consumption-labor decisions. In a sticky-price equilibrium, however, a spillover onto consumption-saving decisions would produce an additional departure from a macro-finance separation.

2.1. Spillover onto the intratemporal rate of substitution

The key property of our home production habits is that in equilibrium the aggregate production function and market clearing imply at all dates the following relationship between surplus levels[14]

$$\frac{Z_t}{Z} = \left(\frac{S_t}{S}\right)^{1+\xi_2}$$

so their respective effects on the intratemporal marginal rate of substitution can offset for an appropriate choice of parameter ξ_2.

[12]We are not denying the possibility that a more volatile discount factor better fits quantity dynamics (in particular hump-shaped dynamics, as argued by Boldrin et al., 2001). However, we argue that the first step of the modeling exercise of incorporating volatile discount factors in a macro model should be to keep the spillovers on quantities contained. We can then allow for an arbitrary spillover and a role of habits in the determination of quantity dynamics.

[13]More precisely, under a macro-finance separation the New Keynesian part of the model entirely determines shock-exposure elasticities, while the Campbell-Cochrane habits control shock-price elasticities. The RBC part has only an indirect impact on shock-price elasticities via the precautionary effect of habits; see also Lopez et al. (2015).

[14]The equilibrium law of motion reduces to $\hat{z}_{t+1}/(1+\xi_2) = \rho_s \hat{z}_t/(1+\xi_2) + \Lambda[\hat{z}_t/(1+\xi_2)]\varepsilon_{t+1}^c = \sum_{j=0}^{\infty} \rho_s^j \Lambda[\hat{z}_{t-j}/(1+\xi_2)]\varepsilon_{t-j+1}^c$, which implies $\hat{z}_t/(1+\xi_2) = \hat{s}_t$ in the appropriate equivalence class for stochastic processes.

Parameter ξ_2 controls the size of the spillover of the surplus levels onto equilibrium quantities. In fact, letting $\xi_2 = 0$, the optimal intratemporal rate of substitution between consumption and labor,

$$-\frac{\partial U_t/\partial N_t}{\partial U_t/\partial C_t} = \chi \frac{A_t^{1-\gamma} C_t^{\gamma}}{(1-N_t)^{\gamma}} \left(\frac{S_t}{Z_t}\right)^{\gamma}$$

reduces to the one under power utility.

Home consumption habits are needed to have the substitution effect towards home consumption dominate the income effect of a negative market consumption shock, so households choose not to absorb the consumption movement by increasing significantly their labor effort.

2.2. Spillover onto the intertemporal rate of substitution

Time-varying risk aversion may spill over onto the equilibrium allocation if it affects the rate of substitution between consumption and saving, $r_t = -\ln E_t M_{t+1}$. In our Gaussian external-habit setting, the dynamic IS equation balances an intertemporal substitution motive and a precautionary savings motive as[15]

$$r_t = -\ln(\beta) - \frac{\gamma(1-\rho_s-\xi_1/\gamma)}{2} + \gamma E_t \Delta c_{t+1} - \xi_1 \hat{s}_t$$

where parameter ξ_1 controls the spillover onto consumption-saving decisions, as in Campbell and Cochrane (1999) and Wachter (2006).

2.3. Macro-finance separation: flexible prices

We formalize the last results in the following proposition:[16]

Proposition 1. *Given the spillover parameter $\xi \equiv [\xi_1; \xi_2] \in \mathbb{R}^2$, for any $\xi_1 \in \mathbb{R}$ and for any value of the preference parameter $\gamma \in \mathbb{R}_+$, there is a unique value of parameter $\xi_2 = 0$ such that the flexible-price competitive equilibrium is macro-financially separate, for all $\xi_1 \in \mathbb{R}$.*

The online appendix provides additional details on the proof of proposition 1 and shows how under macro-finance separation our habit structure can motivate the original consumption-based asset pricing model of Campbell and Cochrane (1999) as the outcome of a generic production economy.

2.4. Macro-finance separation: sticky prices

Once we activate further rigidities, such as sticky prices, and we discuss the equilibrium separation requirements a crucial question is what is the empirically relevant monetary policy in place. A natural choice is a Taylor rule that responds to inflation and some detrended version of output, in which case the competitive equilibrium is separate whenever the flexible-price equilibrium

[15] In the derivation we assume the conditional homoskedasticity of consumption growth, so $var_t(c_{t+1}) = var(\varepsilon_t^c)$, which is consistent, for example, with a macro-finance separation or with a solution for consumption based on a first-order approximation of the structural equations.

[16] Appendix C and proposition 4 analyze the internal habit specification and the associated conditions for separation.

is, with the additional requirement of no intertemporal spillovers. The following proposition formalizes this results and extends the macro-finance separation results of proposition 1 to the sticky-price setting:

Proposition 2. *For any value of the preference parameter $\gamma \in \mathbb{R}_+$, there is a unique value of parameter $\xi = [0; 0]$ such that the sticky-price competitive equilibrium is macro-financially separate.*

The online appendix describes the competitive equilibrium and proves proposition 2. Intuitively, if the flexible-price equilibrium is macro-financially separate, the output gap, $c_t - c_t^n$, is a sufficient statistic for aggregate marginal costs and inflation; since the only remaining source of financial spillovers is the dynamic IS equation, we then require a zero spillover parameter ξ_1.

3. Term structures of equity and interest rates

We work with the cashflow processes implied by the macro-financially separate competitive equilibrium under a Taylor rule to study the asset pricing implications of our New Keynesian production economy with Campbell-Cochrane external habits. These processes are characterized by the structural relations (7), (8), (9), and the equilibrium process for the logarithm of aggregate consumption

$$c_t = c_t^n + \psi_c u_t,$$

with $\psi_c = \gamma(1 - \widetilde{\beta}\rho_u)/\{(1 - \widetilde{\beta}\rho_u)[\gamma(1 - \rho_u) + \phi_y] + \kappa(\phi_\pi - \rho_u)\}$ and the flexible-price consumption process $c_t^n = a_t$.

3.1. Equilibrium cashflows

The New Keynesian framework models endogenously a difference between the real and the nominal term structures and a difference between aggregate consumption and market dividends. Table 1 and figure 2 summarize their main differences by representing the anticipated reaction of the main cashflow processes to macroeconomic shocks.

Four properties of the equilibrium cashflow processes are worth emphasizing. First, bond payoffs do not display mean growth and equity payoffs do. Second, short-run shocks to the growth rate of technology do not have a contemporaneous effect on bond payoffs but increase equity payoffs. Third, a positive long-run shock increases both consumption and marginal costs, with a negative overall contemporaneous effect on market dividends and a positive one on inflation. Finally, ex-ante consumption and dividend growth are positive when the conditional mean of technology growth is above average and so is the ex-ante value of inflation.

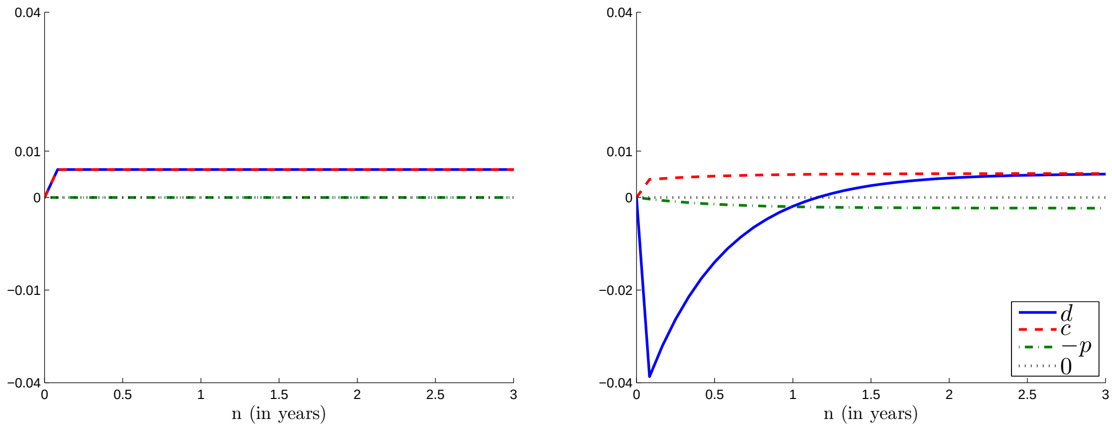
The intuition behind these properties of cashflows stems from the equilibrium equation

$$r_t = r_t^n - \gamma(1 - \rho_u)\psi_c u_t,$$

where r_t^n represents the natural rate of real interest rates. A positive movement in expected technology growth prompts households, who expect future growth, to command a higher interest on savings but the real rate increases less than the natural rate as a consequence of the monetary frictions; incentives to save remain too low, so demand and output go above potential and exert upward pressure on marginal costs. This cost effect depresses corporate profits while causing

Asset	Cashflow process	Deterministic growth	Loading on u_t	Loading on σe^a_{t+1}	Loading on σe^u_{t+1}
Consumption equity	Δc_{t+1}	μ	$C_c \in (0, 1)$	1	> 0
Market equity	Δd_{t+1}	μ	$> \gamma C_c$	1	< 0
Nominal bond	$-\pi_{t+1}$	0	< 0	0	< 0
Real bond	0	0	0	0	0

Table 1: Dynamics of the cashflow processes that determine the prices of four assets: an equity claim to the aggregate market consumption good (consumption equity), an equity claim to aggregate dividends (market equity), a claim to a unit of the numeraire (nominal bond), and a claim to a unit of consumption (real bond). The cashflow process that determines consumption equity is consumption growth, market equity is determined by dividend growth, nominal bonds by negative inflation and real bonds pay a constant real cashflow with trivial dynamics. The cashflow loadings are calculated for a nontrivial degree of price rigidities.



(a) *Short-run shock-exposure elasticities.*

(b) *Long-run shock-exposure elasticities.*

Figure 2: Percentage change in cashflows over a given horizon after 1 standard deviation short-run and long-run shocks to technology growth arriving next month.

inflationary pressure as firms try to reset prices to realign markups to the desired level. Markups are then expected to jump back up as the excessive production gets corrected, and hence dividends are expected to grow more than consumption, while positive inflation persists for a while.

Nominal rigidities in turn exacerbate these effects, as inflation becomes more stable with stronger rigidities, and hence markups must absorb a larger share of the shocks that hit the economy, with a stronger contractionary effect on corporate profits. Note how the New Keynesian model explains endogenously the stylized fact that dividend growth is much more volatile than consumption growth ex ante. This property is true also ex-post in the presence of mean reversion in the states that drive quantities (technology and the conditional mean of technology growth). Note in fact how the last two columns of table 1 imply that a negative correlation between short-run and long-run shocks would dampen the volatility of consumption growth but amplify dividend growth fluctuations. Therefore, we have an endogenous mechanism by which dividends are a levered version of consumption, as routinely assumed in endowment-economy equilibrium asset pricing models.

3.2. Equilibrium term structures

Short-run shocks command a positive price, and a negative enough correlation between short-run and long-run shocks implies that long-run shocks command a negative price. In this context, a cashflow exposure to long-run shocks greater than the corresponding discount rate exposure implies an insurance effect. Therefore, table 1 shows how consumption claims and bonds (nominal and real) have an upward-sloping term structure of risk premia and dividend claims a downward-sloping term structure, all starting from a strictly positive level. Moreover, the negative price of long-run shocks implies that there is a strictly positive inflation risk premium at all horizons.[17]

To formalize these results, we start by describing the stochastic discount factor,

$$\begin{aligned} m_{t+1} &= -\ln(\beta) - \gamma\Delta c_{t+1} - \gamma\Delta s_{t+1} \\ &= -\ln(\beta) - \gamma E_t\Delta c_{t+1} + \gamma(1-\rho_s)\hat{s}_t - x_t(E_{t+1} - E_t)c_{t+1} \end{aligned}$$

where $x_t \equiv \gamma[1 + \Lambda(s_t)]$ is the price of risk. We then solve for the term structures of the different cashflow claims by relying on the essentially-affine approximation proposed by Lopez, Lopez-Salido and Vazquez-Grande (2015), which isolates the first-order components of equilibrium asset prices, performs comparably to numerical solution methods, and is particularly appropriate in our context because the underlying model of cashflows is solved to the first order and would therefore not allow for an accurate computation of higher-order terms.

We set up the system in the form of Lopez et al.,

$$\begin{bmatrix} \Delta c_{t+1} \\ \Delta d_{t+1} \\ -\pi_{t+1} \end{bmatrix} = \begin{bmatrix} \mu \\ \mu \\ 0 \end{bmatrix} + \begin{bmatrix} C_c \\ C_d \\ C_{-p} \end{bmatrix} \zeta_t + \begin{bmatrix} D_c \\ D_d \\ D_{-p} \end{bmatrix} \varepsilon_{t+1}$$
$$\zeta_{t+1} = A\zeta_t + B\varepsilon_{t+1}$$

where $\zeta_t = u_t$ and $\varepsilon_t \sim Niid(0, I_2)$. We subsequently apply their essentially-affine approximation to solve for the no-arbitrage price of a claim to some cashflow d that will realize in n periods, $P^{(n)}_{d,t} = E_t(M_{t,t+n}D_{t+n})$ and the associated holding-period expected excess return $E_t(R^{e,(n)}_{d,t+1}) = E_t(M_{t+1})E_t(P^{(n-1)}_{d,t+1}/P^{(n)}_{d,t})$, which take the approximate equilibrium log form

$$r^{e,(n)}_{d,t+1} = E_t r^{e,(n)}_{d,t+1} + V_{d,n-1,t}\varepsilon_{t+1}$$

where d denotes the four different cashflow processes (consumption, corporate profits, the numeraire, and the inverse of the price level), and where the stochastic vector

$$V_{d,n-1,t} \equiv \underbrace{D_d}_{\substack{\text{short-run}\\ \text{cashflow}\\ \text{risk}}} + \underbrace{B^{(n-1)}_{d,\zeta}B}_{\substack{\text{long-run cashflow}\\ \text{and discount rate risk}}} + \underbrace{B^{(n-1)}_{d,s}\Lambda_t D_c}_{\substack{\text{habit-related}\\ \text{discount rate}\\ \text{risk}}} \tag{12}$$

[17]The online appendix shows how these results are robust to a departure from macro-financial separation that activates nontrivial investment choices ($\xi_3 > 0$).

represents the quantity of risk in the nth cashflow strip, with coefficients

$$B_{d,\zeta}^{(n)} = (C_d - \gamma C_c)(I - A)^{-1}(I - A^n)$$

$$B_{d,s}^{(n)} = \gamma(1 - \rho_s) + \rho_s B_{d,s}^{(n-1)} - \frac{1 - \rho_s}{\gamma^2}\left(B_{d,s}^{(n-1)} - \gamma\right)^2 - \frac{D_c(D_d + B_{d,\zeta}^{(n-1)}B - B_{d,s}^{(n-1)}D_c)'}{S}\left(B_{d,s}^{(n-1)} - \gamma\right)$$

with $B_{d,s}^{(0)} = 0$. Appendix D sketches the essentially-affine approximation we use, which is covered extensively by Lopez et al. (2015).

The resulting closed-form approximate solution provides insight into the determinants of the term structures of risk premia on different cashflow claims. The first term in equation (12) owes entirely to the one-month ahead volatility in cashflows; the second term captures the effect that news about the conditional mean of technology growth have on tomorrow's prices through their effect on future cashflows and discount rates; the third term reflects the effect of movements in risk aversion on tomorrow's prices through their effect on long-run discount rates as habits slowly grow closer to consumption.

3.3. Calibration

Table 2 lists all deep parameters in the model and their calibration. We calibrate all parameters of the production side of the economy using standard values in the New Keynesian literature as in Galí (2008). Quasi-Frisch's elasticity of labor supply is 1, consistent with steady-state hours $N = 1/3$. The labor share in value added is $1 - \alpha = 2/3$. The elasticity of substitution in the CES aggregator is $\varepsilon = 6$ and the average duration of prices is $(1 - \eta)^{-1} = 9$ months. The Taylor rule coefficients are $\phi_\pi = 1.5$ and $\phi_y = 0.5/12$ (Taylor, 1999). The spillover parameter ξ is set to zero, so we work under macro-finance separation.

We calibrate all parameters relating with the pricing kernel as in Campbell and Cochrane (1999). Like in the original analysis, this choice of parameters allows to capture the observed persistence in market dividend-price ratios, a realistic maximum Sharpe ratio of around 0.4 as well as, crucially, a reasonable equity premium of 6.5% that is nonetheless substantially lower than the premium commanded by short-term equities, which can reach up to 12% on average, consistent with observed strip returns. This feature is the actual meaning of the downward-sloping term structure of equity documented by Binsbergen et al. (2012a). We calibrate the subjective discount rate, β, to match an average monthly real rate r of 0.94% per year.

To calibrate the remaining parameters, which pin down the model's dynamics, we choose a parametrization that matches a few moments at annual frequency of consumption and dividend growth using annual data on personal consumption expenditure in nondurable goods and services and on nonfinancial corporate profits over the period 1929-2014 from the Bureau of Economic Analysis, which we express in real terms through the core PCE price index.[18] We start by estimating via maximum likelihood an $ARMA(1,1)$ structure and recover the implied monthly AR coefficient.

[18]While we use the same dataset as Bansal and Yaron (2004), we differ in our choice to match corporate profits rather than CRSP dividends; corporate profits allow to exploit a common data source while remaining fully consistent with the model, in which corporate profits and dividends coincide. Results are similar if we use a dividend series (e.g., using CRSP value-weighted returns) instead.

	Parameter		Value
New Keynesian block	γ	Utility curvature in market and home consumption	2
	$1/\varphi$	Quasi-Frisch's labor supply elasticity	1
	β	Subjective discount factor	.9991
	$1-\alpha$	Labor share in value added	2/3
	ε	Elasticity of substitution in Dixit-Stiglitz aggregator	6
	$1/(1-\eta)$	Average price duration (in months)	9
	ϕ_π	Policy response coefficient to inflation movements	1.5
	ϕ_y	Policy response coefficient to output movements	.5/12
Habit block	ξ_1	Financial spillover onto the intertemporal rate of substitution	0
	ξ_2	Financial spillover onto the intratemporal rate of substitution	0
	ρ_s	Habit persistence	.9940
Exogenous block	μ	Mean technology growth	.0030
	ρ_u	Persistence of the conditional mean of technology growth	.8470
	σ	Conditional volatility of technology	.0181
	ϕ	Relative volatility of the conditional mean of technology	.1311
	ρ	Correlation between short-run and long-run shocks	$-.9433$

β matches an average real interest rate of .94% per year.
ρ_s matches an average market equity premium of 6.53% per year.
μ matches an average annual consumption growth of 3.60%.
ρ_u matches an average AR root in an ARMA(1,1) representation of annual consumption growth of .136.
$[\sigma; \phi; \rho]$ match a volatility of annual consumption growth of 2.49%, a volatility of annual dividend growth of 33.07% and a correlation between consumption and dividend growth of .580.

Table 2: Deep parameters and their calibration (monthly frequency). Data for real consumption growth and real dividend growth use annual BEA data over the period 1929-2014 for personal consumption expenditure in nondurables and services and for nonfinancial corporate profits before taxes, and are deflated by the core PCE price index. Monthly simulated data are aggregated to an annual frequency and are matched to the corresponding data moments.

We pick the remaining parameters to match the volatility of annual consumption growth (2.49%), the volatility of annual dividend growth (33.07%) and the correlation between consumption and dividend growth (0.580).[19] Finally, it is worth noting that our baseline calibration implies a correlation between consumption growth and inflation of −0.31 at a quarterly frequency, in line with the empirical moment extracted by Binsbergen et al. (2012b).

3.4. Results

Figure 3 reports the average term structure of equilibrium risk premia, volatilities and Sharpe ratios of consumption and market equities and of real and nominal interest rates. Figure 1b additionally reports the term structure of hold-to-maturity returns, which are (conditionally) linear combinations of holding-period returns. The average premium on the market portfolio is 6.53% (annualized, monthly basis), considerably less than the short end of the term structure of the equity premium, and compares with a slightly lower premium on the consumption portfolio of 5.97%.[20] The inflation risk premium is sizeable and positive at all maturities, starting at zero and increasing steadily up to slightly more than 1% per year at a 40-year horizon. Figure 3 also shows how only long-duration cashflow strips are mean-variance efficient.

Figure 4 plots the term structures conditional on different values of the state that drives them (surplus consumption). Bad surplus consumption states, which associate with high risk aversion, scale up level, slope and curvature of the term structures. Good surplus consumption states associate with virtually flat term structures.

Moreover, we reproduce the main appealing properties of Campbell and Cochrane (1999), including countercyclical financial market volatility and risk premia, as well as the long-horizon predictability of excess stock returns.

3.4.1. A 3-factor decomposition: Level, short-run and long-run slope

Risk premia are the product of the systematic exposure of each strip on the structural shock and the price of a unit exposure to the structural shock, $x_t D_c$. We can therefore decompose risk premia

$$\ln E_t R^{e,(n)}_{d,t+1} = x_t D_c V'_{d,n-1,t}$$

into three determinants—a level factor $x_t D_c D'_d$, a factor that controls the short end of the curve, $x_t D_c (B_\zeta^{(n-1)} B)'$ and a factor that controls the long end of the curve, $x_t D_c (B_s^{(n-1)} \Lambda_t D_c)'$.

[19] Another reason we depart in our baseline calibration from a trend-stationary technology process is that it implies a trivial martingale component in the discount factor $m^P_{t+1} = 0$ because there are no permanent shocks to the marginal utility of wealth. Alvarez and Jermann (2005) argued forcefully for a model of the marginal utility of wealth to include nontrivial permanent and transient components.

[20] Just like the equity premium, also the variance risk premium remains at a level similar to Campbell and Cochrane (1999), with a 30-day expected variance under the risk-neutral and the physical measures of 29.6 and 29.3, respectively (see Lopez et al., 2015, for a closed-form approximate expression for the variance risk premium, which shows its positiveness and its determinants). Thus, despite an implicit variance with the right magnitude, the implied variance risk premium of 0.3 (in percentages squared) is an order of magnitude smaller than the empirical estimates in the literature (e.g., Bollerslev, Tauchen and Zhou, 2009).

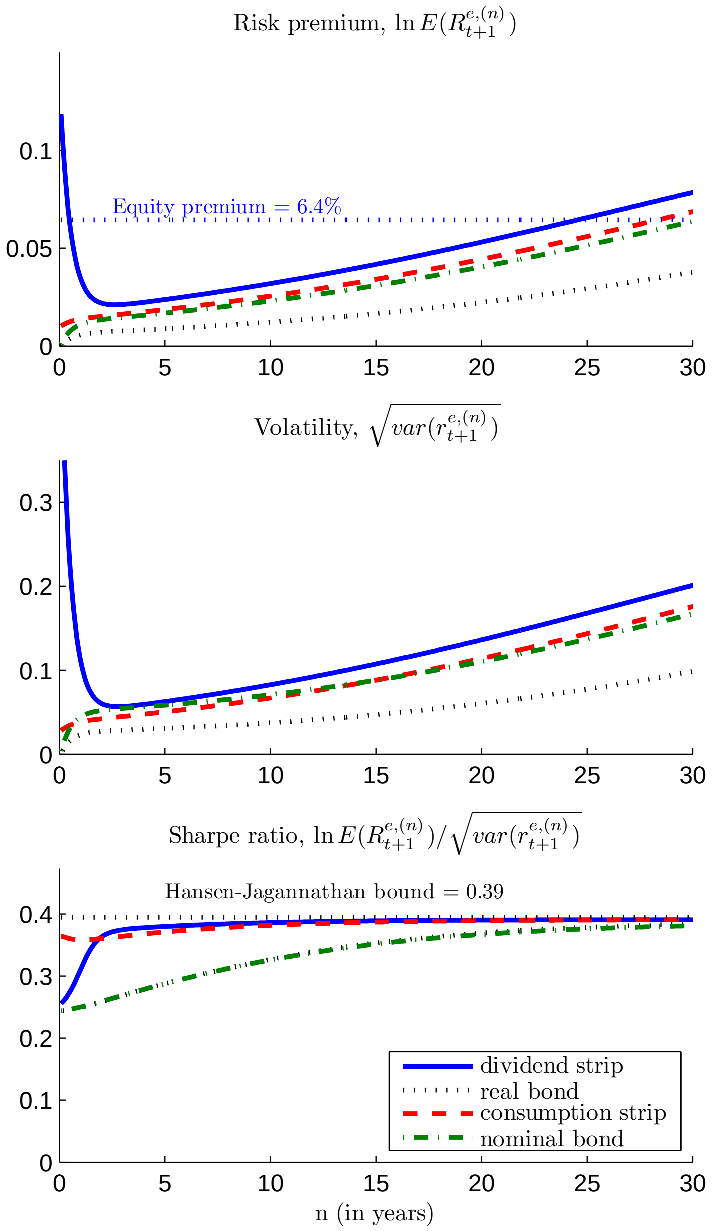
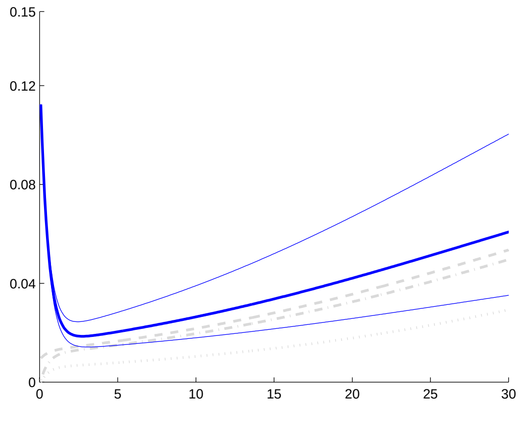
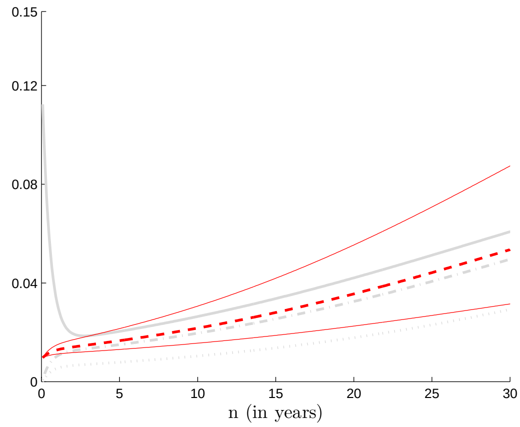


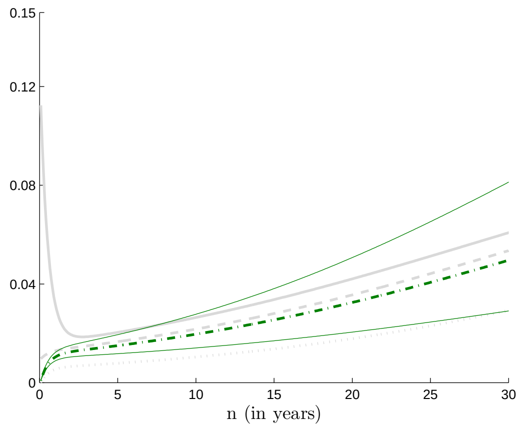
Figure 3: Unconditional term structures of equity and interest rates under macro-finance separation. Different lines associate with term structures of different cashflow claims: real bonds (dotted), nominal bonds (dash-dotted), consumption equity (dashed) and market equity (solid line).



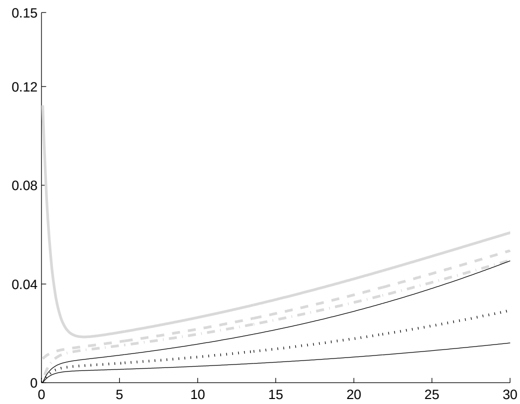
(a) *Dividend strips*.



(b) *Consumption strips*.



(c) *Nominal bonds*.



(d) *Real bonds*.

Figure 4: State dependence of the term structures of holding-period risk premia (median and interquartile range).

Level. The short end of the term structures depends primarily on the loadings on short-term cashflow risk through vector D_d, which controls the level of the term structures, whose initial value is

$$cov_t(-m_{t+1}, \Delta d_{t+1}) = x_t D_c D_d'$$

This level factor in the term structures of risk premia is depicted in figure 5a under our baseline calibration.

The level of the term structure of dividend strips can be very high because of the high leverage in corporate profits, which fluctuate more than consumption, as nominal rigidities force firms to act on real wages rather than on prices to absorb the economic shocks, and because of their positive correlation with the priced shock (consumption news). The first dividend strip tends to have a dramatically low payoff precisely in those states in which households are hit by negative consumption shocks. Moreover, the 1-month interest rate is strictly positive by the positive correlation between inflation and consumption news.

Short-run slope. A conditional mean of technology growth above average tomorrow signals good future cashflows (which increase prices) but also lower future marginal utility (which decreases prices as households want to anticipate consumption). This discount rate effect dominates the cashflow effect for all claims considered except for market equities, whose future prices therefore increase after a positive long-run shock and the more so the longer the strip duration. Since positive long-run shocks tend to arrive together with bad consumption news, it follows that this effect generates a negative slope in the term structure of equity and upward slopes in the remaining term structures.

In particular, we are able to generate a downward-sloping short end in the term structure of market equity for any calibration such that $B^{(n)}_{\zeta,d}B$ is sufficiently negative. In fact, for dividend claims the exposure to long-run shocks commands a price

$$cov_t(-m_{t+1}, B^{(n)}_{\zeta,d}\zeta_{t+1}) = x_t(C_d - \gamma C_c)\frac{1-\rho_u^n}{1-\rho_u}(\rho + \psi_c\phi)\phi\sigma^2$$

which is a negative number under $\rho < -\psi_c\phi$ for a sufficiently large degree of price rigidities. Namely, the risk premium of long-run shocks commanded by dividend strips is a negative and convex function of maturity, and the analogous factor in consumption strips and zero-coupon bonds (real and nominal) is a positive and concave function of maturity, a shown in figure 5b under our baseline calibration.

Long-run slope. The loading of tomorrow's yields on surplus consumption captures the properties of the premium commanded by long-duration claims; all term structures display an upward slope at the long end, a property that is driven by the perfectly negative correlation between shocks to consumption and to the price of risk. Tomorrow's price of long-duration claims is low, and hence holding-period returns are low, precisely in those states of the world in which surplus consumption is low, as households forecast lower future marginal utility as their habits adjust to the lower consumption level and hence require compensation to shift resources forward in time.

In particular, the loadings of yields on surplus consumption converge to the positive number $B^{(\infty)}_{s,d} = \gamma$ for any dividend process, with the exception of a knife-edge case (see Lopez et al., 2015), with the speed of convergence controlled by the persistence of habits. Since shouldering surplus-consumption shocks is equivalent to shouldering consumption shocks, the habit-related loading of infinite-duration zero-coupon cashflow claims commands a strictly positive price

$$cov_t(-m_{t+1}, B^{(\infty)}_{s,d}\hat{s}_{t+1}) = \gamma^2 \Lambda_t (1 + \Lambda_t) ||D_c||^2$$

Figure 5c plots these loadings under our baseline calibration listed in table 2.

3.4.2. Dynamic value decomposition: Borovicka-Hansen elasticities

The 3-factor decomposition of the one-month ahead volatility in strip returns is deeply linked with the shock-exposure and shock-price elasticities proposed by Borovicka and Hansen (2014) as measures to quantify the exposure of cashflows over alternative horizons to shocks and the corresponding compensation commanded by investors. In particular, Lopez et al. (2015) show that one can write holding-period risk premia as

$$\ln E_t R^{e,(n)}_{d,t+1} = \underbrace{\varepsilon^{(n)}_{g,t}}_{\substack{\text{discount rate shock-}\\ \text{exposure elasticity}}} - \underbrace{\varepsilon^{(n)}_{p,t}}_{\substack{\text{discount rate shock-}\\ \text{price elasticity}}} + \underbrace{var_t(m_{t+1})}_{\substack{\text{precautionary}\\ \text{motive}}}$$

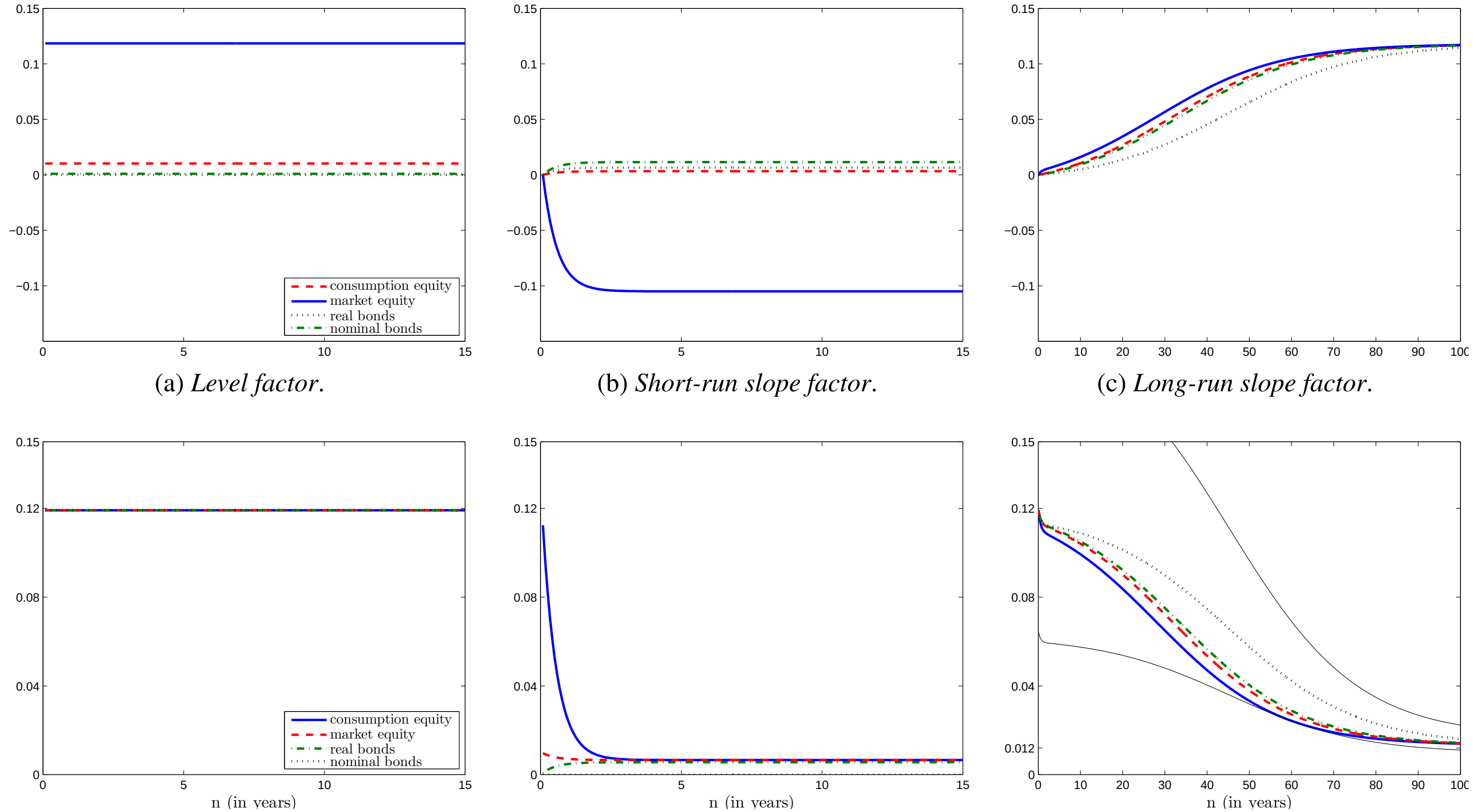
where $\varepsilon^{(n)}_{g,t}$ and $\varepsilon^{(n)}_{p,t}$ denote the elasticities of expected future cashflows and of expected future returns to a marginal increase in exposure at $t+1$ along direction $\alpha_t = x_t D_c$. Therefore, holding-period risk premia are equivalent to a strictly positive level factor (households require some compensation to save when facing uncertainty around future marginal utility), plus the elasticity of future dividends on positive consumption news (cashflow effect) less the elasticity of future investors' compensation on consumption news (discount rate effect).

A marginal increase in exposure with the same direction as a (scaled) consumption shock recovers what movement in expected cashflows and returns associates with that shock. Figure 2 plots these elasticities. On the one hand, positive consumption news associates with positive and partially mean-reverting dividend and consumption news as well as with disinflationary news. On the other hand, positive consumption news associate with lower marginal utility in the near future driven by higher future growth as well as with higher marginal utility in the very long run owing to a habit level slowly growing towards the higher consumption level. Tomorrow's cashflow and discount rate effects combine to explain holding-period risk premia, which will increase with positive cashflow news and decrease with positive discount rate news that will depress tomorrow's prices.

3.4.3. Diagnostics

To gain further insight into the properties of our model of the stochastic discount factor we study the diagnostic decompositions of the discount factor proposed by Alvarez and Jermann (2005) and Hansen and Scheinkman (2009).

In the context of an essentially-affine approximation, Lopez et al. (2015) show how the martin-



(a) *Level factor*. (b) *Short-run slope factor*. (c) *Long-run slope factor*.

(d) *Precautionary motive*. (e) *Discount rate shock-exposure elasticities*. (f) *Discount rate shock-price elasticities*.

Figure 5: 3-factor decomposition (upper panel) and Borovicka-Hansen dynamic value decomposition (bottom panel) of holding-period risk premia of different zero-coupon cashflow claims. The bottom panel plots annualized shock-exposure and shock-price elasticities after a marginal increase in exposure along the direction $x_t D_c$. Thin solid lines in the plots of shock-price elasticities represents the interquartile range for the elasticities of real bonds. Decompositions are such that holding-period risk premia (figure 1a) = [(a)+(b)+(c)] = [(d)+(e)-(f)].

gale component of the stochastic discount factor is

$$m_{t+1}^P = \begin{cases} -\frac{1}{2}x_t^2\|D_c\|^2 - x_t D_c \varepsilon_{t+1}, & \text{if } \phi = \xi_1 = 0 \\ -\frac{1}{2}\gamma^2\sigma^2 \begin{bmatrix} 1 \\ \frac{\phi}{1-\rho_u} \end{bmatrix}' \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{\phi}{1-\rho_u} \end{bmatrix} - \gamma\sigma \begin{bmatrix} 1 \\ \frac{\phi}{1-\rho_u} \end{bmatrix}' \Sigma\varepsilon_{t+1} & \text{elsewhere} \end{cases}$$

which is discountinuous at $\phi = \xi_1 = 0$, has trivial properties only under trend-stationary technology, and implies the approximate entropy ratio

$$\begin{aligned} \omega_t &= \frac{var_t(m_{t+1}^P)}{var_t(m_{t+1})} \\ &= \begin{cases} 1 & \text{if } \phi = \xi_1 = 0 \\ \frac{\gamma\sigma^2\left[1+\frac{2\rho\phi}{1-\rho_u}+\left(\frac{\phi}{1-\rho_u}\right)^2\right]}{(1-\rho_s)(1-2\hat{s}_t)}, & \text{elsewhere} \end{cases} \end{aligned} \tag{13}$$

The martingale component of the stochastic discount factor reveals a permanent component in the marginal utility of consumption such that shocks to surplus consumption (if $\phi = \xi_1 = 0$) or shocks to the predictable component of consumption (if $\phi \neq 0$ or $\xi_1 \neq 0$) have a permanent effect on the marginal utility of wealth even though both risk aversion and the predictable component of consumption are stationary.

Consider two extreme cases, $\phi = 0$ (random-walk technology) and $[\phi; \rho] = [1 - \rho_u; -1]$ (trend-stationary technology). The case of trend-stationary technology implies $m_{t+1}^P = 0$ because there are no permanent shocks to the marginal utility of wealth. The case of random-walk technology, combined with a zero spillover parameter $\xi_1 = 0$, implies a variance ratio (13) constant at unity, and hence a trivial transient component of the stochastic discount factor. This property is appealing in that it satisfies a diagnostic property advocated by Alvarez and Jermann (2005); however, it would predict an always flat real bond term structure as well as no time-variation in the relative importance of the permanent and transient components, which seems at odds with the return forecastability literature (see Lettau and Ludvigson, 2010; Koijen et al., 2010; and Lopez et al., 2015 for more details).[21]

Therefore, intermediate parametrizations that display unit-root dynamics with some amount of mean reversion produce a model of the stochastic discount factor that displays three key realistic features: a time-varying permanent component, a time-varying transient component, and time-variation in the relative importance of the permanent and transient components.

[21] If the real bond loadings on surplus consumption are on the stable path, it is extremely difficult to have an average entropy ratio close to one, as advocated by Alvarez and Jermann (2005); our baseline calibration produces an entropy ratio of 2.4%. The finding in Alvarez and Jermann of real and nominal variance ratios close to one rests however on proxies for the unobservable infinite-horizon zero-coupon bonds; in our model one can show that using a 20-year bond as a proxy for the infinite-duration bond associates with entropy ratios much closer to unity (67.5%); the high persistence of surplus consumption is responsible for the low speed of convergence of the loadings, as shown in figure 5c. The same is true if we consider nominal payoffs and a decomposition of the nominal stochastic discount factor.

3.4.4. Varying the degree of nominal rigidity

Figure 6 shows the effect of price stickiness and highlights its role in generating an initially downward-sloping term structure of market equity and in flattening the bond yield curve. Equilibrium risk premia and volatilities on zero-coupon equities shift upwards as the degree of nominal price rigidity increases, whereas the opposite occurs for zero-coupon nominal bonds. In the limiting case as nominal rigidities disappear (price duration = 1 month) there is no endogenous difference between the real and nominal bond term structures and between the term structures of consumption and market equity.

The effect on the term structures is mainly driven by cashflows, as stickier prices make dividends more volatile (which exacerbates the negative slope in the term structure of equity) and the conditional mean of consumption growth and inflation more stable (which flattens the term structure of nominal interest rates and reduces the inflation risk premium). Note how a similar flattening of the term structure occurs also for zero-coupon real bonds, driven by the weaker discount rate effect.

Finally, it is worth noting the highly nonlinear effect of increasing the degree of price rigidities (or symmetrically of decreasing the anti-inflationary stance), which stems from the convexity of the equilibrium coefficients (e.g., ψ_c) on the key parameters.

3.4.5. Varying the monetary policy stance

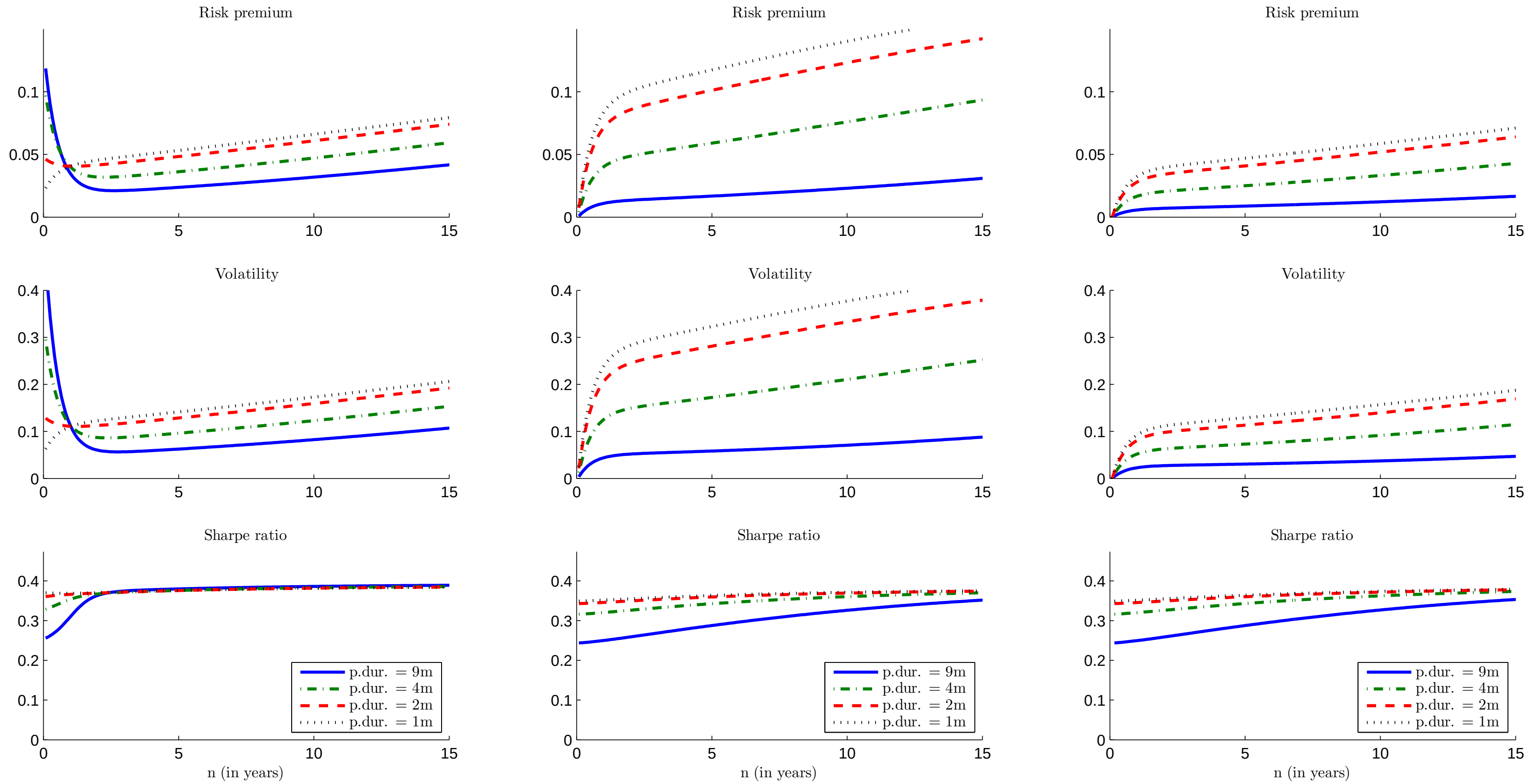
Figure 7 shows the endogenous effect of monetary policy on the term structures of equity and interest rates. The effect of a weaker anti-inflationary stance (lower Taylor rule coefficients ϕ_π and ϕ_y) is similar to the effect of larger nominal rigidities, except for the opposite effect on the inflation risk premium. Equilibrium risk premia and volatilities on zero-coupon equities shift upwards as the policy responds less aggressively to inflation, whereas the opposite occurs for zero-coupon nominal bonds.

The effect on the term structures is mainly driven by cashflows, as a less aggressive anti-inflationary stance makes dividends and inflation more volatile (which exacerbates the negative slope in the term structure of equity and increases the inflation risk premium) and the conditional mean of consumption growth more stable (which flattens the term structure of nominal and real interest rates via a weaker discount rate effect).

4. Conclusion

We incorporate risk premia variation arising from Campbell-Cochrane external habits in a standard macro model with nominal rigidities. We propose a method to break the apparent tradeoff between either matching the dynamics of macroeconomic variables or asset pricing dynamics in nonlinear habit models. The notion of macro-finance separation (and arbitrarily small departures from it) is shown to be useful for incorporating large discount rate variation in a DSGE framework while preserving the model's ability to fit quantities.

We derive testable implications for the term structures of equity and interest rates that conform with recent capital market evidence, including a downward-sloping term structure of equity returns and volatilities, upward-sloping term structures of nominal and real interest rates, and a positive inflation risk premium. The model can be easily extended to study the reaction of capital markets

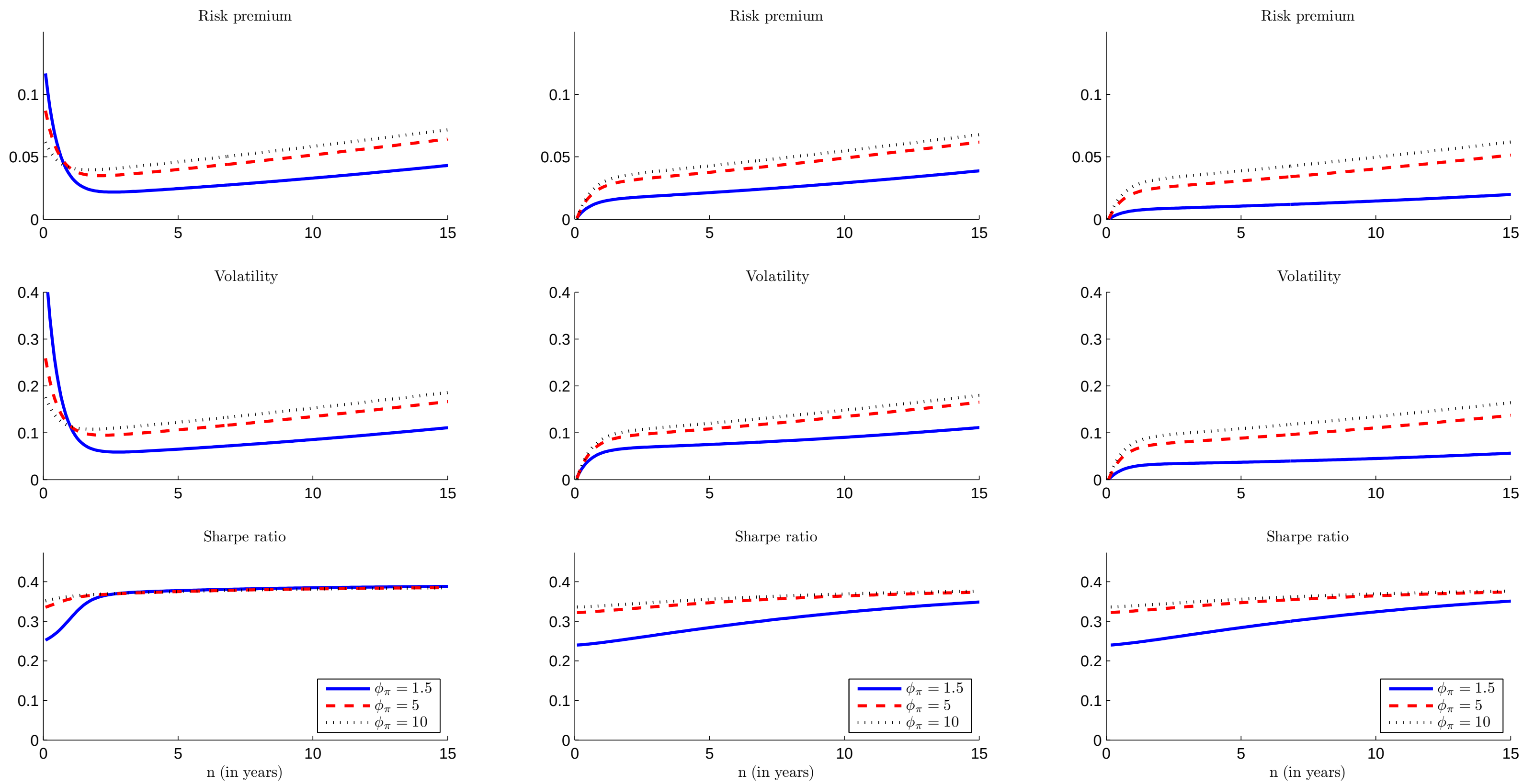


(a) *Dividend strips*.

(b) *Nominal bonds*.

(c) *Real bonds*.

Figure 6: Term structure of dividend strips, nominal interest rates and real interest rates for different degrees of price stickiness under macro-finance separation. Different lines represent different calibrations for the average price duration: one month (dotted), six months (dashed), nine months (dash-dotted) and twelve months (solid line).



(a) *Dividend strips*. (b) *Nominal bonds*. (c) *Real bonds*.

Figure 7: Term structure of dividend strips, nominal interest rates and real interest rates for different policy rule parametrizations. Different lines represent different calibrations for the anti-inflationary stance.

to unexpected monetary news, which our model is naturally able to address as it displays time-varying risk premia within a model of quantities that is appropriate to study the effect of monetary disturbances.

Our framework remained parsimonious along many dimensions that can easily be generalized. In particular, further work could relax the two-shock structure and the univariate price of risk. The introduction of demand shocks may help to mitigate the correlation puzzle (e.g., Albuquerque, Eichenbaum, Papanikolau and Rebelo, 2015, for a recent exposition).

Finally, we work under a full macro-finance separation, which is likely an unnecessarily strong requirement; namely, small departures from macro-finance separation are most likely empirically valid descriptions of the data, as they would for example include the case of stochastic capital accumulation. An estimated model that builds on our framework could identify such spillovers, and hence the parameter vector ξ. In this context, the essentially-affine approximation by Lopez et al. (2015) is particularly convenient in estimation in that it permits the use of linear filtering techniques.

Appendix

A. Campbell-Cochrane habit specification in a production economy

The law of motion of surplus consumption assumed by Campbell and Cochrane (1999) in their endowment economy with random-walk consumption can be cast in three *equivalent* specifications:

$$\hat{s}_{t+1} = \rho_s \hat{s}_t + \Lambda(\hat{s}_t)(E_{t+1} - E_t)c_{t+1} \tag{A.1a}$$

$$= \rho_s \hat{s}_t + \Lambda(\hat{s}_t)(\Delta c_{t+1} - \mu) \tag{A.1b}$$

$$= \rho_s \hat{s}_t + \Lambda E_t(\Delta c_{t+1} - \mu) + \Lambda(\hat{s}_t)(E_{t+1} - E_t)c_{t+1} \tag{A.1c}$$

where $\mu = E(\Delta c)$. The equality breaks down however once we allow for a predictable component in consumption growth, consistent with a generic production economy.[22] To understand what specification we should retain in a production economy, note how there is a strong reason to prefer specification (A.1a) owing to its implications for the risk-free rate and for the relationship between consumption and the habit level.

A.1. Local structure and predeterminedness

As shown by Campbell and Cochrane (1999) and Lynch and Randall (2011), specifications (A.1b) and (A.1c) imply the local habit structure

$$x^c_{t+1} = xc + c_{t+1} - \sum_{j=0}^{\infty} \rho_s^j \Delta c_{t-j+1} + O(\|\varepsilon\|^2)$$

$$= xc + \sum_{j=0}^{\infty} \theta_j \Delta c_{t-j+1} + O(\|\varepsilon\|^2)$$

[22] For example, in a recent study of Campbell-Cochrane habits with non-random-walk cashflows, Lynch and Randall (2011) adopt specification (A.1c).

where $\theta_j \equiv 1 - \rho_s^j$ and $xc \equiv \ln(1 - S)$, so the consumption habit is a slow moving average of past consumption growth such that consumption growth moves transitorily consumption away from habits. Specification (A.1a) implies a habit structure that also depends on what people expect to consume,

$$\begin{aligned} x_{t+1}^c &= xc + c_{t+1} - \sum_{j=0}^{\infty} \rho_s^j \varepsilon_{t-j+1}^c + O(\|\varepsilon\|^2) \\ &= xc + \sum_{j=0}^{\infty} E_{t-j} \Delta c_{t-j+1} + \sum_{j=0}^{\infty} \theta_j \varepsilon_{t-j+1}^c + O(\|\varepsilon\|^2) \end{aligned}$$

so the consumption habit is the sum of past anticipated consumption movements and of a slow moving average of past consumption shocks, which receive their full weight only asymptotically ($\lim_{j \to \infty} \theta_j = 1$); only unanticipated movements in consumption move consumption away from habits. Surplus consumption is thus basically detrended consumption.

Since $\theta_0 = 0$, the habit level is locally predetermined, $x_{t+1}^c = E_t x_{t+1}^c$, under all specifications.

A.2. Relationship between consumption and the habit level

Specifications (A.1a), (A.1b) and (A.1c) imply the respective relationship between consumption and the habit level (see the online appendix for more details)

$$\begin{aligned} \frac{\partial x_t^c}{\partial c_t} &= 1 - \frac{\Lambda(\hat{s}_{t-1})}{\exp(-s_t) - 1} \frac{(E_t - E_{t-1}) \mathcal{M}_t^c}{\mathcal{M}_t^c} \\ \frac{\partial x_t^c}{\partial c_t} &= 1 - \frac{\Lambda(\hat{s}_{t-1})}{\exp(-s_t) - 1} \\ \frac{\partial x_t^c}{\partial c_t} &= 1 - \frac{\Lambda(\hat{s}_{t-1})}{\exp(-s_t) - 1} + \frac{\Lambda(\hat{s}_{t-1}) - \Lambda}{\exp(-s_t) - 1} \frac{E_{t-1} \mathcal{M}_t^c}{\mathcal{M}_t^c} \end{aligned}$$

with $\mathcal{M}_t^c$ the shadow value of surplus consumption.

It follows that, in the steady state, consumption habits move strictly positively with consumption, $\partial x^c / \partial c = 1$, under specification (A.1a) but they are unrelated with consumption, $\partial x^c / \partial c = 0$, under specifications (A.1b) and (A.1c), a property that leads to the critique by Ljungqvist and Uhlig (2015), who look at the second derivative $\partial^2 x_t^c / \partial c_t^2$ and note that in a neighborhood of the steady state the habit process can move strictly negatively with consumption. The reason specification (A.1a) bypasses Ljungqvist and Uhlig's critique is that the equilibrium expression for the ex-ante value of consumption is no longer a structural relation in a production economy but an outcome of optimization.

A.3. No risk-free rate puzzle

The respective equilibrium risk-free rates under specifications (A.1a), (A.1b) and (A.1c) are[23]

$$r_t = r + \gamma E_t(\Delta c_{t+1} - \mu) \tag{A.2a}$$
$$r_t = r + x_t E_t(\Delta c_{t+1} - \mu) \tag{A.2b}$$
$$r_t = r + x E_t(\Delta c_{t+1} - \mu) \tag{A.2c}$$

where $x_t \equiv \gamma(1 + \Lambda_t)$ is the price of risk. As shown by equations (A.2b) and (A.2c), specifications (A.1b) and (A.1c) imply a distorted dynamic IS equation relative to a power-utility specification that would imply a risk-free rate puzzle. Note in fact how a large price of risk $x = \gamma/S$ is necessary to generate a large equity premium; the parametrization $S < 1$ is the element that amplifies the coefficient of risk aversion (see the online appendix) while remaining neutral on the risk-free rate, and that thereby allows for breaking the tradeoff between solving the equity premium and the risk-free rate puzzles in the habit framework. We therefore discard specifications (A.1b) and (A.1c) on the ground that they would kill the central idea of the Campbell-Cochrane habits. We thus retain specification (A.1a) and the associated dynamic IS equation (A.2a).

A.4. Home consumption habits

Our home consumption habits can produce a macro-finance separation, and hence break the quantity puzzle, because the same state drives both surplus market and home consumption, so the respective effects on consumption-labor decisions can offset one another.

The local microfoundations of our home consumption habit parallel those of the market consumption habit. We choose the steady-state coefficient Z to minimize the distance

$$\iota = \min_{Z \in (0,1)} \left\| \frac{Z}{1-Z}\frac{1-S}{S}\varepsilon^c_{t+1} - \varepsilon^h_{t+1} \right\|^2$$

of the home consumption habit from local predeterminedness near the steady state. Therefore, the home consumption habit can be written locally as

$$\begin{aligned} x^h_{t+1} &= xh + h_{t+1} - \sum_{j=0}^{\infty} \rho_s^j \varepsilon^h_{t-j+1} + O(\|\iota\xi, \iota\phi, \varepsilon\|^2) \\ &= xh + \sum_{j=0}^{\infty} E_{t-j}\Delta h_{t-j+1} + \sum_{j=0}^{\infty} \theta_j \varepsilon^h_{t-j+1} + O(\|\iota\xi, \iota\phi, \varepsilon\|^2) \end{aligned}$$

with $xh \equiv \ln(1 - Z)$, where ϕ indexes the policy in place and is such that $\phi = 0$ implies the flexible-price equilibrium, where ξ indexes the distance from macro-finance separation ($\xi = 0$). Note that the habit is locally predetermined when $\iota = 0$ even as we move arbitrarily far from the macro-financially separate, flexible-price equilibrium; for example, $\iota = 0$ whenever market

[23] For simplicity, we turn off the spillover parameter ξ_1 as it adds nothing to the argument.

and home consumption shocks are perfectly correlated as under the commonly used one-shock technology structure.

The home consumption habit is the sum of past anticipated home consumption movements and of a slow moving average of past home consumption innovations, which receive their full weight only asymptotically ($\lim_{j\to\infty}\theta_j = 1$); only unexpected movements in home consumption move home consumption away from habits, which coincide with home consumption in the long run. Surplus home consumption is thus basically detrended home consumption.

Finally, home consumption habits relate with home consumption via

$$\frac{\partial x_t^h}{\partial h_t} = 1 + \frac{1-N_t}{N_t}\frac{(1-\alpha)(1+\xi_2)\Lambda(s_{t-1})}{\exp(-z_t)-1} \times \frac{(E_t - E_{t-1})\mathcal{M}_t^h}{\mathcal{M}_t^h}$$

and hence the habit moves strictly positively with home consumption in the steady state, $\partial x^h/\partial h = 1$.

B. Capital accumulation

In this section we allow for nontrivial capital accumulation driven by

$$K_{t+1} = (1-\delta)K_t + \Phi\Big(\frac{I_t}{K_t}\Big)K_t$$

where δ is the depreciation rate and capital is costly adjusted according to the function

$$\Phi\Big(\frac{I_t}{K_t}\Big) = \frac{e^{\mu}-1+\delta}{1-\xi_3^2} + \frac{\widetilde{\delta}^{\frac{1}{\xi_3}}}{1-\frac{1}{\xi_3}}\Big(\frac{I_t}{K_t}\Big)^{1-\frac{1}{\xi_3}}, \quad \text{with} \quad \widetilde{\delta} \equiv \frac{e^{\mu}-1+\delta}{1+\frac{1}{\xi_3}}$$

The parametric form of capital adjustment costs is standard (e.g., Jermann, 1998; Boldrin et al., 2001; Binsbergen et al., 2012b) and is calibrated to imply the steady-state relations $I/K = \widetilde{\delta}$, $\Phi(I/K) = I/K$, $\Phi'(I/K) = 1$ and $-\Phi''(I/K)I/K = 1/\xi_3$; note that we allow the steady-state investment-capital ratio to depend on ξ_3 to avoid an unappealing discontinuity at $\xi_3 = 0$. Through the adjustment cost curvature, $1/\xi_3$, investment is determined by Tobin's Q, which in our frictionless setting equals the expected discounted value of future market dividends (Hayashi, 1982); since the discounting is done via the Campbell-Cochrane pricing kernel, surplus consumption can spill over onto investment.

Therefore, the spillover of the time-varying risk aversion on quantities is now controlled by parameter $\xi = [\xi_1;\xi_2;\xi_3]$ with $\xi_3 \geq 0$. As usual, parameter ξ_1 controls the spillover on the consumption-saving tradeoff and parameter ξ_2 controls the spillover on the consumption-labor tradeoff. Additionally, parameter ξ_3 controls the spillover on consumption-investment decisions; the absence of this type of spillover implies zero investment.

The only adjustment of the baseline model in this setting is the shape of the shock structure that drives surplus home consumption, which we now specify in terms of the function

$$f_h[H_t] = \ln[A_t^{\alpha}(A_t - H_t)^{1-\alpha} - \widetilde{\delta}e^{\alpha\mu t}Q_t^{\xi_3}K_t^{1-\alpha}]^{\frac{1}{1-\alpha}}$$

Surplus home consumption is no longer driven just by shocks to home consumption but is now also driven by shocks to the market value of the capital stock owned by consumers. This specification is arbitrarily close to the baseline specification for a curvature ξ_3 close to zero, and it is necessary to control the spillover on the intratemporal rate of substitution. In fact, market clearing, $Y_t = C_t + I_t$, and the optimality condition for investment

$$I_t = \widetilde{\delta} Q_t^{\xi_3} K_t, \qquad \xi_3 < \infty$$

imply that, in equilibrium, $(1-\alpha)(E_{t+1} - E_t) f_h[H_{t+1}] = (E_{t+1} - E_t) c_{t+1}$ and therefore

$$\hat{z}_t = (1 + \xi_2)\hat{s}_t$$

as required to control the intratemporal spillover.

We can therefore state in proposition 3 the requirements for macro-finance separation in the context of nontrivial capital accumulation:

Proposition 3. *Given the spillover parameter* $\xi \equiv [\xi_1; \xi_2; \xi_3] \in \mathbb{R}^3$, *for any value of the preference parameter* $\gamma \in \mathbb{R}_+$,

(a) *there is a unique value of parameter* $[\xi_2; \xi_3] = [0; 0]$ *such that the flexible-price competitive equilibrium is macro-financially separate, for any* $\xi_1 \in \mathbb{R}$*;*
(b) *there is a unique value of parameter* $\xi = [0; 0; 0]$ *such that the sticky-price competitive equilibrium under a Taylor rule in inflation and output is macro-financially separate.*

Starting from a macro-financially separate equilibrium we can then allow for an arbitrarily small spillover by varying ξ; in particular, movements in ξ_3 allow for positive investment as a function of Q and for an equilibrium arbitrarily close to the equilibrium with deterministic capital ($\xi_3 = 0$).

C. Internal habits

The marginal utilities of consumption and home consumption when consumers internalize the endogeneity of habits are

$$\frac{\partial U_t^{\text{int.}}}{\partial C_t} = C_t^{-\gamma} S_t^{1-\gamma} + C_t^{-1} \Lambda(\hat{s}_{t-1})(E_t - E_{t-1})\mathcal{M}_t^c$$
$$\frac{\partial U_t^{\text{int.}}}{\partial N_t} = -\chi A_t H_t^{-\gamma} Z_t^{1-\gamma} + (1-\alpha)(1+\xi_2)\frac{\Lambda(s_{t-1})}{N_t}(E_t - E_{t-1})\mathcal{M}_t^h$$

where $\mathcal{M}_t^c$ and $\mathcal{M}_t^h$ are the shadow values of surplus market and home consumption, respectively. The online appendix details the derivation. On the one hand, a positive market (home) consumption shock means a lower marginal value of market (home) consumption; on the other hand, a positive market (home) consumption shock increases the habit level and thereby increases the marginal value of market (home) consumption. When habits are internal, households take into account also the second effect and they also become sensitive to unexpected movements in the shadow value of the surplus levels, which depend on current and future market and home consumption.

If habits are internal, people balance their static habit motives as well as their precautionary savings and intertemporal substitution motives only if $\gamma = 1$; a unitary elasticity of intertemporal substitution has the effect of the two habits on the marginal utility of consumption exactly offset, so the marginal utility of consumption (and hence the stochastic discount factor) reduces to the one under power utility,

$$\frac{\partial U_t}{\partial C_t} = C_t^{-1}$$

Proposition 4. *Given the spillover parameter $\xi \equiv [\xi_1; \xi_2; \xi_3] \in \mathbb{R}^3$, for any value of the preference parameter $\gamma \in \mathbb{R}_+$, the Pareto optimum is macro-financially separate if and only if $\gamma = 1$, for any $\xi \in \mathbb{R}^3$.*[24]

The macro-financially separate Pareto optimum displays the same low and stable risk premia as under a power-utility specification; all time-variation in risk premia is symptomatic of the presence of an externality, as Pareto optimal asset prices associate with log-utility investors. It follows that the Pareto optimal dynamic IS equation is

$$r_t^{\text{int.}} = -\ln(\beta) - \frac{1 - \rho_s - \xi_1}{2} S^2 + E_t \Delta c_{t+1}$$

Against this background, if we take to the logical extreme the critique by Lettau and Uhlig (2000), a necessary diagnostic requirement for a model with habits (or, more generally, for any model that incorporates risk premia variation into a macro model) to be deemed admissible is that it can be calibrated to display a macro-finance separation. In this context, internal habits have dramatically different asset pricing implications than external habits; namely, the internal-habit economy would reduce to the power-utility model not only in terms of quantity implications but also in terms of asset pricing implications. The trivial asset pricing implications of the macro-financially separate internal-habit specification are the reason we favor the external-habit specification.

D. Essentially-affine approximation

The practical approximation proceeds in three steps. See Lopez et al. (2015) for a treatment in greater generality and detail, and for a comparison of its quality with alternative solution methods.

D.1. First step

Cashflows. Loglinearize the first-order conditions driving quantities and solve for the approximate quantity dynamics

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + C_c \zeta_t + D_c \varepsilon_{t+1} + O(\|\zeta_t, \varepsilon_{t+1}\|^2) \\ \Delta d_{t+1} &= \mu_d + C_d \zeta_t + D_d \varepsilon_{t+1} + O(\|\zeta_t, \varepsilon_{t+1}\|^2) \end{aligned}$$

[24]Since under $\gamma = 1$ the stochastic discount factor under internal habits reduces to the one under power-utility, parameter ξ_3 can be left unrestricted to produce a macro-finance separation.

where c is log consumption, d is the log of an arbitrary cashflow process, and where the state ζ_t that drives quantities follows the $VAR(1)$ process

$$\zeta_{t+1} = A\zeta_t + B\varepsilon_{t+1}$$

with $\varepsilon_t \sim Niid(0, I)$ a vector of shocks.

Discount rates. The stochastic discount factor is

$$\begin{aligned} m_{t+1} &= \ln(\beta) - \gamma\mu_c - \gamma C_c\zeta_t + \gamma(1-\phi)\hat{s}_t - x_t D_c\varepsilon_{t+1} + O(\|\zeta_t, \varepsilon_{t+1}\|^2) \\ &= -r_t - \frac{1}{2}x_t^2\|D_c\|^2 - x_t D_c\varepsilon_{t+1} + O(\|\zeta_t, \varepsilon_{t+1}\|^2) \end{aligned}$$

where the residual term comes from the approximate equation for consumption growth and the last equality is by the no-arbitrage relation $r_t = -\ln E_t M_{t+1}$. The time-varying price of risk follows a nonlinear process $x : \hat{s}_t \mapsto x(\hat{s}_t) = \frac{\gamma}{S}(1 - 2\hat{s}_t)^{1/2}$ that is responsible for the absence of a closed-form solution to the problem, which would otherwise take an exponential-affine form. We therefore approximate the endogenous and nonlinear dynamics of the price of risk as

$$x_t = x(0) + x'(0)\hat{s}_t + O(\|\hat{s}_t\|^2) \tag{D.1}$$

$$x_t^2 = x(0)^2 + 2x(0)x'(0)\hat{s}_t + O(\|\hat{s}_t\|^2) \tag{D.2}$$

where, since in the Campbell-Cochrane specification $x''(0)x(0) + x'(0)^2 = 0$, the residual in equation (D.2) is exactly zero.

Thus approximated, the price of risk has an essentially-affine form and thereby allows for an exponential-affine solution for equilibrium yields, since all sources of stochastic volatility owe to the time-varying price of risk and since the risk-free rate is exactly affine in the state vector.

D.2. Second step

Guess the exponential-affine solution for yields $y_{d,t}^{(n)} \equiv -\frac{1}{n}\ln(P_t^{(n)}/D_t)$,

$$y_{d,t}^{(n)} = -\frac{1}{n}A^{(n)} - \frac{1}{n}B_\zeta^{(n)}\zeta_t - \frac{1}{n}B_s^{(n)}\hat{s}_t + O(\|\zeta_t, \hat{s}_t, \varepsilon_{t+1}\|^2)$$

and verify it by the fundamental no-arbitrage pricing formula $0 = \ln E_t(M_{t+1}R_{t+1}^i)$.

In fact, given the Gaussianity of log returns $r_{d,t+1}^{(n)} \equiv \Delta d_{t+1} - (n-1)y_{d,t+1}^{(n-1)} + ny_{d,t}^{(n)}$, we have

$$\begin{aligned} 0 &= E_t m_{t+1} + dp_t^{(n)} - E_t dp_{t+1}^{(n-1)} + E_t\Delta d_{t+1} + \frac{1}{2}var_t(m_{t+1} - dp_{t+1}^{(n-1)} + \Delta d_{t+1}) \\ &= \ln(\beta) + \mu_d - \gamma\mu_c - A^{(n)} + A^{(n-1)} + [C_d - \gamma C_c - B_\zeta^{(n)} + B_\zeta^{(n-1)}A]\zeta_t + [\gamma(1-\phi) - B_s^{(n)} + B_s^{(n-1)}\phi]\hat{s}_t \\ &\quad + \frac{1}{2}\|D_c\|^2 x_t^2 + \frac{1}{2}V_{n-1,t}V_{n-1,t}' - x_t D_c V_{n-1,t}' + O(\|\zeta_t, \hat{s}_t, \varepsilon_{t+1}\|^2) \end{aligned}$$

where $V_{n-1,t} = D_d + B_\zeta^{(n-1)} B - B_s^{(n-1)} D_c + x_t B_s^{(n-1)} D_c/\gamma$. Therefore, using equations (D.1) and (D.2),

$$\begin{aligned}
0 = {} & \ln(\beta) + \mu_d - \gamma\mu_c - A^{(n)} + A^{(n-1)} + \frac{1}{2}\|V_{0,n-1} - x(0)(D_c - V_{1,n-1})\|^2 \\
& + [\gamma(1-\phi) - B_s^{(n)} + B_s^{(n-1)}\phi + x(0)x'(0)\|D_c - V_{1,n-1}\|^2 - x'(0)V_{0,n-1}(D_c - V_{1,n-1})']\hat{s}_t \\
& + [C_d - \gamma C_c - B_\zeta^{(n)} + B_\zeta^{(n-1)} A]\zeta_t + O(\|\zeta_t, \hat{s}_t, \varepsilon_{t+1}\|^2)
\end{aligned}$$

which identifies the exponential-affine solution as the solution to the Riccati equations

$$\begin{aligned}
A^{(n)} &= A^{(n-1)} + \ln(\beta) + \mu_d - \gamma\mu_c + \frac{1}{2}\|V_{0,n-1} - x(0)(D_c - V_{1,n-1})\|^2 \\
B_\zeta^{(n)} &= B_\zeta^{(n-1)} A + C_d - \gamma C_c \\
B_s^{(n)} &= B_s^{(n-1)}\phi + \gamma(1-\phi) + x(0)x'(0)\|D_c - V_{1,n-1}\|^2 - x'(0)V_{0,n-1}(D_c - V_{1,n-1})'
\end{aligned}$$

with

$$\begin{aligned}
V_{0,n-1} &= D_d + B_\zeta^{(n-1)} B - B_s^{(n-1)} D_c \\
V_{1,n-1} &= \frac{1}{\gamma} B_s^{(n-1)} D_c
\end{aligned}$$

These closed-form expressions allow for computing the entire term structure of yields, $y_{d,t}^{(n)}$, from a simulated path of the state vector $[\zeta_t; \hat{s}_t]$ up to a remainder of order at least $O(\|\zeta_t, \hat{s}_t, \varepsilon_{t+1}\|^2)$.

D.3. Third step

Finally, we can use the lognormal no-arbitrage pricing formula to compute

$$\begin{aligned}
\ln E_t R_{t+1}^{e,(n)} &= x_t D_c V'_{n-1,t} \\
E_t r_{t+1}^{e,(n)} &= x_t D_c V'_{n-1,t} - \frac{1}{2} V_{n-1,t} V'_{n-1,t}
\end{aligned}$$

To simulate x_t and thereby a sample path for risk premia and return volatilities we use the exact dynamics $x(\hat{s}_t)$.

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ONLINE APPENDIX

I. Flexible-price equilibria

This section characterizes the Pareto optimum and the flexible-price equilibrium.

I.1. Pareto optimum (internal habits, flexible prices)

The Pareto optimum can be characterized as the solution to a social planner problem. However, we appeal to the welfare theorems and decentralize the economy to build intuition and gain insight into the consumption and labor margins.

I.1.1. Consumers

Internal-habit consumers maximize the intertemporal objective

$$\max \quad U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - X_t^c)^{1-\gamma} - 1}{1-\gamma} + \chi \frac{(H_t - X_t^h)^{1-\gamma} - 1}{1-\gamma} \right)$$

subject to the budget constraint and the structural habit equations, $H_t = A_t(1 - N_t)$, and

$$C_t - X_t^c = C_t S_t, \qquad \hat{s}_{t+1} = \rho_s \hat{s}_t + \Lambda(\hat{s}_t)(E_{t+1} - E_t)\ln(C_{t+1})$$

$$H_t - X_t^h = H_t Z_t, \qquad \hat{z}_{t+1} = \rho_s \hat{z}_t + (1-\alpha)(1+\xi_2)\Lambda[\hat{z}_t/(1+\xi_2)](E_{t+1} - E_t)\ln(A_{t+1} - H_{t+1})$$

Optimality requires that the joint evolution of the processes satisfies

$$\frac{\partial U_t^{\text{int.}}}{\partial C_t} = C_t^{-\gamma} S_t^{1-\gamma} + \frac{\Lambda(s_{t-1})}{C_t}(E_t - E_{t-1})\mathcal{M}_t^c \tag{I.1}$$

$$\mathcal{M}_t^c = C_t^{1-\gamma} S_t^{1-\gamma} + \beta E_t \mathcal{M}_{t+1}^c [\rho_s + \Lambda'(s_t)\varepsilon_{t+1}^c] \tag{I.2}$$

$$\frac{\partial U_t^{\text{int.}}}{\partial N_t} = -\chi A_t H_t^{-\gamma} Z_t^{1-\gamma} + (1-\alpha)(1+\xi_2)\frac{\Lambda(s_{t-1})}{N_t}(E_t - E_{t-1})\mathcal{M}_t^h \tag{I.3}$$

$$\mathcal{M}_t^h = \chi H_t^{1-\gamma} Z_t^{1-\gamma} + \beta E_t \mathcal{M}_{t+1}^h [\rho_s + \Lambda'(s_t)\varepsilon_{t+1}^c] \tag{I.4}$$

where $\mathcal{M}_t^c$ and $\mathcal{M}_t^h$ are Lagrange multipliers associated with the market and home consumption habit equations that affect the marginal utility of wealth with a time-varying loading.

I.1.2. Firms

Firms maximize period profits, $Y_t - W_t N_t$, subject to the production technology, $Y_t = A_t N_t^{1-\alpha}$, which results in the optimality condition

$$W_t = (1-\alpha)\frac{Y_t}{N_t}$$

I.1.3. Equilibrium

After imposing market clearing, $Y_t = C_t$, we can characterize the Pareto optimum by the equality between the intratemporal rate of substitution and the marginal product of labor,

$$-\frac{\partial U_t^{\text{int.}}/\partial N_t}{\partial U_t^{\text{int.}}/\partial C_t} = (1-\alpha)\frac{C_t}{N_t}$$

It is straightforward to verify how a unitary elasticity of intertemporal substitution, $\gamma = 1$, produces constant shadow values of surplus market and home consumption. Under this parametrization we have

$$M^{\text{int.}}_{t+1} = \beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-1}$$
$$-\frac{\partial U^{\text{int.}}_t/\partial N_t}{\partial U^{\text{int.}}_t/\partial C_t} = \frac{\chi C_t}{1-N_t}$$

so all intertemporal and intratemporal effects of the habit are absent. The condition $\gamma = 1$ is therefore sufficient to grant a macro-finance separation when habits are internal, for any value of the spillover parameter ξ_2.

Moreover, the condition $\xi_2 = 1$ is sufficient to grant a macro-finance separation to a first-order approximation under balanced growth, for any value of the elasticity of intertemporal substitution.

I.2. Flexible-price equilibrium (external habits)

Optimality requires that the joint evolution of the processes satisfies

$$\frac{\partial U_t}{\partial C_t} = C_t^{-\gamma} S_t^{-\gamma}$$
$$\frac{\partial U_t}{\partial N_t} = -\chi A_t H_t^{-\gamma} Z_t^{-\gamma}$$
$$-\frac{\partial U_t/\partial N_t}{\partial U_t/\partial C_t} = (1-\alpha)\frac{C_t}{N_t}$$

Thus, the competitive equilibrium is characterized by

$$\chi\frac{N_t}{(1-N_t)^{\gamma}}\hat{S}_t^{-\gamma\xi_2} = (1-\alpha)\Big(\frac{C_t}{A_t}\Big)^{1-\gamma}$$

up to an irrelevant constant. The competitive equilibrium is macro-financially separate if and only if $\xi_2 = 0$.

II. Competitive equilibrium under a Taylor rule

The full model driving quantities is, to a first-order approximation,

$$\begin{aligned}
\pi_t &= \widetilde{\beta} E_t \pi_{t+1} + \kappa(c_t - c^n_t) - \gamma\lambda\xi_2(s_t - s^n_t)\\
r_t - r^n_t &= \gamma E_t(\Delta c_{t+1} - \Delta c^n_{t+1}) - \xi_1(s_t - s^n_t)\\
r_t - r^n_t &= i_t - r^n_t - E_t\pi_{t+1}\\
i_t &= \phi_\pi \pi_t + \phi_y(c_t - a_t) + \phi_s \hat{s}_t\\
&= r^n_t + \phi_\pi\pi_t + \phi_y(c_t - c^n_t) + \phi_s(s_t - s^n_t) + v_t
\end{aligned}$$

where $v_t = -r^n_t + \phi_s s^n_t$ and where we allow for a hypothetical reaction to risk premia by monetary policy to gain better insight into the role of the Taylor rule. The state equations are

$$\begin{aligned}
a_{t+1} &= a_t + \mu + u_t + \sigma e^a_{t+1}\\
u_{t+1} &= \rho_u u_t + \phi\sigma e^u_{t+1}\\
s_{t+1} &= (1-\rho_s)s + \rho_s s_t + \Lambda_t(E_{t+1} - E_t)c_{t+1}
\end{aligned}$$

To verify the guessed stationarity of $\widehat{mc}_t$ and hence of $[\pi_t; c_t - c^n_t]$, pose the linear parametric forms $c_t - c^n_t =$

$\psi_{cs}(s_t - s_t^n) + \psi_c u_t$ and $\pi_t = \psi_{\pi s}(s_t - s_t^n) + \psi_\pi u_t$, and verify them as

$$\begin{aligned}\psi_{cs} &= \frac{\lambda\gamma\xi_2(\phi_\pi - \rho_s) - (\xi_1 + \phi_s)(1 - \widetilde{\beta}\rho_s)}{(1 - \widetilde{\beta}\rho_s)[\gamma(1 - \rho_s) + \phi_y] + \kappa(\phi_\pi - \rho_s)} \\ \psi_{\pi s} &= -\frac{\lambda\gamma\xi_2[\gamma(1 - \rho_s) + \phi_y] + (\xi_1 + \phi_s)\kappa}{(1 - \widetilde{\beta}\rho_s)[\gamma(1 - \rho_s) + \phi_y] + \kappa(\phi_\pi - \rho_s)}\end{aligned} \tag{II.1}$$

and

$$\begin{aligned}\psi_c &= \frac{\gamma(1 - \widetilde{\beta}\rho_u)}{(1 - \widetilde{\beta}\rho_u)[\gamma(1 - \rho_u) + \phi_y] + \kappa(\phi_\pi - \rho_u)} \\ \psi_\pi &= \frac{\gamma\kappa}{(1 - \widetilde{\beta}\rho_u)[\gamma(1 - \rho_u) + \phi_y] + \kappa(\phi_\pi - \rho_u)}\end{aligned}$$

which is the unique solution of the model economy as long as the system is determined. Macro-finance separation holds only if $\phi_s = -\xi_1$ and $\xi_2 = 0$, in which case solution (II.1) reduces to $\psi_{cs} = \psi_{\pi s} = 0$.

Consumption equity cashflows. Equilibrium aggregate consumption grows at rate

$$\begin{aligned}\Delta c_{t+1} &= \mu + [1 - (1 - \rho_u)\psi_c]u_t + \sigma e_{t+1}^a + \psi_c\phi\sigma e_{t+1}^u \\ &= \mu + C_c u_t + \sigma e_{t+1}^a + \psi_c\phi\sigma\varepsilon_{t+1}^u\end{aligned}$$

where $C_c \equiv [\phi_y(1 - \widetilde{\beta}\rho_u) + \kappa(\phi_\pi - \rho_u)]/\{[\gamma(1 - \rho_u) + \phi_y](1 - \widetilde{\beta}\rho_u) + \kappa(\phi_\pi - \rho_u)\} \in (0, 1)$.

Market equity cashflows. Equilibrium aggregate profits, $P_tD_t = P_tC_t - W_tN_t$, which firms pay out as dividends, are, up to a first-order approximation around the undistorted steady state,

$$d_t = c_t - \frac{1 - \alpha}{\alpha}\widehat{mc}_t$$

where average marginal costs are the inverse of average markups. Corporate profits increase with output and decrease with marginal costs. Therefore,

$$\begin{aligned}\Delta d_{t+1} &= \Delta c_{t+1} - \frac{\gamma(1 - \alpha) + \alpha + \varphi}{\alpha}[\Delta c_{t+1} - \Delta a_{t+1}] \\ &= \mu + C_d u_t + \sigma e_{t+1}^a - \frac{\gamma(1 - \alpha) + \varphi}{\alpha}\psi_c\phi\sigma e_{t+1}^u\end{aligned}$$

where $C_d \equiv 1 + \gamma^2(2/\alpha - 1)(1 - \rho_u)(1 - \widetilde{\beta}\rho_u)/\{[\gamma(1 - \rho_u) + \phi_y](1 - \widetilde{\beta}\rho_u) + \kappa(\phi_\pi - \rho_u)\} > 1$.

Nominal bond cashflows. The payoff at time $t + n$ for a n-period zero-coupon nominal bond is a unit of money, whose real value is $1/P_{t+n}$, i.e., the dividend grows at rate $\Delta d_{t+1} = \ln(1/P_{t+1}) - \ln(1/P_t) = -\pi_{t+1}$ with

$$\begin{aligned}-\pi_{t+1} &= -\psi_\pi\rho_u u_t - \psi_\pi\phi\sigma e_{t+1}^u \\ &= C_{-p}u_t - \frac{\kappa}{1 - \widetilde{\beta}\rho_u}\psi_c\phi\sigma e_{t+1}^u\end{aligned}$$

with $C_{-p} \equiv -\gamma\kappa\rho_u/\{[\gamma(1 - \rho_u) + \phi_y](1 - \widetilde{\beta}\rho_u) + \kappa(\phi_\pi - \rho_u)\} < 0$.

Real bond cashflows. The payoff at time $t + n$ for a n-period zero-coupon real bond is a unit of numeraire, i.e., the log dividend is $d_t = 0$.

III. Relationship between market and home consumption, and the habit levels

III.1. Market consumption habits

As shown by Campbell and Cochrane (1999), we can write the derivative of utility with respect to consumption as

$$\frac{\partial U_t}{\partial C_t} = C_t^{-\gamma} F_t^c$$

$$F_t^c = S_t^{-\gamma}\Big[1 - E_t \sum_{j=0}^{\infty} \beta^j \Big(\frac{C_{t+j}S_{t+j}}{C_t S_t}\Big)^{-\gamma} \frac{\partial X_{t+j}^c}{\partial C_t}\Big] \tag{III.1}$$

$$= S_t^{-\gamma}\Big[1 - \frac{\partial X_t^c}{\partial C_t}\Big] - \beta E_t \Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma} \Big[\frac{\partial X_{t+1}^c}{\partial C_t} + \frac{\partial X_{t+1}^c}{\partial X_t^c}\frac{\partial X_t^c}{\partial C_t}\Big] W_{t+1}^c \tag{III.2}$$

$$\text{with} \qquad W_t^c = S_t^{-\gamma} + \beta E_t \Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma} \frac{\partial X_{t+1}^c}{\partial X_t^c} W_{t+1}^c$$

where we used

$$\frac{\partial X_{t+j}^c}{\partial C_t} = \frac{dX_{t+1}^c}{dC_t} \prod_{h=2}^{j} \frac{\partial X_{t+h}^c}{\partial X_{t+h-1}^c}$$

$$\frac{dX_{t+1}^c}{dC_t} = \frac{\partial X_{t+1}^c}{\partial C_t} + \frac{\partial X_{t+1}^c}{\partial X_t^c}\frac{\partial X_t^c}{\partial C_t}$$

because the law of motion of surplus consumption defines an implicit function $X_{t+1}^c = X_{t+1}^c(X_t^c, C_{t+1}, C_t)$, so C_t affects X_{t+1}^c directly and via X_t^c. From the law of motion

$$\ln\Big(1 - \frac{X_{t+1}^c}{C_{t+1}}\Big) = \rho_s \ln\Big(1 - \frac{X_t^c}{C_t}\Big) + \Lambda\Big[\ln\Big(1 - \frac{X_t^c}{C_t}\Big)\Big]\Big(\ln C_{t+1} - E_t \ln C_{t+1}\Big)$$

we can easily derive

$$\frac{\partial X_{t+1}^c}{\partial X_t^c} = \frac{C_{t+1}S_{t+1}}{C_t S_t}[\rho_s + \Lambda'(s_t)\varepsilon_{t+1}^c] \tag{III.3}$$

$$\frac{\partial X_{t+1}^c}{\partial C_t} = -(1 - S_t)\frac{C_{t+1}S_{t+1}}{C_t S_t}[\rho_s + \Lambda'(s_t)\varepsilon_{t+1}^c] \tag{III.4}$$

and we can verify that $\mathcal{M}_t^c = C_t^{1-\gamma} S_t W_t^c$ in equation (I.2). Thus, we can plug equations (III.3) and (III.4) in expression (III.2) and, since $\partial U_t^{\text{int.}}/\partial C_t = C_t^{-\gamma} F_t^c$, we use equation (I.1) to deduce

$$\frac{\partial x_t^c}{\partial c_t} = 1 - \frac{\Lambda(s_{t-1})}{\exp(-s_t) - 1} \times \frac{(E_t - E_{t-1})\mathcal{M}_t^c}{\mathcal{M}_t^c} \tag{III.5}$$

It follows that consumption habits move strictly positively with consumption in the steady state, $\partial x^c/\partial c = 1$.

III.2. Home consumption habits

Analogously for the internal home consumption habit,

$$\frac{\partial U_t}{\partial H_t} = \chi H_t^{-\gamma} F_t^h$$

$$F_t^h = Z_t^{-\gamma}\Big[1 - E_t \sum_{j=0}^{\infty} \beta^j \Big(\frac{H_{t+j} Z_{t+j}}{H_t Z_t}\Big)^{-\gamma} \frac{\partial X_{t+j}^h}{\partial H_t}\Big]$$

$$= Z_t^{-\gamma}\Big[1 - \frac{\partial X_t^h}{\partial H_t}\Big] - \beta E_t \Big(\frac{H_{t+1}}{H_t}\Big)^{-\gamma} \Big[\frac{\partial X_{t+1}^h}{\partial H_t} + \frac{\partial X_{t+1}^h}{\partial X_t^h} \frac{\partial X_t^h}{\partial H_t}\Big] W_{t+1}^h$$

$$\text{with} \qquad W_t^h = Z_t^{-\gamma} + \beta E_t \Big(\frac{H_{t+1}}{H_t}\Big)^{-\gamma} \frac{\partial X_{t+1}^h}{\partial X_t^h} W_{t+1}^h$$

From the law of motion

$$\ln\Big(1 - \frac{X_{t+1}^h}{H_{t+1}}\Big) = \rho_s \ln\Big(1 - \frac{X_t^h}{H_t}\Big) + \Lambda_l \Big[\ln\Big(1 - \frac{X_t^h}{H_t}\Big)\Big]\Big[\ln[A_{t+1}^{\alpha/(1-\alpha)}(A_{t+1} - H_{t+1})] - E_t \ln[A_{t+1}^{\alpha/(1-\alpha)}(A_{t+1} - H_{t+1})]\Big]$$

we can easily derive

$$\frac{\partial X_{t+1}^h}{\partial X_t^h} = \frac{H_{t+1} Z_{t+1}}{H_t Z_t}[\rho_s + \Lambda'(s_t)\varepsilon_{t+1}^c]$$

$$\frac{\partial X_{t+1}^h}{\partial H_t} = -(1 - Z_t)\frac{H_{t+1} Z_{t+1}}{H_t Z_t}[\rho_s + \Lambda'(s_t)\varepsilon_{t+1}^c]$$

and we can verify that $\mathcal{M}_t^h = \chi H_t^{1-\gamma} Z_t W_t^h$ in equation (I.4). Thus, since $\partial U_t^{\text{int.}}/\partial H_t = \chi H_t^{-\gamma} F_t^h$, we use equation (I.3) to find

$$\frac{\partial x_t^h}{\partial h_t} = 1 + \frac{1 - N_t}{N_t} \frac{(1-\alpha)(1+\xi_2)\Lambda(s_{t-1})}{\exp(-z_t) - 1} \times \frac{(E_t - E_{t-1})\mathcal{M}_t^h}{\mathcal{M}_t^h}$$

It follows that home consumption habits move strictly positively with home consumption in the steady state, $\partial x^h/\partial h = 1$.

IV. Coefficient of risk aversion

We follow Swanson (2012) and compute the coefficient of risk aversion of a consumer faced with a mean-zero, variance-σ, state-independent gamble, which she can avoid by paying a one-time fee $\mu_t(\sigma)$. In our context, the indirect utility function of a consumer with generic budget constraint $\mathcal{A}_{t+1} = (1 + r_t)\mathcal{A}_t + W_t N_t + D_t - C_t$, where $\mathcal{A}_t$ is the household's beginning-of-period asset, is

$$V(\mathcal{A}_t; \zeta_t) = \max_{\substack{[C_t; N_t]: \\ C_t + \mathcal{A}_{t+1} = (1+r_t)\mathcal{A}_t + W_t N_t + D_t}} \frac{(C_t - X_t^c)^{1-\gamma} - 1}{1-\gamma} + \frac{\chi[A_t(1 - N_t) - X_t^h]^{1-\gamma} - 1}{1-\gamma} + \beta E_t V(\mathcal{A}_{t+1}; \zeta_{t+1})$$

where ζ_t denotes all state variables that drives the economy. Swanson (2012) shows how in a context of expected utility the household's coefficient of absolute risk aversion to the gamble, $R(\mathcal{A}_t; \zeta_t) \equiv \lim_{\sigma \to 0} \frac{\mu(\mathcal{A}_t; \zeta_t, \sigma)}{\sigma^2/2}$, equals

$$R(\mathcal{A}_t; \zeta_t) = \frac{-E_t V_{11}(\mathcal{A}_{t+1}; \zeta_{t+1})}{E_t V_1(\mathcal{A}_{t+1}; \zeta_{t+1})}$$

IV.1. External habits

By the optimality conditions and the envelope theorem, the steady-state coefficient of absolute risk aversion is

$$\frac{-V_{11}}{V_1} = \frac{\gamma}{S} \frac{r/C}{1 + \chi(\frac{HZ}{CS})^{1-\gamma}}$$

Relative to the case without habits ($S = Z = 1$), the coefficient of risk aversion scales up dramatically, as $S < 1$. Relative to case of an endowement economy ($\chi = 0$), the coefficient scales down as people can use the labor margin to absorb economic shocks (Swanson, 2012).

Moreover, we can express the coefficient of relative risk aversion as either

$$\frac{-V_{11}\mathcal{W}}{V_1} = \frac{\gamma}{S} \frac{1}{1 + \chi(\frac{HZ}{CS})^{1-\gamma}}$$

where $\mathcal{W}_t = E_t(\sum_{j=0}^{\infty} M_{t,t+j} C_{t+j})$ defines the wealth portfolio. In our baseline calibration, the steady-state coefficient of relative risk aversion is 10, a reasonable amount that stands in contrast with the value of 35 in the original calibration by Campbell and Cochrane (1999). The main reason for such a lower risk aversion coefficient is the fact that in a production economy people can use the labor margin to absorb economic shocks, which scales down the steady-state price of risk γ/S. Importantly, however, this property does not compromise the model's ability to capture large average risk premia.

IV.2. Internal habits

By the optimality conditions and the envelope theorem, we have

$$\frac{-V_{11}}{V_1} = \gamma \frac{r/C}{1 + \chi(\frac{HZ}{CS})^{1-\gamma}} > \gamma \frac{r/C}{1 + \chi(\frac{H}{C})^{1-\gamma}}$$

which is larger than the steady-state coefficient of absolute risk aversion in the model without habits because of the typical ratio $Z/S > 1$. For typical calibrations, however, the difference is immaterial and in any event the coefficient of risk aversion when habits are internal is substantially smaller than the coefficient under external habit formation.

V. Frisch's elasticity of labor supply

The optimal labor choice (4) implies the elasticity of hours worked to the wage rate, given a constant marginal utility of wealth,

$$\left.\frac{\partial \ln(N_t)}{\partial \ln(W_t/P_t)}\right|_{V_{1,t}} = \frac{Z_t}{\gamma} \frac{1 - N_t}{N_t}$$

Thus, in the steady state Frisch's elasticity scales down by a factor $Z \in (0, 1)$ relative to the no-habit case, whereas over time the elasticity drops in a recession ($Z_t = S_t^{1+\xi_2}$) as if people became very averse to fluctuations in labor during a downturn; this property follows from the fact that in a downturn people become particularly sensitive to fluctuations in both market and home consumption.

In the text we also define the quasi-Frisch's elasticity as the elasticity of hours worked to the wage rate, given a constant marginal utility of wealth and a constant surplus home-consumption ratio,

$$\left.\frac{\partial \ln(N_t)}{\partial \ln(W_t/P_t)}\right|_{V_{1,t},Z_t} = \frac{1}{\gamma} \frac{1 - N_t}{N_t}$$

VI. Replicating Campbell-Cochrane in a production economy

In line with Campbell and Cochrane (1999), consider flexible prices and a random-walk specification, $\phi = 0$, in our external-habit framework under macro-finance separation, $\xi = 0$. The absence of investment and the random-walk specification of technology reduce the production economy to a particularly simple structure:

$$\begin{aligned}\Delta c_{t+1} &= \mu + \sigma e^{a}_{t+1} \\ r_t &= -\ln(\beta) + \gamma\mu - \frac{\gamma(1-\rho_s)}{2} \\ m_{t+1} &= \ln(\beta) - \gamma\mu + \gamma(1-\rho_s)\hat{s}_t - x_t\sigma e^{a}_{t+1}\end{aligned}$$

where $x_t = \gamma[1 + \Lambda(\hat{s}_t)]$.

The outcome is observationally equivalent to the model by Campbell and Cochrane.

VII. Home production vs. standard leisure

The presence of preferences that include home production rather than standard leisure induces a difference between the textbook New Keynesian model in Galí (2008) and our specification, which reduces to a scale factor in consumption:

Galí (2008): $$c_t = \left(a_t + \frac{\gamma(1-\widetilde{\beta}\rho_u)}{(1-\widetilde{\beta}\rho_u)[\gamma(1-\rho_u)+\phi_y] + \kappa(\phi_\pi - \rho_u)} u_t\right) \frac{1+\varphi}{\gamma(1-\alpha)+\alpha+\varphi} \qquad \text{with } \gamma = 1$$

LLV 2015: $$c_t = \left(a_t + \frac{\gamma(1-\widetilde{\beta}\rho_u)}{(1-\widetilde{\beta}\rho_u)[\gamma(1-\rho_u)+\phi_y] + \kappa(\phi_\pi - \rho_u)} u_t\right)$$

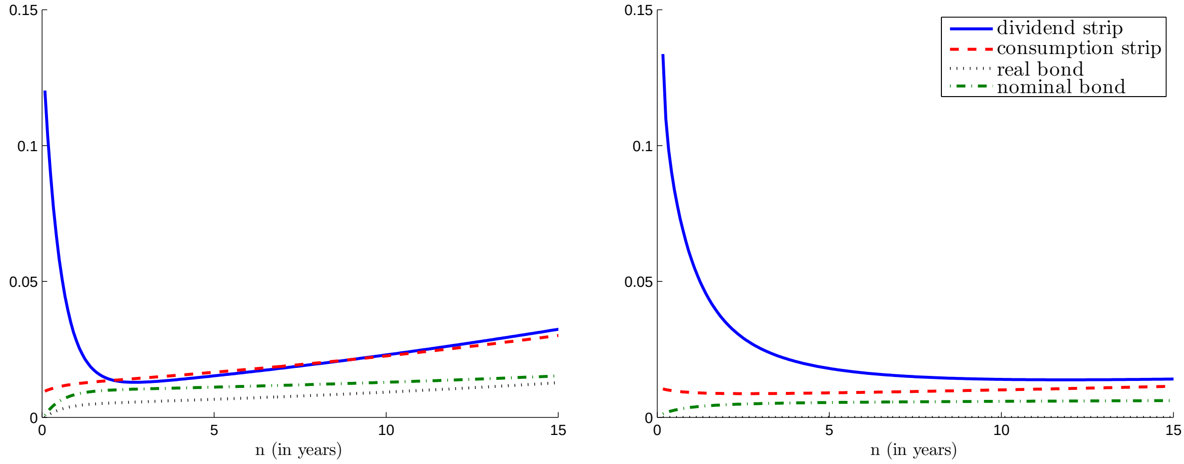
This difference is necessary to allow the economy to be compatible with balanced growth for any parameter $\gamma > 0$ in the case with home production. All first-order differences dissappear under the choice of a unit EIS, $\gamma = 1$.

VIII. Robustness of results to nontrivial capital accumulation

Nontrivial capital accumulation, flexible prices. The volatility of dividends relative to consumption is increasing in the investment-output ratio, which in turn is increasing in the adjustment cost curvature, ξ_3. With flexible prices, we are unable to produce enough leverage in dividends while simultaneously matching the volatility of real private nonresidential fixed investment (BEA-NIPA). The volatility of dividends is too low for a level of adjustment costs that produces a realistic volatility in investment growth, which is just another way to state the quantity puzzle; we can generate enough volatility in corporate profits only if capital adjustment costs are sufficiently low (ξ_3 is large) and therefore if the spillover of surplus consumption on quantities is large. Such a large departure from macro-finance separation induces a large financial spillover on investment via the elasticity of investment to Q.

Moreover, even if we were to accept such a counterfactually large volatility of investment, the mere presence of nontrivial capital cannot deliver the desired term structure properties. Adding capital without sticky prices adds a state that could in theory provide insurance, and so explain the term structure of equity but only for low adjustment costs (large ξ_3). However, capital accumulation breaks macro-finance separation and also adds surplus consumption as a state that drives quantities; for a low degree of adjustment costs, dividends become much riskier and the insurance effect of capital is overwhelmed by the risk effect of surplus consumption, so the term structure of equity slopes upwards.

Nontrivial capital accumulation, sticky prices. The continuity of the model's solution in ξ_3 implies that when adjustment costs are high the documented properties of the term structures of equity and interest rates are robust. We match a volatility of annual investment growth 3 times as large as that of consumption growth for a coefficient $\xi_3 = .3$. Figure H.8 plots the equilibrium term structures. We can produce a larger volatility in investment for lower adjustment costs but the positive spillover of surplus consumption on inflation becomes particularly important for larger values of ξ_3 and the term structure of nominal bonds would slope downwards. Moreover, note that the nonlinearities that surplus consumption injects in quantities become larger with ξ_3 as we move away from macro-finance separation, so the accuracy of the loglinearized solution deteriorates.



(a) *Holding-period risk premia.* (b) *Hold-to-maturity risk premia.*

Figure H.8: Average term structures of risk premia (in percent per year) with sticky prices and stochastic capital accumulation ($\xi_3 = .3$).

IX. Pricing levered consumption

An alternative definition of market equity can be formulated in terms of a claim to a levered version of consumption,

$$d_t = const. + \ell c_t$$

Table I.3 shows how this type of equity displays a downward-sloping term structure of returns for $\ell \geq \gamma$. The claim is particularly risky because of its perfect correlation with consumption, while the long-duration claims contain an insurance component, as the cashflow effect of the loading on long-run technology dominates the discount rate effect of long-run technology.

Asset	Cashflow process	Deterministic growth	Loading on u_t	Loading on σe^a_{t+1}	Loading on σe^u_{t+1}
Unlevered consumption	Δc_{t+1}	μ	$C_c \in (0,1)$	1	> 0
Corporate profits	Δd_{t+1}	μ	$> \gamma C_c$	1	< 0
Levered consumption	$\ell \Delta c_{t+1}$	μ	$> \gamma C_c$	ℓ	> 0

Table I.3: Dynamics of the cashflow processes that determine the prices of three types of equity: a claim to aggregate market consumption, a claim to aggregate corporate profits, and a claim to levered market consumption. The cashflow loadings are calculated for a leverage parameter $\ell > \gamma$ and for a nontrivial degree of price rigidities.