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Zheng-Guang Wu [a b] , Ju H. Park [a] , Hongye Su [b] & Jian Chu [b]

[a] Department of Electrical Engineering, Yeungnam University, 214-1 Dae-Dong, Kyongsan 712-749, Republic of Korea

[b] National Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Yuquan Campus, Hangzhou Zhejiang 310027, P.R. China

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Zheng-Guang Wu[ab], Ju H. Park[a*], Hongye Su[b] and Jian Chu[b]

[a]*Department of Electrical Engineering, Yeungnam University, 214-1 Dae-Dong, Kyongsan 712-749, Republic of Korea;* [b]*National Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Yuquan Campus, Hangzhou Zhejiang 310027, P.R. China*

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This article investigates the problem of non-fragile synchronisation control for complex networks with time-varying coupling delay and missing data. A stochastic variable satisfying the Bernoulli random binary distribution is utilised to model the missing data. Based on the Gronwall's inequality, an exponential synchronisation condition is obtained ensuring the exponential mean-square stability of the error system. Then a sufficient condition for designing the non-fragile synchronisation controller is proposed. Finally, a simulation example is given to show the efficiency of the proposed design methods.

Keywords: complex dynamical networks; synchronisation; non-fragile control; missing data

1. Introduction

Complex dynamical networks (CDNs) which consist of interacting dynamical entities with an interplay between dynamical states and interaction patterns have attracted extensive attention in the past decades due to the fact that many systems in nature can be modelled by CNDs, for example, power grids, communication networks and the World Wide Web (Liu, Wang, Liang, and Liu 2008, 2009, 2012; Liu, Zhao, and Hill 2010). As an important collective behaviour of CDNs, synchronisation has been widely studied (Wu and Chua 1995; Wang and Chen 2002; Cao, Li, and Wei 2006; Duan, Wang, Chen, and Huang 2009; Lu and Ho 2010, 2011; Wang, Liu, Liu, and Shi 2010; Yang and Cao 2010a; Wang, Wang, and Liu 2010b; Yang, Cao, and Lu 2011; Xiang and Wei 2011; Lee, Park, Ji, Kwon, and Lee 2012). In most of the work on the synchronisation of CDNs, it is assumed that the network can work well during all the time, that is, signals can always be transmitted successfully. However, in practical applications, the network may fail occasionally, that is, there may be a non-zero probability that the signals contain missing data. For example, on 26 December 2006, tens of submarine fibre cables were destroyed in the earthquake of Taiwan; as a result, the Internet from the Chinese Mainland to North America, Europe, Southeast Asia and some other places were paralysed (Wang, Xiao, and Wang 2010a). Very recently, in Wang, Xiao, and Guan (2010c) and Wang, Xiao, and Wang (2010d) the global synchronisation of CDNs with network failures has been studied based on the description of the switching system. The networked synchronisation control problem of CDNs with time-varying delay has been investigated in Wang, Zhang, Wang, and Yang (2010e), where both the data packet dropouts and network-induced delays have been taken into account in the synchronisation controller design. In Huang, Ho, Lu, and Kurths (2012) the partial synchronisation problem of stochastic CDNs has been investigated. Unlike the existing results, the CDNs considered in Huang et al. (2012) suffers from a class of communication constraint, that is, only part of nodes' states can be transmitted.

It should be pointed out that in the existing results on synchronisation control of CDNs, an absolutely indispensable assumption is that the designed controller is exactly implemented (Li, Zhang, Hu, and Nie 2011; Shen, Wang, and Liu 2012). However, in practical, as a part of a closed-loop system, the designed controller should be able to tolerate some uncertainty in its coefficients due to the fact that the uncertainty is not avoided, and it may be caused by many reasons, such as finite word length in digital systems, the imprecision inherent in analogue systems and the need for additional tuning of parameters in the final controller implementation, and thus it is required that there exists a non-zero (although possibly small)

*Corresponding author. Email: jessie@ynu.ac.kr

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margin of tolerance around the controller designed, that is, to design a non-fragile controller such that the controller is insensitive to uncertainties (Keel and Bhattacharyya 1997). The non-fragile control problem has been widely investigated by many researchers (Keel and Bhattacharyya 1997; Yue and Lam 2005; Xu, Lam, Yang, and Wang 2006; Li and Jia 2009; Shu, Lam, and Xiong 2009). For example, in Shu et al. (2009) the non-fragile exponential stabilisation for a class of discrete-time linear systems with missing data in actuators, and the non-fragile exponential stabilisation for a class of discrete-time linear systems with missing data in actuators. Yue and Lam (2005) have studied the design problem of non-fragile guaranteed cost controller for uncertain descriptor systems with delays and a controller has been designed to guarantee that the closed-loop system is regular, impulse-free and exponentially stable and an upper bound of the guaranteed cost function. Although the importance of the non-fragile control problem has been widely recognised, no related results have been established for CDNs. Correspondingly, it is meaningful and interesting to study the non-fragile synchronisation of CDNs. The main purpose of this article, therefore, is to shorten such a gap by making the first attempt to deal with the non-fragile synchronisation control for CNDs with time-varying coupling delay and missing data.

In this article, we pay attention to the problem of non-fragile synchronisation control for complex networks with time-varying coupling delay and missing data, which is modelled by a stochastic variable satisfying the Bernoulli random binary distribution. An exponential synchronisation condition is proposed and the design method of the non-fragile synchronisation controller is also given in terms of an LMI approach. Finally, the feasibility of the proposed design methods is shown by a simulation example.

Notation: The notations used throughout this article are fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space and the set of all $m\times n$ real matrices, respectively. The notation $X > Y$ ($X \geqslant Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive definite (positive semidefinite). I and 0 represent the identity matrix and a zero matrix, respectively. The superscript 'T' represents the transpose and $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix. $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix. $\Pr\{\alpha\}$ means the occurrence probability of the event α and $\Pr\{\alpha|\beta\}$ means the occurrence probability of α conditional on β. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$, respectively, mean the expectation of the stochastic variable x and the expectation of the stochastic variable x conditional on the stochastic variable y. $\otimes$ denotes the notation of the Kronecker product. For an arbitrary matrix B and two symmetric matrices A and C,

$$\begin{bmatrix} A & B \\ * & C \end{bmatrix}$$

denotes a symmetric matrix, where '*' denotes the term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Preliminaries

Consider the following CDN consisting of N identical coupled nodes with each node being an n-dimensional dynamical system:

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} G_{ij} A x_j(t-\tau(t)) + u_i(t), \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i(t)$ and $u_i(t)$ are, respectively, the state variable and the control input of the node i, $A = (a_{ij})_{n\times n} \in \mathbb{R}^{n\times n}$ is the constant inner coupling matrix of the nodes and $G = (G_{ij})_{N\times N}$ is the the outer-coupling matrix of the network. If there is a connection from node j to node i ($i \neq j$), then $G_{ij} = 1$, otherwise, $G_{ij} = 0$ ($i \neq j$). The diagonal elements of matrix G are defined by

$$G_{ii} = -\sum_{j=1, j\neq i}^{N} G_{ij}, \quad i = 1, 2, \ldots, N. \tag{2}$$

The scalar $\tau(t)$ denotes the time-varying delay satisfying

$$0 < \tau(t) \leqslant \tau, \quad \dot{\tau}(t) \leqslant \mu \tag{3}$$

where $\tau > 0$ and $\mu > 0$ are known constants. f: $\mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector-valued function and satisfies the following sector-bounded condition (Wang, Liu, and Liu 2008):

$$[f(x) - f(y) - U(x-y)]^{\mathrm{T}}[f(x) - f(y) - V(x-y)] \leqslant 0, \quad \forall x, y \in \mathbb{R}^n \tag{4}$$

where U and V are constant matrices of appropriate dimensions.

Let $e_i(t) = x_i(t) - s(t)$ be the synchronisation error, where $s(t) \in \mathbb{R}^n$ is the state trajectory of the unforced isolate node $\dot{s}(t) = f(s(t))$. It can be seen that $\dot{e}_i(t) = \dot{x}_i(t) - f(s(t))$. Substituting (1) into $\dot{e}_i(t)$ and noting (2), we can get the following error dynamics of CND (1):

$$\dot{e}_i(t) = g(e_i(t)) + \sum_{j=1}^{N} G_{ij} A e_j(t - \tau(t)) + u_i(t),\quad i = 1, 2, \ldots, N \qquad (5)$$

where $g(e_i(t)) = f(x_i(t)) - f(s(t))$. It should be pointed out that $g(e_i(t))$ is dependent on $x_i(t)$ and $s(t)$. However, in order to avoid cumbersome notations, we will use the simpler symbol $g(e_i(t))$ instead of $g(x_i(t), s(t))$ (Wang, Ho, and Liu 2005).

In this article, we consider the following controller forms:

$$u_i^c(t) = (K_i + \Delta K_i(t))e_i(t), \quad i = 1, 2, \ldots, N \qquad (6)$$

where K_i is controller gain matrix to be determined, $u_i^c(t) \in \mathbb{R}^n$ is the output of the controller and ΔK_i represents the possible controller gain fluctuation. It is assumed that ΔK_i has the following structure:

$$\Delta K_i = H_i \Delta_i(t) E_i \qquad (7)$$

where $\Delta_i(t) \in \mathbb{R}^{k \times l}$ is an unknown time-varying matrix satisfying

$$\Delta_i(t)^{\mathrm{T}} \Delta_i(t) \leqslant I \qquad (8)$$

and H_i and E_i are known constant matrices.

The data-missing phenomenon probably occurs when the control signal $u_i^c(t)$ is transmitted. In this article, we model the data loss phenomena via a stochastic approach (Shen, Wang, Shu, and Wei 2008; Wang, Ho, Liu, and Liu 2009; Wang et al. 2010a,b). Then the control input with data missing can be described as

$$u_i(t) = \alpha(t) u_i^c(t), \quad i = 1, 2, \ldots, N \qquad (9)$$

where

$$\Pr\{\alpha(t) = 1\} = \alpha, \quad \Pr\{\alpha(t) = 0\} = 1 - \alpha$$

where $\alpha \in [0, 1]$ is a known constant.

By substituting (6) and (9) into (5), we obtain

$$\dot{e}_i(t) = g(e_i(t)) + \sum_{j=1}^{N} G_{ij} A e_j(t - \tau(t)) + \alpha(t)(K_i + \Delta K_i(t))e_i(t), \quad i = 1, 2, \ldots, N. \qquad (10)$$

It is clear that (10) can be rewritten as

$$\dot{e}(t) = \bar{g}(e(t)) + (G \otimes A)e(t - \tau(t)) + \alpha(t)(K + H\Delta(t)E)e(t) \qquad (11)$$

where

$$K = \mathrm{diag}\{K_1, K_2, \ldots, K_N\},$$
$$H = \mathrm{diag}\{H_1, H_2, \ldots, H_N\},$$
$$\Delta(t) = \mathrm{diag}\{\Delta_1(t), \Delta_2(t), \ldots, \Delta_N(t)\},$$
$$E = \mathrm{diag}\{E_1, E_2, \ldots, E_N\},$$
$$e(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_N(t) \end{bmatrix}, \quad \bar{g}(e(t)) = \begin{bmatrix} g(e_1(t)) \\ g(e_2(t)) \\ \vdots \\ g(e_N(t)) \end{bmatrix}.$$

We give the following definition and lemmas which will be useful in the sequel.

Definition 2.1: The CDN (1) is said to be exponentially mean-square synchronised if the error system (11) is exponentially mean-square stable, i.e. there exist two constants $\alpha > 0$ and $\beta > 0$ such that

$$\mathbb{E}\{\|e(t)\|^2\} \leqslant \alpha e^{-\beta t} \sup_{-\tau \leqslant \theta \leqslant 0} \|e(\theta)\|^2. \qquad (12)$$

Lemma 2.2 (Park, Ko, and Jeong 2011)**:** *For any matrix* $\begin{bmatrix} M & S \\ * & M \end{bmatrix} \geqslant 0$, *scalars* $\tau > 0$, $\tau(t) > 0$ *satisfying* $0 < \tau(t) \leqslant \tau$, *vector function* $\dot{x}(t + \cdot) : [-\tau, 0] \to \mathbb{R}^n$ *such that the concerned integrations are well defined, then*

$$-\tau \int_{t-\tau}^{t} \dot{x}(\alpha)^{\mathrm{T}} M \dot{x}(\alpha) \mathrm{d}\alpha \leqslant \varpi(t)^{\mathrm{T}} \Omega \varpi(t) \qquad (13)$$

where

$$\varpi(t) = \begin{bmatrix} x(t)^{\mathrm{T}} & x(t - \tau(t))^{\mathrm{T}} & x(t - \tau)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$
$$\Omega = \begin{bmatrix} -M & M - S & S \\ * & -2M + S + S^{\mathrm{T}} & -S + M \\ * & * & -M \end{bmatrix}.$$

Lemma 2.3 (Xie 1996)**:** *Let* $L = L^{\mathrm{T}}$, H *and* E *be real matrices of appropriate dimensions with* F *satisfying* $F^{\mathrm{T}}F \leqslant I$. *Then,* $L + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0$, *if and only if there exists a scalar* $\rho > 0$ *such that* $L + \rho^{-1}HH^{\mathrm{T}} + \rho E^{\mathrm{T}}E < 0$ *or equivalently*

$$\begin{bmatrix} L & H & \rho E^{\mathrm{T}} \\ * & \rho I & 0 \\ * & * & \rho I \end{bmatrix} < 0. \qquad (14)$$

Our goal is to design a set of controllers (6) such that the CDN (1) is exponentially mean-square synchronised. In other words, we want to find a gain matrix K such that the error system (11) is exponentially mean-square stable.

3. Main results

In this section, we first provide two sufficient conditions on exponential mean-square stability of the error system (11). Then, we investigate the design problem of exponential mean-square synchronisation for CND (1). Before presenting the main results, for the sake of presentation simplicity, we denote

$$\bar{U} = \frac{(I_N \otimes U)^{\mathrm{T}}(I_N \otimes V)}{2} + \frac{(I_N \otimes V)^{\mathrm{T}}(I_N \otimes U)}{2},$$

$$\bar{V} = -\frac{(I_N \otimes U)^{\mathrm{T}} + (I_N \otimes V)^{\mathrm{T}}}{2}.$$

Theorem 3.1: *The error system* (11) *is exponentially mean-square stable if there exist matrices* $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $\left[\begin{smallmatrix} Z & S \\ * & Z \end{smallmatrix}\right] \geqslant 0$ *and scalars* $\lambda > 0$ *and* $\delta > 0$ *such that*

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & S & \Xi_{14} & \Xi_{15} & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & 0 & \Xi_{25} & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 \\ * & * & * & -\lambda I & \tau Z & 0 \\ * & * & * & * & -Z & 0 \\ * & * & * & * & * & -Z \end{bmatrix} < 0 \qquad (15)$$

where

$$\begin{aligned}
\Xi_{11} &= \alpha P(K + H\Delta(t)E) + \alpha(K + H\Delta(t)E)^{\mathrm{T}}P + Q_1 \\
&\quad + Q_2 - Z - \lambda\bar{U} + \delta I, \\
\Xi_{12} &= P(G \otimes A) + Z - S, \\
\Xi_{14} &= P - \lambda\bar{V}, \\
\Xi_{15} &= \tau\alpha(K + H\Delta(t)E)^{\mathrm{T}}Z, \\
\Xi_{16} &= \tau\sqrt{\alpha(1-\alpha)}(K + H\Delta(t)E)^{\mathrm{T}}Z, \\
\Xi_{22} &= -(1-\mu)Q_2 - 2Z + S + S^{\mathrm{T}}, \\
\Xi_{23} &= -S + Z, \\
\Xi_{25} &= \tau(G \otimes A)^{\mathrm{T}}Z, \\
\Xi_{33} &= -Q_1 - Z.
\end{aligned}$$

Proof: Consider the following Lyapunov functional for the system (11):

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \qquad (16)$$

where

$$\begin{aligned}
V_1(e_t) &= e(t)^{\mathrm{T}}Pe(t), \\
V_2(e_t) &= \int_{t-\tau}^{t} e(s)^{\mathrm{T}}Q_1 e(s)\mathrm{d}s, \\
V_3(e_t) &= \int_{t-\tau(t)}^{t} e(s)^{\mathrm{T}}Q_2 e(s)\mathrm{d}s, \\
V_4(e_t) &= \tau\int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}(s)^{\mathrm{T}}Z\dot{e}(s)\mathrm{d}s\,\mathrm{d}\theta.
\end{aligned}$$

Define the infinitesimal operator $\mathbb{L}$ of $V(e_t)$ as follows (Gao, Wu, and Shi 2009):

$$\mathbb{L}V(e_t) = \lim_{h\to 0^+}\frac{1}{h}\{\mathbb{E}\{V(e_{t+h})|e_t\} - V(e_t)\}. \qquad (17)$$

Taking the derivative of (16) along the solution of system (11) yields

$$\begin{aligned}
\mathbb{E}\{\mathbb{L}V_1(t)\} &= 2\mathbb{E}\{e(t)^{\mathrm{T}}P\dot{e}(t)\} \\
&= 2\mathbb{E}\{e(t)^{\mathrm{T}}P(\bar{g}(e(t)) + (G \otimes A)e(t-\tau(t)) \\
&\quad + \alpha(K + H\Delta(t)E)e(t))\}, \qquad (18)
\end{aligned}$$

$$\mathbb{E}\{\mathbb{L}V_2(t)\} = \mathbb{E}\{e(t)^{\mathrm{T}}Q_1e(t) - e(t-\tau)^{\mathrm{T}}Q_1e(t-\tau)\}, \qquad (19)$$

$$\begin{aligned}
\mathbb{E}\{\mathbb{L}V_3(t)\} &= \mathbb{E}\{e(t)^{\mathrm{T}}Q_2e(t) \\
&\quad - (1-\dot{\tau}(t))e(t-\tau(t))^{\mathrm{T}}Q_2e(t-\tau(t))\} \\
&\leqslant \mathbb{E}\{e(t)^{\mathrm{T}}Q_2e(t) \\
&\quad - (1-\mu)e(t-\tau(t))^{\mathrm{T}}Q_2e(t-\tau(t))\}, \qquad (20)
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}\{\mathbb{L}V_4(t)\} &= \mathbb{E}\left\{\tau^2\dot{e}(t)^{\mathrm{T}}Z\dot{e}(t) - \tau\int_{t-\tau}^{t}\dot{e}(s)^{\mathrm{T}}Z\dot{e}(s)\mathrm{d}s\right\} \\
&\leqslant \mathbb{E}\{\tau^2(\bar{g}(e(t)) + (G\otimes A)e(t-\tau(t)) \\
&\quad + \alpha(K + H\Delta(t)E)e(t))^{\mathrm{T}}Z(\bar{g}(e(t)) \\
&\quad + (G\otimes A)e(t-\tau(t)) + \alpha(K+H\Delta(t)E)e(t)) \\
&\quad + \tau^2\alpha(1-\alpha)e(t)^{\mathrm{T}}(K+H\Delta(t)E)^{\mathrm{T}} \\
&\quad \times Z(K+H\Delta(t)E)e(t) + \varpi(t)^{\mathrm{T}}\Omega\varpi(t)\}, \qquad (21)
\end{aligned}$$

where Lemma 2.2 is applied and

$$\varpi(t) = \begin{bmatrix} e(t)^{\mathrm{T}} & e(t-\tau(t))^{\mathrm{T}} & e(t-\tau)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$

$$\Omega = \begin{bmatrix} -Z & Z-S & S \\ * & -2Z+S+S^{\mathrm{T}} & -S+Z \\ * & * & -Z \end{bmatrix}.$$

On the other hand, it follows from (4) that for any $\lambda > 0$

$$y(t) = \lambda\begin{bmatrix} e(t) \\ \bar{g}(e(t)) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \bar{U} & \bar{V} \\ * & I \end{bmatrix}\begin{bmatrix} e(t) \\ \bar{g}(e(t)) \end{bmatrix} \leqslant 0. \qquad (22)$$

Thus,

$$\mathbb{E}\{\mathbb{L}V(t)\} \leqslant \mathbb{E}\{\mathbb{L}V_1(t) + \mathbb{L}V_2(t) + \mathbb{L}V_3(t) - y(t)\} \leqslant \mathbb{E}\left\{\begin{bmatrix} \varpi(t) \\ \bar{g}(e(t)) \end{bmatrix}^{\mathrm{T}} \Lambda \begin{bmatrix} \varpi(t) \\ \bar{g}(e(t)) \end{bmatrix}\right\}$$

where

$$\Lambda = \begin{bmatrix} \hat{\Xi}_{11} & \Xi_{12} & S & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & 0 \\ * & * & \Xi_{33} & 0 \\ * & * & * & -\lambda I \end{bmatrix} + \begin{bmatrix} \Xi_{15} \\ \Xi_{25} \\ 0 \\ \tau Z \end{bmatrix} Z \begin{bmatrix} \Xi_{15} \\ \Xi_{25} \\ 0 \\ \tau Z \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \Xi_{16} \\ 0 \\ 0 \\ 0 \end{bmatrix} Z \begin{bmatrix} \Xi_{16} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$$

and

$$\hat{\Xi}_{11} = \alpha P(K + H\Delta(t)E) + \alpha(K + H\Delta(t)E)^{\mathrm{T}} P + Q_1 + Q_2 - Z - \lambda\bar{U}.$$

By using the Schur complement, it can be obtained immediately from (15) that

$$\mathbb{E}\{\mathbb{L}V(t)\} \leqslant -\delta\mathbb{E}\{\|e(t)\|^2\}. \tag{23}$$

Noticing (16) and (23), we have

$$\rho\mathbb{E}\{\|e(t)\|^2 \leqslant \mathbb{E}\{V(t)\} \leqslant \mathbb{E}\{V(0)\} - \delta\int_0^t \mathbb{E}\{\|e(s)\|^2\}\mathrm{d}s \tag{24}$$

where $\rho = \lambda_{\min}(P)$, which implies that

$$\mathbb{E}\{\|e(t)\|^2 \leqslant \frac{\mathbb{E}\{V(0)\}}{\rho} - \frac{\delta}{\rho}\int_0^t \mathbb{E}\{\|e(s)\|^2\}\,\mathrm{d}s. \tag{25}$$

Finally, similar to Shen et al. (2012), it follows from Gronwall's inequality that

$$\mathbb{E}\{\|e(t)\|^2\} \leqslant \frac{\mathbb{E}\{V(0)\}}{\rho} e^{-\frac{\delta}{\rho}t} \tag{26}$$

which, from Definition 2.1, means that the system (11) is exponentially mean-square stable. This completes the proof. □

It is noted that (15) is not an LMI and thus is unsolvable using standard numerical tools. Based on Lemma 2.3, we can get the following LMI condition, which can be applied for checking the stability of system (11).

Theorem 3.2: *The error system* (11) *is exponentially mean-square stable if there exist matrices* $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $\begin{bmatrix} Z & S \\ * & Z \end{bmatrix} \geqslant 0$ *and scalars* $\lambda > 0$, $\delta > 0$ *and* $\varepsilon > 0$ *such that*

$$\begin{bmatrix} \tilde{\Xi}_{11} & \Xi_{12} & S & \Xi_{14} & \tilde{\Xi}_{15} & \tilde{\Xi}_{16} & \alpha PH & \varepsilon E^{\mathrm{T}} \\ * & \Xi_{22} & \Xi_{23} & 0 & \Xi_{25} & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\lambda I & \tau Z & 0 & 0 & 0 \\ * & * & * & * & -Z & 0 & \tau\alpha ZH & 0 \\ * & * & * & * & * & -Z & \tau\sqrt{\alpha(1-\alpha)}ZH & 0 \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{27}$$

where Ξ_{12}, Ξ_{14}, Ξ_{22}, Ξ_{23}, Ξ_{25} *and* Ξ_{33} *are given in* Theorem 3.1, *and*

$$\tilde{\Xi}_{11} = \alpha PK + \alpha K^{\mathrm{T}} P + Q_1 + Q_2 - Z - \lambda\bar{U} + \delta I,$$
$$\tilde{\Xi}_{15} = \tau\alpha K^{\mathrm{T}} Z,$$
$$\tilde{\Xi}_{16} = \tau\sqrt{\alpha(1-\alpha)}K^{\mathrm{T}} Z.$$

Proof: Rewriting (15), we can get

$$\begin{bmatrix} \tilde{\Xi}_{11} & \Xi_{12} & S & \Xi_{14} & \tilde{\Xi}_{15} & \tilde{\Xi}_{16} \\ * & \Xi_{22} & \Xi_{23} & 0 & \Xi_{25} & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 \\ * & * & * & -\lambda I & \tau Z & 0 \\ * & * & * & * & -Z & 0 \\ * & * & * & * & * & -Z \end{bmatrix} + \begin{bmatrix} \alpha PH \\ 0 \\ 0 \\ 0 \\ \tau\alpha ZH \\ \tau\sqrt{\alpha(1-\alpha)}ZH \end{bmatrix} \times \Delta(t)\begin{bmatrix} E & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} E^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta(t) \times \begin{bmatrix} \alpha H^{\mathrm{T}} P^{\mathrm{T}} & 0 & 0 & 0 & \tau\alpha H^{\mathrm{T}} Z^{\mathrm{T}} & \tau\sqrt{\alpha(1-\alpha)}H^{\mathrm{T}} Z^{\mathrm{T}} \end{bmatrix} < 0. \tag{28}$$

Based on Lemma 2.3, we can find the above inequality is equivalent to (27). This completes the proof. □

In the following, we will develop a design result based on Theorem 3.1.

Theorem 3.3: *The CDN* (1) *is exponentially mean-square synchronised if there exist matrices* $P = \mathrm{diag}\{P_1, P_2, \ldots, P_N\} > 0$, $Q_1 > 0$, $Q_2 > 0$, $\begin{bmatrix} Z & S \\ * & Z \end{bmatrix} \geqslant 0$,

$X = \mathrm{diag}\{X_1, X_2, \ldots, X_N\}$, *and scalars* $\lambda > 0$ *and* $\varepsilon > 0$ *such that*

$$\begin{bmatrix} \hat{\Xi}_{11} & \Xi_{12} & S & \Xi_{14} & \hat{\Xi}_{15} & \hat{\Xi}_{16} & \alpha PH & \varepsilon E^{\mathrm{T}} \\ * & \Xi_{22} & \Xi_{23} & 0 & \hat{\Xi}_{25} & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\lambda I & \tau P & 0 & 0 & 0 \\ * & * & * & * & -2P+Z & 0 & \tau\alpha PH & 0 \\ * & * & * & * & * & -2P+Z & \tau\sqrt{\alpha(1-\alpha)}PH & 0 \\ * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{29}$$

where

$$\hat{\Xi}_{11} = \alpha X + \alpha X^{\mathrm{T}} + Q_1 + Q_2 - Z - \lambda\bar{U},$$
$$\hat{\Xi}_{15} = \tau\alpha X^{\mathrm{T}},$$
$$\hat{\Xi}_{16} = \tau\sqrt{\alpha(1-\alpha)}X^{\mathrm{T}},$$
$$\hat{\Xi}_{25} = \tau(G\otimes A)^{\mathrm{T}}P,$$

and the other parameters follow the same definitions as those in Theorem 3.1. *Furthermore, the desired controllers gain matrices are given by*

$$K_i = P_i^{-1}X_i, \quad i = 1, 2, \ldots, N. \tag{30}$$

Proof: By Lemma 2.3, it can be easily obtained from (29) that there exists a small enough $\delta > 0$ such that

$$\begin{bmatrix} \Lambda_{11} & \Xi_{12} & S & \Xi_{14} & \hat{\Xi}_{15} & \hat{\Xi}_{16} \\ * & \Xi_{22} & \Xi_{23} & 0 & \hat{\Xi}_{25} & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 \\ * & * & * & -\lambda I & \tau P & 0 \\ * & * & * & * & -2P+Z & 0 \\ * & * & * & * & * & -2P+Z \end{bmatrix} + \begin{bmatrix} \alpha PH \\ 0 \\ 0 \\ 0 \\ \tau\alpha PH \\ \tau\sqrt{\alpha(1-\alpha)}PH \end{bmatrix} \Delta(t)\begin{bmatrix} E & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} E^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta(t) \times \begin{bmatrix} \alpha H^{\mathrm{T}}P^{\mathrm{T}} & 0 & 0 & 0 & \tau\alpha H^{\mathrm{T}}P^{\mathrm{T}} & \tau\sqrt{\alpha(1-\alpha)}H^{\mathrm{T}}P^{\mathrm{T}} \end{bmatrix} < 0 \tag{31}$$

where $\Lambda_{11} = \hat{\Xi}_{11} + \delta I$. On the other hand, define matrix $J = \mathrm{diag}\{I, I, I, I, PZ^{-1}, PZ^{-1}\}$ and $X = PK$.

Then, pre- and post-multiplying (15) with J and J^{T}, respectively, we obtain that (15) is equivalent to

$$\begin{bmatrix} \Lambda_{11} & \Xi_{12} & S & \Xi_{14} & \hat{\Xi}_{15} & \hat{\Xi}_{16} \\ * & \Xi_{22} & \Xi_{23} & 0 & \hat{\Xi}_{25} & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 \\ * & * & * & -\lambda I & \tau P & 0 \\ * & * & * & * & -PZ^{-1}P & 0 \\ * & * & * & * & * & -PZ^{-1}P \end{bmatrix} + \begin{bmatrix} \alpha PH \\ 0 \\ 0 \\ 0 \\ \tau\alpha PH \\ \tau\sqrt{\alpha(1-\alpha)}PH \end{bmatrix} \Delta(t)\begin{bmatrix} E & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} E^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta(t) \times \begin{bmatrix} \alpha H^{\mathrm{T}}P^{\mathrm{T}} & 0 & 0 & 0 & \tau\alpha H^{\mathrm{T}}P^{\mathrm{T}} & \tau\sqrt{\alpha(1-\alpha)}H^{\mathrm{T}}P^{\mathrm{T}} \end{bmatrix} < 0. \tag{32}$$

Noting $Z > 0$, we have $-PZ^{-1}P \leqslant -2P + Z$. Thus, it is clear that if (31) holds, then (32) holds, which implies that (15) holds. This completes the proof. □

Remark 1: In Theorem 3.3, the non-fragile controllers are designed such that CDN (1) is exponentially mean-square synchronised. It is noted that the obtained condition is formulated by LMI which can be checked easily. To the best of our knowledge, it is the first time to deal with the non-fragile synchronisation control for CNDs with time-varying coupling delay and missing data.

4. Numerical example

In this section, we illustrate the effectiveness of the methods proposed in this article via an example, which is borrowed from Li et al. (2011). Consider CND (1) with three nodes. The outer-coupling matrix is assumed to be $G=(G_{ij})_{N\times N}$ with

$$G=\begin{bmatrix}-1 & 0 & 1\\ 0 & -1 & 1\\ 1 & 1 & -2\end{bmatrix}.$$

The time-varying delay is chosen as $\tau(t)=0.2+0.05\sin(10t)$. A straightforward calculation gives $\tau=0.25$ and $\mu=0.5$. The nonlinear function f is taken as

$$f(x_i(t))=\begin{bmatrix}-0.5x_{i1}+\tanh(0.2x_{i1})+0.2x_{i2}\\ 0.95x_{i2}-\tanh(0.75x_{i2})\end{bmatrix}.$$

It is easy to verify that the above nonlinear function f satisfies the sector-bounded condition (4) with

$$U=\begin{bmatrix}-0.5 & 0.2\\ 0 & 0.95\end{bmatrix},\quad V=\begin{bmatrix}-0.3 & 0.2\\ 0 & 0.2\end{bmatrix}.$$

The inner-coupling matrix is set as

$$A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

and

$$H_1=H_2=H_3=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\quad E_1=E_2=E_3=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}.$$

In this example, we choose $\alpha=0.5$. By using the Matlab, we solve LMI (29) and get the following feasible solution:

$$P_1=\begin{bmatrix}13.0142 & -0.2224\\ -0.2224 & 11.0223\end{bmatrix},\quad P_2=\begin{bmatrix}13.0142 & -0.2224\\ -0.2224 & 11.0223\end{bmatrix},\quad P_3=\begin{bmatrix}12.3715 & -0.1988\\ -0.1988 & 10.5999\end{bmatrix}$$

$$X_1=\begin{bmatrix}-26.5095 & -3.3592\\ -0.5756 & -45.3826\end{bmatrix},\quad X_2=\begin{bmatrix}-26.5095 & -3.3592\\ -0.5756 & -45.3826\end{bmatrix},\quad X_3=\begin{bmatrix}-26.9928 & -3.1111\\ -0.4548 & -44.3870\end{bmatrix}$$

$$Q_1=\begin{bmatrix}10.1217 & -0.2482 & -0.3126 & 0.0086 & -0.1983 & -0.0016\\ -0.2482 & 7.7729 & 0.0086 & -0.1965 & -0.0020 & -0.2136\\ -0.3126 & 0.0086 & 10.1217 & -0.2482 & -0.1983 & -0.0016\\ 0.0086 & -0.1965 & -0.2482 & 7.7729 & -0.0020 & -0.2136\\ -0.1983 & -0.0020 & -0.1983 & -0.0020 & 10.0788 & -0.2559\\ -0.0016 & -0.2136 & -0.0016 & -0.2136 & -0.2559 & 7.6567\end{bmatrix}$$

$$Q_2=\begin{bmatrix}10.1307 & -0.2945 & -0.0397 & 0.0018 & -0.3642 & 0.0100\\ -0.2945 & 7.2801 & 0.0018 & -0.0089 & 0.0109 & -0.3184\\ -0.0397 & 0.0018 & 10.1307 & -0.2945 & -0.3642 & 0.0100\\ 0.0018 & -0.0089 & -0.2945 & 7.2801 & 0.0109 & -0.3184\\ -0.3642 & 0.0109 & -0.3642 & 0.0109 & 10.6499 & -0.3285\\ 0.0100 & -0.3184 & 0.0100 & -0.3184 & -0.3285 & 7.5098\end{bmatrix}$$

$$Z=\begin{bmatrix}9.8046 & -0.1546 & 0.1776 & -0.0058 & -0.6736 & 0.0140\\ -0.1546 & 8.6869 & -0.0058 & 0.0988 & 0.0136 & -0.4695\\ 0.1776 & -0.0058 & 9.8046 & -0.1546 & -0.6736 & 0.0140\\ -0.0058 & 0.0988 & -0.1546 & 8.6869 & 0.0136 & -0.4695\\ -0.6736 & 0.0136 & -0.6736 & 0.0136 & 10.1544 & -0.1526\\ 0.0140 & -0.4695 & 0.0140 & -0.4695 & -0.1526 & 8.9728\end{bmatrix}$$

$$S=\begin{bmatrix}2.3151 & -0.0659 & -0.0744 & 0.0018 & 0.6237 & -0.0124\\ -0.0835 & 1.7482 & 0.0023 & -0.0712 & 0.0021 & 0.6012\\ -0.0744 & 0.0018 & 2.3151 & -0.0659 & 0.6237 & -0.0124\\ 0.0023 & -0.0712 & -0.0835 & 1.7482 & 0.0021 & 0.6012\\ 0.5644 & -0.0102 & 0.5644 & -0.0102 & 1.4451 & -0.0425\\ 0.0041 & 0.5614 & 0.0041 & 0.5614 & -0.0725 & 1.0118\end{bmatrix}$$

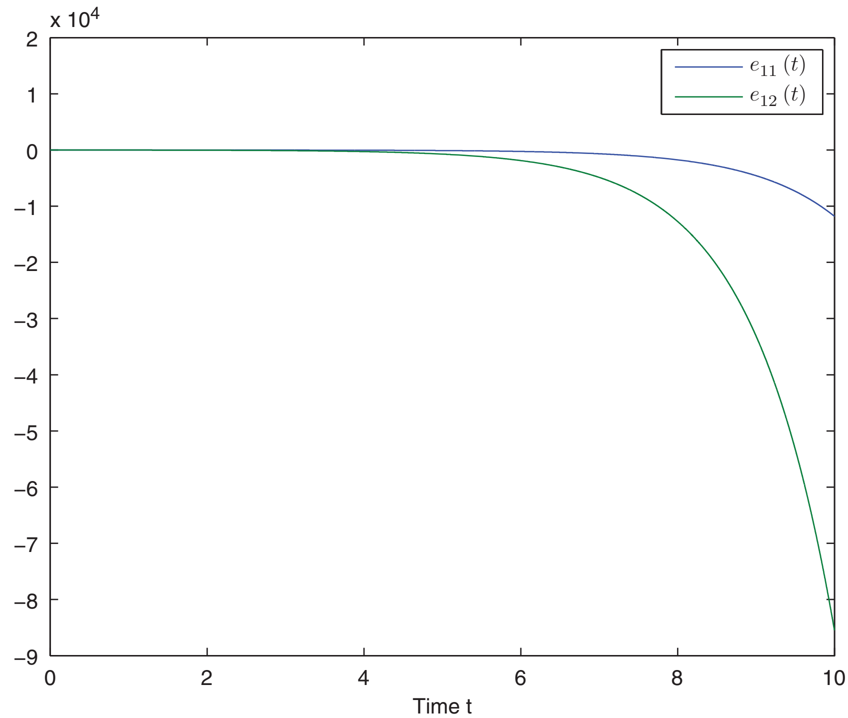

Figure 1. State trajectory $e_1(t)$ of the error system (11) without controllers.

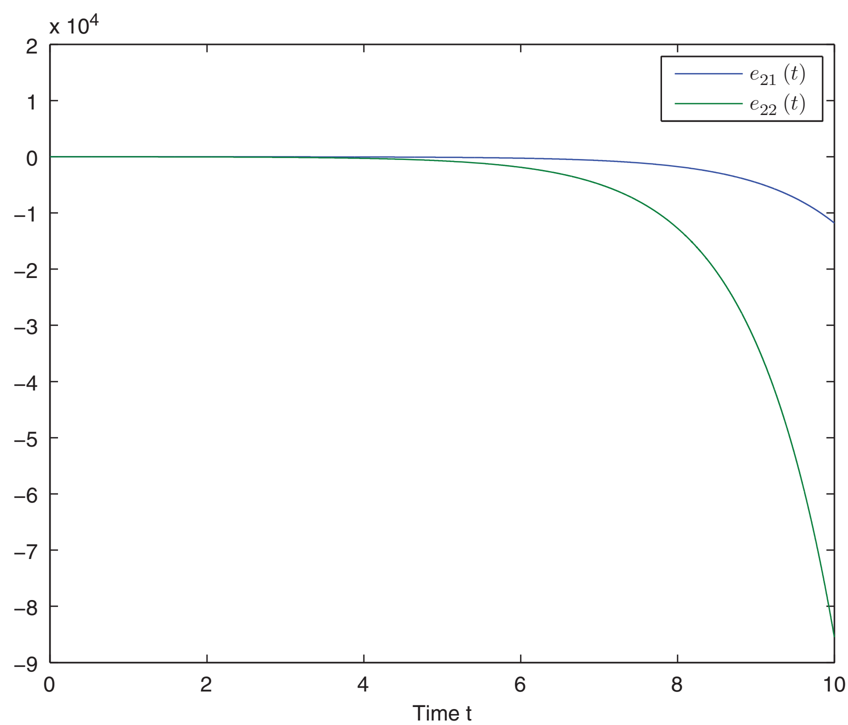

Figure 2. State trajectory $e_2(t)$ of the error system (11) without controllers.

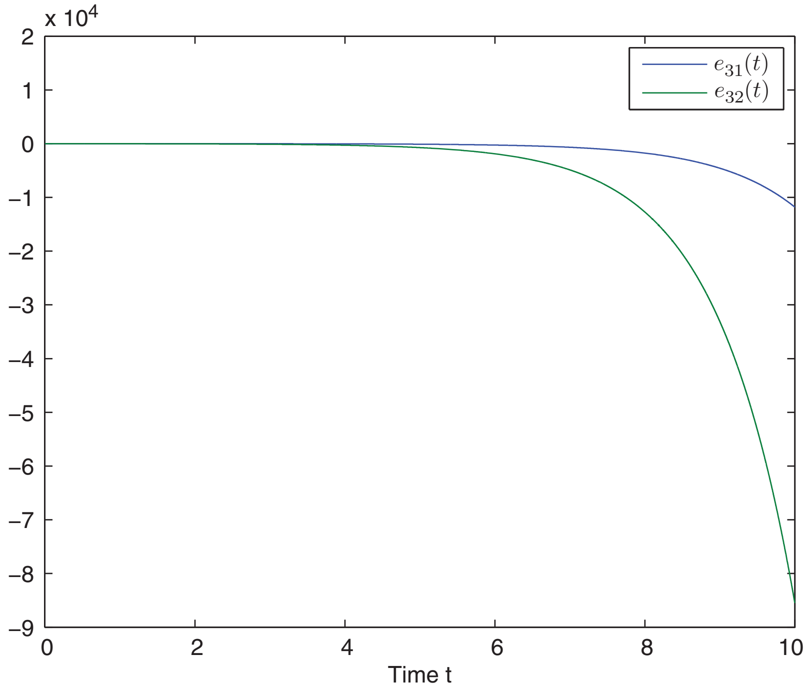

Figure 3. State trajectory $e_3(t)$ of the error system (11) without controllers.

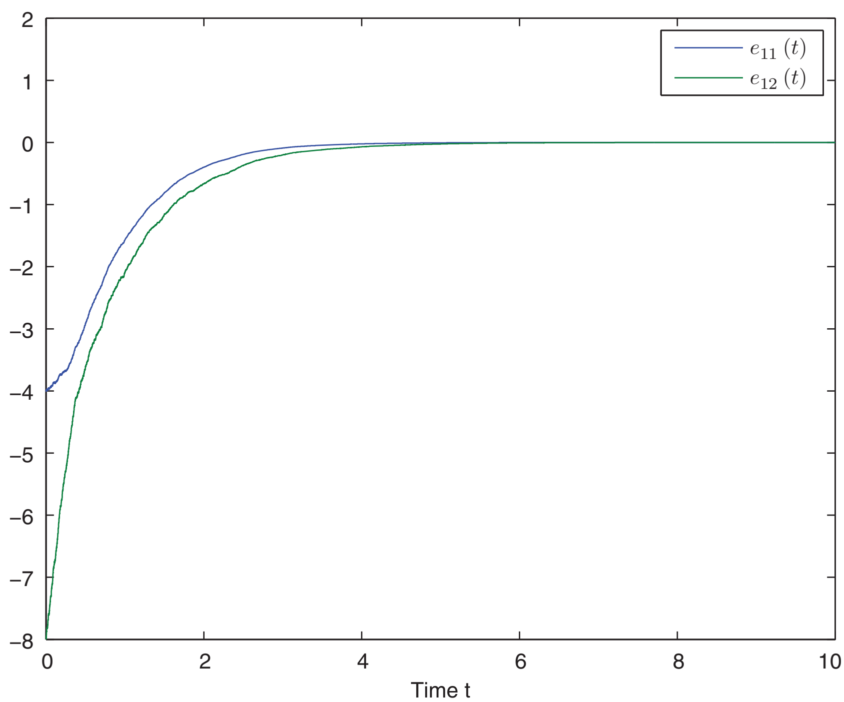

Figure 4. State trajectory $e_1(t)$ of the error system (11) with designed controllers.

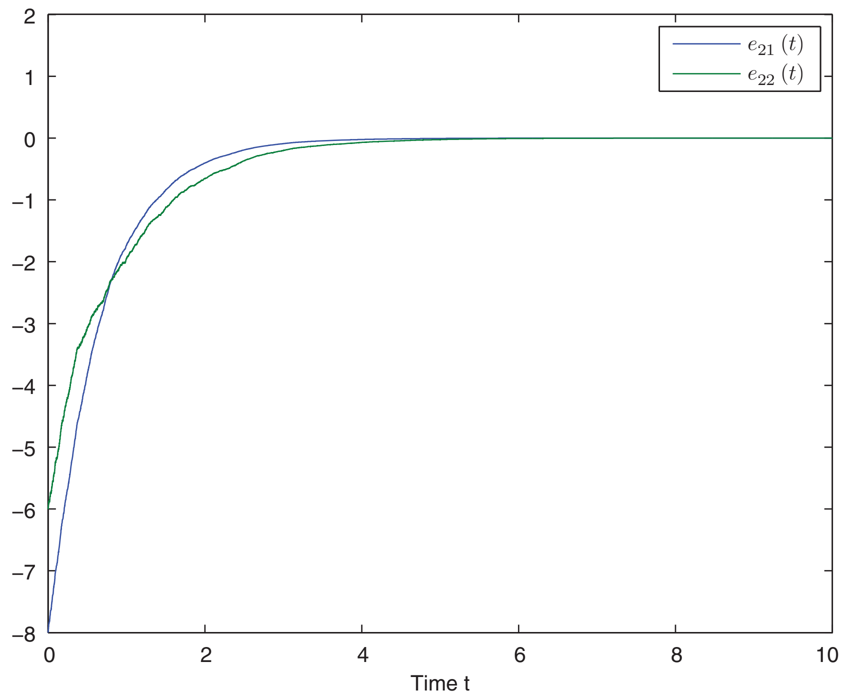

Figure 5. State trajectory $e_2(t)$ of the error system (11) with designed controllers.

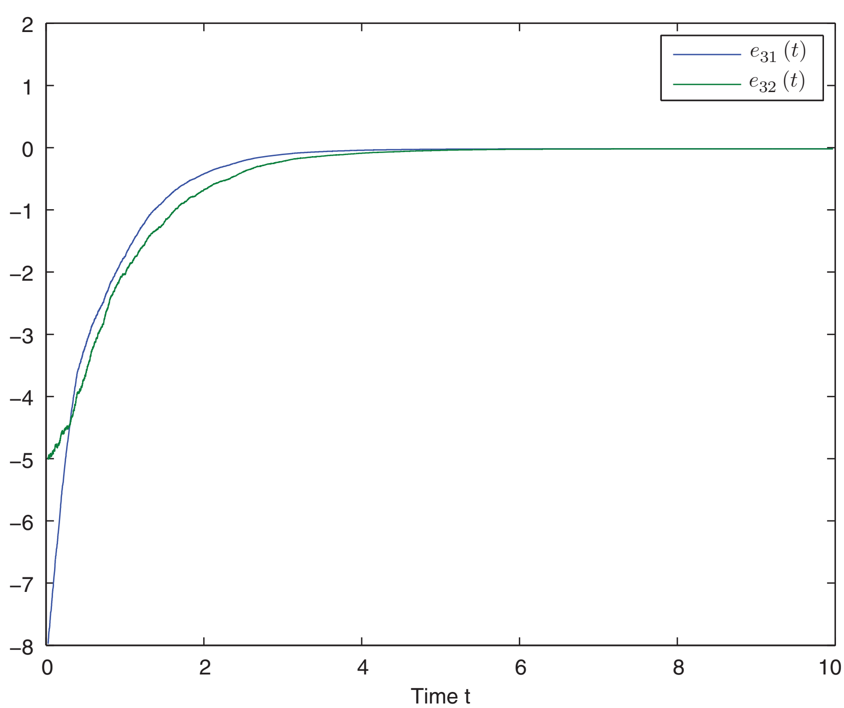

Figure 6. State trajectory $e_3(t)$ of the error system (11) with designed controllers.

and $\lambda = 17.5719$, $\varepsilon = 15.3160$. Then we can obtain the gain matrices of the desired controllers as follows:

$$K_1 = \begin{bmatrix} -2.0386 & -0.3286 \\ -0.0933 & -4.1240 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -2.0386 & -0.3286 \\ -0.0933 & -4.1240 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -2.1832 & -0.3188 \\ -0.0838 & -4.1935 \end{bmatrix}.$$

The state trajectories of the error system (11) without controllers and with the above controllers are given in Figures 1–3 and Figures 4–6, respectively, where $x_1(0) = [2\ \ 1]^{\mathrm{T}}$, $x_2(0) = [-2\ \ 3]^{\mathrm{T}}$, $x_3(0) = [-2\ \ 4]^{\mathrm{T}}$, $s(0) = [6\ \ 9]^{\mathrm{T}}$. It is clear that the error system (11) without controllers is unstable, and the state trajectories of the error system (11) with designed controllers converge to zero, that is, thus the CDN (1) with designed controllers is synchronised.

5. Conclusions

In this article, the non-fragile synchronisation control problem has been considered for complex networks with time-varying coupling delay and missing data. The missing data has been modelled by a stochastic variable satisfying the Bernoulli random binary distribution. An exponential stability condition of the error system has been proposed based on Gronwall's inequality. Based on the condition, a set of non-fragile synchronisation controllers has been designed. A simulation result has demonstrated the successful application of the proposed design methods.

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