NON-DATA-AIDED FREQUENCY OFFSET AND SYMBOL TIMING ESTIMATION FOR BINARY CPM: PERFORMANCE BOUNDS

Jaume Riba and Gregori Vazquez

Department of Signal Theory and Communications. Universitat Politècnica de Catalunya.
Campus Nord, Ed. D5, Jordi Girona 1 i 3, 08034, Barcelona (Spain). Email: jriba,gregori@gps.tsc.upc.es

ABSTRACT

The use of (spectrally efficient) CPM modulations may lead to a serious performance degradation of the classical non-data-aided (NDA) frequency and timing estimators due to the presence of self noise. The actual performance of these estimators is usually much worse than that predicted by the classical Modified Cramér-Rao Bound. In this contribution we apply some well-known results in the field of signal processing to these two important problems of synchronization. In particular we propose and explain the meaning of the Unconditional CRB in the synchronization task. Simulation results for MSK and GMSK, along with the performance of some classical and recently proposed synchronizers, show that the proposed bound (along with the MCRB) is useful for a better prediction of the ultimate performance of the NDA estimators.

1. INTRODUCTION

One of the fundamental tasks of a digital receiver is the estimation of the carrier frequency and symbol timing directly from the received data (see [1] and references therein for an exhaustive review). Frequency and timing synchronization algorithms are typically categorized in Decision-Directed (DD), Data-Aided (DA) and Non-Data_Aided (NDA) methods. While DD and DA schemes offer better tracking and acquisition performance respectively, NDA methods are preferred when the decisions are not available or not reliable, and the data is not known. NDA algorithms offer the additional advantage of being phase-independent, thus avoiding spurious locks and prolonged acquisitions caused by complex iteractions between phase and frequency and/or phase and timing correction algorithms. Additionally, simple symbol-by-symbol decisions cannot be obtained in Continuous Phase Modulations (CPM), for which the DD schemes becomes more complicated. As it is known, CPM modulations are attractive due to their high spectral efficiency and constant envelope.

As in all estimation problems, the computation of the Cramér-Rao Bound (CRB) is of interest. However, in the field of digital communications, the CRB is hard to evaluate. A more manageable performance limit is the so-called Modified CRB (MCRB) proposed in [2], which is generally lower than the true CRB. However, it is difficult to known in advance whether the MCRB is tight enough for use in practical applications. In fact, one can only assess that the MCRB will be attained by DD methods operating at high SNR [3]. The situation is that, most of the times, the performance of the NDA methods [1] is much worse than that predicted by the MCRB, especially in the case of CPM signals of long *frequency response* length. In part, the reason for this optimistic performance prediction is that the MCRB is associated only to the shape of the isolated derivative pulse (with respect the frequency or the timing parameters). In particular, it does not take into account that the pseudo-pulses and their delayed versions overlap among themselves. Estimators ignoring this fact, usually exhibit a high amount of self noise.

This departure is specially dramatic for the cases of: i) frequency estimation of linear and CPM modulations, and ii) frequency and timing estimation of CPM modulations. Therefore, the question arises of determining whether the degraded performance in those cases is caused by inherent theoretical difficulties of, on the contrary, it may be associated to the nature of the estimator. This is especially important in the vast field of synchronization, where most methods have been developed under *ad hoc* basis. In particular, it is of interest to evaluate the inherent impact of the increased spectral efficiency of CPM modulation to the synchronization task.

In this paper, we propose a new bound for frequency and timing estimation that may be useful for complementing the information given by the MCRB. Its application is limited to the class of quadratic NDA estimators. This limitation is justified because: i) most NDA methods proposed in the literature are quadratic; ii) quadratic algorithms have a reduced complexity and, then, they are suited for a real-time implementation; and iii) they usually exhibit a robust performance behavior in low-SNR conditions.

The bound proposed in this paper, which complement previous work by the authors in [4][5][6][7], is obtained from well-known results derived in the field of signal processing [8]. To this end, we formulate a discrete-time linear signal model for binary CPM signals which is identical to that employed to formulate several important problems in the signal processing field, as the problem of direction finding with narrow-band sensor arrays. We focus on the special case

This work was supported by TIC98-0412, TIC98-0703 (CICYT) and CIRIT/Generalitat de Catalunya 1998SGR-00081.

0-7803-6293-4/00/$10.00 ©2000 IEEE.

of Gaussian Minimum Shift Keying (GMSK) modulation, which is the modulation adopted in the GSM European cellular mobile digital system. It is noted that the proposed model is also useful for the formulation of new estimators, as those proposed in [4][5][6][7], but this is omitted here for space reasons.

2. DISCRETE-TIME LINEAR MODEL FOR BINARY CPM

2.1. Laurent expansion

As shown in [9][1], the Laurent expansion allows the exact representation of a binary CPM modulation as the superposition of $J = 2^{L-1}$ linearly modulated signals of symbol interval T. After antialiasing filtering of bandwidth $F_s/2$ and sampling at $F_s = 1/T_s$, the complex envelope is given by:

$$r(k) = \sum_{m=0}^{J-1} \sum_{n=-\infty}^{+\infty} c_{m,n} g_m(kT_s - nT - \tau) e^{j(\theta_o + 2\pi f k T_s)} + w(kT_s) \quad (1)$$

where $g_m(t)$ are the pseudo-pulses; τ, f and θ_o are the timing, frequency and phase errors respectively, $w(kT_s)$ is the AWGN term, and, finally, $c_{m,n}$ are the pseudo-symbols, which are known to be zero-mean, uncorrelated and of unite-power, $E\left[c_{m,n}c^*_{m',n'}\right] = \delta_{m,m'}\delta_{n,n'}$. It is assumed that the number of samples per symbol $N_{ss} = T/T_s$ is integer. Note that the previous model includes also the case of linear modulations (J=1), for which $g_0(t)$ is usually designated such that $g_0 * g_0(kT) = 0$ for $k \neq 0$ (ISI-free condition). However, in the general case (J>1) one obtains that $g_{m'} * g_m(kT) \neq 0$, which means that, at the receiver, one cannot recover the pseudo-symbols $c_{m,n}$ simply by a bank of matched filters. Classical synchronization methods for CPM consider that the signal power is mostly concentrated in the first component corresponding to $g_0(t)$. Then, the resulting algorithms exhibit *self-noise* as a result of the contribution of the remaining pseudo-pulses. This leads to a floor-jitter in their performance, which is the reason why it departs significantly from the theoretical performance lower bounds known in synchronization. Moreover, these bounds are obtained by ignoring two important facts: i) the presence of the pseudo-pulses $g_m(t)$ for m>0, and ii) the auto-interference of the main pulse, $g_0 * g_0(kT) \neq 0$. Fig. 1 shows the pseudo-pulses obtained for the MSK and GMSK modulations, where one can observe the double effect of decreasing BT: i) the length of the main pulse $g_0(t)$ increases, and ii) the energy of the remaining pseudo-pulses increases.

2.2. Vector notation

Several important problems in the signal processing field can be reduced to estimating the parameters in the following model:

$$\mathbf{r} = \mathbf{A}(\lambda)\mathbf{x} + \mathbf{w} \quad (2)$$

where, in our case, λ is the parameter of interest, which may be either $\lambda = f$ or $\lambda = \tau$, depending on the problem considered. $\mathbf{A}(\lambda)$ is the transfer matrix dependent on parameter of interest and, finally, $\mathbf{w}$ is the white noise vector

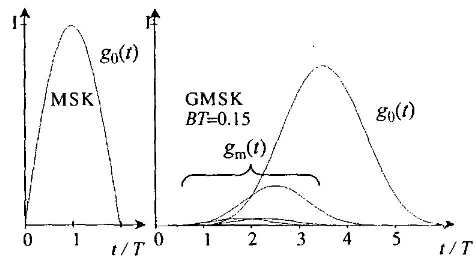

Figure 1: Pseudo-pulses for MSK (BT=∞) and GMSK (BT=0.15).

($E\left[\mathbf{w}\mathbf{w}^H\right] = \sigma^2\mathbf{I}$, $\sigma^2 = 2N_0F_s$). The interest of considering the Laurent expansion is that it allows us to write a model like (2) for the problem of frequency and timing estimation of CPM signals. For this purpose, we use (1) for representing the isolated contribution of the i-th pulse of the m-th shape by the following column vector:

$$\mathbf{a}_{m,i}(\lambda) \doteq e^{j2\pi f(iT - T_o)} \left[g_m(-iT - \tau), g_m(-iT + T_s - \tau)e^{j2\pi f T_s}, \ldots, g_m(-iT + (M-1)T_s - \tau)e^{j2\pi f T_s(M-1)}\right]^{\mathrm{T}} \quad (3)$$

where T_o is a constant that reflects the arbitrary time origin of the problem. The observation interval is limited to M samples, being M arbitrarily fixed by the synchronizer designer. If we now consider that the transmitter sends a burst of L symbols, then, the J-signal composite received data vector $\mathbf{r}$ (observation vector) can be represented using (2) by means of the following *linear transfer matrix* $\mathbf{A}(\theta)$:

$$\begin{aligned} \mathbf{A}(\lambda) &\doteq \left[\ \mathbf{A}_o(\lambda),\ \ \mathbf{A}_1(\lambda),\ \ \ldots,\ \ \mathbf{A}_{J-1}(\lambda)\ \right] \quad (4) \\ \mathbf{A}_m(\lambda) &\doteq \left[\mathbf{a}_{m,o}(\lambda), \mathbf{a}_{m,1}(\lambda), \ldots, \mathbf{a}_{m,L-1}(\lambda)\right] \end{aligned}$$

where the vector $\mathbf{x}$ of signals is:

$$\begin{aligned} \mathbf{x} &\doteq \left[\ \mathbf{x}_0^T\ \ \mathbf{x}_1^T\ \ \ldots\ \ \mathbf{x}_{J-1}^T\ \right]^T \quad (5) \\ \mathbf{x}_m &\doteq \left[x_{m,o}, x_{m,1}, \ldots, x_{m,L-1}\right]^T \\ x_{m,i} &\doteq c_{m,i}e^{j\theta_o} \end{aligned}$$

which holds that

$$\mathbf{\Gamma} = E\left[\mathbf{x}\mathbf{x}^H\right] = \mathbf{I}. \quad (6)$$

Then, it has been shown that the problem of synchronization of CPM signals can be reduced to a well-known estimation problem in the field of signal processing (especially in the problem of direction finding with narrow-band sensor arrays), as formulated in (2). It is noted, however, that for the problem at hand, a single observation of the process $r(t)$ (*snapshot*) is available, while in the problem of direction finding, one usually assumes that a set of N>1 *snapshots* are available, from which the parameters are extracted. With the specific restriction of N=1 in mind for the problem at hand, we are now able to take benefit of the high amount of research performed in this field. For space reasons, in this paper we will concentrate only on the derivation of new performance bounds for the GMSK modulation.

3. NEW CRB FOR QUADRATIC NDA ESTIMATORS

With the motivation of deriving a new bound (tighter than the MCRB) for synchronization that may be used as a more realistic benchmark for NDA methods, we use the results in the filed of signal processing developed in [8][10]. To this end, we propose the use of the so-called Stochastic or Unconditional CRB (UCRB), which is obtained under the assumption that the signals in $\mathbf{x}$ are Gaussian random variables. It is noted, however, that the symbols in digital communication are far from being Gaussian. Therefore, the bound obtained is applicable only to estimators that are (only) based on the sample second order moments of the signal, i.e. to those estimators that are quadratic.

Under the Gaussian assumption, and using the fact that the noise variance (σ^2) and the second order statistics of $\mathbf{x}$ ($\mathbf{\Gamma} = \mathbf{I}$) are known, the likelihood function can be now easily computed [8][10]:

$$\begin{aligned} \Lambda(\mathbf{r}|\lambda) &= C' \exp\left(-\mathbf{r}^H \mathbf{R}^{-1} \mathbf{r}\right) \\ \mathbf{R} &= \mathbf{A}\mathbf{\Gamma}\mathbf{A}^H + \sigma^2 \mathbf{I} \end{aligned} \qquad (7)$$

In some signal processing problems, it is usual to assume that the signal covariance matrix $\mathbf{\Gamma}$ is unknown, and it is considered a parameter that should be estimated jointly with the parameters of interest. Then, it becomes necessary to compute the joint CRB for the parameters of interest, along with σ^2 and $\mathbf{\Gamma}$. Contrarily, in the synchronization problem, it is more reasonable to assume that matrix $\mathbf{\Gamma}$ is known, as given in (6). Note that this assumption is equivalent to assuming a perfect knowledge of the kind of modulation that is arriving at the demodulator. Therefore, the new bound is obtained simply as the inverse of the Fisher information element corresponding to the parameter of interest (the general expression can be found in [8][10]):

$$UCRB(\lambda; \sigma^2, \mathbf{\Gamma}) = \frac{1}{\min\limits_{T_0} \operatorname{Re}\left[\operatorname{tr}\left(\mathbf{R}^{-1}\left\{\frac{\partial}{\partial\lambda}\mathbf{R}\right\}\mathbf{R}^{-1}\left\{\frac{\partial}{\partial\lambda}\mathbf{R}\right\}\right)\right]} \qquad (8)$$

where $\frac{\partial}{\partial\lambda}\mathbf{R} = \mathbf{A}\mathbf{\Gamma}\mathbf{D}_\lambda^H + \mathbf{D}_\lambda\mathbf{\Gamma}\mathbf{A}^H$, and $\mathbf{D}_\lambda = \frac{\partial}{\partial\lambda}\mathbf{A}$. As the selection of the time origin T_0 is arbitrary, the computation of the minimum with respect to T_0 in (8) is necessary (in the case of $\lambda = f$) for obtaining a bound as tight as possible This is also required for the derivation of the MCRB for frequency estimation. For the case of timing estimation, however, the selection of T_0 has no influence on the CRB.

Finally, it is noted that, although the bound given by (8) is derived under the assumption that the noise power is also known, a preliminary study, which is omitted here for space reasons, has shown that this fact has a slight impact on the predicted performance.

4. SIMULATION RESULTS AND CONCLUDING REMARKS

In this section, we present some simulation results for the MSK and GMSK modulations. The bounds are compared with the performance of known estimators taken from [1].

In Figs. 2 and 3, the UCRB is compared with the classical MCRB for the problem of symbol timing estimation of MSK and GMSK (BT=0.3) signals respectively. In

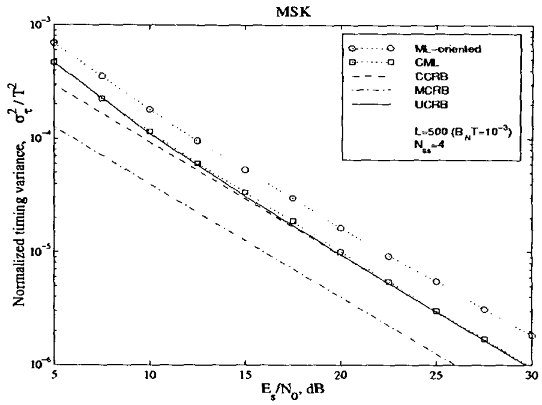

Figure 2: Comparison between the UCRB and the classical MCRB for timing estimation in MSK. The performance of two quadratic TEDs is also shown.

2, the bounds are compared with the performance of two quadratic estimators: the ML-oriented TED [1] and the conditional ML (CML) TED proposed in [5]. It is seen that while no-one reaches the MCRB, the UCRB is nearly attained by the CML TED. Additionally, one can observe that the UCRB reaches asymptotically the Conditional CRB (CCRB), whose use for synchronization of linear modulations was proposed in [4][5]. In Fig. 3, it is seen that the fourth-power, *ad hoc* method proposed in [11] exhibits a performance that lies between the MCRB and the UCRB. In general, for the class of NDA methods that perform more than quadratic operations on the received data, the UCRB (in combination with the MCRB) gives us the information of how much is gained by these higher order operations. Contrarily, it may happen that a fourth (or higher) - order synchronizer does not perform better than the UCRB. The meaning in that case would be that, potentially, there is no reason for this higher order non-linearity, and a quadratic (of reduced complexity) synchronization method can be potentially devised for the problem at hand.

Similar results are obtained for the problem of frequency estimation. For that problem, we simply show in Fig. 4 that the discrepancy between the UCRB and the MCRB increases when the observation length increases. The reason is that, while the MCRB tends to depend inversely on the cube of the observation length, the UCRB depends simply inversely on the observation length, as happens with the timing estimation problem for both limits. Finally, Fig. 5 shows the impact of the normalized Gaussian bandwidth BT, that is predicted by the UCRB. It is worth noting that the MCRB predicts a false impact because it only takes into account the shape of the signal spectrum. This leads to a slight performance penalty for the timing estimation and, surprisingly, to a performance improvement for the frequency estimation. The UCRB, however, predicts a penalty for both problems, although the frequency variance tends to decrease for very small values of BT, due to the fact that,

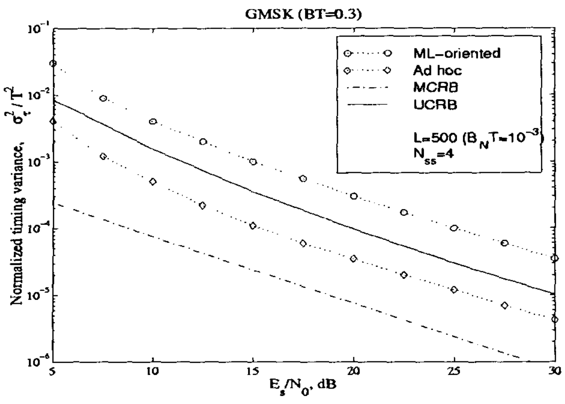

Figure 3: Comparison between the UCRB and the classical MCRB for timing estimation in GMSK. The performance of a quadratic TED (ML-based) and a fourth-power TED (*ad hoc*) is also shown.

in these extreme cases, the signal resembles more to a pure sinusoid.

5. REFERENCES

[1] Umberto Mengali and Aldo D'Andrea. *Synchronization Techniques for Digital Receivers.* Plenum Press, 1997.

[2] Aldo D'Andrea, Umberto Mengali, and Ruggero Reggiannini. "The Modified Cramer-Rao Bound and its Application to Synchronization Problems". *IEEE Transactions on Communications*, 42(2/3/4): pp. 1391–1399, February/March/April 1994.

[3] Marc Moeneclaey. On the True and the Modified Cramer-Rao Bounds for the Estimation of a Scalar Parameter in the Presence of Nuisance Parameters. *IEEE Transactions on Communications*, 46(11): pp. 1536–1544, November 1998.

[4] Jaume Riba, Gregori Vázquez, and Sergio Calvo. "Conditional Maximum Likelihhod Frequency Estimation for Offset Modulations". In *Proc. International Conference on Acoustics, Speech and Signal Processing, ICASSP'98, Seattle (USA)*, volume VI, pages pp. 3425–3428, May 1998.

[5] Jaume Riba and Gregori Vázquez. "Conditional Maximum Likelihhod Timing Recovery". In *Proc. International Conference on Acoustics, Speech and Signal Processing, ICASSP'99, Phoenix (Arizona, USA)*, volume IV, pages pp. 1817–1820, March 1999.

[6] Jaume Riba, Josep Sala, and Gregori Vazquez. Conditional Maximum Likelihood Timing Recovery: Estimators and Bounds. Submited to IEEE Transactions on Signal Processing, 1999.

[7] Gregori Vázquez and Jaume Riba. Vol. 2, chapter 2: Non-Data Aided Digital Synchronization. Submited to Prentice-Hall, 1999.

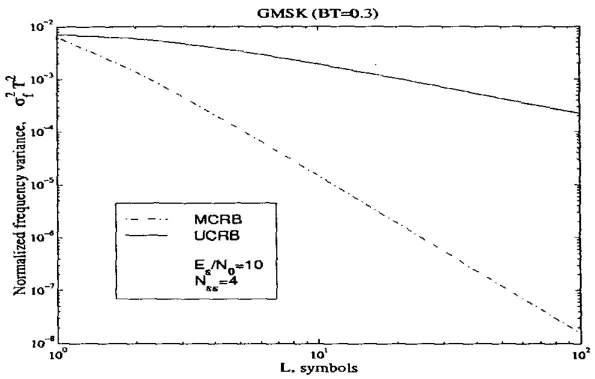

Figure 4: Comparison between the conventional MCRB and the UCRB for frequency estimation.

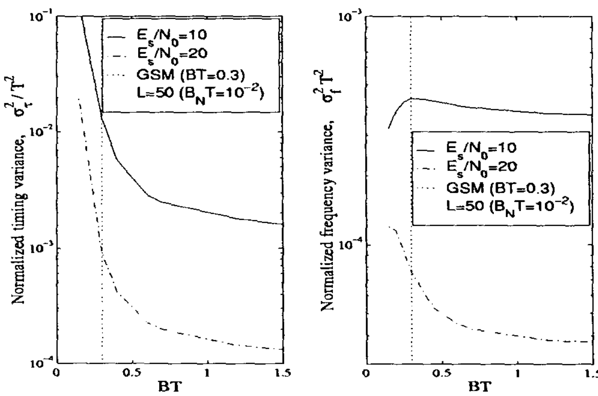

Figure 5: Impact of GMSK normalized Gaussian filter bandwidth, BT, on the UCRB (MSK corresponds to $BT = \infty$).

[8] Petre Stoica and Arye Nehora. "Performance Study of Conditional and Unconditional Direction-Of-Arrival Estimation". *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 38: pp. 1783–1795, October 1990.

[9] Pierre A. Laurent. "Exact and Approximate Construction of Digital Phase Modulations by Superposition of Amplitude Modulated Pulses". *IEEE Transactions on Communications*, 34(2): pp. 150–160, February 1986.

[10] B. Ottersten, M. Viberg, and P. Stoica. *Radar Array Processing.* Springer-Verlag, 1993. Chapter: Exact and Large Sample Maximum Likelihood Techniques for Parameter Estimation and Detection.

[11] Aldo D'Andrea, Umberto Mengali, and Ruggero Reggiannini. A Digital Approach to Clock Recovery in Generalized Minimum Shift Keying. *IEEE Transactions on Vehicular Technology*, 39, 1990.