Non-cooperative Multi-radio Channel Allocation in Wireless Networks

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Abstract— **Channel allocation has been extensively studied in the framework of cellular networks, but the emergence of new system concepts, such as cognitive radio systems, bring this topic into the focus of research again. In this paper, we study the problem of competitive multi-radio multi-channel allocation in wireless networks in detail. We characterize the Nash equilibria in a static game and we conclude that in spite of the non-cooperative behavior of such devices, their channel allocation results in a system-efficient and load-balancing solution. In addition, we consider the fairness properties of the resulting channel allocations and their resistance to the possible coalitions of a subset of devices. Finally, we present three algorithms to achieve a load-balancing Nash equilibrium channel allocation, each of them using a different set of available information. To the best of our knowledge, our paper is the first contribution to this important topic.**

I. Introduction

Wireless networks provide a flexible and cost-efficient method to establish communication between different parties. Each wireless network operates in a frequency band assigned by the authorities that regulate the frequency spectrum in the given country. In general, the communication medium assigned to a given network is shared among the communication devices using some *multiple access* technique.

Frequency Division Multiple Access (FDMA) is one of the widely used techniques to enable several users to share a communication medium that consists of a given frequency band [17], [18]. The basic principle of FDMA is to split up the available bandwidth to distinct sub-bands called *channels*. Assigning the radio transceivers to these channels is commonly referred to as the *channel allocation* problem[1]. Not surprisingly, an efficient channel allocation is a cornerstone of the design of existing wireless networks.

In this paper, we present a game-theoretic analysis of fixed channel allocation strategies of devices using multiple radios. Using a static non-cooperative game, we analyze the scenario of a single collision domain, i.e., if each of the devices can interfere with a transmission of every other device. We derive the Nash equilibria in this game and show that they are system efficient and they achieve a load balancing solution. We also study the fairness issues and the problem of coalition forming in the channel allocation problem. We show that a Nash equilibrium that resists to coalitions of users is necessarily fair as well. Furthermore, we propose three algorithms to achieve the system-efficient Nash equilibrium solutions. The first is a sequential algorithm that needs global coordination; the second is a distributed algorithm that needs a local, but perfect information and the third is a distributed algorithm that is based on imperfect local information. We provide the proof for the convergence properties of these algorithms.

This work is a first step towards the deeper understanding of the non-cooperative behavior of such devices and is applicable in a broad context of wireless communications, with particular attention to the emerging field of cognitive radio systems [10]. To the best of our knowledge, our paper is the first to address the problem of multi-radio channel allocation in competitive networks, and hence it can give a guideline, how to study the impact of selfishness in these novel technologies.

The paper is organized as follows. First, we present related work on channel allocation and channel access in wireless networks in Section II. In Section III, we introduce our system model along with a game-theoretic description of competitive channel allocation. Section IV provides a comprehensive analysis of the Nash equilibria in the channel allocation game. We study fairness issues in Section V. In addition, we provide some results on coalition-proof Nash equilibria in Section VI. In Section VII, we propose three simple algorithms to reach the desired Nash equilibria. Finally, we conclude in Section VIII.

II. Related work

There has been a significant amount of work on channel allocation in wireless networks, notably for cellular networks. Channel allocation schemes in cellular networks can be divided into three categories: fixed channel allocation (FCA), dynamic channel allocation (DCA) and hybrid channel allocation (HCA), which combines the two former methods.

In a fixed channel allocation scheme, the same number of channels are permanently allocated to the radios at the base stations. To study fixed channel allocation, most authors used graph coloring / labelling techniques (e.g., in [19]). The FCA method performs very well under high traffic load, but it cannot adapt to changing traffic conditions or user distributions

To overcome the inflexibility of FCA, many authors proposed dynamic channel allocation (DCA) methods (e.g. as presented in [6], [20]). In contrast to FCA, there is no constant relationship between the base stations in a cell and their respective channels. All channels are available for each base station and they are assigned dynamically as new users arrive. Typically, the available channels are evaluated according to a cost function and the one with the minimum cost is used [7]. Due to its dynamic property, the DCA can adapt to changing traffic demand. Because adaptation implies some

[1]In the literature, the terms channel assignment and frequency assignment are also used for the channel allocation problem.

cost, it performs worse in case of a heavy traffic load. For a comprehensive survey on the topic, we refer the reader to [11].

Due to the emergence of alternative communication technologies, channel allocation schemes became a focus of research again. Mishra *et al.* [14] propose a channel allocation method for wireless local area networks (WLANs) based weighted graph coloring.

Recently, several researchers have considered devices using multiple radios, notably in mesh networks (for a survey on mesh networks, see [2]). In the multi-radio communication context, channel allocation and access also became one of the crucial topics. Related work on multi-radio medium access includes, but not restricted to [1], [3], [16].

In all the related work cited so far, their authors assumed that the radio devices cooperate to achieve a high system performance. This assumption might not hold, as the users of these devices are usually selfish and they want to maximize their own performance without necessarily respecting the system objectives. Game theory provides a straightforward tool to study medium access problems in competitive wireless networks and has been applied to the CSMA/CA protocol [8], [12] and to the Aloha protocol [13]. Furthermore, a fixed channel allocation game was presented in [9] based on graph coloring. For cognitive radio networks, the authors of [15] propose a dynamic channel allocation schemes based on a potential game. In addition, they suggest another technique based on machine learning with different utility functions.

III. System Model and Concepts

We assume an available frequency band divided into orthogonal channels of the same bandwidth using the FDMA method (e.g., 11 orthogonal channels in case of the 802.11a protocol). We also assume that these channels have the same expected channel characteristics. We denote the set of available orthogonal channels by $\mathcal{C}$.

In our model, *pairs of users* want to communicate with each other over a single hop. We assume that each user participates in only one such communication session, hence we denote the set of such communication links by $\mathcal{N}$. Each user owns a device equipped with $k \leq |\mathcal{C}|$ radio transmitters, all having the same communication capabilities. The communication between two devices is bidirectional and they always have some packets to exchange. We assume that each communicating pair is a *selfish player*, whose objective is to maximize its total rate or channel utilization. We will use this term to denote both the communicating pair of users and the communication link between them. In this paper, we further assume that each device can hear the transmissions of every other device if they are using the same channel. This means that the players reside in a *single collision domain*. We make this assumption to avoid the hidden terminal problem described for example in [17], [18].

We assume that there is a mechanism that enables the players to use multiple channels to communicate at the same time (as it is implemented in [1] for example). We denote the number of radios of player i using channel c by $k_{i,c}$ for every $c \in \mathcal{C}$. For the simplicity of presentation, let us denote the set of channels used by player i by $\mathcal{C}_i$, where $\mathcal{C}_i \subset \mathcal{C}$ and $0 \leq |\mathcal{C}_i| \leq k$. We further assume that there is no limitation on the number of radios per channel.

We formulate the multi-radio channel allocation problem as a non-cooperative game as follows. We define the *strategy* of player i as its channel allocation vector:

$$s_i = \{k_{i,1}, \ldots, k_{i,|\mathcal{C}|}\} \quad (1)$$

Hence, its strategy consists in defining the number of radios on each of the channels[2]. The strategy vector of all players defines the strategy matrix S, where the row i of the matrix corresponds to the strategy vector of player i:

$$S = \begin{pmatrix} s_1 \\ \ldots \\ s_{|\mathcal{N}|} \end{pmatrix} \quad (2)$$

Furthermore, we denote the strategy matrix except for the strategy of player i by S_{-i} as shown in (3):

$$S_{-i} = \begin{pmatrix} s_1 \\ \ldots \\ s_{i-1} \\ s_{i+1} \\ \ldots \\ s_{|\mathcal{N}|} \end{pmatrix} \quad (3)$$

Figure 1 presents an example channel allocation with six available channels ($|\mathcal{C}| = 6$), four players ($|\mathcal{N}| = 4$) and each user device equipped by four radios ($k = 4$). Figure 2 presents the strategy matrix that corresponds to this example.

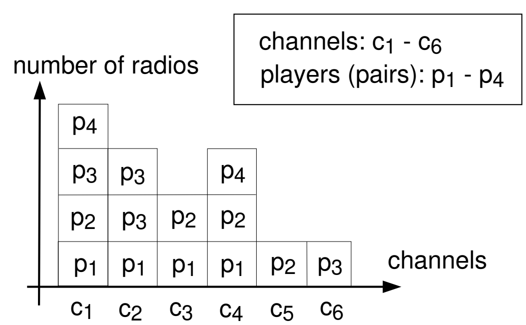

Fig. 1. An example for a channel allocation, where $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$.

The total number of radios employed by player i can be written as $k_i = \sum_c k_{i,c}$. Similarly, we can obtain the number of radios using a particular channel $k_c = \sum_i k_{i,c}$. In Figure 1, each player has a radio on channel c_1, but channel c_5 is occupied only by player p_2. Player p_3 employs two radios on channel c_2 to get more bandwidth on that particular channel. Regarding the number of radios per player, we have $k_{p_1} = k_{p_2} = k_{p_3} = 4$ and $k_{p_4} = 2$, meaning that player p_4 is not using all of his radios.

[2]Note that this number can be zero.

players	c_1	c_2	c_3	c_4	c_5	c_6
p_1	1	1	1	1	0	0
p_2	1	0	1	1	1	0
p_3	1	2	0	0	0	1
p_4	1	0	0	1	0	0

Fig. 2. Strategy matrix of the example in Figure 1.

We assume that the players are rational and their objective is to maximize their *utility* in the network. We denote the utility of player i by U_i. For simplicity, we assume that each player i wants to *maximize his total rate (R_i)* in the system and thus the utility function is the achieved bitrate. We leave the study of other utility functions for future work.

We assume that the total rate on channel c is shared equally among the radio transmitters using that channel. This fair rate allocation is achieved for example by using a reservation-based TDMA schedule on a given channel. A similar result was reported by Bianchi in [5] for the CSMA/CA protocol using optimal backoff window values. Even if the radio transmitters are controlled by selfish users in the CSMA/CA protocol, they can achieve this fair sharing as shown in [8]. We further assume that the *total available bitrate* $R_c(k_c)$ on a channel c (i.e., the sum of the achieved bitrate of all players on channel c) is a non-increasing function of the number of radios k_c deployed on this channel. In fact $R_c(k_c)$ is independent of k_c for a TDMA protocol and for the CSMA/CA protocol using optimal backoff window values [5]. In practice, the backoff window values used in the CSMA/CA protocol implementation (e.g., in the 802.11 standard) are not optimal; and due to packet collisions $R_c(k_c)$ becomes a decreasing function for $k_c > 1$. Since we assume that channels have the same bandwidth and channel characteristics, the rate function does not depend on the channel and thus we can write that $R(k_c)$ for any channel $c \in \mathcal{C}$. If $k_c = 0$, we define $R(0) = 0$; note however that this case has no relevance in our model.

Figure 3 presents the total rate $R(k_c)$ as a function of the number of radios using channel c.

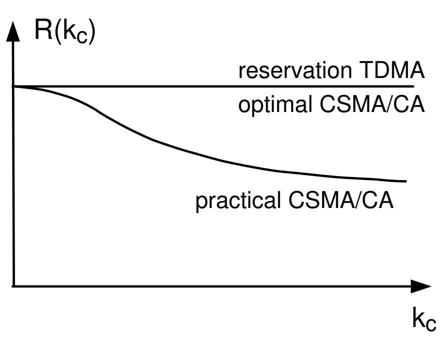

Fig. 3. The total available rate $R(k_c)$ for different MAC protocols.

If player i chooses to operate $k_{i,c}$ radios in a given channel, his rate on this channel can be written as $R_{i,c} = \frac{k_{i,c}}{k_c} \cdot R(k_c)$. We assume that the players do not cheat at the MAC layer as in [8] for example. Thus, we can write that $R_{i,c} > 0$ for all $c \in \mathcal{C}$, where $k_{i,c} > 0$. Recall that in Figure 1, the higher is the number of radios in a given channel, the lower is the rate per radio. Hence, for example for player p_2, we have $R_{2,1} < R_{2,4} < R_{2,3} < R_{2,5}$. We can obtain the total rate R_i for player i by $R_i = \sum_c R_{i,c}$.

In summary, we can write the utility function for player i as:

$$U_i(S) = R_i = \sum_{c \in \mathcal{C}} R_{i,c} = \sum_{c \in \mathcal{C}} \frac{k_{i,c}}{k_c} \cdot R(k_c) \qquad (4)$$

We model the channel allocation problem with a *single stage game*, which corresponds to a fixed channel allocation among the players.

In order to study the strategic interaction of the players, we first introduce the concept of Nash equilibrium.

Definition 1: (Nash Equilibrium – NE): The strategy matrix $S^* = \{s_1^*, \ldots, s_{|\mathcal{N}|}^*\}$ defines a Nash Equilibrium (NE), if for every player i, we have:

$$U_i(s_i^*, S_{-i}^*) \geq U_i(s_i^{'}, S_{-i}^*) \qquad (5)$$

for every strategy $s_i^{'}$.
In other words, in a NE none of the players can unilaterally change its strategy to increase its utility. A NE solution is often inefficient from the system point of view. We characterize the efficiency of the solution by the concept of Pareto-optimality.

Definition 2: (Pareto-Optimality): The strategy matrix S^{po} is Pareto-optimal if $\nexists S^{'}$ such that:

$$U_i(S^{'}) \geq U_i(S^{po}), \forall i \qquad (6)$$

with strict inequality for at least one player i.
This means that in a Pareto-optimal channel allocation S^{po} no player i can improve his utility without decreasing the utility of at least one other player j.

IV. Nash Equilibria

In this section, we study the existence of Nash equilibria in the single collision domain channel allocation game. Note that we omit the proofs of many intermediate results due to space limitations.

It is straightforward to see that if the total number of radios is smaller than or equal to the number of channels, then a flat channel allocation, in which the number of radios per channel does not exceed one, is a Nash equilibrium.

Fact 1: If $|\mathcal{N}| \cdot k \leq |\mathcal{C}|$, then any channel allocation, in which $k_c = 1, \forall c \in \mathcal{C}$ is a Pareto-optimal NE.

For the remainder of the paper, we assume that $|\mathcal{N}| \cdot k > |\mathcal{C}|$, hence the devices have a conflict during the channel allocation process. In the following lemmas, we express necessary conditions for a Nash equilibrium. The first necessary condition shows that the players should use all of their radios.

First we show that a selfish player should use all of his radios in order to maximize his total rate.

Lemma 1: In a NE of the multi-radio channel allocation game, $k_i = k, \forall i$.

In the example presented in Figure 1, Lemma 1 does not hold for players p_4, because it uses only two radios. Hence, the example cannot be a NE.

Proof: We can prove the lemma by contradiction. Assume that there exists a NE, in which player i uses only $k_i < k$ radios. As mentioned previously, in our model we assume that $k \leq |\mathcal{C}|$ and we assumed that in the NE $|\mathcal{C}_i| \leq k_i < k$, thus we necessarily have $|\mathcal{C}_i| < |\mathcal{C}|$. This implies that there always exists a channel $c \notin \mathcal{C}_i$. If the player deploys an additional radio on this channel c, then he increases his utility due to the fact that $R_{i,c} > 0$ for $k_{i,c} = 1$. Hence, we have a contradiction and the original allocation cannot be a NE. ■

Let us now consider a NE strategy matrix in the multi-radio channel allocation game denoted by S^*, where $s_i^* \in S^*$ is the NE strategy of player i (i.e., the i-th row of the matrix). Let us consider two arbitrary channels b and c in this NE strategy allocation. Without loss of generality, we assume that there are more radios using channel b, meaning that $k_b \geq k_c$, and denote their difference by:

$$\delta_{b,c} = k_b - k_c \tag{7}$$

Assume that player i moves one of his radios from channel b to c. Let us define the *benefit of change*, i.e. the difference in the utility of player i, as follows:

$$\begin{aligned} \Delta &= U_i(s_i^{'}, S_{-i}^*) - U_i(s_i^*, S_{-i}^*) \\ &= \frac{k_{i,b}-1}{k_b-1} \cdot R(k_b-1) + \frac{k_{i,c}+1}{k_c+1} \cdot R(k_c+1) \\ &- \frac{k_{i,b}}{k_b} \cdot R(k_b) - \frac{k_{i,c}}{k_c} \cdot R(k_c) \end{aligned} \tag{8}$$

We can show a second necessary condition for a NE, namely that player i has a benefit of moving one radio to a channel, where he has no radios if the difference of the number of radios deployed on the two channels exceeds one.

Lemma 2: If $k_{i,b} > 0$, $k_{i,c} = 0$ and $\delta_{b,c} > 1$ for any player i, then S^* is not a NE channel allocation.

In the example presented in Figure 1, Lemma 2 holds e.g. for player p_1 and the channels $b = c_1$ and $c = c_5$. Hence, the example cannot be a NE.

Proof: Assume that S^* is a NE channel allocation. Suppose that player i moves one of his radios from channel b to c. Using the conditions in the lemma, we can write the benefit of change defined in (8) as:

$$\begin{aligned} \Delta &= \frac{k_{i,b}-1}{k_b-1} R(k_b-1) + \frac{1}{k_c+1} R(k_c+1) - \frac{k_{i,b}}{k_b} R(k_b) \\ &= \frac{k_{i,b}}{k_b-1} R(k_b-1) - \frac{1}{k_b-1} R(k_b-1) \\ &+ \frac{1}{k_c+1} R(k_c+1) - \frac{k_{i,b}}{k_b} R(k_b) \end{aligned}$$

Let us notice that the sum of the first and last terms is always strictly greater than 0, because $\delta_{b,c} > 1$ implies that $k_b > 1$. Hence, it is enough to investigate the sign of the sum of the two other terms. Using (7), we can rewrite the sum of the two middle terms as:

$$\frac{R(k_c+1)}{k_c+1} - \frac{R(k_c-1)}{k_c-1} = \frac{R(k_c+1)}{k_c+1} - \frac{R(k_c+\delta_{b,c}-1)}{k_c+\delta_{b,c}-1}$$

Due to the assumption $\delta_{b,c} > 1$ and the non-increasing rate function $R(\cdot)$, we have:

$$\frac{R(k_c+1)}{k_c+1} - \frac{R(k_c+\delta_{b,c}-1)}{k_c+\delta_{b,c}-1} \geq 0$$

Hence, the benefit of change is positive and thus S^* cannot be a NE. This contradiction concludes the proof. ■

Let us now derive the third necessary condition. This condition shows that player i should again change the position of one of his radios, if he has at least two radios more on channel b than channel c and overall, there are more radios on channel b than on channel c. The rationale of the lemma is that player i can decrease the imbalance of his radios by reallocating them between the channels b and c.

Lemma 3: If $k_{i,b} > 1$, $k_{i,c} = 0$ and $\delta_{b,c} = 1$ for any player i, then S^* is not a NE.

In the example presented in Figure 1, the conditions of Lemma 3 hold for player p_3 and the channels $b = c_2$ and $c = c_3$. Hence, the example cannot be a NE. The proof of the lemma is similar to the previous proof and hence we omit it.

Suppose now that we have two channels b and c such that $k_b = k_c$ (which is equivalent to $\delta_{b,c} = 0$), that means that the total number of radios is the same on the two channels. Assume that we have a player i with $k_{i,b} > k_{i,c} > 0$. Let us define the integer value $\gamma_{i,b,c}$ as:

$$\gamma_{i,b,c} = k_{i,b} - k_{i,c} \tag{9}$$

Using the value $\gamma_{i,b,c}$ introduced above, we can derive the fourth necessary condition. The lemma shows that if there exist these two channels with an equal number of radios and a user has no radio on one of them and more than one radio on the other, than he should reallocate one of his radios to the channel, in which he is not present yet.

Lemma 4: If $\gamma_{i,b,c} \geq 2$, $k_{i,c} = 0$ and $\delta_{b,c} = 0$ for any player i, then S^* is not a NE.

In Figure 1, the conditions of Lemma 3 hold for player p_3 and the channels $b = c_2$ and $c = c_4$. Thus, this is not a NE. The proof of the lemma is similar to the proof of the previous lemmas.

Let us now consider a channel allocation S and let us divide the channels into three sets. We define the set of channels C_{max} with the maximum number of radios, i.e., where $b \in C_{max}$ has $k_b = \max_{l \in \mathcal{C}} k_l$. Similarly, let us define the set of the least occupied channels C_{min}, where $c \in C_{min}$ has $k_c = \min_{l \in \mathcal{C}} k_l$. We denote the set of the remaining channels by C_{rem}. In Figure 1, $C_{max} = \{c_1\}$, $C_{min} = \{c_5, c_6\}$ and $C_{rem} = \{c_2, c_3, c_4\}$.

Using Lemmas 1, 2, 3 and 4, we conclude on a fifth necessary condition. This shows that in a Nash equilibrium, the difference in the total number of radios between any two channels cannot exceed one.

Proposition 1: In a NE S^* in the multi-radio channel allocation game, we have $\delta_{b,c} \leq 1$ for all $b, c \in \mathcal{C}$.
Basically, in the proof we divide the possible scenarios into group and show that either of the lemmas apply to the members of these groups.

Combining our results so far, we can establish a set of necessary and sufficient conditions for the NE.

Theorem 1: Assume that we have $|\mathcal{N}| \cdot k > |\mathcal{C}|$. Then a channel allocation S^* is a NE iff the two following conditions hold:

- $\delta_{b,c} \leq 1$ for any $b, c \in \mathcal{C}$ and
- $k_{i,c} \leq 1$ for any $b, c \in \mathcal{C}$ and $i \in \mathcal{N}$ *except for* players j with $\nexists c \in C_{min}$ such that $k_{j,c} = 0$. For such a player j, the second condition changes as follows: $k_{j,c} \leq 1$ if $c \in C_{max}$ and $\gamma_{i,a,c} \leq 1$ for any channel $a, c \in C_{min}$.

An example of a NE channel allocation is shown in Figure 4, where the second condition of the theorem has an exception for player p_1. Figure 5 presents an example with no exception on the second condition for any player.

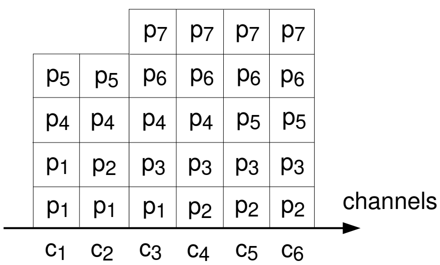

Fig. 4. An example for a NE channel allocation. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 7$ and $k = 4$. Note that the second condition of Theorem 1 has an exception for player p_1.

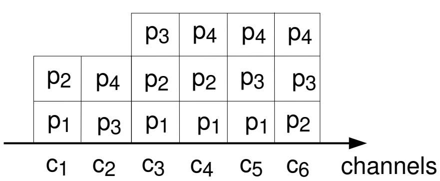

Fig. 5. A NE channel allocation with no exception of the second condition of Theorem 1. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$.

Proof: Let us show that the above conditions are necessary. Proposition 1 established that the first condition is necessary. Lemmas 2, 3 and 4 make the second condition necessary.

Now we prove that these conditions are sufficient as well. According to the first condition of the theorem, the difference between the number of radios on any two channels cannot be more than one, thus the set C_{rem} does not exist. This also means that any change consists in moving some radios from channels in C_{max} to channels in C_{min}. Consequently, the moves can be considered separately.

Let us thus consider the moving of one radio from a channel $b \in C_{max}$ to a channel $c \in C_{min}$ which results in a strategy $s_i^{'}$ for player i. Substituting $\gamma_{i,b,c} = k_{i,b} - k_{i,c} \leq 1$, we can write the benefit of change expressed in (8) as follows:

$$\begin{aligned}\Delta &= \frac{k_{i,c} + \gamma_{i,b,c} - 1}{k_c} R(k_c) + \frac{k_{i,c} + 1}{k_c + 1} R(k_c + 1) \\ &- \frac{k_{i,c} + \gamma_{i,b,c}}{k_c + 1} R(k_c + 1) - \frac{k_{i,c}}{k_c} R(k_c) \\ &= (\gamma_{i,b,c} - 1) \left(\frac{R(k_c)}{k_c} - \frac{R(k_c + 1)}{k_c + 1} \right)\end{aligned}$$

Note that second factor is always positive and hence the difference is non-positive for $\gamma_{i,b,c} \leq 1$. For this value, the strategy matrix S^* defines a NE. ■

Theorem 1 establishes an interesting property about NE: *In fact, all NE channel allocations achieve load-balancing over the channels in* $\mathcal{C}$. In the next theorem, we will show that allowing selfish channel allocation over a wide band of frequencies results in an efficient spectrum utilization.

Theorem 2: Assume that we have $|\mathcal{N}| \cdot k > |\mathcal{C}|$. Then any NE channel allocation S^* is Pareto-optimal.

Proof: The proof is straightforward. In a NE channel allocation S^* we have $k_c > 0$ for each $c \in \mathcal{C}$. Note that in S^*, the sum of the utility of all players $U_{total} = \max_i \sum_i U_i$ which implies both Pareto- and system-optimality. ■

Usually, there is a way to improve the efficiency of the non-cooperative NE solution using a cooperative solution, e.g., the Nash bargaining framework. Note that in this case, as shown in the proof of the theorem, the Pareto-optimal NE channel allocation is also system-optimal. All channels are fully utilized, and hence the system efficiency cannot be further improved. This implies that no cooperative solution can further increase the system's efficiency.

V. Fairness issues

In this section, we study the *fairness* properties of the selfish multi-radio channel allocation game. Fairness is an important aspect of resource allocation problems in general, and of computer networks in particular. We have seen in Section IV that in the selfish multi-radio channel allocation problem, the NE solutions are Pareto-optimal and also system-efficient. Unfortunately, these Pareto-optimal allocations might be highly unfair by giving advantage to some players while neglecting others. For example, in the channel allocation presented in Figure 4 assuming that the rate function $R(\cdot)$ is constant, player p_1 has the total rate $U_1 = \frac{19}{20}$, while player p_4 has the total rate $U_4 = \frac{16}{20}$. In order to study the fairness properties of the NE channel allocations, we use a particular metric called *max-min fairness (MMF)* as defined in [4]:

Definition 3: (Max-Min Fairness – MMF): The strategy matrix S^{mmf} is max-min fair if the utility of player i cannot be increased without decreasing the utility of another player j for which $U_i(S^{mmf}) \geq U_j(S^{mmf})$.

Using this concept, we identify the max-min fair NE channel allocations as expressed in Theorem 3.

Theorem 3: A NE channel allocation S^* is max-min fair if and only if $|C_{min}| \cdot k_c \equiv 0 \pmod{|\mathcal{N}|}$. This implies that $U_i = U_j, \forall i, j \in \mathcal{N}$.

In other words, if the total number of radios in the least allocated channels are equal for every player, the NE allocation is max-min fair. In the proof, we will show that the condition implies an equal utility for each player.

Proof: First, we will prove that $|C_{min}| \cdot k_c \equiv 0 \pmod{|\mathcal{N}|}$ implies $U_i = U_j, \forall i, j \in \mathcal{N}$. Then we will show that the latter condition implies max-min fairness.

Let us now express the utility of any player i in a NE allocation S^*. In the equation below, k_c denotes the number of radios that use channel $c \in C_{min}$.

$$\begin{aligned} U_i &= \frac{\sum_{c \in C_{min}} k_{i,c}}{k_c} R(k_c) + \frac{k - \sum_{c \in C_{min}} k_{i,c}}{k_c + 1} R(k_c + 1) \\ &= \sum_{c \in C_{min}} k_{i,c} (\frac{R(k_c)}{k_c} - \frac{R(k_c + 1)}{k_c + 1}) + k \frac{R(k_c + 1)}{k_c + 1} \end{aligned}$$

Similarly, we can express the utility of another player j:

$$U_j = \sum_{c \in C_{min}} k_{j,c} (\frac{R(k_c)}{k_c} - \frac{R(k_c + 1)}{k_c + 1}) + k \frac{R(k_c + 1)}{k_c + 1}$$

If $|C_{min}| \cdot k_c = |\mathcal{N}| \cdot \kappa$, where κ is an integer, then we have an equal number of radios in the channels in C_{min}, meaning that $\sum_{c \in C_{min}} k_{i,c} = \sum_{c \in C_{min}} k_{j,c} = \kappa$ and vice versa. If and only if every player has an equal number of radios in the channels $c \in C_{min}$, do we have $U_i = U_j$.

Second, we prove by contradiction that the equality of the utilities is necessary to max-min fairness.

Let us suppose that there exist a max-min fair NE channel allocation S^* in which $U_i < U_j$ for some $i, j \in \mathcal{N}$. We know from Lemma 1 that $k_i = k_j = k$, and hence we can interchange the radios of player i with the radios of player j. This results in $U_i > U_j$ while the utilities of other players do not change. Hence, the original channel allocation S^* is not max-min fair. Conversely, if $U_i = U_j$, this implies max-min fairness by definition. ■

From this theorem, we can immediately see that the perfectly balanced channel allocation is also max-min fair.

Corollary 1: The NE S^* in which $C_{min} = C_{max}$ (i.e., $k_b = k_c$, $\forall b, c \in \mathcal{C}$) is max-min fair as well.

We can easily check max-min fairness given the values of $|\mathcal{C}|$, $|\mathcal{N}|$ and k.

$$k_c = \frac{\mathcal{N} \cdot k - |C_{min}|}{\mathcal{C}} \quad (10)$$

Due to the fact that $|C_{min}| \leq |\mathcal{C}|$, we can write that:

$$k_c = \left\lfloor \frac{\mathcal{N} \cdot k}{\mathcal{C}} \right\rfloor \quad (11)$$

We can also compute $|C_{min}|$ as follows:

$$|C_{min}| = \mathcal{N} \cdot k - k_c \cdot |\mathcal{C}| \quad (12)$$

If $|C_{min}| = 0$, then every channel has the same number of radios (i.e., $k_b = k_c, \forall b, c \in \mathcal{C}$). If $|C_{min}| > 0$, then we can easily verify the condition of Theorem 3.

VI. Coalition-proof Nash Equilibria

The definition of NE expresses the resistance to the deviation of a single player. In a realistic situation, it might be possible that several players collude to increase their payoff at the expense of other players. Such a collusion is called a *coalition*. The problem of how these coalitions are formed is a separate research topic itself, thus in this paper we assume that any players can form a coalition. We can generalize the notion of NE for coalitions as follows.

Definition 4: (Coalition-Proof Nash Equilibrium – CPNE): The strategy matrix S^{cpne} defines a coalition-proof Nash equilibrium if there *does not exist* any coalition $\Gamma \in \mathcal{N}$ and any strategy of this coalition $S_\Gamma^{'}$ such that the following set of conditions is true:

$$U_i(S_\Gamma^{'}, S_{-\Gamma}^{cpne}) > U_i(S_\Gamma^{cpne}, S_{-\Gamma}^{cpne}), \forall i \in \Gamma \quad (13)$$

This means that no coalition can deviate form S^{cpne} such that the utility of *all* of its members increases.

The definition of coalition-proof Nash equilibrium is very restrictive. We can define this notion in the broader sense:

Definition 5: (Strong Coalition-Proof Nash Equilibrium – SCPNE): The strategy matrix S^{scpne} defines a coalition-proof Nash equilibrium if there *does not exist* any coalition $\Gamma \in \mathcal{N}$ and any strategy of this coalition $S_\Gamma^{'}$ such that the following set of conditions is true:

$$U_i(S_\Gamma^{'}, S_{-\Gamma}^{scpne}) \geq U_i(S_\Gamma^{scpne}, S_{-\Gamma}^{scpne}), \forall i \in \Gamma \quad (14)$$

with strict inequality for at least one player $i \in \Gamma$.

This means that no coalition can deviate form S^{cpne} such that the utility of *some* of its members increases while the utility of other members do not change. From the definition, we can immediately see the following fact:

Fact 2: If in the NE S^*, we have $C_{min} = C_{max}$ (i.e., $k_b = k_c$, $\forall b, c \in \mathcal{C}$), then S^* is (strong) coalition-proof.

The intuition is that in a channel allocation S^* for which $k_b = k_c$, $\forall b, c \in \mathcal{C}$, any player that changes necessarily decreases his utility, hence S^* is a (strong) coalition-proof NE by definition.

In the remainder of this section, we assume that $C_{min} \neq C_{max}$ and we derive results that highlight the strong coalition-proof NE. First, we prove a necessary condition that enables a given NE allocation to be strong coalition-proof in addition.

Theorem 4: If NE channel allocation S^{scpne} is strong coalition-proof then there *does not exist* two channels $b \in C_{max}$ and $c \in C_{min}$ and two players $i, j \in \mathcal{N}$ such that $k_{i,b} > 0$ and $k_{j,b} > 0$ while $k_{i,c} = 0$ and $k_{j,c} = 0$.

To illustrate the condition of Theorem 4, let us emphasize that the example shown in Figure 5 is not a strong coalition-proof NE.

We provide an example for a strong coalition-proof NE in Figure 6.

Proof: It is easy to see that if the conditions of the theorem do not hold, then i and j can form a coalition and one of them (for example player i) can increase the utility of the other by moving one radio from b to c. Hence it is a necessary condition. ■

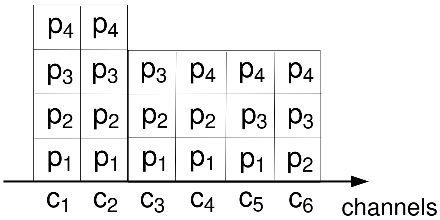

Fig. 6. An example for a strong coalition-proof NE channel allocation, where $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 5$.

We could not prove that the set of conditions in Theorem 4 is sufficient to establish a strong coalition-proof NE, but we could not find a counterexample, where the conditions hold and the channel allocation is not strong coalition-proof NE. Hence, we provide the following conjecture.

Conjecture 1: If there *does not exist* two channels $b \in C_{max}$ and $c \in C_{min}$ and two players $i, j \in \mathcal{N}$ such that $k_{i,b} > 0$ and $k_{j,b} > 0$ while $k_{i,c} = 0$ and $k_{j,c} = 0$ then the NE channel allocation S^{scpne} is strong coalition-proof. Hence the above condition is a sufficient condition.

Nonetheless, we can show that the set of strong coalition-proof NE channel allocations is a subset of the max-min fair channel allocations.

To prove this result, we first prove the following lemmas. Due to lack of space, we omit their proof.

Lemma 5: For any NE channel allocation S^* in which $C_{max} > 0$, we have $k_c < |\mathcal{N}|$.

Note that Lemma 5 applies to any NE channel allocation, not only to the the max-min fair NE.

Lemma 6: For any NE channel allocation S in which $C_{max} > 0$ and S *is not MMF*, we have $k_c + 1 < |\mathcal{N}|$.

Using Lemma 6, we can show in the following theorem that all strong coalition-proof NE channel allocations are max-min fair as well.

Theorem 5: If NE channel allocation S is strong coalition-proof (strictly speaking if the necessary condition expressed in Theorem 4 holds) then it is max-min fair as well.

We again omit the proof of the theorem due to space limitations.

As a summary, Figure 7 shows all channel allocations by properties.

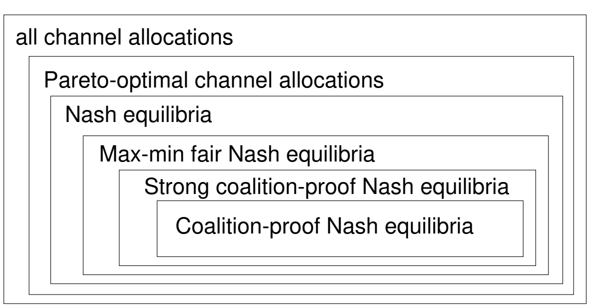

Fig. 7. Summary of channel allocations with different properties.

VII. Convergence to a Nash Equilibrium

We have demonstrated in Section IV that the non-cooperative behavior of the selfish players lead to Pareto-optimal, load balancing Nash equilibria. In this section, we propose three different algorithms, each using a different set of available information to enable the selfish players to converge to one of these Nash equilibria from an arbitrary initial configuration. The three algorithms are the following: 1) centralized algorithm using perfect information, 2) distributed algorithm using perfect information, and 3) distributed algorithm using imperfect (local) information, respectively.

A. Centralized Algorithm Using Perfect Information

We have proven in Theorem 1 that a Nash equilibrium channel allocation has a load-balancing property. In addition, we have shown in Theorem 2 that all the Nash equilibria are Pareto-optimal as well. Now, let us propose a simple centralized algorithm to achieve one of these efficient Nash equilibria.

Algorithm 1 Pareto-optimal NE channel allocation with global coordination and perfect information

```
for i = 1 to |N| do
    for j = 1 to k do
        if k_c = k_l, ∀c, l ∈ C then
            use the radio on a channel c, where k_{i,c} = 0
        else
            use the radio on a channel c, where k_c = min_{l∈C} k_l
        end if
    end for
end for
```

Notice that the algorithm requires the sequential action of the players and hence it needs global coordination. In addition, the players have to have a perfect information about the number of radios on each of the channels. This can be achieved by the global coordination mentioned before or by having an extra radio per device for scanning the channels. Global coordination is unlikely to exist in a wireless networking scenario with selfish players. The second assumption about perfect information might not hold either, because selfish players should allocate all of their radios for communication as shown in Lemma 1. It is possible to model the cost of scanning with one radio instead of using it for communication. The investigation of this issue is part of our future work.

B. Distributed Algorithm Using Perfect Information

In order to overcome the limitations of the centralized algorithm proposed in Section VII-A, we suggest a second algorithm that does not require global coordination, but still assumes a perfect information about the available channels.

We define a *round-based* distributed algorithm that works as follows. First we assume that there exist a random radio assignment of the players over the channels. For simplicity, we exclude the specific Nash equilibria that result in an exception of the second condition of Theorem 1. This means that we assume that no players allocates more than one device on any channel. After the initial channel assignment, each player evaluates the number of radios (which defines the approximate length of the round) on *each of the channels* $c \in \mathcal{C}$ and tries to improve his total rate by reorganizing his radios. Unfortunately, this procedure might result in a continuous reallocation of the radios for all players. An example for such a continuous reorganization is shown in Figure 8, where channel c_6 is empty and thus each player moves his radio from c_1 to c_6. In the next round, the same effect happens and they all move their radios back from c_6 to c_1.

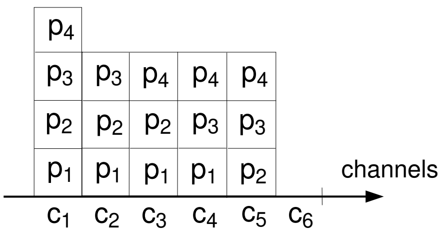

Fig. 8. An example for a channel allocation which results in a continuous reallocation of the radios for all players (i.e., each player moves his radio from c_1 to c_5 and back) if there is no randomized backoff mechanism implemented. In this example, $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$.

To avoid these instable channel allocations, we leverage the technique of backoff mechanism well known in the 802.11 medium access technology [18]. We define a *backoff window* W and each player chooses a random initial value for his *backoff counter* with uniform probability from the set $\{1, \dots, W\}$. Then in every round each player decreases his backoff counter by one and applies the re-allocation of his radios only when the backoff counter reaches zero. After the he changed his channel allocation, he will reset the backoff counter as described previously. We can notice that using the backoff mechanism, the players play a game in a quasi-sequential order.

We provide the pseudo-code for the distributed algorithm described previously in Algorithm 2.

We can prove that Algorithm 2 stabilizes in one of the load-balancing Nash equilibrium channel allocations.

Theorem 6: Algorithm 2 converges to one of the NE channel allocations.

To prove the theorem, let us introduce the notion of *state graph* G. We represent each possible channel allocation as a node in G. We call the NE channel allocations a *stable state* and the other states as *transitional states*. The transition between any two states depends only on the set of nodes which have their backoff counter equal to zero. Hence, our algorithm has the Markov-property.

Proof: To prove the theorem, one has to prove two properties: 1) The algorithm does not stabilize in a transitional state and 2) there exists a path with positive probability from any transitional state to one of the stable states.

The first property is easy to prove, because in each transitional state (non-NE) there exists at least one player, who has

Algorithm 2 Distributed Pareto-optimal NE channel allocation algorithm using perfect information

RandomChannelAllocation()
while there is change **do**
  ChannelUpdate()
  for $i = 1$ to $|\mathcal{N}|$ **do**
    if backoff counter is 0 **then**
      reorganize the radios of i in the to maximize the total rate:
      for $j = 1$ to k **do**
        assume that radio j uses channel b
        move the radio j from b to channel $c_{\min}$ if $\exists c_{\min} \in C$, $c_{\min} = \arg\min_c k_c$ such that $k_{i,c_{\min}} = 0$ and $k_{c_{\min}} < k_b - 1$
      end for
      reset the backoff counter to a new value from the set $\{1, \dots, W\}$
    else
      decrease the backoff counter value by one
    end if
  end for
end while

the motivation to change. The second property holds as well, because we have a positive probability that only one player changes in each round with no repetition of the players until a NE is reached. This special case is exactly the procedure described in Algorithm 1. ■

C. Distributed Algorithm Using Local Information

The distributed algorithm presented in Section VII-B used perfect information. This assumption requires that the either the players share their local information about the channel allocation, or that each of them uses a separate radio for scanning the channels he is not using at the moment. These assumptions might not hold in a selfish networking context. The first assumption requires that the players collaborate, which might not be their best interest. The second assumption contradicts with Lemma 1, which proves that selfish players should use all of their radios for communication[3].

In this subsection, we assume that players have only a local information about the channels, on which they operate a radio. In order to improve their performance, they proceed as follows. The players apply the random backoff mechanism we introduced in Section VII-B. In each round where player i's backoff counter is equal to zero, he calculates the average number of devices on the channels he knows (recall that we denote this set by $\mathcal{C}_i$). We denote the average number of devices on the channels in $\mathcal{C}_i$ by m_i. For each channel $b \in \mathcal{C}_i$ with $k_b - m_i \geq 1$ player i *moves his radio with a uniform random probability to another channel* $c \notin \mathcal{C}_i$. This is the first property of the algorithm with imperfect information.

[3]In our future work, we will model the cost of scanning, which might result in a temporary decrease of the total rate, but with the promise of better total rate in the future.

Similarly to Theorem 6, one can show that the above procedure reaches a stable state. Unfortunately, the available local information might be insufficient for the players to identify if the achieved stable state is Nash equilibrium. We show an example for such a “false Nash equilibrium” in Figure 9.

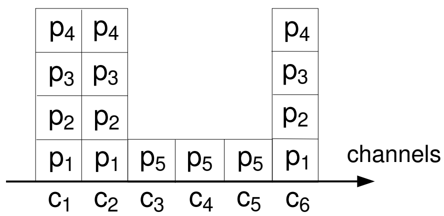

Fig. 9. An example for a stability state using the distributed algorithm with imperfect information. Each player believes that this is a Nash equilibrium due to the insufficient local information. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 5$ and $k = 3$.

In order to solve the problem of inefficient stable states, we introduce the following mechanism: player i checks the number of radios for each of the channels $b \in \mathcal{C}_i$ as suggested above and *with a small probability* ϵ *he moves his radio to another channel* $c \notin \mathcal{C}_i$ even if $0 < k_b - m_i < 1$. He chooses the new channel c with a uniform random probability as presented before. This second property allows us to resolve the inefficient stability states, but in the same time, it will also cause the instability of the Nash equilibria.

We provide the description of our algorithm below. Note that this algorithm now includes both properties: 1) the backoff mechanism and 2) the mechanism to break inefficient stable states.

Due to the second property of our algorithm, its does not perfectly converge to the existing Nash equilibria (more precisely, it converges there with high probability, but it does not stay in a Nash equilibrium solution). Nevertheless, we can observe that the algorithm remains in states that are “close” to Nash equilibria in terms of load-balancing. We demonstrate this intuition by the simulations presented in Section VII-D.

D. Simulation Results for Algorithm 3

We implemented Algorithm 3 in MATLAB 7.0.0.1 and with a special focus on wireless 802.11a protocol (meaning that we have chosen 11 orthogonal channels as a default value for $\mathcal{N}$) In this subsection, we present our simulation results that we we will investigate the performance of algorithm 3 in terms of convergence time and efficiency. In each of the simulations, we assumed a constant rate function $R(\cdot)$, note however, that the algorithm shows similar properties for any decreasing rate function introduced in Section III.

Let us first highlight the best and worst case in terms of the desired load-balancing. The best case is one of the NE channel allocations, while the worst case is characterized by the fact that there exist k channels where each of the players have a radio, while the rest of the channels have no radios at all. In Figure 10, we present the worst case channel allocation

Algorithm 3 Distributed Pareto-optimal NE channel allocation algorithm using local information

RandomChannelAllocation()
while there is change **do**
  ChannelUpdate()
  for $i = 1$ to $|\mathcal{N}|$ **do**
    if backoff counter is 0 **then**
      if $(max_{c \in C_i}(k_c) - min_{c \in C_i}(k_c) > 1)$ **then**
        for $j = 1$ to k **do**
          assume that radio j uses channel b
          if $k_b > m_i$ **then**
            move the radio j from b to $c \notin \mathcal{C}_i$, where c is chosen with uniform random probability from the set $\mathcal{C} \backslash \mathcal{C}_i$
          end if
        end for
      else
        for $j = 1$ to k **do**
          assume that radio j uses channel b
          if $k_b \geq m_i$ **then**
            move the radio j from b to $c \notin \mathcal{C}_i$ with probability ϵ, where c is chosen with uniform random probability from the set $\mathcal{C} \backslash \mathcal{C}_i$
          end if
        end for
      end if
    else
      decrease the backoff counter value by one
    end if
  end for
end while

that is opposed to the best case NE in Figure 5 and we refer to it as *unbalanced (UB).*

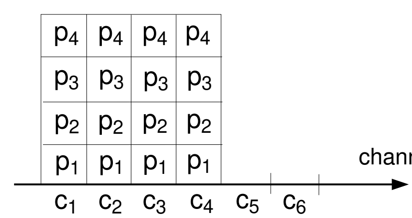

Fig. 10. An example for a worst case channel allocation that is completely unbalanced, as opposed to a NE (best case) shown in Figure 5. Here $|\mathcal{C}| = 6$, $|\mathcal{N}| = 4$ and $k = 4$.

We calculate the average number of radios per channel as $m = \frac{|\mathcal{N}| \cdot k}{|\mathcal{C}|}$. We can compare the utilization of every channel c to the average to achieve the total balance of the channel allocation S:

Definition 6: (Balance:) The *balance* β of a channel allocation S is defined as the sum $\beta(S) = \sum_{c \in |\mathcal{C}|} |k_c - m|$.

The notion of balance allows us to define the efficiency of a given channel allocation as a proportion between the worst case and the best case channel allocations.

Definition 7: (Efficiency:) The *efficiency* ϕ of a channel allocation S is defined as $\phi(S) = \frac{\beta(S_{\mathrm{UB}})-\beta(S)}{\beta(S_{\mathrm{UB}})-\beta(S_{\mathrm{NE}})}$.

Let us emphasize that for any channel allocation S, we have $0 \leq \phi(S) \leq 1$. Furthermore, $\phi(S_{\mathrm{NE}}) = 1$ and $\phi(S_{\mathrm{UB}}) = 0$ as desired by this measure.

Let us now we define the *average efficiency* over time and *efficiency ratio*. To this end, we denote the efficiency in round t by $\phi(t, S)$.

Definition 8: (Average efficiency and efficiency ratio:) The *average efficiency* $\overline{\phi}$ at round T is defined as the sum $\overline{\phi}(T) = \sum_{t=1}^{T} \phi(t, S)$. From this definition, we derive the *efficiency ratio* as $\Phi = \lim_{T\to\infty} \frac{\overline{\phi}(T)}{T}$

Note that the efficiency ratio expresses the performance of the distributed channel allocation algorithm per round over a long period of time. In our simulations, we applied a finite simulation time, hence we measured the efficiency ratio at $T = 10000$.

Finally, let us define the *convergence time* of Algorithm 3 as follows.

Definition 9: (Convergence Time): We define the *convergence time* of Algorithm 3, as the time when the channel allocation efficiency first reaches the value of one (i.e., the efficiency of a NE, $\phi(S_{\mathrm{NE}})$).

Suppose that there are a few players in the game and they use the 802.11a medium access protocol. We assume that the length of one round in the updating algorithm is 10ms. This duration of one round corresponds roughly to the time until these devices all transmit one MAC layer packet, i.e., the time that the devices can learn about other devices in the channel.

For each simulation result, where we present average values, we have performed 100 simulation runs. As mentioned previously, we run each simulation for 10000 rounds that corresponds to 100s according to the assumption above. For the convergence time simulations, we present our results with a 0.95 confidence level on the mean value.

Let us first present an example run for our distributed algorithm with imperfect information in Figure 11. One can notice that the algorithm quickly reaches the NE state and thus the average efficiency converges to one. Also, one can observe that the system sometimes leaves the NE state due to the second property (change a radio on a channel $c \in C_{max}$ in a stable state with probability ϵ), but the system quickly returns to a NE.

First, we investigate the effect of the number of radios per device on the efficiency ratio (shown in Figure 12a) and on the convergence time of the algorithm (presented in Figure 12b). We can observe on the figures that the efficiency is basically one if there are more than two radios per device. This means that the algorithm drives the system into one of the NE state and it stays in these NE for most of the time. For two radios per devices, the effect of changing the channel for even one radio has a significant impact that undermines the stability of the NE more easily. Note however, that even in this case, the efficiency ratio is very high (close to the efficiency of the NE). It is also worth to mention that the higher the number of radios per device, the more channels the players know. Hence, the increasing amount information helps their decisions.

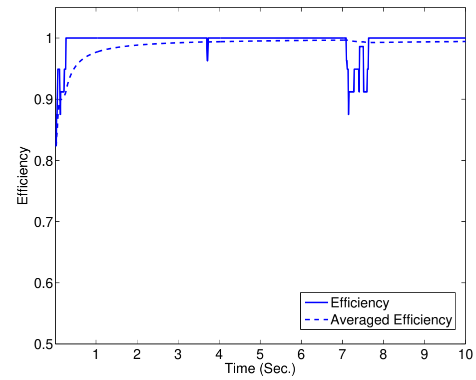

Fig. 11. One simulation run: Efficiency and averaged efficiency vs. time using the values $|\mathcal{C}| = 11$, $|\mathcal{N}| = 10$, $k = 3$ and $W = 15$

Next, we investigate the effect of the number of players, each device having three radios and present our results in Figures 12c and 12d. We can see that our distributed algorithm keeps the system in an efficient state, although the efficiency is slightly lower for particular values of $|\mathcal{N}|$. The reason is that our algorithm is designed to lead the players to *any possible NE*. Indeed, the number of possible NE depends on the value of $|C_{min}|$ as derived in (12). If $|C_{min}| = 0$, there exists only one NE, which is perfectly load-balanced. The higher the absolute value $||C_{min}| - |C_{max}|| = |\frac{|\mathcal{C}|}{2} - |C_{min}||$, the lower the number of possible NE, hence it is more difficult to reach one of them. This explains the higher convergence time in Figure 12d for $|\mathcal{N}| = 11, 15, 22$. One can notice that for $|\mathcal{N}| = 11, 22$, we have $|C_{min}| = 0$, thus the relatively high convergence time. Furthermore, if there exist only a few NE, then the algorithm is likely to break them due to the property caused by ϵ. This results in a lower efficiency for these values. Let us emphasize that even for these cases, our algorithm performs very well resulting in a high efficiency ratio.

In the second set of simulations, we study the effect of the two parameters that introduce the randomness to Algorithm 3. First, we show the effect of ϵ on the efficiency ratio and the convergence time in Figures 13a and 13b. One can observe that the efficiency ratio is very high, but slightly decreases as ϵ increases. The reason is that with a higher ϵ value it is more likely that the algorithm does not stay in a NE, once it has reached it. As a tradeoff, for low ϵ values, the algorithm converges slower, because it might stabilize in "false stable states" for a longer time. It is interesting to observe that some values of ϵ achieve a good balance between efficiency and convergence time, which motivated our choice $\epsilon = 0.0001$ for the default value.

Finally, we study the effect of the size of the backoff window in Figures 13c and 13d. Intuitively, efficiency increases with the backoff window size, because of the decreasing

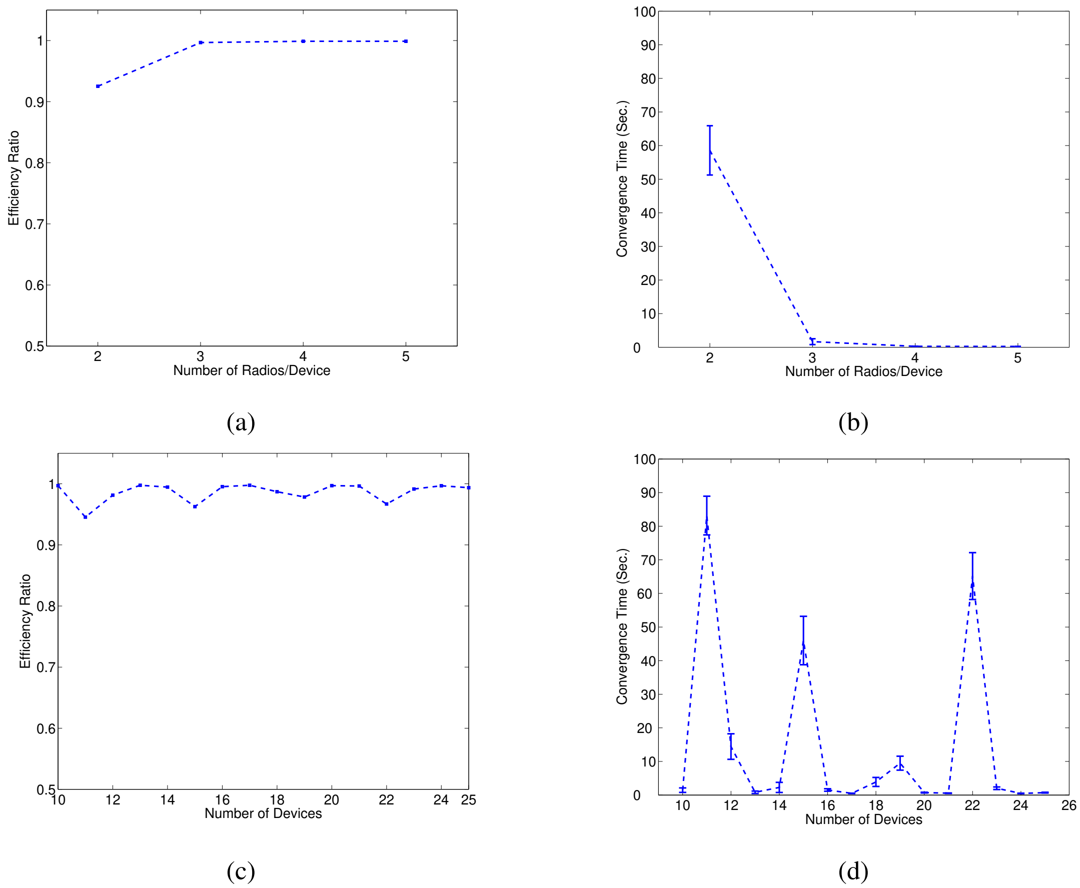

Fig. 12. The effect of the total number of radios: (a) The efficiency ratio and (b) the convergence time as a function of the number of radios per device k. Similarly, we show (c) the efficiency ratio and (d) the convergence time as a function of the number of players $|\mathcal{N}|$. The simulation parameters are $|\mathcal{C}| = 11$, $\epsilon = 10^{-4}$ and $W = 15$. In addition, we used the following default values $|\mathcal{N}| = 10$ and $k = 3$, where they did not correspond to the measured parameter.

number of simultaneous channel changes. Interestingly, this increase is very rapid and the algorithm is very efficient for quite small backoff window values. With a larger backoff window, there is a high chance that our algorithm realizes a sequential procedure similar to the centralized algorithm with perfect information described in Algorithm 1. Note however that setting a very high backoff window value is not reasonable, because it makes the players waiting for an unnecessary long time. Due to the same reason, convergence time drops quickly as the backoff window value increases.

In summary, we can observe that, in spite of the fact that convergence is not theoretically ensured, the proposed distributed algorithm based on imperfect information ensures high system performance and good convergence time.

VIII. Conclusion

In this paper, we have considered the problem of competitive channel allocation among devices using multiple radios. Our main conclusion is that in spite of the non-cooperative behavior of such devices, their Nash equilibrium channel allocations result in a Pareto-optimal solution, which means that they are system-efficient as well. These Nash equilibria are characterized by the fact that the devices occupy the available channels almost evenly. We have also studied fairness and coalitions in the selfish context. Finally, we have provided three algorithms to achieve this efficient Nash equilibrium channel allocation and we study their convergence properties either theoretically or numerically.

In terms of future work, we will pursue our theoretical investigations of selfish multi-radio channel allocation in multiple collision domains. We will pay a particular attention to the application of study of existing fairness metrics in the competitive context. In addition, we will take the cost of channel scanning into consideration. Last but not least, we will study the convergence properties of our algorithms that achieve Nash equilibria in the single collision domain and we will extend them (or design new algorithms) to the general case of multiple collision domains.

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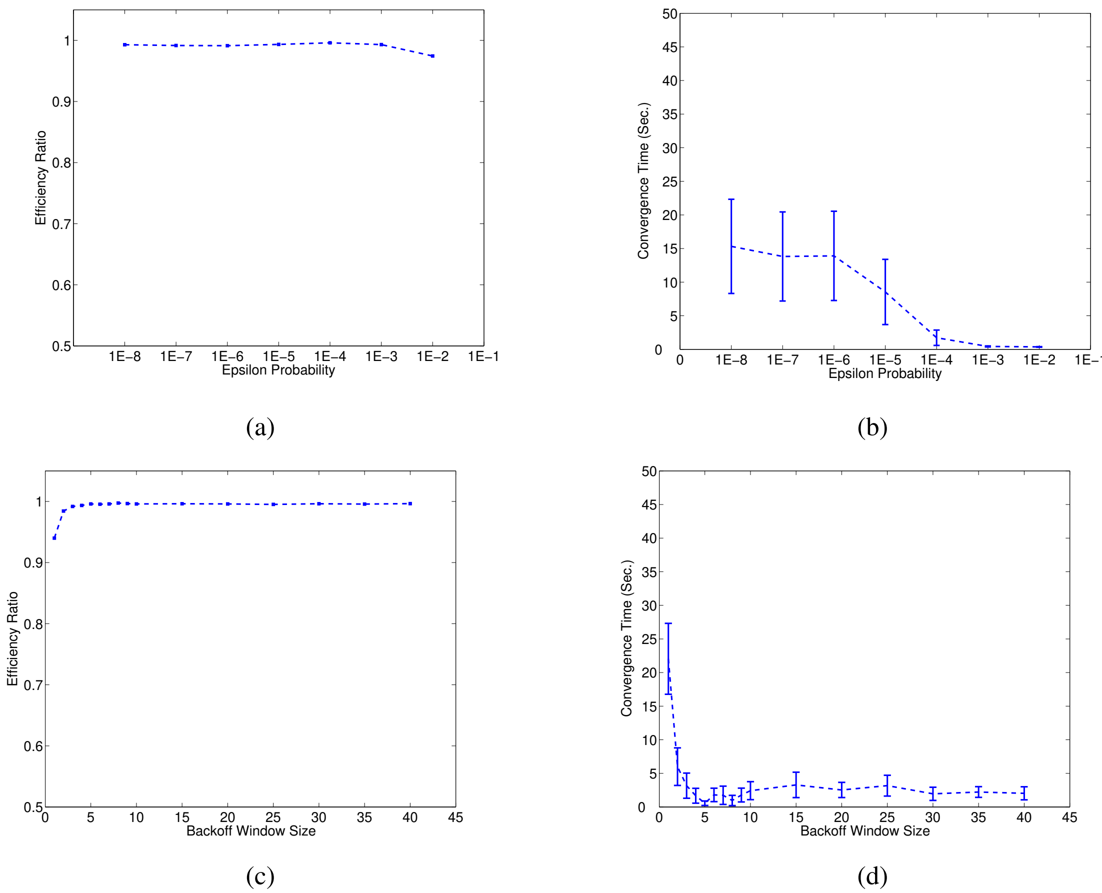

Fig. 13. The effect of randomness parameters: (a) Efficiency ratio and (b) convergence time as a function of ϵ. Furthermore, we present (c) the efficiency ratio and (d) the convergence time as a function of the backoff window size W. The simulation parameters are $|\mathcal{C}| = 11$, $|\mathcal{N}| = 10$ and $k = 3$. In addition, we used the following default values $\epsilon = 10^{-4}$ and $W = 15$, where they did not correspond to the measured parameter.

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