Non-Gaussianity as a Probe of the Physics of the Primordial Universe and the Astrophysics of the Low Redshift Universe

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EXECUTIVE SUMMARY

A new and powerful probe of the origin and evolution of structures in the Universe has emerged and been actively developed over the last decade. In the coming decade, *non-Gaussianity*, i.e., the study of non-Gaussian contributions to the correlations of cosmological fluctuations, will become an important probe of both the early and the late Universe. Specifically, it will play a leading role in furthering our understanding of two fundamental aspects of cosmology and astrophysics:

- The physics of the very early universe that created the primordial seeds for large-scale structures, and
- The subsequent growth of structures via gravitational instability and gas physics at later times.

To date, observations of fluctuations in the Cosmic Microwave Background (CMB) and the Large-Scale Structure of the Universe (LSS) have focused largely on the Gaussian contribution as measured by the two-point correlations (or the power spectrum) of density fluctuations. However, an even greater amount of information is contained in non-Gaussianity and a large discovery space therefore still remains to be explored. Many observational probes can be used to measure non-Gaussianity, including CMB, LSS, gravitational lensing, Lyman-α forest, 21-cm fluctuations, and the abundance of rare objects such as clusters of galaxies and high-redshift galaxies. *Not only does the study of non-Gaussianity maximize the science return from a plethora of present and future cosmological experiments and observations, but it also carries great potential for important discoveries in the coming decade.*

I. BEYOND A SIMPLE APPROXIMATION TO NATURE

The last decade has witnessed tremendous advances in our understanding of the Universe. The measurements of the anisotropies in the CMB temperature and polarization fluctuations by the Cosmic Background Explorer (COBE), the Wilkinson Microwave Anisotropy Probe (WMAP), and many ground-based and sub-orbital experiments, and of the distribution of galaxies by the CfA Redshift Survey, Two-degree Field Galaxy Redshift Survey (2dFGRS), and the Sloan Digital Sky Survey (SDSS), among others, were milestones in modern cosmology. The two-point correlation function (or its Fourier transform, the power spectrum) of temperature and polarization anisotropies, as well as that of the galaxy distribution, have sharpened our view of the Universe (e.g., [1]) - we now know that the Universe is 13.7 ± 0.1 Gyr old, and made of $4.6 \pm 0.2\%$ hydrogen and helium nuclei, $22.8 \pm 1.3\%$ dark matter, and the rest, $72.6 \pm 1.5\%$, is in the form of dark energy. The spatial geometry of the observable Friedmann-Robertson-Walker Universe is spatially flat (Euclidean) to about 1%.

Explaining the CMB and LSS power spectra requires only a handful of numbers: today's expansion rate, the energy density of atoms, dark matter and dark energy, the optical depth resulting from hydrogen reionization, and the amplitude and scale-dependence of the primordial seed fluctuations. However, knowing the values of these parameters does not provide us with a complete understanding of the physical laws governing the Universe: knowing the abundance of dark matter and dark energy does not tell us what they are.

Knowing the amplitude and scale-dependence of the primordial fluctuations does not tell us what created those primordial fluctuations. It is clear that we need more information.

In fact, we *do* have more information: the WMAP temperature map contains 10^6 pixels, and there are 10^5 spectra of galaxies surveyed by SDSS. Yet, cosmologists spent the last decade measuring and interpreting the two-point correlations, which contain only ~ 1000 and 100 numbers for WMAP and SDSS, respectively.

This kind of compression of the data is justified if, and *only if*, the statistical distribution of the observed fluctuations is a Gaussian distribution with random phases. Any information contained in the departure from a perfect Gaussian, *non-Gaussianity*, is not encoded in the power spectrum, but has to be extracted from measurements of higher-order correlation functions. The study and characterization of non-Gaussianity began three decades ago with the first large scale structure surveys, but the main focus until recently has been on two-point correlations and Gaussian fluctuations.

In this White Paper we describe how non-Gaussianity is a particularly potent probe of the fundamental origin and the late time evolution of structures.

II. NON-GAUSSIANITY AS A PROBE OF THE PHYSICS OF THE PRIMORDIAL UNIVERSE

Over the last decade we have accumulated a good deal of observational evidence from CMB and LSS power spectra that the observed structures originated from seed fluctuations in the very early universe. The leading theory explaining the primordial origin of cosmological fluctuations is cosmic inflation [2], a period of accelerated expansion at very early times. During inflation, microscopic quantum fluctuations were stretched to macroscopic scales to provide the seed fluctuations for the formation of large-scale structures like our own Galaxy.

What was the physics responsible for inflation? Many theoretical ideas have been proposed to explain the existence of an early phase of accelerated expansion. Inflation models with the minimum number of degrees of freedom, parameters and tuning needed to solve the flatness and homogeneity problems give a fairly well-defined range of predictions. While the current experimental data has ruled out a good fraction of that range, there remains a substantial range that still fits the data [1].

Learning about the physics of inflation is equivalent to learning about the evolution and interactions of quantum fields in the very early Universe. Measurements of the power spectrum alone have limited potential in revealing this information. The power spectrum is determined by the inflationary expansion rate and its time-dependence which in turn relates to the evolution of the inflationary energy density. However, the power spectrum does not strongly constrain the interactions of the field (or fields) associated with this energy density. The power spectrum is therefore degenerate in terms of the inflationary action that can lead to it - the power spectrum is therefore degenerate in terms of the inflationary action that can lead to it - inflation models with different field interactions can lead to very similar predictions for the power spectrum. Non-Gaussianity is a sensitive probe of the aspects of inflation that are difficult to probe by other means. Specifically, it is a probe of the interactions of the field(s) driving inflation and therefore contains vital information about the fundamental physics operative during inflation.

In many single field slow-roll models the non-Gaussianity is small and likely unobservable

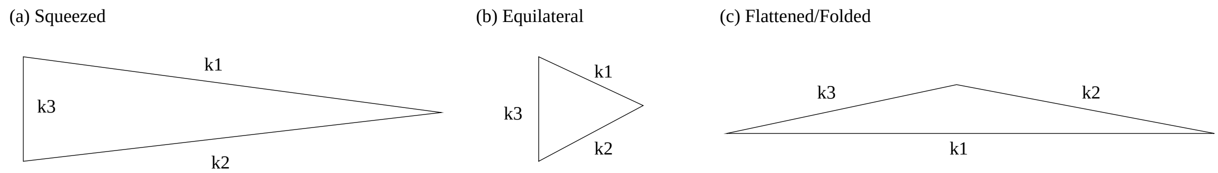

FIG. 1: Bispectrum shapes, $B(k_1, k_2, k_3)$, which can be characterized by triangles formed by three wave vectors. The shape (a) has the maximum signal at the squeezed configuration, $k_3 \ll k_2 \approx k_1$, and can be produced by models of inflation involving multiple fields. The shape (b) has the maximum signal at the equilateral configuration, $k_1 = k_2 = k_3$, and can be produced by non-canonical kinetic terms of quantum fields. The shape (c) has the maximum signal at the flattened configuration, $k_3 \approx k_2 \approx 2k_1$, and can be produced by non-vacuum initial conditions.

by virtue of the inflaton field being weakly coupled. However, a large, detectable amount of non-Gaussianity can be produced when any of the following conditions is violated:

- **Single Field**. There was only one quantum field responsible for driving inflation and for generating the primordial seeds for structures.
- **Canonical Kinetic Energy**. The kinetic energy of the quantum field is such that the speed of propagation of fluctuations is equal to the speed of light.
- **Slow Roll**. The evolution of the field was always very slow compared to the Hubble time during inflation.
- **Initial Vacuum State**. The quantum field was in the preferred adiabatic vacuum state (also sometimes called the "Bunch-Davies vacuum") just before the quantum fluctuations were generated during inflation.

Inflation is expected to produce undetectable levels of primordial non-Gaussianity, only when *all* of the above conditions are satisfied (see, e.g., [3], for a review) - the conditions that inflation models have the minimum number of degrees of freedom, parameters and tuning needed to solve the flatness and homogeneity problem. *Confirming or ruling out this class of inflation models is an important goal.*

Non-Gaussianity is measured by various methods. A standard approach is to measure non-Gaussian correlations, i.e., the correlations that vanish for a Gaussian distribution. The three-point function (or its Fourier transform, the bispectrum) is such a correlation.

The three-point function correlates density or temperature fluctuations at three points in space. Equivalently, the bispectrum, $B(k_1, k_2, k_3)$, correlates fluctuations with three wave vectors (see Figure 1). These three wave vectors form a triangle in Fourier space, and thus there are many triangles one can form and look for. The amount of information captured by the bispectrum is therefore potentially far greater than that of the power spectrum, which correlates only two wave vectors with the same magnitude.

An important theoretical discovery made toward the end of the last decade is that violation of each of the above conditions (single field, canonical kinetic energy, slow roll, and initial vacuum state) results in unique signals with specific triangular shapes: multi-field models, non-canonical kinetic term models, non-adiabatic-vacuum models (e.g., initially excited states), and non-slow-roll models can generate signals in squeezed triangles ($k_3 \ll k_2 \approx k_1$), equilateral triangles ($k_1 = k_2 = k_3$), flattened/folded triangles ($k_3 \approx k_2 \approx 2k_1$), and more complex configurations, respectively (see, e.g., [4–9]). When more than one of the conditions are violated, a linear combination of different shapes would arise [10].

The squeezed configuration in Fourier space is equivalent to the primordial curvature perturbation in position space, $\Phi(\mathbf{x})$ (up to a sign this is the usual Newtonian potential), given by $\Phi(\mathbf{x}) = \phi_g(\mathbf{x}) + f_{NL}\phi_g^2(\mathbf{x})$, where $\phi_g(\mathbf{x})$ is a Gaussian field. This form of non-linearity was first recognized by Ref. [11] within the context of inflation, and the parameter f_{NL} characterizes the amount of non-Gaussianity in this particular configuration. The latest constraint on f_{NL} from the WMAP 5-year data is $f_{NL} = 38 \pm 21$ (68% CL; [12]). While the statistical significance of the signal is still low (about 2-σ level), future experiments such as the Planck CMB satellite and high-redshift galaxy surveys are expected to yield much tighter constraints [13, 14], and might well lead to a convincing detection.

A new method for measuring f_{NL} from galaxy surveys that does not rely on the bispectrum, but uses the fact that the power spectrum of density extrema (where galaxies are formed) on large scales increases (decreases) for a positive (negative) f_{NL} [15] (also see [16] for a generalized result) is particularly promising. More specifically, f_{NL} introduces a scale-dependent modification of the galaxy power spectrum, which increases as $\sim 1/k^2$ as one goes to smaller k (larger spatial scales), and evolves roughly as $(1 + z)$ as a function of redshift. This method yields a competitive limit already from SDSS [17], and there is a realistic chance that one can reach sensitivity at the level of $\Delta f_{NL} \lesssim 1$, e.g., [18–20]. Note that the signature of non-Gaussianity is a smooth feature; thus, wide-field photometric surveys are well suited to study this effect.

These findings suggest that non-Gaussian correlations are a very powerful probe of the physics of inflation. Understanding non-Gaussianity does for inflation what direct detection experiments do for dark matter, or the Large Hadron Collider for the Higgs particles. It probes the interactions of the field sourcing inflation, revealing the fundamental aspects of the physics at very high energies that are not accessible to any collider experiments. For this reason, non-Gaussianity has been a key player in the recent surge in a very productive exchange of ideas between cosmologists and high-energy theorists, and we have every reason to expect that this will continue in a bigger form in the coming decade.

Moreover, recent studies suggest that potential alternatives to inflation scenarios, such as an early contracting phase of the Universe followed by a bounce (rather than expanding), tend to generate large non-Gaussianity. Null detection of non-Gaussianity at the level of $\Delta f_{NL} \lesssim 1$ would rule out all of the alternative models based on a contracting phase currently proposed and reviewed in [21].

While detection of large non-Gaussianity would not rule out inflation, it would rule out the class of models satisfying all of the above conditions simultaneously (single field, canonical kinetic energy, slow roll, and initial vacuum state). The most important aspect of primordial non-Gaussianity is that a convincing detection of the squeezed configuration, f_{NL}, will *rule out all classes of inflationary models based upon a single field* [22]. The shape of the two-point correlation function (characterized by the so-called primordial tilt, n_s, and the running index, α_s) and the existence or absence of primordial gravitational waves, would provide important constraints on large classes of inflationary models, but they would never be able to rule out single-field inflation.

To summarize, non-Gaussian correlations offer a new window into the details of the fundamental physics of the primordial Universe that are not accessible by Gaussian correlations.

III. NON-GAUSSIANITY AS A PROBE OF THE ASTROPHYSICS OF THE LOW-REDSHIFT UNIVERSE

Gaussian fluctuations become non-Gaussian as cosmic structures evolve and go through various non-linear processes. *This property makes non-Gaussianity a sensitive probe of the evolution of cosmic structures and numerous non-linear astrophysical processes of the low-redshift Universe.*

Non-Gaussianity can be used to extract additional information about the gravitational lensing effect [23], the Sunyaev–Zel'dovich effect [24], the cosmic reionization epoch [25], and the Integrated Sachs–Wolfe effect [26, 27], which can be used to constrain the equation of state parameter of dark energy [28].

Non-Gaussianity is a sensitive probe of small non-linear effects that must have existed at the photon decoupling epoch, $z \simeq 1090$, via non-linear general relativistic effects [29], the non-linear evolution of the photon-baryon fluid [30], non-linear perturbations of the electron density at recombination [31, 32], and non-linearities in the radiative transfer such as the non-linear Sachs–Wolfe effect and weak lensing [33].

Non-Gaussianity also offers powerful diagnostics of galaxy formation measuring how galaxies trace the underlying mass distribution (see [34] for a review), as well as of the physics of the Inter Galactic Medium (IGM) measuring how gas traces the underlying mass distribution [35]. New tracers of the underlying mass distribution, the cosmological 21-cm fluctuations (see [36] for a review), will contain far more information in its higher order correlation functions than in the two-point correlations. Further theoretical studies are needed to exploit the rich information available in the 21-cm fluctuations.

IV. HOW TO EXPLOIT NON-GAUSSIANITY IN THE COMING DECADE

The tremendous power of non-Gaussianity for constraining the physics of the primordial Universe and the astrophysics of the low redshift Universe has begun to be fully appreciated toward the end of the last decade. What do we expect over the next decade?

The theoretical discovery that different triangle configurations of the bispectrum are sensitive to different aspects of the physics of inflation was a major achievement of the last decade. So far three distinct configurations (see Figure 1) have been investigated, but it is entirely possible that new physics may be probed by different configurations. Moreover, there is no reason to stop at the three-point function. Recent studies suggest that the four-point function (or its Fourier transform, the trispectrum) gives us additional information about inflation models [37] and potential alternatives [38], beyond what is possible with the three-point function, and the Planck CMB satellite is expected to yield useful limits [39]. Studies of what is possible beyond the three-point function have just begun. More theoretical studies are necessary to fully exploit the potential of non-Gaussianity.

Low redshift non-linear astrophysical phenomena are very rich and important subjects by themselves; however, they may mask the primordial non-Gaussian signatures. While several studies have suggested that the squeezed configuration, f_{NL}, is relatively insensitive to low redshift phenomena, e.g., [40], more studies are required to develop a secure method to extract the primordial non-Gaussianity. The low redshift contamination of the other triangle configurations, as well as to the four-point function, is yet to be studied.

The Galactic emission is non-Gaussian, and its effect must be understood and subtracted. Studies of the WMAP data [1] have shown that the Galactic contamination of f_{NL} is not very large; however, at the level of sensitivity that the Planck satellite is expected to reach, $\Delta f_{NL} \sim 5$, foregrounds would play an important role. Again, the foreground contamination of the other configurations and the four-point function is yet to be studied. The method based upon the galaxy power spectrum [15] is still quite new, and we need more investigations of the systematic errors in this method to fully explot its potential of reaching $\Delta f_{NL} \lesssim 1$.

What kind of observations are needed for measurements of non-Gaussianity? A sensible approach seems to measure non-Gaussianity with a combination of many complementary observables including CMB, LSS, gravitational lensing, Lyman-α forest, 21-cm fluctuations, and the abundance of clusters of galaxies and high-redshift galaxies. Examples of on-going/funded missions include the Planck satellite (CMB), the South Pole Telescope (SPT; CMB, clusters), the Atacama Cosmology Telescope (ACT; CMB, clusters), the SDSS-III (LSS, Lyα forest), the Hobby-Eberly Telescope Dark Energy Experiment (HETDEX; LSS), the Dark Energy Survey (DES; LSS, clusters), the Panoramic Survey Telescope & Rapid Response System (Pan-STARRS; LSS, lensing), and the Extended Röntgen Survey with an Imaging Telescope Array (eROSITA; LSS, clusters). Proposed future missions include the Joint Dark Energy Mission (JDEM; LSS, lensing), the Square Kilometer Array (SKA; LSS, 21-cm), the Large Synoptic Survey Telescope (LSST; LSS, lensing), the Cosmic Inflation Probe (CIP; LSS), and a CMB Polarization Satellite (CMBPol; CMB).

The variety of observations listed above are complementary in very important ways: they probe different spatial scales (CMB probes the largest spatial scales, whereas LSS, Lyα, lensing, 21-cm, and clusters probe small spatial scales that are not accessible with CMB) and fold in a variety of post-inflationary physics. Such a range of observations expands the window opened by the CMB and may uncover an unexpected interplay between cosmological phenomena in the dynamics of evolving structures.

It is overwhelmingly clear that theoretical advances in our understanding of sources of observable non-Gaussianity will enable us to extract much more information, and maximize the science return from a plethora of experiments.

V. CONCLUSION

Non-Gaussianity offers a powerful probe of the physics of the primordial Universe and the non-linear astrophysical processes in the low redshift Universe. Over the last decade we have come to realize the tremendous discovery potential of non-Gaussianity. *Just about every on-going/funded/proposed cosmological observation can be used effectively to measure non-Gaussianity, and possibly revolutionize our understanding of the Universe in the past and present.*

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