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NON–FRAGILE CONTROLLERS FOR A CLASS OF TIME–DELAY NONLINEAR SYSTEMS

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The paper deals with the synthesis of a non-fragile state controller with reduced design complexity for a class of continuous-time nonlinear delayed symmetric composite systems. Additive controller gain perturbations are considered. Both subsystems and interconnections include time-delays. A low-order control design system is first constructed. Then, stabilizing controllers with norm bounded gain uncertainties are designed for the control design system using linear matrix inequalities (LMIs) for both delay-independent and delay-dependent stability approaches. The main result shows that when such a non-fragile low-order controllers are implemented into each local controller of the decentralized controller for the global system, the global closed-loop systems are globally asymptotically stable.

Keywords: decentralized control, large scale complex systems, nonlinear systems, continuous-time systems, delay, reduced-order systems

AMS Subject Classification: 93A14, 93A15, 93B51, 93C10, 93D15

1. INTRODUCTION

Modern control design approaches assume that the stable controller designed off-line will be implemented exactly. Unfortunately, this is not the case in practice. Even small implementation changes in controller parameters may destabilize the close-loop system. Sources of controller parameter uncertainties include finite word length, inherent imprecision in analog-digital and digital-analog conversions, finite resolution measuring equipments, round-off errors in numerical computations, etc. The importance of fragility, i. e. sensitivity of controller parameters for small changes, is underlined in large scale complex systems when implementing low-cost low-order local controllers. It motivates the development of new control design methods which include the solution of the fragility problem mainly for large scale complex systems. Symmetric composite systems represent an important class of these systems.

1.1. Prior work

Recent papers starting with [10, 11, 17] point out the possible fragility of robust controllers. To cope with this difficulty, several types of controller uncertainties have been considered. Fragility within an additive uncertainty for state feedback controllers is considered in [25], while [18] extends this issue on H_∞ robust state feedback control for time-delayed systems. A multiplicative uncertainty for H_∞ controllers is considered in [30]. A guaranteed cost control approach is studied in [33]. LMI results for H_∞ state feedback controller for both types of controller uncertainties are dealt in [22]. Digital controller implementation and fragility issues studies [16]. [4] deals with resilient output stabilization. The most complete recent exposition on this issue is presented in [20]. It includes also the stabilization of nonlinear systems [24].

The motivation for studying plants possessing features of symmetric composite systems arises in very different application areas. Real world system examples can be found in parallel systems such as flow splitting parallel reactors with combined precooling [14], electric power systems operating in parallel [6], industrial manipulators with several degrees of freedom [27], flexible structures [26], space crystal furnace [12], homogeneous interconnected systems such as seismic cables [13], or the problem of formations of vehicles in cyclic pursuit which has been solved using circulant matrices in [21]. Relevant references on applications are presented in [12, 13, 14, 21, 26, 27]. Relevant theoretic results are presented in [1, 2, 3, 7, 8, 15, 19, 29, 31, 32]. Low-order control design for delayed uncertain symmetric composite systems is considered in [7, 8, 14, 15, 19, 29, 31, 32].

One of the new open research directions is the inclusion of the fragility issues into the solution of the resilient stabilization for a class of symmetric composite systems. The present paper mainly extends the results on the fragility with additive uncertainties for centralized control design in [18, 20, 22, 25], and [24]. It extends also the results on the low-order fragile and also non-fragile control design for linear or uncertain symmetric composite systems with nominally linear parts in [2, 3, 4, 7, 31] on continuous-time nonlinear delayed symmetric composite systems.

The preliminary version of this paper was published in [5].

1.2. Outline of the paper

This paper presents the synthesis of a non-fragile state controller with additive perturbations and reduced design complexity for a class of continuous-time nonlinear delayed symmetric composite systems when considering both delay-independent and delay-dependent cases. The paper presents first the construction of a low-order control design system. Then the delay-independent stability and the delay-dependent stability of the overall system, the reduced control design system, and their relation are studied. The non-fragile state controller design is performed by using well known LMIs for the reduced control design system. Finally, it is proved that when such a controller is implemented into each subsystem, then the resulting non-fragile decentralized controllers globally asymptotically stabilize the overall system. The proposed method originally contributes with an essential reduction of control design complexity when considering non-fragile controllers for this class of systems.

2. PROBLEM FORMULATION

2.1. Subsystem and interconnections

Consider a nonlinear symmetric composite system consisting of $N \geq 2$ subsystems, where the ith subsystem is described as follows

$$\begin{aligned} \dot{x}_i(t) = Ax_i(t) + A_{di}x_{di}(t) + Bu_i(t) + h_i(t, x_i) + h_{di}(t, x_{di}) + s_{zi}(t) \\ x_i(t_o) = \Phi_i(t_o) \qquad \forall\, t_o \in [-d, 0] \qquad i = 1, \ldots, N \qquad N \geq 2 \end{aligned} \tag{1}$$

where $x_i(t), u_i(t)$, and $s_{zi}(t)$ are n-, m-, and p_s-dimensional vectors of the subsystem states, control inputs, and interconnection inputs, respectively. $\Phi_i(t_o)$ is a given initial function. The interconnections are described in the form

$$s_{zi}(t) = \sum_{j=1, i\neq j}^{N} \left(L_{ij}y_{zj}(t) + h_{ij}(t, y_{zj}) + L_{dij}y_{dzj}(t) + h_{dij}(t, y_{dzj})\right) \tag{2}$$

where $y_{zj}(t)$ is the p_z-dimensional vector of the interconnection output from the subsystem j which is related to the state vector in the form

$$y_{zj}(t) = C_z x_j(t) \qquad y_{dzj} = C_{dz}x_{dj}(t) \tag{3}$$

$x_{dj}(t) = x_j(t-d)$, d denotes a point time delay. Further, the notation $v_d(t) = v(t-d)$ is used for any signal or vector $v(t)$ throughout this manuscript. The interconnection matrices L_{ij}, L_{dij} have the following structure

$$\begin{aligned} L_{ii} = L_p \qquad L_{dii} = L_{dp} \\ L_{ij} = L_q \qquad L_{dij} = L_{dq} \quad (i \neq j) \end{aligned} \tag{4}$$

$A, A_d, B, C_z, C_{dz}, L_p, L_q, L_{dp}$, and L_{dq} are constant nominal matrices.

Assumption 1. The nonlinearities $h_i(t,\cdot)$, $h_{ij}(t,\cdot)$, $h_{di}(t,\cdot)$, $h_{dij}(t,\cdot)$ are uncertain arbitrarily time-varying piecewise-continuous functions belonging to a class of piecewise-continuous real functions $\boldsymbol{H}_{(*)}$ as follows

$$\begin{aligned} \boldsymbol{H}_i &\stackrel{\text{def}}{=} \{h_i(t,\cdot) : \mathbb{R}^n \to \mathbb{R}^n | h_i(t,\cdot)^T h_i(t,\cdot) \\ &\quad \leq \alpha^2 x_i^T H_p^T H_p x_i\} \subset \boldsymbol{D}_p \\ \boldsymbol{H}_{ij} &\stackrel{\text{def}}{=} \{h_{ij}(t,\cdot) : \mathbb{R}^n \to \mathbb{R}^n | h_{ij}(t,\cdot)^T h_{ij}(t,\cdot) \\ &\quad \leq \alpha^2 x_j^T H^T H x_j\} \subset \boldsymbol{D} \\ \boldsymbol{H}_{di} &\stackrel{\text{def}}{=} \{h_{di}(t,\cdot) : \mathbb{R}^n \to \mathbb{R}^n | h_{di}(t,\cdot)^T h_{di}(t,\cdot) \\ &\quad \leq \sigma^2 x_{di}^T H_{dp}^T H_{dp} x_{di}\} \subset \boldsymbol{D}_{dp} \\ \boldsymbol{H}_{dij} &\stackrel{\text{def}}{=} \{h_{dij}(t,\cdot) : \mathbb{R}^n \to \mathbb{R}^n | h_{dij}(t,\cdot)^T h_{dij}(t,\cdot) \\ &\quad \leq \sigma^2 x_{dj}^T H_d^T H_d x_{dj}\} \subset \boldsymbol{D}_d \end{aligned} \tag{5}$$

over the domains of continuity $\boldsymbol{D}_p, \boldsymbol{D}, \boldsymbol{D}_{dp}, \boldsymbol{D}_d$, where H_p, H, H_{dp}, H_d are given constant matrices and $\alpha > 0$, $\sigma > 0$ are given scalars.

Assumption 2. Suppose that the structure of the unknown nonlinear interconnections has the form

$$\begin{aligned} h_i(t,x_i) &= e(t,x_i)H_p x_i(t) + \sum_{j=1,j\neq i}^{N} e(t,x_j)Hx_j(t) \\ h_{di}(t,x_{di}) &= e_d(t,x_{di})H_{dp}x_{di}(t) + \sum_{j=1,j\neq i}^{N} e_d(t,x_{dj})H_d x_{dj}(t) \end{aligned} \tag{6}$$

where $e(t,x_i) \in [-1,\ 1]$ and $e_p(t,x_{di}) \in [-1,\ 1]$, for all i.

The goal is to find a non-fragile decentralized state controller with additive norm-bounded uncertainties globally asymptotically stabilizing the system (1) – (5). The controller is composed of N local identical controllers of the form

$$u_i(t) = (K + \Delta K_i(t))\, x_i(t) \qquad i = 1,\ldots,N \tag{7}$$

where $x_i(t)$ is the n-dimensional controller state of the subsystem i. $\Delta K_i(t) = DF_i(t)\,E$ are controller gain perturbations, where D and E are given constant matrices. $F_{(\cdot)}(t)$ are unknown arbitrarily time-varying Lebesgue measurable functions satisfying $F_{(\cdot)}(t)^T F_{(\cdot)}(t) \leq I$. K is the controller matrix to be determined. Note that this matrix is identical for all subsystems, thus one can take advantage of the symmetric structure of the large scale composite system to reduce the control design complexity.

2.2. Compacted description

Let the compacted description of the system (1) – (6) be as follows

$$\begin{aligned} \dot{x}(t) &= \overline{A}x(t) + \overline{A}_d x_d(t) + \overline{B}u(t) + \overline{h}(t,x) + \overline{h}_d(t,x_d) \\ x(t_o) &= \overline{\Phi}(t_o) \quad \forall\, t_o \in [-d,0] \end{aligned} \tag{8}$$

where $x(t), u(t)$ are nN-, mN-dimensional vectors of the system states and control inputs, respectively. $\overline{\Phi}(t_o)$ is a given initial function. The nominal matrices are defined as follows

$$\begin{aligned} &\overline{A} = (\overline{A}_{ij}) \qquad \overline{A}_{ii} = A + L_p C_z \qquad \overline{A}_{ij} = L_q C_z \\ &\overline{A}_d = (\overline{A}_{dij}) \qquad \overline{A}_{dii} = A_d + L_{dp} C_z \qquad \overline{A}_{dij} = L_{dq} C_{dz} \\ &\overline{B} = \mathrm{diag}(B,\ldots,B) \end{aligned} \tag{9}$$

The admissible nonlinearities $\overline{h}(t,x)$ and $\overline{h}_d(t,x_d)$ in (6) are uncertain piecewise-continuous functions satisfying the following inequalities

$$\begin{aligned} \overline{\boldsymbol{H}} &\stackrel{\text{def}}{=} \{\overline{h}(t,\cdot) : \mathbb{R}^{Nn} \to \mathbb{R}^n | \overline{h}(t,\cdot)^T \overline{h}(t,\cdot) \\ &\quad \leq \alpha^2 x^T \overline{H}^T \overline{H} x\} \subset \overline{\boldsymbol{D}} \\ \overline{\boldsymbol{H}}_d &\stackrel{\text{def}}{=} \{\overline{h}_d(t,\cdot) : \mathbb{R}^{Nn} \to \mathbb{R}^n | \overline{h}_d(t,\cdot)^T \overline{h}_d(t,\cdot) \\ &\quad \leq \sigma^2 x_d^T \overline{H}_d^T \overline{H}_d x_d\} \subset \overline{\boldsymbol{D}}_d \end{aligned} \tag{10}$$

From Assumption 1, the bounding matrices $\overline{H}$ and $\overline{H}_d$ are N block-partitioned matrices defined as follows

$$\begin{aligned} \overline{H} &= \mathrm{diag}(\overline{H}_1, \dots, \overline{H}_N) & \overline{H}_i &= (H \cdots H \;\; H_p \;\; H \cdots H) \\ \overline{H}_d &= \mathrm{diag}(\overline{H}_{d1}, \dots, \overline{H}_{dN}) & \overline{H}_{di} &= (H_d \cdots H_d \;\; H_{dp} \;\; H_d \cdots H_d) \end{aligned} \tag{11}$$

with H_p located at the ith position in $\overline{H}_i$. Analogously, the same location holds for the position of the H_{dp} in $\overline{H}_{di}$.

Consider the non-fragile stabilizing controller for the system (8) – (11) in the form

$$u(t) = (\overline{K} + \Delta\overline{K}(t))\, x(t) = \mathrm{diag}(K + \Delta K_1(t), \dots, K + \Delta K_N(t))\, x(t) \tag{12}$$

which is a compacted equivalent description of (7), where $\Delta\overline{K}(t) = \overline{D}\overline{F}(t)\overline{E}$ are controller gain perturbations. $\overline{D} = \mathrm{diag}(D, \dots, D)$ and $\overline{E} = \mathrm{diag}(E, \dots, E)$ are constant matrices. $\overline{F}(t) = \mathrm{diag}(F_1(t), \dots, F_N(t))$ are unknown arbitrarily time-varying Lebesgue measurable functions satisfying $\overline{F}(t)^T\overline{F}(t) \leq I$.

2.3. The problem

The goal is to derive a complexity-reduced procedure for designing a non-fragile globally asymptotically stabilizing decentralized state controller (7) with additive gain perturbations for a class of nonlinear state-delayed symmetric composite systems (1) – (6).

Consider two cases as follows

- delay-independent stability approach
- delay-dependent stability approach

3. SOLUTION

3.1. System transformation

The system (8) has the bounding matrices $\overline{H}, \overline{H}_d$ which have the structure of symmetric composite systems. This structural feature can be exploited by using the transformation of states to get two reduced order models. Consider

$$\tilde{x}(t) = Tx(t) \tag{13}$$

by using the $nN \times nN$ nonsingular matrix $T = G^{-1}$.

Suppose a real $sn \times sn$ matrix $T(n, s)$ in the form

$$\begin{aligned} T(n,1) &= I \\ T(n,s) &= \begin{pmatrix} I & 0 & \dots & 0 & I \\ 0 & I & \dots & 0 & I \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I & I \\ -I & -I & \dots & -I & I \end{pmatrix} \qquad s > 1 \end{aligned} \tag{14}$$

where I denotes here $n \times n$ identity matrix. Then T is defined through

$$\begin{aligned} T(i) &= \mathrm{diag}[T(n, N-i)I, \dots, I] \in \mathbb{R}^{Nn\times Nn} \\ G &= T(0)\,T(1)\cdots T(N-1) \qquad i = 0,\dots,N-1 \end{aligned} \tag{15}$$

The constructive way of how to use this transformation is presented in [31].

Lemma 1. Consider the matrix $\overline{H}$ by (11) and any given $J = \mathrm{diag}[J_o,\dots,J_o]$, where J, J_o are $nN \times nN$, $n \times n$ matrices Then, the following equalities

$$\begin{aligned} G^{-1}\overline{H}G &= \mathrm{diag}(H_s,\dots,H_s,\ H_c) \\ G^T\overline{H}G &= \mathrm{diag}(2H_s,\ 6H_s,\dots,N(N-1)H_s,\ NH_c) \\ G^{-1}J(G^{-1})^T &= \mathrm{diag}\Big(\frac{1}{2}J_o,\ \frac{1}{6}J_o,\dots,\frac{1}{N(N-1)}J_o\Big) \\ G^TJG &= \mathrm{diag}(2J_o,\dots,N(N-1)J_o,\ NJ_o) \end{aligned} \tag{16}$$

hold, where $H_s = H_p - H$ and $H_c = H_s + NH$.

Applying now the transformation (13) on the system (8), we get the system

$$\begin{aligned} \dot{\tilde{x}}(t) &= \widetilde{A}\tilde{x}(t) + \widetilde{A}_d\tilde{x}_d(t) + \widetilde{B}u(t) + \tilde{h}(t,\tilde{x}) + \tilde{h}_d(t,\tilde{x}_d) \\ \tilde{x}(t_o) &= \widetilde{\Phi}(t_o) \quad \forall\, t_o \in [-d, 0] \end{aligned} \tag{17}$$

where

$$\begin{aligned} \widetilde{A} &= \mathrm{diag}(A_s,\dots,A_s,\ A_c) \\ \widetilde{A}_d &= \mathrm{diag}(A_{ds},\dots,A_{ds},\ A_{dc}) \\ \widetilde{B} &= \mathrm{diag}(B,\dots,B,\ B) \\ \tilde{h}(t,\tilde{x}) &= \mathrm{diag}(h_s(t,\tilde{x}_1),\dots,h_s(t,\tilde{x}_{N-1}),\ h_c(t,\tilde{x}_N)) \\ \tilde{h}_d(t,\tilde{x}_d) &= \mathrm{diag}(h_{ds}(t,\tilde{x}_{d1}),\dots,h_{ds}(t,\tilde{x}_{d,N-1}),\ h_{dc}(t,\tilde{x}_{dN})) \end{aligned} \tag{18}$$

The individual elements of diagonal blocks in (18) are given as follows (when dropping a subsystem index)

$$\begin{aligned} A_s &= A + (L_p - L_q)C_z \\ A_c &= A_s + NL_qC_z \\ A_{ds} &= A_d + (L_{dp} - L_{dq})C_{dz} \\ A_{dc} &= A_{ds} + NL_{dq}C_{dz} \\ h_s(t,\tilde{x}_i) &= e(t,\tilde{x}_i)(H_p - H)\tilde{x}_i(t) \\ h_c(t,\tilde{x}_N) &= h_s(t,\tilde{x}_N) + e(t,x_N)NH\tilde{x}_N(t) \\ h_{ds}(t,\tilde{x}_{di}) &= e_d(t,\tilde{x}_{di})(H_{dp} - H_d)\tilde{x}_{di}(t) \\ h_{dc}(t,\tilde{x}_{dN}) &= h_{ds}(t,\tilde{x}_{dN}) + e_d(t,\tilde{x}_{dN})NH_d\tilde{x}_{dN}(t) \end{aligned} \tag{19}$$

The transformation (13) yields the relation $x_i = \tilde{x}_i - \tilde{x}_N$ for $i = 1,\dots, N-1$ and $x_N = \tilde{x}_N - \sum_{i=1}^{N-1}\tilde{x}_i$. It is evident that the trajectory of the ith subsystem can be described by the system

$$\begin{aligned} \dot{\hat{x}}_i(t) &= \hat{A}_i\hat{x}_i(t) + \hat{A}_{di}\hat{x}_{di}(t) + \hat{B}\hat{u}(t) + \hat{h}_i(t,\hat{x}_i) + \hat{h}_{di}(t,\hat{x}_{di}) \\ \hat{x}_i(t_o) &= \hat{\Phi}_i(t_o) \quad \forall\, t_o \in [-d,0] \qquad i-1,\ldots,N-1 \end{aligned} \tag{20}$$

where $\hat{x}_i = (\tilde{x}_i^T,\, \tilde{x}_N^T)^T$, $\hat{u}_i = (\tilde{u}_i^T,\, \tilde{u}_N^T)^T$, $\hat{A}_i = \mathrm{diag}(A_s,\ A_c)$, $\hat{A}_{di} = \mathrm{diag}(A_{ds},\ A_{dc})$, $\hat{B} = \mathrm{diag}(B,\ B)$. The nonlinear terms are $\hat{h}_i(t,\hat{x}_i) = \mathrm{diag}(h_s(t,\tilde{x}_i), h_c(t,\tilde{x}_N))$, $\hat{h}_{di}(t,\hat{x}_{di}) = \mathrm{diag}(h_{ds}(t,\tilde{x}_{di}), h_{dc}(t,\tilde{x}_{dN}))$. Therefore, the dynamics of the original overall system can be described by the subsystem model (20) consisting of two parts operating in parallel. A "subsystem state" $\hat{x}_i$ and the "average state" $\hat{x}_N$. The system (20) has identical structure for all subsystems $i = 1,\ldots,N-1$. All phenomena encountered in the whole system can be studied by means of the model (20). It leads finally to two systems of order n. To simplify the notation, denote $x_s(t)$ a generic state for any $\hat{x}_i(t)$ in (20) and $x_c(t) = x_N(t)$. We get the systems

$$\begin{aligned} \dot{x}_s(t) &= A_s x_s(t) + A_{ds}x_{ds}(t) + Bu(t) + h_s(t,x_s) + h_{ds}(t,x_{ds}) \\ \hat{x}_s(t_o) &= \hat{\Phi}_s(t_o) \quad \forall\, t_o \in [-d,0] \\ \dot{x}_c(t) &= A_c x_c(t) + A_{dc}x_{dc}(t) + Bu(t) + h_c(t,x_c) + h_{dc}(t,x_{dc}) \\ \hat{x}_c(t_o) &= \hat{\Phi}_c(t_o) \quad \forall\, t_o \in [-d,0] \end{aligned} \tag{21}$$

where $x_{ds}(t) = x_s(t-d)$ and $x_{dc}(t) = x_c(t-d)$. The transformation T in (13) is non-singular. Its application on the system (8) results in the system (17) with block diagonal structure where the first $N-1$ blocks are identical. This fact is reflected in (20). The generic system (21) is directly based on (20). It means that the dynamic properties of (8) and (21) are equivalent.

Note that each subsystem state of the closed-loop system can be thought as the closed-loop system consisting of the plants (21) and the controller $u(t) = (K + \Delta K(t))\,x(t) = (K+DF(t)\,E)\,x(t)$ simultaneously stabilizing both these plants. D, E are given constant matrices, while $F(t)$ is the uncertainty matrix.

3.2. Reduced control design system

The control design for the systems (21) belongs to the problem of simultaneous stabilization. The structure of the plants (21) has a specific structure, where the difference between both systems in simply given by the differences between $A_c - A_s$. $A_{dc} - A_{ds}$, $h_c(t,\tilde{x}_N) - h_s(t,\tilde{x}_N)$, and $h_{dc}(t,\tilde{x}_{dN} - h_{ds}(t,\tilde{x}_{dN})$ as follows from (19). It offers to use the specific structure of the systems (21) to construct a single system which includes both of these plants as their particular cases. The motivation for such a construction is that the control design methods for a single system are available, while the solution of the problem of simultaneous stabilization is less developed. Moreover, the state dimension of a single system is n, while the system (21) has the dimension $2\times n$. It leads to an additional reduction of the control design complexity.

Define the matrix

$$\frac{N}{2}L_q C_z = H_a \qquad \frac{N}{2}L_{dq}C_{dz} = H_{da} \tag{22}$$

Construct the n-dimensional system as follows

$$\dot{x}_m(t) = A_m x_m(t) + A_{dm} x_{dm}(t) + B u_m(t) + h_m(t, x_m) + h_{dm}(t, x_{dm}) \tag{23}$$

where the matrices are defined by the expressions

$$A_m = A + L_p C_z + \Big(\frac{N}{2} - 1\Big) L_q C_z \qquad A_{dm} = A_d + L_{dp} C_z + \Big(\frac{N}{2} - 1\Big) L_{dq} C_{dz} \tag{24}$$

The admissible nonlinearities in (23) are piecewise-continuous real functions satisfying the following inequalities

$$\begin{aligned} \boldsymbol{H}_m &\stackrel{\text{def}}{=} \{h_m(t,\cdot) : \mathbb{R}^{2n} \to \mathbb{R}^n | h_m(t,\cdot)^T h_m(t,\cdot) \\ &\le \alpha^2 x_m^T H_m^T H_m x_m\} \subset \boldsymbol{D}_m \\ \boldsymbol{H}_{dm} &\stackrel{\text{def}}{=} \{h_{dm}(t,\cdot) : \mathbb{R}^{2n} \to \mathbb{R}^n | h_{dm}(t,\cdot)^T h_{dm}(t,\cdot) \\ &\le \sigma^2 x_{dm}^T H_{dm}^T H_{dm} x_{dm}\} \subset \boldsymbol{D}_{dm} \end{aligned} \tag{25}$$

where $H_m = |H_p| + 0.5(N-1)|H| + |H_a|$ and $H_{dm} = |H_{dp}| + 0.5(N-1)|H_d| + |H_{da}|$.

Consider a non-fragile stabilizing controller for the system (24) − (26) in the form

$$u_m(t) = (K + \Delta K_m(t))\, x_m(t) \tag{26}$$

where $\Delta K_m(t) = D F_m(t)\, E$ are controller gain perturbations. D, E are given constant matrices. $F_m(t)$ are unknown arbitrarily time-varying Lebesgue measurable functions satisfying $F_m(t)^T F_m(t) \le I$.

The closed-loop system (23) − (26) has the form

$$\dot{x}_m(t) = (A_m + BK + D F_m(t)\, E)\, x_m(t) + A_{dm} x_d(t) + h_m(t, x_m) + h_{dm}(t, x_{dm}). \tag{27}$$

The gain matrix K can be determined by the procedure which follows. Note that D, E are given constant matrices, while the uncertainty is included in an unknown time-varying matrix $F_m(t)$ with known bounds. Therefore, the procedures for designing K must include this gain perturbation so that the resulting closed-loop system is asymptotically stable for all admissible gain perturbations.

3.3. Stability analysis

The motivation for the stability analysis is the evaluation of the performance between the overall open-loop system and the reduced control design open-loop system. The results are subsequently used is the synthesis of controllers in the closed-loop systems.

3.3.1. Delay-independent stability

The robust delay-independent stability of the system (23) as introduced for instance in [20], can be established by using the Lyapunov–Krasovskii functional as follows

$$V(t, x_m) = x_m(t)^T P x_m(t) + \int_{t-d}^{t} x_m(s)^T Q x_m(s)\, \mathrm{d}s \tag{28}$$

where $P \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$ are symmetric positive definite matrices.

Using the results of the Lyapunov theory, the requirements on negative definiteness of the derivative of (28) along the trajectories of the system (23) leads to the following result [20]. Denote $X = P^{-1},\ \gamma = \alpha^{-2},\ \eta = \sigma^{-2}$.

Theorem 1. Given symmetric matrices $P > 0,\ Q > 0$, then the system (23) with $u_m = 0$ is robustly delay-independent stable if

$$\begin{aligned} &\min\ \gamma,\ \eta \\ &\ \text{s.t.}\quad X > 0 \\ &Y(A_m) = \begin{pmatrix} A_m X + X A_m^T & XW & XH_m & I & A_{dm} \\ \bullet & -W & 0 & 0 & 0 \\ \bullet & \bullet & -\gamma I & 0 & 0 \\ \bullet & \bullet & \bullet & -I & 0 \\ \bullet & \bullet & \bullet & \bullet & -W + \eta H_{dm}^T H_{dm} \end{pmatrix} < 0 \end{aligned} \tag{29}$$

exists, where $W = \omega^{-1}Q,\ X = \omega P^{-1}$ are matrices, while $\omega > 0,\ \gamma = \alpha^{-2},\ \eta = \sigma^{-2}$ are scalars.

Note that the symbol $\bullet$ in (29) and later on denotes standard symmetric terms used in LMIs.

The subsequent results compares delay-independent stability and robustness issues among the systems (8), (21), and (23) in order to evaluate their performance relations.

Theorem 2. The system (8) is robustly delay-independent stable if and only if both systems (21) are robustly delay-independently stable.

P r o o f. Consider the Lyapunov–Krasovskii functional for the system (8) as follows

$$\overline{V}(t,x) = x(t)^T \overline{P} x(t) + \int_{t-d}^{t} x(s)^T \overline{Q} x(s)\,\mathrm{d}s \tag{30}$$

where $\overline{P} = \mathrm{diag}(P,\ldots,P)$, $\overline{Q} = \mathrm{diag}(Q,\ldots,Q)$. Therefore $\overline{P} \in \mathbb{R}^{Nn\times Nn}$ and $Q \in \mathbb{R}^{Nn\times Nn}$ are symmetric positive definite matrices.

Denote $\overline{Y}(\overline{A})$ the matrix with the same structure as the matrix $Y(A_m)$ when substituting $A_m \to \overline{A}$, $A_{dm} \to \overline{A}_d$, $H_m \to \overline{H}$, $H_{dm} \to \overline{H}_d$ and $\overline{X} = \mathrm{diag}(X,\ldots,X)$. Applying the standard Lyapunov theory with the Lyapunov function (30) to the system (8), we get the LMI stability condition of Theorem 1 with the matrix $\overline{Y}(\overline{A})$. Denote analogously $\overline{Y}(\overline{A}_s)$ and $\overline{Y}(\overline{A}_c)$ through testing negative definiteness of the matrix $Y(A_m)$ when considering the system (21), respectively. Taking into account the transformation $\overline{T} = \mathrm{diag}(T,T,T,T,T,T)$, we get the relation

$$\overline{T}^T \overline{Y}(\overline{A}) \overline{T} = \mathrm{diag}[2Y(A_s),\ldots,N(N-1)Y(A_s),\ NY(A_c)] \tag{31}$$

Hence, the assertion of Theorem 2 follows immediately. □

Theorem 3. Given the systems (21) and (23). If the system (23) is robustly delay-independent stable, then the systems (21) are robustly delay-independent stable.

Proof. Suppose that the system (23) is delay-independent stable. This system includes both systems (21) as two particular cases by construction. The construction leads to the relations

$$\begin{aligned} h_m(t,\, x_m) &= h_s(t,\, x_m) + e(t,\, x_m) H_a x_m(t) \\ &= h_c(t,\, x_m) - e(t,\, x_m) H_a x_m(t) \\ h_{dm}(t,\, x_{dm}) &= h_{ds}(t,\, x_{dm}) + e_d(t,\, x_{dm}) H_{da} x_{dm}(t) \\ &= h_c(t,\, x_{dm}) - e_d(t,\, x_{dm}) H_{dm} x_{dm}(t) \end{aligned} \tag{32}$$

where h_s, h_c, h_{ds}, h_{dc} are given by (19), while H_a, H_{da} are defined by (22). Consider two particular cases of nonlinearities such as $e(t,\, x_m) = e_d(t,\, x_{dm}) = 1$ and $e(t,\, x_m) = e_d(t,\, x_{dm}) = -1$. It leads directly, when taking into account (11), to the relations (32).

The subsequent result is concerned with the robust delay-independent stability of the system (8) provided that the system (23) is robustly delay-independently stable. □

Corollary 1. Suppose that the system (23) is robustly delay-independent stable. Then the system (8) is robustly delay-independent stable.

Proof. The assertion follows immediately from Theorem 2 and Theorem 3. □

3.3.2. Delay-dependent stability

The robust delay-dependent stability of the system (23) as presented for instance in [20], can be established by using the Lyapunov–Krasovskii functional as follows

$$\begin{aligned} V_d(t, x_m) &= x_m(t)^T P x_m(t) + \int_{t-d}^{t} x_m(s)^T Q x_m(s)\,\mathrm{d}s + W_1 + W_2 + W_3 \\ W_1 &= \int_{t-d}^{t} r_1 h_m^T(s) h_m(s)\,\mathrm{d}s + \int_{t-d}^{t}\int_{t+\theta}^{t} r_1 h_{dm}^T(s) h_{dm}(s)\,\mathrm{d}s\mathrm{d}\theta \\ W_2 &= \int_{t-d}^{t}\int_{t+\theta}^{t} r_2 [x_m^T(s) A_m^T A_m x_m(s)\,\mathrm{d}s\mathrm{d}\theta \\ W_3 &= \int_{t-d}^{t}\int_{t+\theta-d}^{t} r_3 [x_m^T(s) A_{dm}^T A_{dm} x_m(s)\,\mathrm{d}s\mathrm{d}\theta \end{aligned} \tag{33}$$

where $P \in \mathbb{R}^{n\times n}$, $Q \in \mathbb{R}^{n\times n}$ are symmetric positive definite matrices and r_1, r_2, r_3 are positive scalars.

Using the results of the Lyapunov theory, the requirements on negative definiteness of the derivative of (33) along the trajectories of the system (23) leads to the following result [20].

Theorem 4. Given the system (23), symmetric matrices $P > 0$, $Q > 0$ and positive scalars r_1, r_2, r_3, d^*. The system (23) with $u_m = 0$ is robustly delay-dependent stable for all $d \in (0,\ d^*]$ if

min γ, η

s.t. $X > 0,\ R > 0,\ \varepsilon_1 > 0,\ \varepsilon_2 > 0,\ \varepsilon_3 > 0,\ \varepsilon_4 > 0$

$$Y_d(A_m) = \begin{pmatrix} W & XA_m^T & XA_{dm}^T & A_{dm} & XH_m^T & 0 & 0 & 0 \\ \bullet & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -\varepsilon_3 I & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -R & XH_{dm}^T & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\eta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\varepsilon_4 I \end{pmatrix} < 0 \tag{34}$$

exists, where

$$\begin{aligned} &X = P^{-1} \quad R = P^{-1}QP^{-1} \quad \varepsilon_1 = (d^* r_1)^{-1} \quad \varepsilon_2 = (d^* r_2)^{-1} \\ &\varepsilon_3 = (d^*[r_1^{-1} + r_2^{-1} + r_3^{-1}])^{-1} \qquad \gamma = \alpha^{-2} \quad \eta = \sigma^{-2} \quad \varepsilon_4 = -d^* r_3 \\ &W = (A_m + A_{dm})X + X(A_m + A_{dm})^T \end{aligned} \tag{35}$$

Remark 1. Theorem 4 holds for non-zero delay d. Zero delay requires to redefine the matrix $Y_d(A_m)$ in (34) so that A_{dm} is zero and $A_m + A_{dm} \to A_m$. Note that the solution of decentralized robust stabilization by using dynamic output feedback for a class of nonlinear interconnected systems with zero delay is presented in [23].

The subsequent result compares delay-dependent stability and robustness issues among the systems (8), (21), and (23) in order to evaluate their performance relations.

Theorem 5. The system (8) is robustly delay-dependent stable if and only if the systems (21) are both robustly delay-dependent stable.

Proof. Consider the Lyapunov–Krasovskii functional for the system (8) as follows

$$\begin{aligned} \overline{V}_d(t,x) &= x(t)^T \overline{P} x(t) + \int_{t-d}^{t} x(s)^T \overline{Q} x(s)\,\mathrm{d}s + \overline{W}_1 + \overline{W}_2 + \overline{W}_3 \\ \overline{W}_1 &= \int_{t-d}^{t} r_1 \overline{h}^T(s)\overline{h}(s)\,\mathrm{d}s + \int_{t-d}^{t}\int_{t+\theta}^{t} r_1 \overline{h}_d^T(s)\overline{h}_d(s)\,\mathrm{d}s\mathrm{d}\theta \\ \overline{W}_2 &= \int_{t-d}^{t}\int_{t+\theta}^{t} r_2[x^T(s)\overline{A}^T\overline{A}x(s)\,\mathrm{d}s\mathrm{d}\theta \\ \overline{W}_3 &= \int_{t-d}^{t}\int_{t+\theta-d}^{t} r_3[x^T(s)\overline{A}_d^T\overline{A}_d x(s)\,\mathrm{d}s\mathrm{d}\theta \end{aligned} \tag{36}$$

where $\overline{P} = \mathrm{diag}(P,\ldots,P)$, $\overline{Q} = \mathrm{diag}(Q,\ldots,Q)$. Therefore $\overline{P} \in \mathbb{R}^{Nn \times Nn}$ and $Q \in \mathbb{R}^{Nn \times Nn}$ are symmetric positive definite matrices and r_1, r_2, r_3 are positive scalars.

Denote $\overline{Y}_d(\overline{A})$ the matrix with the same structure as the matrix $Y_d(A_m)$ when substituting $A_m \to \overline{A}$, $A_{dm} \to \overline{A}_d$, $H_m \to \overline{H}$, $H_{dm} \to \overline{H}_d$, $\overline{X} = \text{diag}(X, \dots, X)$, and $\overline{R} = \text{diag}(R, \dots, R)$. Applying the standard Lyapunov theory with the Lyapunov function (36) to the system (8), we get the LMI stability condition of Theorem 4 with the matrix $\overline{Y}_d(\overline{A})$. Denote analogously $\overline{Y}_d(\overline{A}_s)$ and $\overline{Y}_d(\overline{A}_c)$ the matrix with the structure of $Y_d(A_m)$ when considering the first and the second systems in (21), respectively. Taking into account the transformation $\overline{T}_d = \text{diag}(T, T, T, T, T, T, T, T, T)$, we get the relation

$$\overline{T}_d^T \overline{Y}_d(\overline{A}) \overline{T}_d = \text{diag}[2Y(A_s), \dots, N(N-1)Y(A_s),\, NY(A_c)] \tag{37}$$

Hence, the assertion of Theorem 5 follows immediately. □

Theorem 6. If the system (23) is robustly delay-independent stable, then the systems (21) are robustly delay-dependent stable.

Proof. Suppose that the system (23) is delay-dependent stable. This system includes the systems (21) as two particular cases by construction. The proof continues in the same way of reasoning as the proof of Theorem 3. □

The subsequent result concerns the robust delay-dependent stability of the system (8) assuming that of the system (23).

Corollary 2. Suppose that the system (23) is robustly delay-dependent stable. Then the system (8) is robustly delay-dependent stable.

Proof. The assertion follows immediately from Theorem 5 and Theorem 6. □

3.4. Control design

3.4.1. Delay-independent control design

The robust delay-independent stability of the closed-loop system (23), (26) can be established by the inclusion of the system (27) into the test (29). The system (27) includes gain perturbations which are manipulated into the LMI format given by Theorem 7.8 in [20]. The resulting convex optimization problem presents the following lemma using LMIs.

Lemma 2. Given the system (23), perturbation matrices D, E in the controller (26), and symmetric matrices $P > 0$ and $Q > 0$. Suppose there exist positive scalars

γ and η, matrices $X = X^T > 0$ and M so that the problem

$$\begin{aligned} &\min \gamma,\, \eta \\ &\text{s.t. } X > 0,\ M \\ &Y_c(A_m) = \begin{pmatrix} W_m & XW & WH_m^T & I & A_{dm} & BD & \eta W E^T \\ \bullet & -W & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\gamma I & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & W_h & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\eta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\eta I \end{pmatrix} < 0 \end{aligned} \tag{38}$$

has a feasible solution. The block matrices in (38) are defined as $W_m = A_m X + XA_m^T + BM + M^T B^T$ and $W_h = -W + H_{dm}^T H_{dm}$ by using the variables $X = P^{-1}$ and $W = Q$.
The nominal controller gain matrix is given by

$$K = MX^{-1} \tag{39}$$

Then the system (23) is robustly delay-independently stabilized by the controller (26) with the gain matrix (39).

Lemma 2 is concerned with the control design method which is convenient for direct computations of resilient controller gain matrix. The following theorem states the main result when considering the delay-independent approach.

Theorem 7. Consider the non-fragile state controller design for a nonlinear delayed symmetric composite system defined by equations (1) – (6) with additive perturbations (7). Construct the reduced control design system defined by equations (23) – (25). Select the matrix K satisfying (38) – (39) for the system (23) – (25) and use it in (7). Then the overall closed-loop system (1) – (7) is globally asymptotically stable.

Proof. Consider the closed-loop system (1) – (7) rewritten into the global form (8) – (12). Consider also the closed-loop system (23) – (26). Suppose that the gain matrix K leads to a feasible solution of the problem (38). This matrix is used in the controller (7), (12), and also (26). As a result, the closed-loop system (23) – (26) is robustly delay-independently stable. To verify this property according to Theorem 1, we simply substitute the matrix A_m by the the term $A_m + BK + B\Delta K_m$. A controller is synthesized according to Lemma 2 when applying standard manipulations eliminating uncertainty $F_m(t)$ in ΔK_m given by (26) and by using the slack variable $KX = M$ in (39). Substituting the matrix $\overline{A}$ in (9) by the matrix $\overline{A}_c$ defined by blocks $A_{cii} = A_{ii} + BK + B\Delta K_{ii}$ and $A_{cij} = A_{ij}$, eliminating uncertainties

$F_{(t)}$ in (7), and using the scalars γ, η obtained from Lemma 2. The stability problem for the overall system (1)−(7) is reduced on the test of the matrix inequality

$$\overline{Y}_c(\overline{A}_c) = \begin{pmatrix} \overline{W}_c & \overline{X}\,\overline{W} & \overline{W}\,\overline{H}^T & I & \overline{A}_d & \overline{B}\,\overline{D} & \eta\overline{W}\,\overline{E}^T \\ \bullet & -\overline{W} & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\gamma I & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \overline{W}_h & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\eta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\eta I \end{pmatrix} < 0 \qquad (40)$$

where $\overline{W}_c = \overline{A}_c\overline{X} + \overline{X}\,\overline{A}_c^T + \overline{B}\,\overline{M} + \overline{M}^T\overline{B}^T$ and $\overline{W}_h = -\overline{W} + \psi\overline{H}_d^T\overline{H}_d$. $\overline{X} = \mathrm{diag}(X, \ldots, X)$,$\overline{M} = \mathrm{diag}(M, \ldots, M)$, and $\overline{W}_h = \mathrm{diag}(W_h, \ldots, W_h)$. Applying now the transformation $\overline{T}_c = \mathrm{diag}(T, T, T, T, T, T, T)$ to the quadratic form $\overline{T}_c^T\overline{Y}_c(\overline{A}_c)\overline{T}_c$, Theorems 3 and 2 applied on the closed-loop system (1)−(7), the result is proved. □

3.4.2. Delay-dependent control design

The robust delay-dependent stability of the closed-loop system (23), (26) can be established by the inclusion of the system (27) into the test (34). The system (27) includes gain perturbations which are manipulated into the LMI format in a similar way as in the delay-independent case. The resulting convex optimization problem presents the following lemma using LMIs.

Lemma 3. Given the system (23), perturbation matrices D, E in the controller (26), symmetric matrices $P > 0$, $Q > 0$ and positive scalars r_1, r_2, r_3, d^*. Suppose there exist positive scalars ε_1, ε_2, ε_3, ε_4, γ, $\eta > 0$, and symmetric matrices $X > 0$, $R > 0$ and M so that the problem

$$\min \gamma,\, \eta$$

$$\text{s.t. } X > 0,\ R > 0,\ M,\ \varepsilon_1 > 0,\ \varepsilon_2 > 0,\ \varepsilon_3 > 0,\ \varepsilon_4 > 0,$$

$$Y_{dc}(A_m) = \begin{pmatrix} W_d & XA_m^T & XA_{dm}^T & A_{dm} & XH_m & 0 & 0 & 0 & BD & \eta XE^T \\ \bullet & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma I & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -R & XH_{dm}^T & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\psi I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\varepsilon_4 I & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\eta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\eta I \end{pmatrix} < 0 \qquad (41)$$

has a feasible solution. The block matrix W_d means $W_d = (A_m + A_{dm})X + X(A_m + A_{dm)}^T) + BM + M^TB^T$, while $X, R, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, d^*, \gamma, \eta$ are defined in (35). The gain matrix is given by

$$K = MX^{-1} \qquad (42)$$

Then the system (23) is robustly delay-dependently stabilized by the controller (26) with the gain matrix (42).

The following theorem states the main result when considering the delay-dependent approach.

Theorem 8. Consider the non-fragile state controller design with additive perturbations (7) for a nonlinear delayed symmetric composite system defined by equations (1) – (6). Construct the reduced control design system defined by equations (23) – (25). Select the matrix K satisfying (41) – (42) for the system (23) – (25) and use it in (7). Then the overall closed-loop system (1) – (7) is globally asymptotically stable.

P r o o f . The way of reasoning is identical with the proof of Theorem 7 when using Lemma 3 instead of Lemma 2. Considering given $X, M, \gamma, \eta, \varepsilon_1, \ldots, \varepsilon_4$ by (35), the stability problem for the overall system reduced in this case on the test of the matrix inequality

$$\overline{Y}_{dc}(\overline{A}_c) = \begin{pmatrix} \overline{W}_d & \overline{X}\,\overline{A}^T & \overline{X}\,\overline{A}_d^T & \overline{A}_d & \overline{X}\,\overline{H} & 0 & 0 & 0 & \overline{B}\,\overline{D} & \eta\overline{X}\,\overline{E}^T \\ \bullet & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma I & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\overline{R} & \overline{X}\,\overline{H}_d^T & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\psi I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\varepsilon_4 I & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\eta I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\eta I \end{pmatrix} < 0 \tag{43}$$

where $\overline{W}_d = (\overline{A} + \overline{A}_d)\overline{X} + \overline{X}(\overline{A} + \overline{A}_d)^T + \overline{B}\,\overline{M} + \overline{M}^T\overline{B}^T$ $\overline{X} = \mathrm{diag}(X, \ldots, X)$, $\overline{M} = \mathrm{diag}(M, \ldots, M)$, and $\overline{R} = \mathrm{diag}(R, \ldots, R)$. Applying now the transformation $\overline{T}_{dc} = \mathrm{diag}(T, T, T, T, T, T, T, T, T, T)$ to the quadratic form $\overline{T}_{dc}^T \overline{Y}_{dc}(\overline{A}_c)\overline{T}_{dc}$, Theorems 5 and 6, and Corollary 2 applied on the closed-loop system (1) – (7), the result is proved. □

4. CONCLUSION

The main paper contribution consists of a new method of the low-order non-fragile control design for a class of nonlinear delayed symmetric composite systems. Particular structural properties of this class of large scale systems are used for the construction of low-order design systems. The non-fragile controller is designed for the design system to guarantee the global asymptotic stability of the closed-loop design system. The controller is used as a set of local identical controllers in the overall system. Both delay-independent and delay-dependent control design methods are presented. The stability analysis is performed by discussion the stabilization of low complexity systems which guarantee that of the overall system.

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