NON-DARCIAN SEEPAGE STABILITY ANALYSIS OF NON-NEWTONIAN FLUID

by

Pei-Tao QIU[a,b,c], Zhan-Qing CHEN[a,b*], Hai PU[a,b], and Lian-Ying ZHANG[c]

[a] State Key Laboratory for GeoMechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou, China
[b] School of Mechanics and Civil Engineering, China University of Mining and Technology, Xuzhou, China
[c] School of Civil Engineering, Xuzhou University of Technology, Xuzhou, China

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In this paper, the two-phase fluid composed of fine geologic particles and water is considered to be a non-Newtonian fluid, and the seepage dynamics model of the two-phase medium in the rock-soil structure is constructed. Based on the hypothesis, the boundary conditions and solving methods of the model are given and the critical conditions of the model instability are also discussed in detail. It is shown that the existence of the equilibrium state of the kinetic model is determined by the power exponent, the effective fluidity and the non-Darcian flow factor.

Key words: *seepage instability, piping, non-Darcian, non-Newtonian*

Introduction

According to statistics from the China Damby Dam Safety Management Center [1] and the US Geological Survey Cost [2], nearly 40% of dam failure cases are caused by piping failure. Research on the mechanism of piping failure caused by seepage loss has been done by scholars from various countries and has achieved fruitful results. The development of seepage can be traced back to the famous Darcy's law, which was obtained by French hydrologist Darcy in 1856. It is used to describe the linear relationship between hydraulic gradient and seepage velocity [3]. For the piping, it was Terzaghi [4] who first proposed the concept of piping when testing the sand-proof model of sheet pile cofferdam. Istomina defined the piping as the movement or loss of fine particles along the pores of the skeleton particles under the action of seepage, and according to the principle that the weight of the movable particles in the water and the percolation of the particles are balanced, the soil is obtained. Critical slope formula for body tube collapse [5]. Some follow-up scholars began to analyze the mechanism of seepage instability from the aspect of particle size grading. Kenney [6] argues that the boundary between pipe soil and non-pipe soil is 29% fine (not dense) and 24% (compact). Aberg [7] proposed a particle-void ratio chain model related to the compactness and grading curve, and established the boundary particle size calculation formula between the skeleton particles and the movable filler particles. In recent years, with the rapid development of computers, the numerical simulation method has been greatly developed in terms of geotechnical engineering [8, 9]. Jeyisanker *et al.* [10] uses a new particle model to analyze the effects of water transients and

* Corresponding author, e-mail: chenzhanqing@vip.163.com

steady-state seepage through randomly filled coarse-grained soils. The Navier-Stokes equation uses finite-difference discretization and is used to predict critical water flow velocities. Navas [11] proposed a numerical finite element coupled transient model to analyze the effects of different precipitation velocities, permeability, and particle size on seepage stability in earth dams. Based on the results of model tests and finite element analysis, Takahashi [12] applied centrifuge technology to composite fluid dynamics and geotechnical engineering problems to analyze the stability of the foundation under seepage. Wang *et al*. [13] developed a numerical model based on the meshless Galerkin method to simulate the progress of the seepage channel of the dam, which can reproduce experimental data obtained from physical models, such as critical head and progress time. Cheng used a semi-analytical CFD-DEM model to analyze the migration of fine particles and the erosion of pipe-lines in graded soil voids. The seepage characteristics of fine and coarse materials associated with slopes and dams [14]. Yang *et al*. [15], Yang [16], and Yang *et al*. [17, 18] has made a series of original achievements in generalized calculus and special functions, which provides a new method for solving non-linear problems. Shi *et al*. [19] found that the failure mode of fine-grained soil is fluid, because the effective stress is close to zero. However, for coarse-grained soils, the failure mode is due to the presence of effective stresses.

The previous researches on the instability of seepage are based on the Darcian or non-Darcian law of water being Newtonian fluid. In fact, in the seepage process of rock and soil, the gradual loss of fine particles leads to the expansion of the internal water conduit, and gradually destroys the pore structure and skeleton structure inside the rock and soil, resulting in the gradual change of the permeability characteristics of the rock and soil [20]. Since the two-phase fluid composed of such fine particles and water belongs to a non-Newtonian fluid, the permeability characteristics of the liquid phase and the solid phase medium depend on the fracture structure and solid phase volume fraction of the rock and soil. When the fine particles are lost to a certain extent, the rock and soil structure is not enough to withstand the external load under the current conditions and instantaneously destabilizes, and a large number of geotechnical particles protrude with the water to form a through-flow passage. Therefore, this paper considers that the two-phase fluid composed of fine geologic particles and water belongs to non-Newtonian fluid, and the seepage dynamics model of the two-phase medium in rock and soil structure is constructed. Based on the assumptions, the boundary conditions and solving methods of the model are given. The critical conditions of the model instability are discussed. The effects of power exponent, effective fluidity and non-Darcian flow factor on the seepage velocity are analyzed.

Seepage dynamics model

The occurrence of piping generally includes two aspects. One is the structure of the soil itself, that is, there are enough coarse particles to form pores larger than the diameter of the fine particles (internal factors, ie geometric conditions), and the penetration is large enough (external causes, *i. e.* hydraulic conditions). Under the hydraulic conditions that satisfy the occurrence of piping, in the first two-stages of piping formation, as the fine particles and coarse particles are continuously taken away by the water flow, the pores are continuously increased, and the Reynolds coefficient of the water flow is large. Influence of force, seepage velocity and pressure gradient no longer satisfy Darcy's law. In addition, the water flow carrying sediment shows obvious non-Newtonian fluid characteristics, that is, the shear stress and the angular strain rate are non-linear. Therefore, to analyze the problem of instability of the piping structure, the following assumptions must be made on the actual engineering problems:

– the permeation force at the ends of the percolation can drive the fine particles to roll or move between the pores,

– the cement sand particles are continuous media filled uniformly in the water body, and the seepage characteristics satisfy the Forchheimer relationship,
– the cement sand mixture is a non-Newtonian fluid, which satisfies the characteristics of the power law type fluid, that is, the relationship between the shear stress and the angular strain rate is $\tau = C\gamma^n$ (the pseudoplastic fluid when $n < 1$, and the expansive fluid when $n > 1$), where C and n are, respectively, the consistency coefficient and power exponent, and
– the seepage channels formed during the formation of the piping are smooth and parallel.

As shown in fig. 1, a dynamic model is established to analyze the third-stage seepage stability of the piping formation. The stability of the seepage is determined by the mass-conservation equation, the momentum conservation equation and the fluid compression equation of the non-Darcian flow of the water-sand mixture in the smooth parallel seepage channel:

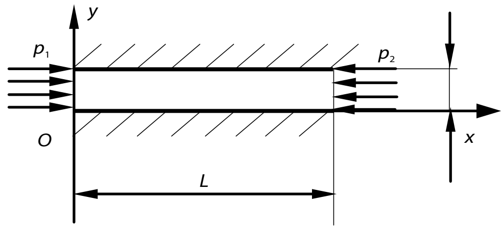

Figure 1. Schematic diagram of seepage dynamics model

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial(\rho V)}{\partial x} = 0 \\ \rho c_a \dfrac{\partial V}{\partial t} = -\dfrac{\partial p}{\partial x} - \dfrac{\mu_e}{k_e} V^n - \rho\beta V^2 \\ \rho = \rho_0 \left[1 + c_f (p - p_0)\right] \end{cases} \tag{1}$$

where p is the fluid pressure, V – the seepage velocity, ρ – the mass density of the mixture, μ_e – the equivalent viscosity, k_e – the effective permeability, β – the non-Darcian flow factor, c_f – the fluid compression coefficient, p_0 – the standard atmospheric pressure, and ρ_0 – the mass density of the fluid at the standard atmospheric pressure. The effective permeability and equivalent viscosity are determined by the width of the percolation passage, b, and the constitutive parameters of the non-Darcian fluid, *i. e.*

The boundary conditions:

$$\begin{cases} p\big|_{x=L} = p_1 \\ p\big|_{x=L} = p_2 \end{cases} \tag{2}$$

The initial conditions:

$$\begin{cases} p\big|_{t=0} = p_1 + \dfrac{p_2 - p_1}{L} x, x \in \left[0, L\right] \\ V\big|_{t=0} = V_{st} \end{cases} \tag{3}$$

The equilibrium state of the power system (1) is determined by the following equation:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} = 0 : \dfrac{\partial(\rho V)}{\partial x} = 0 \\ \dfrac{\partial V}{\partial t} = 0 : \dfrac{\partial p}{\partial x} + \dfrac{\mu_e}{k_e} V^n + \rho\beta V^2 = 0 \\ \dfrac{\partial p}{\partial t} = 0 \end{cases} \tag{4}$$

For the flow of incompressible cement sand mixture, eq. (4) can be simplified. Therefore, the pressure, p_{st}, the seepage velocity, V_{st}, and the mass density, ρ_{st}, in the equilibrium state satisfy:

$$\begin{cases} p_{st} = p_1 + \dfrac{p_2 - p_1}{L}x \\ \rho_{st} = \rho_0 \\ \dfrac{p_2 - p_1}{L} + \dfrac{\mu_e}{k_e}V_{st}^n + \rho\beta V_{st}^2 = 0 \end{cases} \tag{5}$$

It can be seen from eq. (5) that in the equilibrium state, the pressure is linearly distributed, and the mass density and the percolation velocity are uniformly distributed. Since the third equation in expression (5) is a transcendental equation, the analytical expression of V_{st} is generally not obtained. We can use the Runge-Kutta method to get a numerical solution of V_{st}. The specific method is:

In the first step, we find the extreme point of the function:

$$f(y) = \frac{\mu_e}{k_e}y^n + \rho\beta y^2 + \frac{p_2 - p_1}{L}$$

due to $f'(y) = 0$, and have:

$$\frac{n\mu_e}{k_e}y^{n-1} + 2\rho\beta y = 0 \tag{6}$$

The non-zero root of the algebraic equation $f'(y) = 0$:

$$y^* = \left(-\frac{2\rho I_e\beta}{n}\right)^{\frac{1}{n-2}} \tag{7}$$

In the second step, the steady-state value of the seepage velocity is obtained by the dichotomy.

Setting $y_1 = 0$, and $y_2 = y^*$, and using the dichotomy to find the zero point y^{**} of the equation:

$$f(y) = \frac{\mu_e}{k_e}y^n + \rho\beta y^2 + \frac{p_2 - p_1}{L}$$

in the interval $[y_1, y_2]$, we find that y^{**} is equivalent to the root of the third equation in expression (8), *i. e.*, $V_{st} = y^{**}$.

Existence of seepage equilibrium

The stability of the non-Darcian flow of a power-law non-Newtonian fluid in a fracture is determined by the maximum value of the function $f(y)$, *i. e.*, when the function $f(y^*) \geq 0$, $f(y)$ has a zero point, that is, the percolation velocity has a stable value, and the seepage is stable at this time. When $f(y^*) < 0$, the function $f(y)$ has no zero point, that is, the third equation in expression (8) has no real root. At this time, regardless of the initial conditions, the seepage does not reach equilibrium, and the seepage is instability. Below, a mathematical expression for the instability condition of seepage is established.

Based on the previous discussion, we can express the seepage stability condition:

$$f'(y) = 0 \tag{8}$$

Bringing eq. (7) into eq. (8), the condition of seepage instability can be written:

$$\rho\beta\left(-\frac{2\rho\beta I_e}{n}\right)^{\frac{2}{n-2}}+\frac{1}{I_e}\left(-\frac{2\rho\beta I_e}{n}\right)^{\frac{n}{n-2}}-\frac{p_1-p_2}{L}<0 \tag{9}$$

Simplify eq, (9), we obtain:

$$(n-2)\left(-\frac{2\rho\beta I_e}{n}\right)^{\frac{n}{n-2}}>-\frac{2I_e\left(p_1-p_2\right)}{L} \tag{10}$$

Considering $n < 2$, we can rewrite eq. (10):

$$\left(-\frac{2\rho\beta I_e}{n}\right)^{\frac{n}{n-2}}<\frac{2I_e\left(p_1-p_2\right)}{(2-n)L} \tag{11}$$

Taking the logarithm on both sides of eq. (11), we get:

$$\ln(-\beta)>\frac{n-2}{n}\ln\left[-\frac{2I_e(p_1-p_2)}{(n-2)L}\right]-\ln\left(\frac{2\rho I_e}{n}\right) \tag{12}$$

Further simplification:

$$\frac{2}{n}\ln(I_e)+\ln(-\beta)>-\ln\left(\frac{2\rho}{n}\right)+\frac{n-2}{n}\ln\left(\frac{2}{2-n}\frac{p_1-p_2}{L}\right) \tag{13}$$

Based on the previous stability analysis, we divide the 2-D parameter space described in eq. (13) into two parts, which are the stability zone and the instability zone of the seepage. The boundary of two zones can be expressed:

$$\frac{2}{n}\ln(I_e^*)+\ln(-\beta^*)=-\ln\left(\frac{2\rho}{n}\right)+\frac{n-2}{n}\ln\left(\frac{2}{2-n}\frac{p_1-p_2}{L}\right) \tag{14}$$

The main physical and mechanical parameters of the seepage dynamics model in this figure are set as shown in tab. 1. When the power exponent is 0.4, the pseudoplastic fluid is characterized. When the power exponent is 1, the Newtonian fluid is characterized and the power exponent is 1.6 to characterize the expansive fluid.

Table 1. Main physical and mechanical parameters of seepage dynamics model

L [m]	ρ [gcm^{-3}]	p_1 [MPa]	p_2[MPa]	C_a [%]
0.20	1.02	0.60	0.02	$3.89 \cdot 10^4$

It can be seen from fig. 2 that the power exponent, n, the effective fluidity, I_e, and the non-Darcian flow factor, β, determine the existence of the seepage equilibrium state.

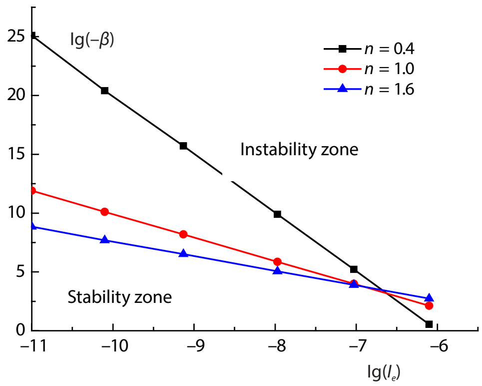

Figure 2. Power law fluid seepage critical instability curve

Conclusion

In our work, the two-phase fluid composed of fine geologic particles and water is considered to be a non-Newtonian fluid, and the seepage dynamics model of the two-phase medium in the rock-soil structure is constructed.

Based on the hypothesis, the boundary conditions and solving methods of the model are given. The critical conditions of the model instability are discussed. The effects of power exponent, effective fluidity and non-Darcian flow factor on seepage velocity are analyzed.

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Nomenclature

c_a – acceleration coefficient, [%]
k_e – the effective permeability, [ms^{-1}]
L – the broken rock particle size, [m]
p_1 – left boundary pressure value, [MPa]
p_2 – right boundary pressure value, [MPa]
V – seepage velocity, [ms^{-1}]

Greek symbols

β – the non-Darcian flow factor, [m^{-1}]
μ_e – the dynamic viscosity, [Pa·s]
ρ – mass density of the mixture, [gcm^{-3}]
ρ_0 – mass density of fluid, [gcm^{-3}]

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