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NON-EXISTENCE OF REAL HYPERSURFACES WITH PARALLEL STRUCTURE JACOBI OPERATOR IN NONFLAT COMPLEX SPACE FORMS

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ABSTRACT. We prove the nonexistence of real hypersurfaces in nonflat complex space forms whose Jacobi operator associated to the structure vector field is parallel. In order to prove this result we also obtain the nonexistence of several classes of non homogeneous real hypersurfaces in complex projective space.

1. Introduction. Let $\mathbf{C}M^m(c)$, $m \geq 2$, $c \neq 0$, be a nonflat complex space form endowed with the metric g of constant holomorphic sectional curvature c. For the sake of simplicity, we will use $c = 4\varepsilon$, $\varepsilon = 1$ or $\varepsilon = -1$. When $\varepsilon = 1$ we will call it the complex projective space, $\mathbf{C}P^m$, and when $\varepsilon = -1$, the complex hyperbolic space, $\mathbf{C}H^m$. Let M be a connected real hypersurface in $\mathbf{C}M^m(c)$ without boundary. Let J denote the complex structure of $\mathbf{C}M^m(c)$ and N a locally defined unit normal vector field on M. Then $-JN = \xi$ is a tangent vector field to M called the structure vector field on M. The study of real hypersurfaces in nonflat complex space forms is a classical topic in differential geometry. The classification of homogeneous real hypersurfaces in the case of complex projective space, $\mathbf{C}P^m$ was obtained by Takagi, see [**6, 11**–**13**], and is given by the following list:

A_1: Geodesic hyperspheres.

A_2: Tubes over totally geodesic complex projective spaces $\mathbf{C}P^k$, $0 < k < m-1$.

B: Tubes over complex quadrics and $\mathbf{R}P^m$.

C: Tubes over the Segre embedding of $\mathbf{C}P^1$x$\mathbf{C}P^n$, where $2n+1 = m$ and $m \geq 5$.

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D: Tubes over the Plucker embedding of the complex Grassmann manifold $G(2,5)$. In this case $m = 9$.

E: Tubes over the canonical embedding of the Hermitian symmetric space $SO(10)/U(5)$. In this case $m = 15$. In the case of complex hyperbolic space $\mathbf{C}H^m$, the classification of homogeneous real hypersurfaces is not completed yet, but we have the following examples, see [**1, 7**]:

A_0: Horospheres.

A_1: Geodesic hyperspheres.

A_2: Tubes over totally geodesic $\mathbf{C}H^k$, $0 < k < m-1$.

B: Tubes over totally geodesic real hyperbolic space $\mathbf{R}H^m$.

Jacobi fields along geodesics of a given Riemannian manifold $(\widetilde{M}, \tilde{g})$ satisfy a very well-known differential equation. This classical differential equation naturally inspires the so-called Jacobi operator. That is, if $\widetilde{R}$ is the curvature operator of $\widetilde{M}$, and X is any tangent vector field to $\widetilde{M}$, the Jacobi operator (with respect to X) at $p \in M$, $\widetilde{R}_X \in \operatorname{End}(T_p\widetilde{M})$, is defined as $(\widetilde{R}_X Y)(p) = (\widetilde{R}(Y,X)X)(p)$ for all $Y \in T_p\widetilde{M}$, being a self-adjoint endomorphism of the tangent bundle $T\widetilde{M}$ of $\widetilde{M}$. Clearly, each tangent vector field X to $\widetilde{M}$ provides a Jacobi operator with respect to X.

The study of Riemannian manifolds by means of their Jacobi operators has been developed following several ideas. For instance, in [**2**], it is pointed out that (locally) symmetric spaces of rank 1 (among them complex space forms) satisfy that all the eigenvalues of $\widetilde{R}_X$ have constant multiplicities and are independent of the point and the tangent vector X. The converse is a well-known problem that has been studied by many authors, although it is still open.

Let M be a real hypersurface in a nonflat complex space form $\mathbf{C}M^m(c)$, and let ξ be the structure vector field on M. We will call the Jacobi operator on M with respect to ξ the structure Jacobi operator on M. In [**5**], the authors obtain a characterization of class A real hypersurfaces as those ones in $\mathbf{C}M^m(c)$ such that the structure Jacobi operator and the shape operator commute. See also [**4**]. In [**3**], the authors classify, under certain additional conditions, real hypersurfaces of $\mathbf{C}P^m$ whose structure Jacobi operator is parallel in a certain sense in the direction of ξ. They obtain class A real hypersurfaces and a non homogeneous real hypersurface. In this paper we study the parallelism of the structure Jacobi operator of a real hypersurface of a nonflat complex space form. In Section 3 we prove the nonexistence of three distinct classes of nonhomogeneous real hypersurfaces in complex projective space. These results are used in Section 4 to obtain the main result of this paper by

Theorem 1. *There exist no real hypersurfaces in a nonflat complex space form* $\mathbf{C}M^m(4\varepsilon)$, $\varepsilon \neq 0$, $m \geq 3$, *whose structure Jacobi operator is parallel.*

2. Preliminaries. Throughout this paper, all manifolds, vector fields, etc., will be considered of class C^∞ unless otherwise stated. Let M be a connected real hypersurface in $\mathbf{C}M^m(4\varepsilon)$, $m \geq 2$, without boundary. Let N be a locally defined unit normal vector field of M. Let ∇ be the Levi-Civita connection on M and (J, g) the Kaehlerian structure of $\mathbf{C}M^m(4\varepsilon)$. For any vector field X tangent to M we write $JX = \phi X + \eta(X)N$ and $-JN = \xi$. Then (ϕ, ξ, η, g) is an almost contact metric structure on M. That is, we have

$$\begin{aligned} \phi^2 X &= -X + \eta(X)\xi, \quad \eta(\xi) = 1, \\ g(\phi X, \phi Y) &= g(X, Y) - \eta(X)\eta(Y). \end{aligned} \tag{2.1}$$

for any tangent vectors X, Y to M. From (2.1) we obtain

$$\phi\xi = 0, \quad \eta(X) = g(X, \xi). \tag{2.2}$$

From the parallelism of J we get

$$(\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi \tag{2.3}$$

and

$$\nabla_X \xi = \phi AX \tag{2.4}$$

for any X, Y tangent vectors to M, where A denotes the Weingarten endomorphism of the immersion. As the ambient space has holomorphic sectional curvature 4ε, the equations of Gauss and Codazzi are given respectively by

$$\begin{aligned} R(X, Y)Z = \varepsilon\{&g(Y, Z)X - g(X, Z)Y + g(\phi Y, Z)\phi X \\ &- g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z\} \\ &+ g(AY, Z)AX - g(AX, Z)AY, \end{aligned} \tag{2.5}$$

and

$$(2.6)\quad (\nabla_X A)Y - (\nabla_Y A)X = \varepsilon\{\eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi\},$$

for any tangent vectors X, Y, Z to M, where R is the curvature tensor of M. In the sequel we will need the following result, see [**8, 10**]:

Theorem A. *Let M be a real hypersurface of $\mathbf{C}M^m(4\varepsilon)$, $\varepsilon \neq 0$, $m \geq 2$. Then the following are equivalent*:

1. *M is locally congruent to one of the homogeneous hypersurfaces of class* A.
2. *$\phi A = A\phi$.*

We will also denote by $\mathbf{D}$ the distribution on M given by all vectors orthogonal to ξ at any point of M.

3. Some nonexistence results. To be used in the proof of the Theorem we will prove the following propositions

Proposition 3.1. *There are no real hypersurfaces M in $\mathbf{C}P^m$, $m \geq 3$, such that the Weingarten endomorphism of M is given by $A\xi = \alpha\xi + U$, $AU = \xi$, $A\phi U = -(1/\alpha)\phi U$, $AX = 0$, where U is a unit tangent vector field in $\mathbf{D}$, X is any tangent vector field to M orthogonal to* Span $\{\xi, U, \phi U\}$ *and α is a certain nonzero smooth function on M.*

Proof. We see that Ker (A) is a holomorphic distribution. Now from the Codazzi equation we have

(3.1)
$$\begin{aligned} -1 = g((\nabla_X A)\xi - (\nabla_\xi A)X, \phi X) &= g(\nabla_X(\alpha\xi + U), \phi X) \\ &= \alpha g(\nabla_X \xi, \phi X) + g(\nabla_X U, \phi X) \\ &= g(\nabla_X U, \phi X) \end{aligned}$$

for any unit $X \in \ker A$. On the other hand, by (2.3) and (2.6) we get

$$(3.2)\quad \begin{aligned} 0 &= g((\nabla_X A)\phi U - (\nabla_{\phi U} A)X, X) = -g(\nabla_X (1/\alpha)\phi U, X) \\ &= -(1/\alpha)g((\nabla_X \phi)U, X) - (1/\alpha)g(\phi\nabla_X U, X) \\ &= (1/\alpha)g(\nabla_X U, \phi X) \end{aligned}$$

The result follows from (3.1) and (3.2). $\square$

Proposition 3.2. *There exist no real hypersurfaces M in $\mathbf{C}P^m$, $m \geq 4$, such that the Weingarten endomorphism is given by $A\xi = \xi + U$, $AU = \xi$, $A\phi U = -\phi U$, where U is a unit vector field in $\mathbf{D}$ and there exist two nonzero holomorphic distributions $\mathbf{D}_1$ and $\mathbf{D}_0$ such that their direct sum is the orthogonal complement of* Span $\{\xi, U, \phi U\}$ *and $AX = -X$, $A\phi X = -\phi X$, $AZ = 0 = A\phi Z$ for any unit $X \in \mathbf{D}_1$, $Z \in \mathbf{D}_0$.*

Proof. For any unit $X \in \mathbf{D}_1$, the Codazzi equation gives us $(\nabla_{X+U}A)U - (\nabla_U A)(X+U) = 0$. That is,

$$-\phi X - A\nabla_X U + \nabla_U X + A\nabla_U X = 0. \tag{3.3}$$

If we take the scalar product of (3.3) and X, we have

$$g(\nabla_X U, X) = 0 \tag{3.4}$$

If we take the scalar product of (3.3) and ξ, bearing in mind (3.4), we get

$$g(\nabla_U X, U) = 0. \tag{3.5}$$

From the Codazzi equation we also have $(\nabla_{X+\phi U}A)U - (\nabla_U A)(X + \phi U) = 2\xi$, and this gives

(3.6)

$$-\phi X + U - A\nabla_X U - A\nabla_{\phi U} U + \nabla_U X + \nabla_U \phi U + A\nabla_U X + A\nabla_U \phi U = 2\xi.$$

If we take the scalar product of (3.6) and ξ, from (2.3), (3.4) and (3.5) we get

$$g(\nabla_U \phi U, U) = 1 \tag{3.7}$$

and, if we take the scalar product of (3.6) and U, from (2.4) and (3.5) we obtain

$$g(\nabla_U \phi U, U) = -2 \tag{3.8}$$

The result follows from (3.7) and (3.8). $\square$

Proposition 3.3. *There exist no real hypersurfaces M in $\mathbf{C}P^m$, $m \geq 3$, such that the Weingarten endomorphism is given by $A\xi = \xi + \beta U$, $AU = \beta\xi + (\beta^2 - 1)U$, $A\phi U = -\phi U$, $AX = -X$ for any tangent vector X orthogonal to* Span$\{\xi, U, \phi U\}$, *where β is a nonvanishing smooth function defined on M.*

Proof. Let X be the unit tangent vector field orthogonal to Span$\{\xi, U, \phi U\}$. By the Codazzi equation applied to X and U, we get $g((\nabla_X A)U - (\nabla_U A)X, U) = 0$. This gives us

$$g(X, \nabla_U U) = (2/\beta)X(\beta) \tag{3.9}$$

Similarly, $g((\nabla_X A)U - (\nabla_U A)X, \xi) = 0$ implies

$$g(X, \nabla_U U) = (1/\beta)X(\beta) \tag{3.10}$$

From (3.9) and (3.10) we get

$$g(X, \nabla_U U) = X(\beta) = 0. \tag{3.11}$$

Also we get $g((\nabla_X A)U - (\nabla_U A)X, X) = 0$, and this yields $\beta^2 g(\nabla_X U, X) = 0$. Thus,

$$g(\nabla_X U, X) = 0 \tag{3.12}$$

As $g(\nabla_U U, \xi) = g(\nabla_U U, U) = 0$, from (3.11) we have

$$\nabla_U U = g(\nabla_U U, \phi U)\phi U \tag{3.13}$$

Now by the Codazzi equation $(\nabla_{X+U} A)\xi - (\nabla_\xi A)(X+U) = -\phi X - \phi U$. From (3.11) this yields

$$\begin{aligned} &\beta\nabla_X U + U(\beta)U + \beta\nabla_U U - \phi X + (\beta^2 - 1)\phi U + \nabla_\xi X \\ &- \xi(\beta)\xi - \xi(\beta^2 - 1)U - (\beta^2 - 1)\nabla_\xi U + A\nabla_\xi X + A\nabla_\xi U = 0. \end{aligned} \tag{3.14}$$

If we take the scalar product of (3.14) and X we obtain $-g(U, \nabla_X X) + g(\nabla_U U, X) - \beta g(\nabla_\xi U, X) = 0$. From (3.11) and (3.12) this gives

$$g(\nabla_\xi U, X) = 0. \tag{3.15}$$

The Codazzi equation yields $(\nabla_{X+\phi U}A)\xi-(\nabla_\xi A)(X+\phi U)=-\phi X+U$. If we develop this equality, we obtain from (3.11)

$$\begin{aligned}-\phi X+\beta\nabla_X U+U+(\phi U)(\beta)U+\beta\nabla_{\phi U}U-\beta\xi\\ -\beta^2U+\nabla_\xi X+\nabla_\xi\phi U+A\nabla_\xi X+A\nabla_\xi\phi U=0.\end{aligned}\tag{3.16}$$

Taking the scalar product of (3.16) and U and bearing in mind (3.15), we obtain

$$(1-2\beta^2)+(\phi U)(\beta)+\beta^2 g(\nabla_\xi\phi U,U)=0.\tag{3.17}$$

The scalar product of (3.16) and U and (3.15) yield

$$g(\nabla_\xi\phi U,U)=4.\tag{3.18}$$

From (3.17) and (3.18) we obtain

$$(1+2\beta^2)+(\phi U)(\beta)=0.\tag{3.19}$$

Now, from the Codazzi equation, we have

$$(\nabla_{\phi U+U}A)\xi-(\nabla_\xi A)(\phi U+U)=U-\phi U.$$

This gives us

$$\begin{aligned}&(\phi U)(\beta)U+\beta\nabla_{\phi U}U-\beta\xi-(\beta^2-1)U\\ &\quad+\nabla_\xi\phi U+A\nabla_\xi\phi U+\beta\nabla_U U+U(\beta)U\\ &\quad+(\beta^2-1)\phi U-\xi(\beta)\xi-\xi(\beta^2-1)U\\ &\quad-(\beta^2-1)\nabla_\xi U+A\nabla_\xi\phi U+A\nabla_\xi U=0.\end{aligned}\tag{3.20}$$

Taking the scalar product of (3.20) and U we have $(\phi U)(\beta)+U(\beta)-\beta^2+1+\beta^2 g(\nabla_\xi\phi U,U)-\xi(\beta^2-1)-\beta^2=0$. From (3.18) and (3.19), we get

$$U(\beta)=2\beta\xi(\beta).\tag{3.21}$$

Now we take the scalar product of (3.20) and ξ and, bearing in mind (3.18), from (3.2) we get

$$\xi(\beta)=U(\beta)=0.\tag{3.22}$$

From the Codazzi equation $(\nabla_U A)\xi - (\nabla_\xi A)U = -\phi U$. Developing this equality, from (3.21) and (3.22) we have $\beta\nabla_U U + (\beta^2-1)\phi U - (\beta^2-1)\nabla_\xi U + A\nabla_\xi U = 0$. Taking the scalar product of this expression and ϕU we obtain, from (3.18)

$$g(\nabla_U U, \phi U) = 5\beta - (1/\beta). \tag{3.23}$$

Now we also have $(\nabla_{U+\xi} A)\xi - (\nabla_\xi A)(U+\xi) = -\phi U$. If we develop this equality and take the scalar product with ϕU, we have

$$\beta^2 = 1/5. \tag{3.24}$$

Now from (3.19) and (3.24), $1 + 2\beta^2 = 0$, which is impossible. □

4. Proof of the theorem. From (2.5), $(\nabla_X R_\xi)Y = 0$ gives us

(4.1)
$$\begin{aligned} &-\varepsilon(g(Y,\phi AX)\xi + g(\xi,Y)\phi AX) + g(\nabla_X A\xi,\xi)AY + g(A\xi,\phi AX)AY \\ &\quad + g(A\xi,\xi)(\nabla_X A)Y - g(Y,\nabla_X A\xi)A\xi - g(AY,\xi)\nabla_X A\xi = 0, \end{aligned}$$

for any X, Y tangent vectors to M. First suppose that ξ is an eigenvector: $A\xi = \alpha\xi$. Let us take $Y \in \mathbf{D}$ in (4.1). Then we get

$$\varepsilon A\phi Y + \alpha A\phi AY = 0. \tag{4.2}$$

And, if we take $Y = \xi$ in (4.1), we have

$$\varepsilon\phi AX + \alpha A\phi AX = 0 \tag{4.3}$$

for any X tangent to M. Equations (4.2) and (4.3) imply $\phi A = A\phi$. Thus, from Theorem A, M must be locally congruent to a hypersurface of type A. In the case of $\varepsilon = 1$, if we take a unit X tangent to M such that $AX = \cot rX$ we have $(\nabla_X R_\xi)\xi = -\cot r^3\phi X$. Thus R_ξ cannot be parallel. In the case of $\varepsilon = -1$ we obtain a similar result. Now suppose that ξ is not principal. Thus in a certain neighborhood of a point p we must find a unit $U \in \mathbf{D}$ an a certain nonzero smooth function β such that $A\xi = \alpha\xi + \beta U$ for a smooth function α on M. If in (4.1) we take $Y = \phi U$, $X = \xi$ and the scalar product with ξ we get $0 = -\varepsilon\beta - \alpha\beta g(A\phi U, \phi U)$. This implies

$$\alpha \neq 0 \quad \text{and} \quad g(A\phi U, \phi U) = -\varepsilon/\alpha. \tag{4.4}$$

If $Y \in \mathbf{D} \cap \operatorname{Span}\{U, \phi U\}^{\perp}$ from (4.1), taking $X = \xi$ and the scalar product with ξ we obtain $0 = \alpha\beta g(AY, \phi U)$. Thus

$$g(AY, \phi U) = 0. \tag{4.5}$$

Similarly, we have

$$g(AY, \phi Y) = 0 \tag{4.6}$$

for any $Y \in \mathbf{D} \cap \operatorname{Span}\{U, \phi U\}^{\perp}$. If in (4.1) we take $Y = U$, $X = \xi$ and the scalar product with ξ we get $\alpha\beta g(AU, \phi U) = 0$. Thus from (4.4) we get

$$g(AU, \phi U) = 0. \tag{4.7}$$

From (4.4), (4.5) and (4.7), we obtain

$$A\phi U = -(\varepsilon/\alpha)\phi U. \tag{4.8}$$

From (4.1), for any X tangent to M we get $-\varepsilon g(U, \phi AX) + \beta^2 g(U, \phi AX) - \alpha g(AU, \phi AX) = 0$. Thus, $\varepsilon A\phi U - \beta^2 A\phi U + \alpha A\phi AU = 0$. Thus from (4.8) we get $-((1 - \varepsilon\beta^2)/\alpha)\phi U + \alpha A\phi AU = 0$. Taking the scalar product of this equality and ϕU we have

$$g(AU, U) = -(\varepsilon - \beta^2)/\alpha. \tag{4.9}$$

If in (4.1) we take $Y = \xi$, for any X tangent to M we obtain

$$g(A\xi, \phi AX)A\xi = \varepsilon\phi AX + \alpha A\phi AX. \tag{4.10}$$

Taking $X = \phi U$, from (4.8) we get

$$AU = \beta\xi - ((\epsilon - \beta^2)/\alpha)U. \tag{4.11}$$

Now if we take $\mathbf{D}_U = \mathbf{D} \cap \operatorname{Span}\{U, \phi U\}^{\perp}$, $\mathbf{D}_U$ is a holomorphic distribution on M. Moreover it is invariant by A. Now take an eigenvector $X \in \mathbf{D}_U$ such that $AX = \lambda X$. From (4.1), (4.8) and (4.11) we obtain that either $\lambda = 0$ or $\lambda = -\varepsilon/\alpha$ and $A\phi X = \lambda\phi X$. Let us suppose that there exists $X \in \mathbf{D}_U$, such that $AX = 0$. Then $A\phi X = 0$. Then, from the Codazzi equation, $(\nabla_X A)\phi X - (\nabla_{\phi X} A)X = -2\varepsilon\xi = A[\phi X, X]$. Taking the scalar product of this equation and ξ

we have $-2\varepsilon = g([\phi X, X], \alpha\xi + \beta U) = \beta g([\phi X, X], U)$. If we now take the product of that equation and U we obtain $0 = g([\phi X, X], AU) = -((\varepsilon - \beta^2)/\alpha)g([\phi X, X], U)$. If $\varepsilon = -1$, we should have $\beta^2 + 1 = 0$ which is impossible. If $\varepsilon = 1$, $\beta^2 = 1$. Thus changing, if necessary, U by $-U$, we can suppose that $\beta = 1$. Thus if for any $X \in \mathbf{D}_U$, $AX = 0$, the result follows from Proposition 3.1.

If we suppose that there exists $X \in \mathbf{D}_U$ such that $AX = (-\varepsilon/\alpha)X$, then the principal distribution $\mathbf{D}_1 = \{Y \in \mathbf{D}_U / AY = (-\varepsilon/\alpha)Y\}$ is holomorphic. Thus, for any $X, Y \in \mathbf{D}_1$, the Codazzi equation gives us $(\nabla_X A)Y - (\nabla_Y A)X = 2\varepsilon g(X, \phi Y)\xi$. Developing this expression, we get

$$2\varepsilon g(X, \phi Y)\xi = \varepsilon\big((1/\alpha)[Y, X] - X(1/\alpha)Y + Y(1/\alpha)X\big) + A[Y, X]. \tag{4.12}$$

Taking the scalar product of (4.12) and ξ we obtain

$$g([Y, X], U) = (-2/\alpha^2\beta)g(X, \phi Y), \tag{4.13}$$

and taking the scalar product of (4.12) and U, we get

$$g([Y, X], U) = (-2\varepsilon/\beta)g(X, \phi Y). \tag{4.14}$$

If $\varepsilon = 1$, (4.13) and (4.14) give $\alpha^2 = 1$ and, if we change ξ by $-\xi$, if necessary, we have $\alpha = 1$. From Propositions 3.2 and 3.3 the result follows. If $\varepsilon = -1$, (4.13) and (4.14) imply $\alpha^2 + 1 = 0$. This is impossible and finishes the proof. $\square$

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