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Non-existence of Lightlike Submanifolds of an Indefinite Trans-Sasakian Manifold with a Non-metric (ϕ, θ)-Connection

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Abstract

Non-metric (ϕ, θ)-connection was defined by this author [14]. Non-metric θ-connection is an example of this connection such that $\phi = \bar{g}$. In this paper, we study two types of 1-lightlike submanifolds M, so called lightlike hypersurface and half lightlike submanifold, of an indefinite trans-Sasakian manifold $\bar{M}$ with a non-metric (ϕ, θ)-connection. We prove that there exist no such two types of 1-lightlike submanifolds.

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Keywords: non-metric (ϕ, θ)-connections, lightlike hypersurface, half lightlike submanifold

1 Introduction

A linear connection $\bar{\nabla}$ on a semi-Riemannian manifold $(\bar{M}, \bar{g})$ is called a *non-metric* (ϕ, θ)*-connection* [14] if it satisfies

$$(\bar{\nabla}_X \bar{g})(Y, Z) = -\,\phi(X, Y)\theta(Z) - \phi(X, Z)\theta(Y), \tag{1.1}$$

for any vector fields X, Y and Z on $\bar{M}$, where ϕ is a $(0, 2)$-type tensor field and θ is a 1-form associated with a vector field ζ by $\theta(X) = \bar{g}(X, \zeta)$.

In case $\phi = \bar{g}$, we say that $\bar{\nabla}$ is a *non-metric θ-connection* [12, 13]. The semi-symmetric non-metric connection [1] and the quarter-symmetric non-metric connection [2, 4, 8, 9, 16] are two important examples of the non-metric θ-connection. These two special connections are important for the mathematical study and the applications to physics.

Oubina [15] introduced the notion of an indefinite trans-Sasakian manifold with an indefinite trans-Sasakian structure $(J, \zeta, \theta, \bar{g})$ of type (α, β). Indefinite Sasakian, Kenmotsu and cosymplectic manifolds are three important kinds of indefinite trans-Sasakian manifold such that

$$\alpha = 1, \;\; \beta = 0, \qquad \alpha = 0, \;\; \beta = 1, \qquad \alpha = 0, \;\; \beta = 0.$$

The theory of lightlike submanifolds is an important topic of research in differential geometry due to its application in mathematical physics, especially in the general relativity. 1-lightlike submanifold is a particular case of r-lightlike submanifold such that $r = 1$. Much of its geometry will be immediately generalized in a formal way to arbitrary r-lightlike submanifolds. Moreover the theory of 1-lightlike submanifold is more simple than that of r-lightlike submanifold. For this reason, we study only 1-lightlike submanifolds.

For a 1-lightlike submanifold (M, g) of a semi-Riemannian manifold $(\bar{M}, \bar{g})$, (2.32) in [6, Section 4.2] and (4.13) in [5] indicates that the induced connection ∇ of M is a non-metric (ϕ, θ)-connection. Motivated by the existing lightlike geometry of non-metric (ϕ, θ)-connection, the objective of this paper is to study two types of 1-lightlike submanifolds of an indefinite trans-Sasakian manifolds with a non-metric (ϕ, θ)-connection, in which the 1-form θ and its associated vector field ζ, defined by (1.1), is identical with the 1-form θ and the vector field ζ of the indefinite almost contact structure $(J, \zeta, \theta, \bar{g})$, respectively. Moreover, the tensor field ϕ in (1.1) is identical with the fundamental 2-form ϕ associated with the structure tensor J of $\bar{M}$ such that

$$\phi(X, Y) = \bar{g}(X, JY). \tag{1.2}$$

We prove that there exist no such two types of 1-lightlike submanifolds of an indefinite trans-Sasakian manifold $\bar{M}$ with a non-metric (ϕ, θ)-connection.

2 Non-existence of lightlike hypersurfaces

An odd-dimensional semi-Riemannian manifold $(\bar{M}, \bar{g})$ is said to be an *indefinite almost contact metric manifold* if there exists a structure set $\{J, \zeta, \theta, \bar{g}\}$, where J is a (1, 1)-type tensor field, ζ is a vector field which is called the *structure vector field* and θ is a 1-form such that

$$J^2X = -X + \theta(X)\zeta, \;\; \bar{g}(JX, JY) = \bar{g}(X, Y) - \epsilon\theta(X)\theta(Y), \;\; \theta(\zeta) = 1, \tag{2.1}$$

for any vector fields X and Y on $\bar{M}$, where $\epsilon = 1$ or -1 according as ζ is spacelike or timelike respectively. From (2.1), we see that $J\zeta = 0$, $\theta \circ J = 0$ and $\theta(X) = \epsilon\bar{g}(X, \zeta)$. In this case, the set $\{J, \zeta, \theta, \bar{g}\}$ is called an *indefinite almost contact metric structure* of $\bar{M}$. In the entire discussion of this article, we shall assume that ζ is unit spacelike, without loss of generality.

An indefinite almost contact metric manifold $(\bar{M}, \bar{g})$ is called an *indefinite trans-Sasakian manifold* [15] if there exist smooth functions α and β such that

$$(\bar{\nabla}_X J)Y = \alpha\{\bar{g}(X, Y)\zeta - \theta(Y)X\} + \beta\{\bar{g}(JX, Y)\zeta - \theta(Y)JX\}, \tag{2.2}$$

for any vector fields X and Y on $\bar{M}$. We say that $\{J, \zeta, \theta, \bar{g}\}$ is an *indefinite trans-Sasakian structure of type* (α, β). Replacing Y by ζ to (2.2), we get

$$\bar{\nabla}_X \zeta = -\alpha JX + \beta(X - \theta(X)\zeta). \tag{2.3}$$

Let (M, g) be a lightlike hypersurface of an indefinite Kaehler manifold $(\bar{M}, \bar{g})$ with a non-metric (ϕ, θ)-connection. It is known that the normal bundle $TM^\perp$ of M is a vector subbundle of the tangent bundle TM, of rank 1, and coincides with the radical distribution $Rad(TM) = TM \cap TM^\perp$. A complementary vector bundle $S(TM)$ of $TM^\perp$ in TM is non-degenerate distribution on M, which is called a *screen distribution* on M, such that

$$TM = TM^\perp \oplus_{orth} S(TM), \tag{2.4}$$

where $\oplus_{orth}$ denotes the orthogonal direct sum. We denote such a lightlike hypersurface by $M = (M, g, S(TM))$. Denote by $F(M)$ the algebra of smooth functions on M and by $\Gamma(E)$ the $F(M)$ module of smooth sections of a vector bundle E over M. Also denote by $(2.1)_i$ the i-th equation of the three equations in (2.1). We use same notations for any others. It is known [6] that, for any null section ξ of $TM^\perp$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a unique null section N of a unique vector bundle $tr(TM)$ in $S(TM)^\perp$ satisfying

$$\bar{g}(\xi, N) = 1, \quad \bar{g}(N, N) = \bar{g}(N, X) = 0, \quad \forall\, X \in \Gamma(S(TM)).$$

We call $tr(TM)$ and N the *transversal vector bundle* and the *null transversal vector field* of M with respect to the screen distribution respectively. Then the tangent bundle $T\bar{M}$ of $\bar{M}$ is decomposed as follow:

$$T\bar{M} = TM \oplus tr(TM) = \{TM^\perp \oplus tr(TM)\} \oplus_{orth} S(TM). \tag{2.5}$$

In the sequel, let X, Y, Z and W be the vector fields on M, unless otherwise specified. Let P be the projection morphism of TM on $S(TM)$ with respect to the decomposition (2.4). From the decompositions (2.4) and (2.5), the local

Gauss and Weingartan formulas of M and $S(TM)$ are given by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X,Y)N, \tag{2.6}$$
$$\bar{\nabla}_X N = -A_N X + \tau(X)N; \tag{2.7}$$
$$\nabla_X PY = \nabla^*_X PY + C(X,PY)\xi, \tag{2.8}$$
$$\nabla_X \xi = -A^*_\xi X - \sigma(X)\xi, \tag{2.9}$$

respectively. where ∇ and ∇^* are the induced linear connections on TM and $S(TM)$ respectively, B and C are the local second fundamental forms on TM and $S(TM)$ respectively, A_N and A^*_ξ are the shape operators on TM and $S(TM)$ respectively, and τ and σ are 1-forms on TM.

The induced connection ∇ on M is not metric and satisfies

$$\begin{aligned}(\nabla_X g)(Y,Z) = & \; B(X,Y)\eta(Z) + B(X,Z)\eta(Y) \\ & - \theta(Y)\phi(X,Z) - \theta(Z)\phi(X,Y),\end{aligned} \tag{2.10}$$

where η is a 1-form on TM such that $\eta(X) = \bar{g}(X,N)$. In general, by (2.5) the structure vector field ζ of $\bar{M}$ is decomposed as follow:

$$\zeta = \omega + a\xi + bN,$$

where ω is a smooth vector fields on $S(TM)$, and a and b are smooth functions given by $a = \theta(N)$ and $b = \theta(\xi)$.

Now we quote the following result by Jin [10]:

Lemma 2.1. *Let M be a lightlike hypersurface of an indefinite almost contact metric manifold $\bar{M}$. Then the distributions $J(TM^\perp)$ and $J(tr(TM))$ are vector subbundles of $S(TM)$, of rank 1.*

Theorem 2.2. *There exist no lightlike hypersurfaces of indefinite trans-Sasakian manifolds with non-metric (ϕ, θ)-connections.*

Proof. By Lemma 2.1, we see that $J\xi$ and JN belongs to $S(TM)$. Consider two vector fields V and U on $S(TM)$ and two 1-forms v and u such that

$$V = -J\xi, \quad U = -JN, \quad v(X) = g(X,V), \quad u(X) = g(X,U). \tag{2.11}$$

Replacing Y by ζ, ξ, N, V and U to (1.2) by turns and using (2.11), we have

$$\phi(X,\zeta) = 0, \quad \phi(X,\xi) = -v(X), \quad \phi(X,N) = -u(X), \tag{2.12}$$
$$\phi(X,V) = -b\theta(X), \qquad \phi(X,U) = \eta(X) - a\theta(X). \tag{2.13}$$

From the fact that $B(X,Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$, we know that B is independent of the choice of the screen distribution $S(TM)$ and satisfies $B(X,\xi) = -bv(X)$. From this equation, (2.6) and (2.9), we obtain

$$\bar{\nabla}_X \xi = -A^*_\xi X - \sigma(X)\xi - bv(X)N. \tag{2.14}$$

Using (1.1), (2.14) and the fact that $B(X,Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$, we obtain

$$B(X,Y) = g(A^*_\xi X, Y) + b\phi(X,Y) - \{\theta(Y) - b\eta(Y)\}v(X). \tag{2.15}$$

For any vector field X of M, by using (2.5), JX is expressed as follow:

$$JX = FX + v(X)N, \tag{2.16}$$

where FX is the tangential component of JX. Taking the scalar product with ζ, V and U to (2.16) by turns and using (2.1), (2.11) and $\bar{g}(JX,\zeta) = 0$, we get

$$\theta(FX) = -av(X), \quad v(FX) = b\theta(X), \quad u(FX) = a\theta(X) - \eta(X). \tag{2.17}$$

Applying $\bar{\nabla}_X$ to $(2.11)_1$ and using (2.2), (2.6), (2.14) and (2.16), we have

$$\begin{aligned} \nabla_X V &= F(A^*_\xi X) - \sigma(X)V - bv(X)U \\ &\quad + b\{\alpha X + \beta FX\} - \beta v(X)\{\omega + a\xi\}, \\ B(X,V) &= v(A^*_\xi X). \end{aligned}$$

On the other hand, taking $Y = V$ to (2.15) and using $(2.13)_1$, we have

$$B(X,V) = v(A^*_\xi X) - b^2\theta(X).$$

From the last two equations, we obtain $b^2\theta(X) = 0$. Taking $X = \xi$ to this result, we get $b = 0$. This implies that ζ is tangent to M.

Applying $\bar{\nabla}_X$ to $(2.11)_2$ and using (2.2), (2.6), (2.7) and (2.16), we have

$$\begin{aligned} \nabla_X U &= F(A_N X) + \tau(X)U + a\{\alpha X + \beta FX\} \qquad (2.18) \\ &\quad - \{\alpha\eta(X) + \beta u(X)\}\zeta, \\ B(X,U) &= v(A_N X) + \beta a v(X). \qquad (2.19) \end{aligned}$$

Replacing Y by ζ to (2.6) and using (2.3) and (2.15), we obtain

$$\nabla_X \zeta = -\alpha FX + \beta\{X - \theta(X)\zeta\}. \tag{2.20}$$

Applying ∇_X to $g(U,\zeta) = 0$ and using (2.10), $(2.13)_2$, $(2.17)_{1,3}$, (2.18), (2.19) and (2.20), we have $\eta(X) = a\theta(X)$. Taking $X = \xi$ to this result, we have $1 = 0$. It is a contradiction. Thus there exist no lightlike hypersurfaces of indefinite trans-Sasakian manifolds admitting non-metric (ϕ, θ)-connections.

3 Non-existence of half lightlike submanifolds

Let (M,g) be a half lightlike submanifold of an indefinite Kaehler manifold $(\bar{M},\bar{g})$ with a non-metric (ϕ, θ)-connection. Then $Rad(TM)$ is a subbundle of TM, of rank 1. There exist complementary non-degenerate vector bundles $S(TM)$ and $S(TM^{\perp})$ of $Rad(TM)$ in TM and $TM^{\perp}$ respectively, which are called the *screen* and *co-screen* distributions on M, such that

$$TM = Rad(TM) \oplus_{orth} S(TM), \; TM^{\perp} = Rad(TM) \oplus_{orth} S(TM^{\perp}). \tag{3.1}$$

We denote such a half lightlike submanifold by $M = (M, g, S(TM), S(TM^{\perp}))$. Choose $L \in \Gamma(S(TM^{\perp}))$ as a unit spacelike vector field, no loss of generality. Consider the orthogonal complementary distribution $S(TM)^{\perp}$ to $S(TM)$ in $T\bar{M}$. Certainly, $Rad(TM)$ and $S(TM^{\perp})$ are vector subbundles of $S(TM)^{\perp}$. As the co-screen distribution $S(TM^{\perp})$ is non-degenerate, we have

$$S(TM)^{\perp} = S(TM^{\perp}) \oplus_{orth} S(TM^{\perp})^{\perp},$$

where $S(TM^{\perp})^{\perp}$ is the orthogonal complementary to $S(TM^{\perp})$ in $S(TM)^{\perp}$. For any null section ξ of $Rad(TM)$, there exists a uniquely defined lightlike vector bundle $ltr(TM)$ and a null vector field N of $ltr(TM)$ satisfying

$$\bar{g}(\xi, N) = 1, \; \bar{g}(N, N) = \bar{g}(N, X) = \bar{g}(N, L) = 0, \quad \forall\, X \in \Gamma(S(TM)).$$

We call N, $ltr(TM)$ and $tr(TM) = S(TM^{\perp}) \oplus_{orth} ltr(TM)$ the *lightlike transversal vector field, lightlike transversal vector bundle* and *transversal vector bundle* of M with respect to $S(TM)$ respectively [7]. Thus $T\bar{M}$ is decomposed as

$$\begin{aligned} T\bar{M} &= TM \oplus tr(TM) = \{Rad(TM) \oplus tr(TM)\} \oplus_{orth} S(TM) \\ &= \{Rad(TM) \oplus ltr(TM)\} \oplus_{orth} S(TM) \oplus_{orth} S(TM^{\perp}). \end{aligned} \tag{3.2}$$

The local Gauss and Weingartan formulas of M and $S(TM)$ are given by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N + D(X, Y)L, \tag{3.3}$$
$$\bar{\nabla}_X N = -A_{_N} X + \tau(X)N + \rho(X)L, \tag{3.4}$$
$$\bar{\nabla}_X L = -A_{_L} X + \mu(X)N + \nu(X)L; \tag{3.5}$$
$$\nabla_X PY = \nabla^*_X PY + C(X, PY)\xi, \tag{3.6}$$
$$\nabla_X \xi = -A^*_{\xi} X - \sigma(X)\xi, \tag{3.7}$$

where ∇ and ∇^* are induced linear connections on TM and $S(TM)$ respectively, B and D are called the *local second fundamental forms* of M, C is called the *local second fundamental form* on $S(TM)$. $A_{_N}$, A^*_{ξ} and $A_{_L}$ are linear operators on TM and τ, ρ, μ, ν and σ are 1-forms on TM.

Using (1.1) and (3.3), we have

$$\begin{aligned} (\nabla_X g)(Y, Z) &= B(X, Y)\eta(Z) + B(X, Z)\eta(Y) \\ &\quad - \theta(Y)\phi(X, Z) - \theta(Z)\phi(X, Y). \end{aligned} \tag{3.8}$$

In general, the vector vector field ζ of $\bar{M}$ is decomposed by

$$\zeta = \omega + a\xi + bN + cL,$$

where ω is a smooth vector field on $S(TM)$, and a, b and c are smooth functions defined by $a = \theta(N)$, $b = \theta(\xi)$ and $c = \theta(L)$.

Now we quote the following result by Jin [11]:

Lemma 3.1. *Let M be a half lightlike submanifold of an indefinite almost contact metric manifold $\bar{M}$. Then the distributions $J(TM^{\perp})$, $J(tr(TM))$ and $J(S(TM^{\perp}))$ are vector subbundles of $S(TM)$, of rank* 1.

Theorem 3.2. *There exist no half lightlike submanifolds of indefinite trans-Sasakian manifolds admitting non-metric (ϕ, θ)-connections.*

Proof. By Lemma 3.1, we see that $J\xi$, JN and JL belongs to $S(TM)$. Consider three vector fields V, U and W, and three 1-forms u, v and w such that

$$V = -J\xi, \qquad U = -JN, \qquad W = -JL, \tag{3.9}$$
$$v(X) = g(X, V), \quad u(X) = g(X, U), \quad w(X) = g(X, W). \tag{3.10}$$

Replacing Y by ξ, N, L, V, U and W to (1.2) by turns, we have

$$\phi(X, \xi) = -v(X), \quad \phi(X, N) = -u(X), \quad \phi(X, L) = -w(X), \tag{3.11}$$
$$\phi(X, V) = -b\theta(X), \qquad \phi(X, U) = \eta(X) - a\theta(X), \tag{3.12}$$
$$\phi(X, W) = -c\theta(X).$$

From the facts that $B(X, Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$ and $D(X, Y) = \bar{g}(\bar{\nabla}_X Y, L)$, we know that B and D are independent of the choice of $S(TM)$ and satisfy

$$B(X, \xi) = -bv(X), \qquad D(X, \xi) = -\lambda(X),$$

where we set $\lambda(X) = bw(X) + cv(X) + \mu(X)$. From (3.3) and (3.7), we obtain

$$\bar{\nabla}_X \xi = -A^*_\xi X - \sigma(X)\xi - bv(X)N - \lambda(X)L. \tag{3.13}$$

Also, using (1.1), (3.5) and (3.13) and the facts that

$$B(X, Y) = g(A^*_\xi X, Y) + b\phi(X, Y) - \{\theta(Y) - b\eta(Y)\}v(X), \tag{3.14}$$
$$D(X, Y) = g(A_L X, Y) + c\phi(X, Y) - \theta(Y)w(X) - \eta(Y)\mu(X). \tag{3.15}$$

For any vector field X of M, by using (3.2), JX is expressed as follow:

$$JX = FX + v(X)N + w(X)L, \tag{3.16}$$

where FX is the tangential component of JX. Taking the scalar product with ζ, V, U and W to (3.16) by turns and using (2.1) and (3.9), we have

$$\theta(FX) = -av(X) - cw(X), \qquad v(FX) = b\theta(X), \tag{3.17}$$
$$u(FX) = a\theta(X) - \eta(X), \qquad w(FX) = c\theta(X). \tag{3.18}$$

Applying $\bar{\nabla}_X$ to $(3.9)_1$ and using (2.2), (3.3), (3.13) and (3.16), we have

$$\begin{aligned}\nabla_X V &= F(A^*_\xi X) - \sigma(X)V - bv(X)U - \lambda(X)W \\ &\quad + b\{\alpha X + \beta FX\} - \beta v(X)\{\omega + a\xi\}, \\ B(X,V) &= v(A^*_\xi X).\end{aligned}$$

Taking $Y = V$ to (3.14) and using $(3.12)_1$, we have

$$B(X,V) = v(A^*_\xi X) - b^2\theta(X).$$

From the last two equation, we have $b^2\theta(X) = 0$. Taking $X = \xi$, we get $b = 0$.

Applying $\bar{\nabla}_X$ to $(3.9)_3$ and using (3.3), (3.5), (3.9) and (3.16), we have

$$\begin{aligned}\nabla_X W &= F(A_{\scriptscriptstyle L} X) + \mu(X)U + \nu(X)W \\ &\quad + c\{\alpha X + \beta FX\} - \beta w(X)\{\omega + a\xi\}, \\ D(X,W) &= w(A_{\scriptscriptstyle L} X).\end{aligned}$$

Taking $Y = W$ to (3.15) and using $(3.12)_3$, we have

$$D(X,W) = w(A_{\scriptscriptstyle L} X) - c^2\theta(X).$$

From the last two equations, we obtain $c^2\theta(X) = 0$. Thus we get $c = 0$. As $b = c = 0$, the structure vector field ζ of $\bar{M}$ is tangent to M.

Applying $\bar{\nabla}_X$ to $(3.9)_2$ and using (2.2), (3.4), (3.9) and (3.16), we have

$$\begin{aligned}\nabla_X U &= F(A_{\scriptscriptstyle N} X) + \tau(X)U + \rho(X)W \qquad (3.19)\\ &\quad + a\{\alpha X + \beta FX\} - \{\alpha\eta(X) + \beta u(X)\}\zeta, \\ B(X,U) &= v(A_{\scriptscriptstyle N} X) + \beta a v(X). \qquad (3.20)\end{aligned}$$

Replacing Y by ζ to (3.3) and using (2.3), we obtain

$$\nabla_X \zeta = -\alpha FX + \beta\{X - \theta(X)\zeta\}. \qquad (3.21)$$

Applying ∇_X to $g(U,\zeta) = 0$ and using (3.8), $(3.12)_2$, $(3.17)_1$, $(3.18)_1$, (3.19), (3.20) and (3.21), we have $\eta(X) = a\theta(X)$. Taking $X = \xi$ to this, we have $1 = 0$. It is a contradiction. Thus there exist no half lightlike submanifolds of indefinite trans-Sasakian manifolds admitting non-metric (ϕ, θ)-connections.

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