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Non-hypersingular boundary integral equations for 3-D non-planar crack dynamics

T. Tada, E. Fukuyama, R. Madariaga

Abstract We derive a non-hypersingular boundary integral equation, in a fully explicit form, for the time-domain analysis of the dynamics of a 3-D non-planar crack, located in an infinite homogeneous isotropic medium. The hypersingularities, existent in the more straightforward expression, are removed by way of a technique of regularization based on integration by parts. The variables are denoted in terms of a local Cartesian coordinate system, one of the axes of which is always held locally perpendicular to the potentially curved surface of the crack. Also given, in a fully explicit form, are the expressions for the off-fault stress and displacement field, as well as the special form of the equations for the case in which the fault is planar.

1 Introduction

In seismology, numerical modeling of the dynamics of rupture propagation on faults with a realistic geometry is essential in the efforts to better understand the complex nature of earthquake phenomena. However, numerical analysis of rupture on non-planar faults has faced many technical difficulties. In the present paper, we present a new theoretical framework, based on the boundary integral equation method (BIEM), to describe the dynamics of 3-D cracks with arbitrary geometry, which is expected to provide an important basis for future advances in the practical numerical modeling of non-planar cracks.

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The BIEM has been extensively applied to various classes of 2-D and 3-D crack-analysis problems (e.g. Beskos 1997). According to the displacement discontinuity method, which has produced the most successful results to date, the stress field over the model space is expressed as a convolution, in time and space, of the slip along the crack and a set of integration kernels. Then a limiting process is so applied that the receiver point approaches the crack face, producing a set of boundary integral equations (BIEs) that relate the traction on the crack surface to the slip on it.

The traction BIEs, thus derived, are hypersingular, and are not immediately amenable to numerical implementation. One of the most popular and successful methods to circumvent the hypersingularities is the approach of regularization. This consists in rewriting, most often through integration by parts, the hypersingular integrals in an equivalent form which involves only weakly singular integrals, at most integrable in the sense of Cauchy principal values.

The integration by parts technique allowed Sládek and Sládek (1984) and Nishimura and Kobayashi (1989) to derive a non-hypersigular BIE for 3-D non-planar cracks, in the Laplace domain and the frequency domain, respectively. These formulations, however, could deal only with the transient response of stationary cracks. Later, an alternative technique, based on path-independent conservation integrals, was used by Zhang and Achenbach (1989) and Zhang (1991) to derive regularized elastodynamic BIEs for 3-D non-planar cracks, in the frequency domain and the time domain respectively. However, it was not evident how their BIEs could be implemented to non-planar crack problems, because they were written in a global Cartesian coordinate system, which does not necessarily agree with the curved surface of the crack, on which slip is defined.

Because of this disadvantage, these earlier BIEs for the 3-D non-planar crack have been confined to conceptual expressions, and they have been reduced to a numerically implementable form only for the special case of the planar crack. Numerical study of the dynamics of the 3-D planar crack has been conducted by Zhang and Gross (1993), who used Zhang's (1991) equations, as well as in an independent work by Fukuyama and Madariaga (1995, 1998), who used the integration by parts technique. Meanwhile, there have been alternative formulations of the elastodynamic BIEs for the time-domain analysis of the 3-D planar crack. Das' (1980) pioneering formulation was based on the traction method, in which the displacement field was expressed as a convolution, in space and time, of the traction on the crack and a set of integration kernels. Geubelle and Rice (1995) followed the displacement discontinuity method, but they formulated their BIEs based on a Fourier transform in space.

A recent study by Krysl and Belytschko (1999) illustrates an original method for a time-domain elastodynamic analysis of a 3-D crack of arbitrary geometry, based on the Element-Free Galerkin method which obviates the need for re-meshing.

In the present paper, we enlarge Fukuyama and Madariaga's (1998) integration by parts technique to derive, for the first time, a non-hypersingular time-domain BIE for the 3-D non-planar crack in a fully explicit form. In order to represent the BIE in terms of the one opening and two shear components of the slip on the crack surface, we introduce a local Cartesian coordinate system first used by Tada and Yamashita (1997). The present article comprises: (1) the BIE describing the traction-slip relation for the 3-D non-planar crack; (2) a similar expression for the off-fault stress field, and its special form for the planar crack; and (3) a similar expression for the off-fault displacement field, and its special form for the planar crack.

2 Representation theorem

We start from the representation theorem of elasticity that expresses the elastic displacement field over the entire medium in terms of the slip distribution along the crack. Assuming that the medium is at rest with no slip for time $t \leq 0$, and that the traction is continuous across the crack, we have, for a crack located in an infinite homogeneous isotropic elastic medium,

$$u_k(\mathbf{x},t) = -\int_\Gamma \mathrm{d}S(\xi) \int_0^t \mathrm{d}\tau \Delta u_i(\xi,\tau) c_{ijpq} n_j(\xi) \times \frac{\partial}{\partial x_q} G_{kp}(\mathbf{x}, t-\tau; \xi, 0) \ , \quad (1)$$

where $u_k(\mathbf{x},t)$ is the displacement in the k-th direction at receiver point $\mathbf{x}$ and time t, Γ the crack surface, ξ the source point on Γ, $\Delta u_i(\xi,\tau)$ the slip across the crack in the i-th direction at location ξ and time τ as defined by the relative displacement of one (positive) side of Γ with reference to the other (negative) side, c_{ijpq} the elastic constants, and $\mathbf{n}(\xi)$ the unit vector normal to the crack surface at location ξ pointing from the negative to the positive side of Γ (Fig. 1). $G_{kp}(\mathbf{x}, t-\tau; \xi, 0)$ is the displacement Green's function, representing the displacement in the k-th direction at receiver point $\mathbf{x}$ and time $t-\tau$ due to a unit force in the p-th direction applied at source point ξ and time 0. Summation over repeated indices is implied and $\delta_{\alpha\beta}$ denotes the Kronecker's delta.

Equation (1) can be rewritten as

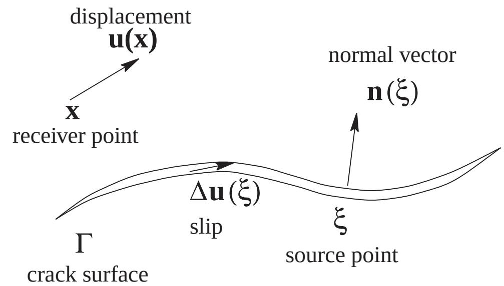

Fig. 1. Nomenclature for symbols

$$u_k(\mathbf{x},t) = -\int_\Gamma \mathrm{d}S(\xi) \int_0^t \mathrm{d}\tau \Delta u_i(\xi,\tau) n_j(\xi) \Sigma_{ij/k}(\mathbf{x}, t-\tau; \xi, 0) \ , \quad (2)$$

where, if the medium is isotropic,

$$\Sigma_{ij/k}(\mathbf{x}, t-\tau; \xi, 0) \equiv c_{ijpq} \frac{\partial}{\partial x_q} G_{pk}(\mathbf{x}, t-\tau; \xi, 0) = \lambda \delta_{ij} \partial_p G_{kp} + \mu(\partial_i G_{jk} + \partial_j G_{ik}) \ , \quad (3)$$

with λ and μ being the Lamé constants, is the stress Green's function, representing the ij-component of the stress at receiver point $\mathbf{x}$ and time $t-\tau$ due to a unit force in the k-th direction applied at source point ξ and time 0. The Green's functions satisfy the equations of motion

$$\frac{\partial}{\partial x_j} \Sigma_{ij/k} = \rho \frac{\partial^2}{\partial t^2} G_{ik} \ , \quad (4)$$

with ρ denoting the density.

In the following, an abbreviated notation

$$\partial_i \equiv \frac{\partial}{\partial x_i}, \quad \Delta u_{i,j} \equiv \frac{\partial}{\partial \xi_j} \Delta u_i(\xi,\tau) \quad (5)$$

will be used for partial derivatives with respect to space, and time derivatives will be denoted by dots, especially by:

$$\ddot{G}_{ij} \equiv \frac{\partial^2}{\partial t^2} G_{ij}(\mathbf{x}, t-\tau; \xi, 0), \quad \Delta \ddot{u}_i \equiv \frac{\partial^2}{\partial \tau^2} \Delta u_i(\xi,\tau) \ . \quad (6)$$

Also, c_L and c_T are the P- and S-wave velocities respectively. Roman subscripts are supposed to run over 1, 2 and 3, while Greek subscripts run over 1 and 2, and summation over repeated indices will be implied wherever necessary. Use is also made of the reciprocity relations

$$\frac{\partial}{\partial \xi_k} G_{ij}(\mathbf{x}, t-\tau; \xi, 0) = -\frac{\partial}{\partial x_k} G_{ij}(\mathbf{x}, t-\tau; \xi, 0) \quad (7)$$

$$\frac{\partial}{\partial \tau} G_{ij}(\mathbf{x}, t-\tau; \xi, 0) = -\frac{\partial}{\partial t} G_{ij}(\mathbf{x}, t-\tau; \xi, 0) \ . \quad (8)$$

3 3-D Green's functions

For simplicity, we use notations

$$r \equiv ||\mathbf{x} - \xi||, \quad \gamma_i \equiv (x_i - \xi_i)/r \quad (9)$$

for the length and orientation cosines of the source-receiver vector, and

$$p^2 \equiv \frac{c_T^2}{c_L^2} = \frac{\mu}{\lambda + 2\mu} \quad . \tag{10}$$

The displacement Green's function G_{ij} is given by

$$G_{ij} = \partial_i \partial_j J + \delta_{ij} G_T \tag{11}$$

or

$$\begin{aligned} G_{11} &= \partial_1^2 J + G_T \qquad G_{23} = \partial_2 \partial_3 J \\ G_{22} &= \partial_2^2 J + G_T \qquad G_{31} = \partial_3 \partial_1 J \quad , \\ G_{33} &= \partial_3^2 J + G_T \qquad G_{12} = \partial_1 \partial_2 J \end{aligned} \tag{12}$$

with

$$J \equiv \frac{1}{4\pi\rho} \left[\frac{1}{r} \int_0^{t-\tau-r/c_L} \mathrm{d}s \cdot s\delta(t - \tau - r/c_L - s) - \frac{1}{r} \int_0^{t-\tau-r/c_T} \mathrm{d}s \cdot s\delta(t - \tau - r/c_T - s) \right] \tag{13}$$

$$G_T \equiv \frac{1}{4\pi\mu r} \delta(t - \tau - r/c_T) \tag{14}$$

Combining the symmetry relation (12) with the equations of motion

$$c_T^{-2} \ddot{G}_T = (\partial_1^2 + \partial_2^2 + \partial_3^2) G_T \tag{19}$$

$$p^2 c_T^{-2} \ddot{G}_L = (\partial_1^2 + \partial_2^2 + \partial_3^2) G_L \quad , \tag{20}$$

we obtain the identities:

$$\partial_3^2 G_{33} = p^2 c_T^{-2} \ddot{G}_L - (\partial_1^2 + \partial_2^2)(G_{33} - G_T + G_L) \tag{21}$$

$$\partial_3^2 G_{11} = c_T^{-2} \ddot{G}_T + \partial_1^2 (G_{33} - G_T) - (\partial_1^2 + \partial_2^2) G_T \tag{22}$$

$$\partial_3^2 G_{22} = c_T^{-2} \ddot{G}_T + \partial_2^2 (G_{33} - G_T) - (\partial_1^2 + \partial_2^2) G_T \quad , \tag{23}$$

which shall be utilized later in order to remove terms containing the derivative ∂_3^2. Although alternative forms of these equations of motion are possible, we choose to use the specific form given by Eq. (21) through Eq. (23), because these render the final BIE in the simplest form.

4 Planar crack

By way of introduction, we start with the simple case of the planar crack. The representation theorem states that:

$$\begin{aligned} u_1(\mathbf{x}, t) &= -\int_\Gamma \mathrm{d}S(\xi) \int_0^t \mathrm{d}\tau (\Delta u_1 \Sigma_{13/1} + \Delta u_2 \Sigma_{23/1} + \Delta u_3 \Sigma_{33/1}) \\ &= -\int_\Gamma \mathrm{d}S(\xi) \int_0^t \mathrm{d}\tau \{ \Delta u_1 (\mu \partial_1 G_{13} + \mu \partial_3 G_{11}) + \Delta u_2 (\mu \partial_2 G_{13} + \mu \partial_3 G_{12}) + \Delta u_3 [\lambda \partial_1 G_{11} + \lambda \partial_2 G_{12} + (\lambda + 2\mu) \partial_3 G_{13}] \} \\ &= -\mu \int_\Gamma \mathrm{d}S(\xi) \int_0^t \mathrm{d}\tau \left[\Delta u_1 \partial_3 (2G_{11} - G_T) + \Delta u_2 \cdot 2\partial_3 G_{12} + \Delta u_3 \partial_1 \left(2G_{33} - 2G_T + \frac{1 - 2p^2}{p^2} G_L \right) \right] \end{aligned} \tag{24}$$

for the elastodynamic case and

$$J \equiv \frac{p^2 - 1}{8\pi\mu} r \tag{15}$$

$$G_T \equiv \frac{1}{4\pi\mu r} \tag{16}$$

for the elastostatic case. Explicit forms of the Green's functions are given in Appendix A. Of particular importance in the present article is the linear combination

$$\begin{aligned} G_L &\equiv G_{11} + G_{22} + G_{33} - 2G_T \\ &= \frac{1}{4\pi\mu r} p^2 \delta(t - \tau - r/c_L) \end{aligned} \tag{17}$$

for the dynamic case and

$$G_L \equiv G_{11} + G_{22} + G_{33} - 2G_T = \frac{1}{4\pi\mu r} p^2 \tag{18}$$

in the static case.

$$\begin{aligned} u_3(\mathbf{x}, t) = &-\int_\Gamma \mathrm{d}S(\xi) \int_0^t \mathrm{d}\tau \{ \Delta u_1 (\mu \partial_1 G_{33} + \mu \partial_3 G_{13}) \\ &+ \Delta u_2 (\mu \partial_2 G_{33} + \mu \partial_3 G_{23}) \\ &+ \Delta u_3 [\lambda \partial_1 G_{13} + \lambda \partial_2 G_{23} + (\lambda + 2\mu) \partial_3 G_{33}] \} \\ = &-\mu \int_\Gamma \mathrm{d}S(\xi) \int_0^t \mathrm{d}\tau \left[(\Delta u_1 \partial_1 + \Delta u_2 \partial_2)(2G_{33} - G_T) \right. \\ &\left. + \Delta u_3 \partial_3 \left(2G_{33} + \frac{1 - 2p^2}{p^2} G_L \right) \right] \quad , \end{aligned} \tag{25}$$

where we made use of the symmetry relation (12). The equation for u_2 is obtained by considering the symmetry between the x_1- and x_2-coordinates. Accordingly,

$$\sigma_{33}(\mathbf{x},t) = \mu\left[\frac{1}{p^2}\partial_3 u_3 + \frac{1-2p^2}{p^2}(\partial_1 u_1 + \partial_2 u_2)\right]$$
$$= -\mu^2 \int_\Gamma \mathrm{d}S(\boldsymbol{\xi}) \int_0^t \mathrm{d}\tau\left[(\Delta u_1\partial_1 + \Delta u_2\partial_2)\cdot 2\partial_3\left(2G_{33} - G_T + \frac{1-2p^2}{p^2}G_L\right)\right.$$
$$\left. - 4\Delta u_3(\partial_1^2 + \partial_2^2)\left(G_{33} - G_T + \frac{1-p^2}{p^2}G_L\right) + \frac{1}{p^2}c_T^{-2}\Delta u_3 \ddot{G}_L\right]$$
$$= -\mu^2 \int_\Gamma \mathrm{d}S(\boldsymbol{\xi}) \int_0^t \mathrm{d}\tau\left[\Delta u_{\alpha,\alpha}\cdot 2\partial_3\left(2G_{33} - G_T + \frac{1-2p^2}{p^2}G_L\right) - \Delta u_{3,\alpha}\cdot 4\partial_\alpha\left(G_{33} - G_T + \frac{1-p^2}{p^2}G_L\right)\right]$$
$$- \mu^2 c_T^{-2} \int_\Gamma \mathrm{d}S(\boldsymbol{\xi}) \int_0^t \mathrm{d}\tau \Delta\ddot{u}_3 \frac{1}{p^2}G_L \ , \tag{26}$$

where Eqs. (12), (20) and (21) have been used and integration by parts has been carried out, so as to prevent hypersingular integrals from appearing in a later BIE. With the limiting process $\mathbf{x} \to \mathbf{s} \in \Gamma$, σ_{33} approaches the normal traction T_3 across the crack at location $\mathbf{s}$ and time t:

$$T_3(\mathbf{s},t) = \sigma_{33}(\mathbf{s},t) \ . \tag{27}$$

With the 3-D Green's functions behaving as $1/r$ in the limit of $r \to 0$, the integrals that involve the first-order spatial derivatives of G_{33}, G_T and G_L are of the order of $1/r^2$ as $r \to 0$ and thus are integrable in the sense of Cauchy principal values.

Likewise, making use of Eqs. (12), (19) and (22) and integrating by parts,

$$T_\alpha(\mathbf{s},t) = -\mu^2 \int_\Gamma \mathrm{d}S(\boldsymbol{\xi}) \int_0^t \mathrm{d}\tau\left[\Delta u_{\beta,\beta}\partial_\alpha(4G_{33} - 3G_T)\right.$$
$$- \Delta u_{\alpha,\beta}\partial_\beta G_T + \Delta u_{3,\alpha}$$
$$\left.\cdot 2\partial_3\left(2G_{33} - G_T + \frac{1-2p^2}{p^2}G_L\right)\right]$$
$$- \mu^2 c_T^{-2} \int_\Gamma \mathrm{d}S(\boldsymbol{\xi}) \int_0^t \mathrm{d}\tau \Delta\ddot{u}_\alpha G_T \ . \tag{29}$$

Equations Eqs. (27) and (29) constitute a set of non-hypersingular BIEs for the 3-D planar crack that express the traction in terms of the slip.

This is the theory underlying Fukuyama and Madariaga's (1998) formulation. Substituting the explicit forms of

$$\sigma_{31}(\mathbf{x},t) = \mu(\partial_1 u_3 + \partial_3 u_1)$$
$$= -\mu^2 \int_\Gamma \mathrm{d}S(\boldsymbol{\xi}) \int_0^t \mathrm{d}\tau\left\{(\Delta u_1\partial_1 + \Delta u_2\partial_2)\partial_1(4G_{33} - 3G_T) + \Delta u_1[-(\partial_1^2 + \partial_2^2)G_T + c_T^{-2}\ddot{G}_T]\right.$$
$$\left. + \Delta u_3 \cdot 2\partial_1\partial_3\left(2G_{33} - G_T + \frac{1-2p^2}{p^2}G_L\right)\right\}$$
$$= -\mu^2 \int_\Gamma \mathrm{d}S(\boldsymbol{\xi}) \int_0^t \mathrm{d}\tau\left[\Delta u_{\beta,\beta}\partial_1(4G_{33} - 3G_T) - \Delta u_{1,\beta}\cdot\partial_\beta G_T + \Delta u_{3,1}\cdot 2\partial_3\left(2G_{33} - G_T + \frac{1-2p^2}{p^2}G_L\right)\right]$$
$$- \mu^2 c_T^{-2} \int_\Gamma \mathrm{d}S(\boldsymbol{\xi}) \int_0^t \mathrm{d}\tau \Delta\ddot{u}_1 G_T \ . \tag{28}$$

The tangential traction T_1 across the crack at location $\mathbf{s}$ is given by $T_1(\mathbf{s},t) = \sigma_{31}(\mathbf{s},t)$. This, along with a similar equation for T_2, may be generalized to

the Green's functions into Eqs. (27) and (29) and making use of the following identity (Fukuyama and Madariaga 1995; Appendix B to the present article):

$$\int_{\Gamma} dS(\xi) \frac{1}{r} \Delta\ddot{u}_i(\xi, t - r/c)$$
$$= 2\pi c \Delta\dot{u}_i(\mathbf{s}, t) - c \int_{\Gamma} dS(\xi) \frac{\gamma_\alpha}{r} \Delta\dot{u}_{i,\alpha}(\xi, t - r/c) \quad (\text{if } \mathbf{s} \in \Gamma) \tag{30}$$

with

$$r \equiv ||\mathbf{s} - \xi|| \tag{31}$$
$$\gamma_i \equiv (s_i - \xi_i)/r \ , \tag{32}$$

we eventually arrive at:

$$T_3(\mathbf{s}, t) = -\frac{3\mu}{\pi} \int_{\Gamma} dS(\xi) \frac{\gamma_\alpha}{r^2} \int_1^p d\nu \cdot \nu \Delta u_{3,\alpha}(\xi, t - \nu r/c_T) - \frac{\mu}{\pi} \int_{\Gamma} dS(\xi) \frac{\gamma_\alpha}{r^2} \Delta u_{3,\alpha}(\xi, t - r/c_T)$$
$$- \frac{\mu}{\pi}(1 - 2p^2) \int_{\Gamma} dS(\xi) \frac{\gamma_\alpha}{r^2} \Delta u_{3,\alpha}(\xi, t - r/c_L) + \frac{\mu}{4\pi c_T} \frac{(1 - 2p^2)^2}{p} \int_{\Gamma} dS(\xi) \frac{\gamma_\alpha}{r} \Delta\dot{u}_{3,\alpha}(\xi, t - r/c_L)$$
$$- \frac{\mu}{2pc_T} \Delta\dot{u}_3(\mathbf{s}, t) \tag{33}$$

$$T_\alpha(\mathbf{s}, t) = \frac{3\mu}{\pi} \int_{\Gamma} dS(\xi) \frac{\gamma_\alpha}{r^2} \int_1^p d\nu \cdot \nu \Delta u_{\beta,\beta}(\xi, t - \nu r/c_T) + \frac{5\mu}{4\pi} \int_{\Gamma} dS(\xi) \frac{\gamma_\alpha}{r^2} \Delta u_{\beta,\beta}(\xi, t - r/c_T)$$
$$- \frac{\mu}{\pi} p^2 \int_{\Gamma} dS(\xi) \frac{\gamma_\alpha}{r^2} \Delta u_{\beta,\beta}(\xi, t - r/c_L) + \frac{\mu}{4\pi c_T} \int_{\Gamma} dS(\xi) \frac{\gamma_\alpha}{r} \Delta\dot{u}_{\beta,\beta}(\xi, t - r/c_T)$$
$$- \frac{\mu}{4\pi} \int_{\Gamma} dS(\xi) \frac{\gamma_\beta}{r^2} \Delta u_{\alpha,\beta}(\xi, t - r/c_T) - \frac{\mu}{2c_T} \Delta\dot{u}_\alpha(\mathbf{s}, t) \ . \tag{34}$$

Equations (33) and (34) coincide with Eq. (A1) and Eq. (3) (or (C21)) of Fukuyama and Madariaga (1998) respectively.

Static case

The elastostatic counterpart of Eqs. (33) and (34) can be obtained by simply dropping the time dependence:

$$T_3(\mathbf{s}) = -\frac{\mu}{2\pi}(1 - p^2) \int_{\Gamma} dS(\xi) \frac{\gamma_\alpha}{r^2} \Delta u_{3,\alpha}(\xi) \tag{35}$$

$$T_\alpha(\mathbf{s}) = -\frac{\mu}{4\pi} \int_{\Gamma} dS(\xi) \left[(1 - 2p^2) \frac{\gamma_\alpha}{r^2} \Delta u_{\beta,\beta}(\xi) + \frac{\gamma_\beta}{r^2} \Delta u_{\alpha,\beta}(\xi) \right] \ . \tag{36}$$

Our Eq. (35) coincides with Eq. (5) of Fukuyama and Madariaga (1995), while our Eq. (36) is a simplification over their Eq. (6).

5 Local Cartesian coordinate system

For the non-planar crack case, we define a local Cartesian coordinate system, one of the axes of which is always held locally perpendicular to the crack surface (Tada and Yamashita 1997). This normal axis shall be denoted by x_n, and the other two, locally tangential to the crack surface, shall be named x_s and x_t, in such a way that (x_n, x_s, x_t) forms a right-handed system (Fig. 2). The choice of x_s and x_t has one degree of freedom corresponding to rotation. We also denote the unit vectors in the three orthogonal directions by $\mathbf{n}$, $\mathbf{s}$ and $\mathbf{t}$ respectively. The local and global coordinate systems are mutually interchangeable by transformation formulae including:

$$\Delta u_i(\xi, \tau) = n_i(\xi)\Delta u_n(\xi, \tau) + s_i(\xi)\Delta u_s(\xi, \tau) + t_i(\xi)\Delta u_t(\xi, \tau) \tag{37}$$

$$\partial_n = n_i(\xi)\partial_i, \quad \partial_s = s_i(\xi)\partial_i, \quad \partial_t = t_i(\xi)\partial_i \ . \tag{38}$$

$$\begin{aligned} G_{nn} &= n_i(\xi)n_j(\xi)G_{ij} & G_{st} &= s_i(\xi)t_j(\xi)G_{ij} \\ G_{ss} &= s_i(\xi)s_j(\xi)G_{ij} & G_{tn} &= t_i(\xi)n_j(\xi)G_{ij} \\ G_{tt} &= t_i(\xi)t_j(\xi)G_{ij} & G_{ns} &= n_i(\xi)s_j(\xi)G_{ij} \end{aligned} \ . \tag{39}$$

This system of local Cartesian coordinates allows us to formulate the BIEs in such a way that all the spatial

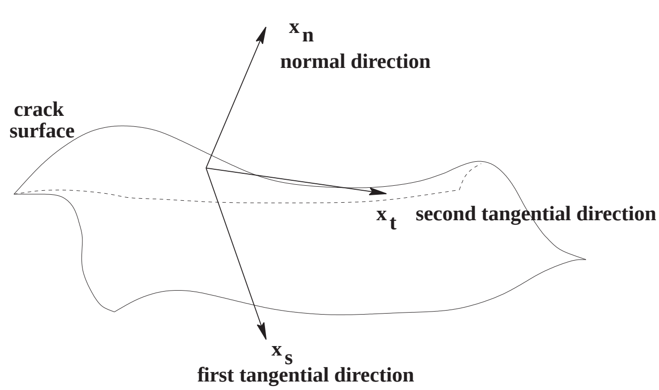

Fig. 2. Local Cartesian coordinate system

derivatives of the slip $\Delta\mathbf{u}$ are those with respect to x_s and x_t. This constitutes an advantage over the use of the global coordinates (x_1, x_2, x_3) as in Zhang (1991), since the slip $\Delta\mathbf{u}$ is defined only along the crack surface.

It should be noted that the symmetry relations (12), as well as the equations of motion (19) through Eq. (23), hold true after replacing the indices $1, 2, 3$ with n, s, t respectively. Hereafter we use the subscript z which runs over n, s or t.

6 Non-planar crack

In the non-planar 3-D crack case, the representation theorem states that:

$$
\begin{aligned}
u_k(\mathbf{x},t) &= -\int_\Gamma \mathrm{d}S(\xi)\int_0^t \mathrm{d}\tau\Delta u_i(\xi,\tau)n_j(\xi)\Sigma_{ij/k}(\mathbf{x},t;\xi,\tau) = -\int_\Gamma \mathrm{d}S(\xi)\int_0^t \mathrm{d}\tau(\Delta u_n\Sigma_{nn/k}+\Delta u_s\Sigma_{sn/k}+\Delta u_t\Sigma_{tn/k})\\
&= -\int_\Gamma \mathrm{d}S(\xi)\int_0^t \mathrm{d}\tau[\Delta u_n(n_k\Sigma_{nn/n}+s_k\Sigma_{nn/s}+t_k\Sigma_{nn/t})+\Delta u_s(n_k\Sigma_{sn/n}+s_k\Sigma_{sn/s}+t_k\Sigma_{sn/t})\\
&\quad+\Delta u_t(n_k\Sigma_{tn/n}+s_k\Sigma_{tn/s}+t_k\Sigma_{tn/t})]\\
&= -\int_\Gamma \mathrm{d}S(\xi)\int_0^t \mathrm{d}\tau\Delta u_n\{n_k[(\lambda+2\mu)\partial_n G_{nn}+\lambda\partial_s G_{sn}+\lambda\partial_t G_{tn}]+s_k[(\lambda+2\mu)\partial_n G_{ns}+\lambda\partial_s G_{ss}+\lambda\partial_t G_{ts}]\\
&\quad+t_k[(\lambda+2\mu)\partial_n G_{nt}+\lambda\partial_s G_{st}+\lambda\partial_t G_{tt}]\}-\int_\Gamma \mathrm{d}S(\xi)\int_0^t \mathrm{d}\tau\{\Delta u_s[n_k(\mu\partial_n G_{sn}+\mu\partial_s G_{nn})\\
&\quad+s_k(\mu\partial_n G_{ss}+\mu\partial_s G_{ns})+t_k(\mu\partial_n G_{st}+\mu\partial_s G_{nt})]+\Delta u_t[n_k(\mu\partial_n G_{tn}+\mu\partial_t G_{nn})+s_k(\mu\partial_n G_{ts}+\mu\partial_t G_{ns})\\
&\quad+t_k(\mu\partial_n G_{tt}+\mu\partial_t G_{nt})]\}\\
&= -\mu\int_\Gamma \mathrm{d}S(\xi)\int_0^t \mathrm{d}\tau\Delta u_n\left[n_k\partial_n\left(2G_{nn}+\frac{1-2p^2}{p^2}G_L\right)+(s_k\partial_s+t_k\partial_t)\left(2G_{nn}-2G_T+\frac{1-2p^2}{p^2}G_L\right)\right]\\
&\quad-\mu\int_\Gamma \mathrm{d}S(\xi)\int_0^t \mathrm{d}\tau\{(\Delta u_s\partial_s+\Delta u_t\partial_t)n_k(2G_{nn}-G_T)+[\Delta u_s s_k\partial_n(2G_{ss}-G_T)+\Delta u_t t_k\partial_n(2G_{tt}-G_T)]\\
&\quad+2(\Delta u_s t_k+\Delta u_t s_k)\partial_n G_{st}\}\ ,
\end{aligned}
\tag{40}
$$

where we made use of the symmetry relation (12). Accordingly,

$$
\begin{aligned}
\sigma_{kl}(\mathbf{x},t) &= \lambda\delta_{kl}\partial_p u_p+\mu(\partial_k u_l+\partial_l u_k)\\
&= -\mu^2\int_\Gamma \mathrm{d}S(\xi)\int_0^t \mathrm{d}\tau\Delta u_n\left\{(a_{kl}\partial_n+d_{kl}\partial_s+e_{kl}\partial_t)\partial_n\left(2G_{nn}+\frac{1-2p^2}{p^2}G_L\right)\right.\\
&\quad\left.+(d_{kl}\partial_n+b_{kl}\partial_s+f_{kl}\partial_t)\partial_s\left(2G_{nn}-2G_T+\frac{1-2p^2}{p^2}G_L\right)+(e_{kl}\partial_n+f_{kl}\partial_s+c_{kl}\partial_t)\partial_t\left(2G_{nn}-2G_T+\frac{1-2p^2}{p^2}G_L\right)\right\}\\
&\quad-\mu^2\int_\Gamma \mathrm{d}S(\xi)\int_0^t \mathrm{d}\tau\big[(\Delta u_s\partial_s+\Delta u_t\partial_t)(a_{kl}\partial_n+\mathrm{d}_{kl}\partial_s+e_{kl}\partial_t)(2G_{nn}-G_T)+\Delta u_s(d_{kl}\partial_n+b_{kl}\partial_s+f_{kl}\partial_t)\partial_n(2G_{ss}-G_T)\\
&\quad+\Delta u_t(e_{kl}\partial_n+f_{kl}\partial_s+c_{kl}\partial_t)\partial_n(2G_{tt}-G_T)+2(\Delta u_s e_{kl}+\Delta u_t\mathrm{d}_{kl})\partial_s\partial_t(G_{nn}-G_T)+2(\Delta u_s f_{kl}+\Delta u_t b_{kl})\partial_n\partial_t(G_{ss}-G_T)\\
&\quad+2(\Delta u_s c_{kl}+\Delta u_t f_{kl})\partial_n\partial_s(G_{tt}-G_T)\big]\ ,
\end{aligned}
\tag{41}
$$

with

$$
\begin{aligned}
a_{kl}(\xi) &\equiv [(1-2p^2)/p^2]\delta_{kl}+2n_k(\xi)n_l(\xi),\\
b_{kl}(\xi) &\equiv [(1-2p^2)/p^2]\delta_{kl}+2s_k(\xi)s_l(\xi),\\
c_{kl}(\xi) &\equiv [(1-2p^2)/p^2]\delta_{kl}+2t_k(\xi)t_l(\xi),\\
d_{kl}(\xi) &\equiv n_k(\xi)s_l(\xi)+s_k(\xi)n_l(\xi)\\
e_{kl}(\xi) &\equiv n_k(\xi)t_l(\xi)+t_k(\xi)n_l(\xi)\\
f_{kl}(\xi) &\equiv s_k(\xi)t_l(\xi)+t_k(\xi)s_l(\xi)\ .
\end{aligned}
\tag{42}
$$

Making use of the symmetry relation (12) and the equations of motion (19) through Eq. (23) and integrating by parts, this can be rewritten as

$$\sigma_{kl}(\mathbf{x},t) = -\mu^2 \int_\Gamma dS(\xi) \int_0^t d\tau \Big\{ - a_{kl}(\Delta u_{n,s}\partial_s + \Delta u_{n,t}\partial_t)\left(2G_{nn} - 2G_T + \frac{1}{p^2}G_L\right)$$

$$+ [b_{kl}\Delta u_{n,s}\partial_s + c_{kl}\Delta u_{n,t}\partial_t + f_{kl}(\Delta u_{n,s}\partial_t + \Delta u_{n,t}\partial_s)]\left(2G_{nn} - 2G_T + \frac{1-2p^2}{p^2}G_L\right)$$

$$+ 2(d_{kl}\Delta u_{n,s} + e_{kl}\Delta u_{n,t})\partial_n\left(2G_{nn} - G_T + \frac{1-2p^2}{p^2}G_L\right)\Big\}$$

$$- \mu^2 \int_\Gamma dS(\xi) \int_0^t d\tau \{(\Delta u_{s,s} + \Delta u_{t,t})[a_{kl}\partial_n(2G_{nn} - G_T) + (d_{kl}\partial_s + e_{kl}\partial_t)(4G_{nn} - 3G_T)]$$

$$+ \Delta u_{s,s}\partial_n[b_{kl}(2G_{ss} - G_T) + 2c_{kl}(G_{tt} - G_T)] + \Delta u_{t,t}\partial_n[c_{kl}(2G_{tt} - G_T)$$

$$+ 2b_{kl}(G_{ss} - G_T)] + \Delta u_{s,t}f_{kl}\partial_n(4G_{ss} - 3G_T) + \Delta u_{t,s}f_{kl}\partial_n(4G_{tt} - 3G_T)$$

$$- (\Delta u_s d_{kl} + \Delta u_t e_{kl})(_{,s}\partial_s +_{,t}\partial_t)G_T\} - \mu^2 c_T^{-2} \int_\Gamma dS(\xi) \int_0^t d\tau[\Delta\ddot{u}_n a_{kl} G_L + (\Delta\ddot{u}_s \mathrm{d}_{kl} + \Delta\ddot{u}_t e_{kl})G_T] \ . \tag{43}$$

Defining the multiplying coefficients

$$\begin{aligned}
A_z(\mathbf{s},\xi) &\equiv n_k(\mathbf{s})z_l(\mathbf{s})a_{kl}(\xi) \\
&= 2n_k(\mathbf{s})z_l(\mathbf{s})n_k(\xi)n_l(\xi) + [(1-2p^2)/p^2]\delta_{zn} \\
B_z(\mathbf{s},\xi) &\equiv n_k(\mathbf{s})z_l(\mathbf{s})b_{kl}(\xi) \\
&= 2n_k(\mathbf{s})z_l(\mathbf{s})s_k(\xi)s_l(\xi) + [(1-2p^2)/p^2]\delta_{zn} \\
C_z(\mathbf{s},\xi) &\equiv n_k(\mathbf{s})z_l(\mathbf{s})c_{kl}(\xi) \\
&= 2n_k(\mathbf{s})z_l(\mathbf{s})t_k(\xi)t_l(\xi) + [(1-2p^2)/p^2]\delta_{zn} \\
D_z(\mathbf{s},\xi) &\equiv n_k(\mathbf{s})z_l(\mathbf{s})\mathrm{d}_{kl}(\xi) \\
&= n_k(\mathbf{s})z_l(\mathbf{s})(n_k(\xi)s_l(\xi) + s_k(\xi)n_l(\xi)) \\
E_z(\mathbf{s},\xi) &\equiv n_k(\mathbf{s})z_l(\mathbf{s})e_{kl}(\xi) \\
&= n_k(\mathbf{s})z_l(\mathbf{s})(n_k(\xi)t_l(\xi) + t_k(\xi)n_l(\xi)) \\
F_z(\mathbf{s},\xi) &\equiv n_k(\mathbf{s})z_l(\mathbf{s})f_{kl}(\xi) \\
&= n_k(\mathbf{s})z_l(\mathbf{s})(s_k(\xi)t_l(\xi) + t_k(\xi)s_l(\xi)) \ ,
\end{aligned} \tag{44}$$

we have, after taking the limit $\mathbf{x} \to \mathbf{s} \in \Gamma$, the following equation for the z-component of the traction across the crack at location $\mathbf{s}$ and time t:

$$T_z(\mathbf{s},t) = n_k(\mathbf{s})z_l(\mathbf{s})\sigma_{kl}(\mathbf{s},t)$$

$$= -\mu^2 \int_\Gamma dS(\xi) \int_0^t d\tau \Big\{ - A_z(\Delta u_{n,s}\partial_s + \Delta u_{n,t}\partial_t)\left(2G_{nn} - 2G_T + \frac{1}{p^2}G_L\right)$$

$$+ [B_z\Delta u_{n,s}\partial_s + C_z\Delta u_{n,t}\partial_t + F_z(\Delta u_{n,s}\partial_t + \Delta u_{n,t}\partial_s)]\left(2G_{nn} - 2G_T + \frac{1-2p^2}{p^2}G_L\right)$$

$$+ 2(D_z\Delta u_{n,s} + E_z\Delta u_{n,t})\partial_n\left(2G_{nn} - G_T + \frac{1-2p^2}{p^2}G_L\right)\Big\}$$

$$- \mu^2 \int_\Gamma dS(\xi) \int_0^t d\tau \{(\Delta u_{s,s} + \Delta u_{t,t})[A_z\partial_n(2G_{nn} - G_T) + (D_z\partial_s + E_z\partial_t)(4G_{nn} - 3G_T) + 2B_z\partial_n(G_{ss} - G_T)$$

$$+ 2C_z\partial_n(G_{tt} - G_T)] + (\Delta u_{s,s}B_z + \Delta u_{t,t}C_z)\partial_n G_T + \Delta u_{s,t}F_z\partial_n(4G_{ss} - 3G_T) + \Delta u_{t,s}F_z\partial_n(4G_{tt} - 3G_T)$$

$$-(\Delta u_s D_z + \Delta u_t E_z)(_{,s}\partial_s +_{,t}\partial_t)G_T\Big\} - \mu^2 c_T^{-2} \int_\Gamma dS(\xi) \int_0^t d\tau[\Delta\ddot{u}_n A_z G_L + (\Delta\ddot{u}_s D_z + \Delta\ddot{u}_t E_z)G_T] \ . \tag{45}$$

The right hand side of this equation is non-hypersingular, in that the integral terms involve at most first-order spatial derivatives of G_{ij}, G_T and G_L which behave as $1/r^2$ as $r \to 0$, and are hence integrable in the sense of Cauchy principal values. Equation (45) is our non-hypersingular BIE for the 3-D non-planar crack problem, expressing the traction components in terms of the slip components. In the special case of the planar crack, it can be easily shown that (45) reduces to Eqs. (27) and (29).

Substituting the explicit forms of the Green's functions into Eq. (45), we finally have, after some algebra,

$$T_z(\mathbf{s},t) = -\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}\int_1^p d\nu\cdot\nu[-30W_{4z}+6U_{1z}-6U_{2z}-36U_{3z}+30W_{10z}-18U_{5z}-6U_{6z}-12U_{7z}](\xi,t-\nu r/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[-12W_{4z}+2U_{1z}-2U_{2z}-14U_{3z}+12W_{10z}-7U_{5z}-2U_{6z}-5U_{7z}-U_{8z}+U_{9z}](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[12p^2W_{4z}+(1-2p^2)U_{1z}-(1-4p^2)U_{2z}-2(1-8p^2)U_{3z}-12p^2W_{10z}+6p^2U_{5z}+2p^2U_{6z}$$
$$+4p^2U_{7z}](\xi,t-r/c_L)-\frac{\mu}{4\pi c_T}\int_\Gamma dS(\xi)\frac{1}{r}[-2\dot{W}_{4z}-2\dot{U}_{3z}+2\dot{W}_{10z}-\dot{U}_{5z}-\dot{U}_{7z}-\dot{U}_{8z}+\dot{U}_{9z}](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi c_T}p\int_\Gamma dS(\xi)\frac{1}{r}[2p^2\dot{W}_{4z}+\dot{U}_{1z}-(1-2p^2)(\dot{U}_{2z}+2\dot{U}_{3z})-2p^2\dot{W}_{10z}](\xi,t-r/c_L)$$
$$-\frac{\mu}{4\pi c_T^2}\int_\Gamma dS(\xi)\frac{1}{r}[\Delta\ddot{u}_n(\xi,t-r/c_L)p^2A_z+(\Delta\ddot{u}_sD_z+\Delta\ddot{u}_tE_z)(\xi,t-r/c_T)]\ , \tag{46}$$

where

$$U_{1z}(\xi,\tau)\equiv(\Delta u_{n,s}\gamma_s+\Delta u_{n,t}\gamma_t)A_z$$
$$U_{2z}(\xi,\tau)\equiv\Delta u_{n,s}B_z\gamma_s+\Delta u_{n,t}C_z\gamma_t$$
$$+(\Delta u_{n,s}\gamma_t+\Delta u_{n,t}\gamma_s)F_z \tag{47}$$
$$U_{3z}(\xi,\tau)\equiv(\Delta u_{n,s}D_z+\Delta u_{n,t}E_z)\gamma_n$$
$$W_{4z}(\xi,\tau)\equiv(U_{1z}-U_{2z}-2U_{3z})\gamma_n^2$$

are combinations of the spatial derivatives of the normal (or opening) component of the slip and

$$U_{5z}(\xi,\tau)\equiv(\Delta u_{s,s}+\Delta u_{t,t})A_z\gamma_n$$
$$U_{6z}(\xi,\tau)\equiv(\Delta u_{s,s}+\Delta u_{t,t})(B_z+C_z)\gamma_n$$
$$U_{7z}(\xi,\tau)\equiv(\Delta u_{s,s}+\Delta u_{t,t})(D_z\gamma_s+E_z\gamma_t)$$
$$+(\Delta u_{s,t}+\Delta u_{t,s})F_z\gamma_n$$
$$U_{8z}(\xi,\tau)\equiv(\Delta u_{s,s}B_z+\Delta u_{t,t}C_z)\gamma_n$$
$$U_{9z}(\xi,\tau)\equiv(\Delta u_sD_z+\Delta u_tE_z)(_{,s}\gamma_s+_{,t}\gamma_t)$$
$$W_{10z}(\xi,\tau)\equiv(\Delta u_{s,s}+\Delta u_{t,t})(A_z\gamma_n^2+B_z\gamma_s^2+C_z\gamma_t^2 \tag{48}$$
$$+2D_z\gamma_n\gamma_s+2E_z\gamma_n\gamma_t)\gamma_n$$
$$+2(\Delta u_{s,t}\gamma_s^2+\Delta u_{t,s}\gamma_t^2)F_z\gamma_n$$

are combinations of the spatial derivatives of the shear components of the slip. In the special case of the planar crack on the x_1x_2-plane, Eq. (46) reduces to a set of equations equivalent to Eqs. (33) and (34).

Static case
The elastostatic counterpart of Eq. (46) may be obtained by simply dropping the time dependence:

$$T_z(\mathbf{s}) = -\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[3(1-p^2)W_{4z}+p^2U_{1z}+p^2U_{2z}$$
$$+2(1-p^2)U_{3z}-3(1-p^2)W_{10z}+(2-3p^2)U_{5z}$$
$$+(1-p^2)U_{6z}+(1-2p^2)U_{7z}-U_{8z}+U_{9z}](\xi)\ . \tag{49}$$

In the special case of the planar crack on the x_1x_2-plane, Eq. (49) reduces to Eqs. (35) and (36).

7
Off-fault stress field
The stress field outside the crack is expressed in a form parallel to Eq. (46):

$$\sigma_{kl}(\mathbf{s},t) = -\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}\int_1^p dv\cdot v[-30W_{4kl}+6U_{1kl}-6U_{2kl}-36U_{3kl}+30W_{10kl}-18U_{5kl}-6U_{6kl}-12U_{7kl}](\xi,t-vr/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[-12W_{4kl}+2U_{1kl}-2U_{2kl}-14U_{3kl}+12W_{10kl}-7U_{5kl}-2U_{6kl}-5U_{7kl}-U_{8kl}+U_{9kl}](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[12p^2W_{4kl}+(1-2p^2)U_{1kl}-(1-4p^2)U_{2kl}-2(1-8p^2)U_{3kl}-12p^2W_{10kl}$$
$$+6p^2U_{5kl}+2p^2U_{6kl}+4p^2U_{7kl}](\xi,t-r/c_L)-\frac{\mu}{4\pi c_T}\int_\Gamma dS(\xi)\frac{1}{r}[-2\dot{W}_{4kl}-2\dot{U}_{3kl}+2\dot{W}_{10kl}$$
$$-\dot{U}_{5kl}-\dot{U}_{7kl}-\dot{U}_{8kl}+\dot{U}_{9kl}](\xi,t-r/c_T)-\frac{\mu}{4\pi c_T}p\int_\Gamma dS(\xi)\frac{1}{r}[2p^2\dot{W}_{4kl}+\dot{U}_{1kl}-(1-2p^2)(\dot{U}_{2kl}+2\dot{U}_{3kl})$$
$$-2p^2\dot{W}_{10kl}](\xi,t-r/c_L)-\frac{\mu}{4\pi c_T^2}\int_\Gamma dS(\xi)\frac{1}{r}[\Delta\ddot{u}_n(\xi,t-r/c_L)p^2a_{kl}+(\Delta\ddot{u}_s d_{kl}+\Delta\ddot{u}_t e_{kl})(\xi,t-r/c_T)]\ ,\qquad(50)$$

where W_{4kl}, U_{1kl} and other similar symbols are defined by equations parallel to Eqs. (47) and (48) where the subscript z should be replaced by kl and the coefficients in majuscule A_z, B_z etc. should be replaced by those in minuscule a_{kl}, b_{kl} etc. Of interest here is the special form of Eq. (50) for the planar 3-D crack lying on the x_1x_2-plane, which is given by:

$$\sigma_{11}(\mathbf{s},t) = -\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}\int_1^p dv\cdot v[12\Delta u_{3,1}\gamma_1(5\gamma_3^2-1)+12\Delta u_{\alpha,\alpha}\gamma_3(5\gamma_1^2-1)](\xi,t-vr/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[4\Delta u_{3,1}\gamma_1(6\gamma_3^2-1)+4\Delta u_{\alpha,\alpha}\gamma_3(6\gamma_1^2-1)-2\Delta u_{1,1}\gamma_3](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[-4p^2\Delta u_{3,1}\gamma_1(6\gamma_3^2-1)+2(1-2p^2)\Delta u_{3,2}\gamma_2-2\Delta u_{\alpha,\alpha}\gamma_3(12p^2\gamma_1^2+1-4p^2)](\xi,t-r/c_L)$$
$$-\frac{\mu}{4\pi c_T}\int_\Gamma dS(\xi)\frac{1}{r}[4\Delta\dot{u}_{3,1}\gamma_1\gamma_3^2+4\Delta\dot{u}_{\alpha,\alpha}\gamma_3\gamma_1^2-2\Delta\dot{u}_{1,1}\gamma_3](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi c_T}p\int_\Gamma dS(\xi)\frac{1}{r}[-4p^2\Delta\dot{u}_{3,1}\gamma_1\gamma_3^2+2(1-2p^2)\Delta\dot{u}_{3,2}\gamma_2-2\Delta\dot{u}_{\alpha,\alpha}\gamma_3(2p^2\gamma_1^2+1-2p^2)](\xi,t-r/c_L)$$
$$-\frac{\mu}{4\pi c_T^2}(1-2p^2)\int_\Gamma dS(\xi)\frac{1}{r}\Delta\ddot{u}_3(\xi,t-r/c_L)\qquad(51)$$

$$\sigma_{12}(\mathbf{s},t) = -\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}\int_1^p dv\cdot v[6(\Delta u_{3,1}\gamma_2+\Delta u_{3,2}\gamma_1)(5\gamma_3^2-1)+12\Delta u_{1,2}\gamma_3(5\gamma_1^2-1)+12\Delta u_{2,1}\gamma_3(5\gamma_2^2-1)](\xi,t-vr/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[2(\Delta u_{3,1}\gamma_2+\Delta u_{3,2}\gamma_1)(6\gamma_3^2-1)+\Delta u_{1,2}\gamma_3(24\gamma_1^2-5)+\Delta u_{2,1}\gamma_3(24\gamma_2^2-5)](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[-(\Delta u_{3,1}\gamma_2+\Delta u_{3,2}\gamma_1)(12p^2\gamma_3^2+1-4p^2)-4p^2\Delta u_{1,2}\gamma_3(6\gamma_1^2-1)-4p^2\Delta u_{2,1}\gamma_3(6\gamma_2^2-1)](\xi,t-r/c_L)$$
$$-\frac{\mu}{4\pi c_T}\int_\Gamma dS(\xi)\frac{1}{r}[2(\Delta\dot{u}_{3,1}\gamma_2+\Delta\dot{u}_{3,2}\gamma_1)\gamma_3^2+\Delta\dot{u}_{1,2}\gamma_3(4\gamma_1^2-1)+\Delta\dot{u}_{2,1}\gamma_3(4\gamma_2^2-1)](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi c_T}p\int_\Gamma dS(\xi)\frac{1}{r}[-(\Delta\dot{u}_{3,1}\gamma_2+\Delta\dot{u}_{3,2}\gamma_1)(2p^2\gamma_3^2+1-2p^2)-4p^2(\Delta\dot{u}_{1,2}\gamma_1^2+\Delta\dot{u}_{2,1}\gamma_2^2)\gamma_3](\xi,t-r/c_L)\qquad(52)$$

$$\sigma_{31}(\mathbf{s},t) = -\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}\int_1^p d\nu\cdot\nu[12\Delta u_{3,1}\gamma_3(5\gamma_3^2-3)+12\Delta u_{\alpha,\alpha}\gamma_1(5\gamma_3^2-1)](\xi,t-\nu r/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[2\Delta u_{3,1}\gamma_3(12\gamma_3^2-7)+\Delta u_{\alpha,\alpha}\gamma_1(24\gamma_3^2-5)+\Delta u_{1,\alpha}\gamma_\alpha](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[-2\Delta u_{3,1}\gamma_3(12p^2\gamma_3^2+1-8p^2)-4p^2\Delta u_{\alpha,\alpha}\gamma_1(6\gamma_3^2-1)](\xi,t-r/c_L)$$
$$-\frac{\mu}{4\pi c_T}\int_\Gamma dS(\xi)\frac{1}{r}[2\Delta\dot{u}_{3,1}\gamma_3(2\gamma_3^2-1)+\Delta\dot{u}_{\alpha,\alpha}\gamma_1(4\gamma_3^2-1)+\Delta\dot{u}_{1,\alpha}\gamma_\alpha](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi c_T}p\int_\Gamma dS(\xi)\frac{1}{r}[-2\Delta\dot{u}_{3,1}\gamma_3(2p^2\gamma_3^2+1-2p^2)-4p^2\Delta\dot{u}_{\alpha,\alpha}\gamma_1\gamma_3^2](\xi,t-r/c_L)$$
$$-\frac{\mu}{4\pi c_T^2}\int_\Gamma dS(\xi)\frac{1}{r}\Delta\ddot{u}_1(\xi,t-r/c_T) \tag{53}$$

$$\sigma_{33}(\mathbf{s},t) = -\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}\int_1^p d\nu\cdot\nu[-12\Delta u_{3,\alpha}\gamma_\alpha(5\gamma_3^2-1)+12\Delta u_{\alpha,\alpha}\gamma_3(5\gamma_3^2-3)](\xi,t-\nu r/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[-4\Delta u_{3,\alpha}\gamma_\alpha(6\gamma_3^2-1)+2\Delta u_{\alpha,\alpha}\gamma_3(12\gamma_3^2-7)](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[4\Delta u_{3,\alpha}\gamma_\alpha(6p^2\gamma_3^2+1-2p^2)-2\Delta u_{\alpha,\alpha}\gamma_3(12p^2\gamma_3^2+1-8p^2)](\xi,t-r/c_L)$$
$$-\frac{\mu}{4\pi c_T}\int_\Gamma dS(\xi)\frac{1}{r}[-4\Delta\dot{u}_{3,\alpha}\gamma_\alpha\gamma_3^2+2\Delta\dot{u}_{\alpha,\alpha}\gamma_3(2\gamma_3^2-1)](\xi,t-r/c_T)$$
$$-\frac{\mu}{4\pi c_T}p\int_\Gamma dS(\xi)\frac{1}{r}[4\Delta\dot{u}_{3,\alpha}\gamma_\alpha(p^2\gamma_3^2+1-p^2)-2\Delta\dot{u}_{\alpha,\alpha}\gamma_3(2p^2\gamma_3^2+1-2p^2)](\xi,t-r/c_L)$$
$$-\frac{\mu}{4\pi c_T^2}\int_\Gamma dS(\xi)\frac{1}{r}\Delta\ddot{u}_3(\xi,t-r/c_L)\ . \tag{54}$$

These equations were also derived by Aochi, Fukuyama and Matsu'ura (1999a, b). As $\gamma_3 \to 0$, Eqs. (53) and (54) reduce to equations equivalent to Eqs. (34) and (33), respectively. Other components, not listed here, may be obtained by considering the symmetry between the coordinates x_1 and x_2.

Static case

The elastostatic counterpart of Eq. (50) may be obtained by simply dropping the time dependence:

$$\sigma_{kl}(\mathbf{s}) = -\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[3(1-p^2)W_{4kl}+p^2U_{1kl}+p^2U_{2kl} + 2(1-p^2)U_{3kl}-3(1-p^2)W_{10kl}+(2-3p^2)U_{5kl} + (1-p^2)U_{6kl}+(1-2p^2)U_{7kl}-U_{8kl}+U_{9kl}](\xi)\ . \tag{55}$$

The special form of Eq. (55) for the planar 3-D crack lying on the x_1x_2-plane is:

$$\sigma_{11}(\mathbf{s}) = -\frac{\mu}{2\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[(1-p^2)\Delta u_{3,1}\gamma_1(1-3\gamma_3^2) + (1-2p^2)\Delta u_{3,2}\gamma_2-(1-p^2)\Delta u_{\alpha,\alpha}\gamma_3(1+3\gamma_1^2) + \Delta u_{2,2}\gamma_3](\xi) \tag{56}$$

$$\sigma_{12}(\mathbf{s}) = -\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}\{\Delta u_{3,1}\gamma_2[p^2-3(1-p^2)\gamma_3^2] + \Delta u_{3,2}\gamma_1[p^2-3(1-p^2)\gamma_3^2] + \Delta u_{1,2}\gamma_3[1-2p^2-6(1-p^2)\gamma_1^2] + \Delta u_{2,1}\gamma_3[1-2p^2-6(1-p^2)\gamma_2^2]\}(\xi) \tag{57}$$

$$\sigma_{31}(\mathbf{s}) = -\frac{\mu}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[2(1-p^2)\Delta u_{3,1}\gamma_3(1-3\gamma_3^2) + 2(1-p^2)\Delta u_{\alpha,\alpha}\gamma_1(1-3\gamma_3^2) - \Delta u_{2,2}\gamma_1 + \Delta u_{1,2}\gamma_2](\xi) \quad (58)$$

$$\sigma_{33}(\mathbf{s}) = -\frac{\mu}{2\pi}(1-p^2)\int_\Gamma dS(\xi)\frac{1}{r^2}[\Delta u_{3,\alpha}\gamma_\alpha(1+3\gamma_3^2) + \Delta u_{\alpha,\alpha}\gamma_3(1-3\gamma_3^2)] \ . \quad (59)$$

As $\gamma_3 \to 0$, Eqs. (58) and (59) reduce to Eqs. (36) and (35), respectively.

8 Off-fault displacement field

The displacement field outside the crack is obtained by substituting the explicit form of the Green's functions into Eq. (40):

$$u_k(\mathbf{s},t) = -\frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}\int_1^p d\nu\cdot\nu[30W_{13k} - 18U_{11k} - 6U_{12k} + 30W_{15k} - 6U_{14k}](\xi, t-\nu r/c_T)$$
$$-\frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[12W_{13k} - 8U_{11k} - 2U_{12k} + 12W_{15k} - 3U_{14k}](\xi, t-r/c_T)$$
$$-\frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[-12p^2W_{13k} - (1-8p^2)U_{11k} - (1-4p^2)U_{12k} - 12p^2W_{15k} + 2p^2U_{14k}](\xi, t-r/c_L)$$
$$-\frac{1}{4\pi c_T}\int_\Gamma dS(\xi)\frac{1}{r}[2\dot{W}_{13k} - 2\dot{U}_{11k} + 2\dot{W}_{15k} - \dot{U}_{14k}](\xi, t-r/c_T)$$
$$-\frac{1}{4\pi c_T}p\int_\Gamma dS(\xi)\frac{1}{r}[-2p^2\dot{W}_{13k} - (1-2p^2)(\dot{U}_{11k}+\dot{U}_{12k}) - 2p^2\dot{W}_{15k}](\xi, t-r/c_L) \ , \quad (60)$$

where

$$U_{11k}(\xi,\tau) \equiv \Delta u_n n_k \gamma_n$$

$$U_{12k}(\xi,\tau) \equiv \Delta u_n(s_k\gamma_s + t_k\gamma_t)$$
$$W_{13k}(\xi,\tau) \equiv (U_{11k}+U_{12k})\gamma_n^2 = \Delta u_n\gamma_n^2\gamma_k$$

$$U_{14k}(\xi,\tau) \equiv \Delta u_s(n_k\gamma_s + s_k\gamma_n) + \Delta u_t(n_k\gamma_t + t_k\gamma_n)$$

$$W_{15k}(\xi,\tau) \equiv (\Delta u_s\gamma_s + \Delta u_t\gamma_t)\gamma_n\gamma_k \quad (61)$$

are combinations of the slip components. Of interest here is the special form of Eq. (60) for the planar 3-D crack lying on the x_1x_2-plane, which is given by:

$$u_1(\mathbf{s},t) = -\frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}\int_1^p d\nu\cdot\nu[6\Delta u_3\gamma_1(5\gamma_3^2-1) + 6\Delta u_1\gamma_3(5\gamma_1^2-1) + 30\Delta u_2\gamma_1\gamma_2\gamma_3](\xi, t-\nu r/c_T)$$
$$-\frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[2\Delta u_3\gamma_1(6\gamma_3^2-1) + 3\Delta u_1\gamma_3(4\gamma_1^2-1) + 12\Delta u_2\gamma_1\gamma_2\gamma_3](\xi, t-r/c_T)$$
$$-\frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[-\Delta u_3\gamma_1(12p^2\gamma_3^2 + 1 - 4p^2) - 2p^2\Delta u_1\gamma_3(6\gamma_1^2-1) - 12p^2\Delta u_2\gamma_1\gamma_2\gamma_3](\xi, t-r/c_L)$$
$$-\frac{1}{4\pi c_T}\int_\Gamma dS(\xi)\frac{1}{r}[2\Delta\dot{u}_3\gamma_1\gamma_3^2 + \Delta\dot{u}_1\gamma_3(2\gamma_1^2-1) + 2\Delta\dot{u}_2\gamma_1\gamma_2\gamma_3](\xi, t-r/c_T)$$
$$-\frac{1}{4\pi c_T}p\int_\Gamma dS(\xi)\frac{1}{r}[-\Delta\dot{u}_3\gamma_1(2p^2\gamma_3^2 + 1 - 2p^2) - 2p^2\Delta\dot{u}_1\gamma_3\gamma_1^2 - 2p^2\Delta\dot{u}_2\gamma_1\gamma_2\gamma_3](\xi, t-r/c_L) \quad (62)$$

$$u_3(\mathbf{s},t) = -\frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}\int_1^p d\nu \cdot \nu[6\Delta u_3\gamma_3(5\gamma_3^2-3)+6\Delta u_\alpha\gamma_\alpha(5\gamma_3^2-1)](\xi, t-\nu r/c_T)$$
$$-\frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[4\Delta u_3\gamma_3(3\gamma_3^2-2)+3\Delta u_\alpha\gamma_\alpha(4\gamma_3^2-1)](\xi, t-r/c_T)$$
$$-\frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[-\Delta u_3\gamma_3(12p^2\gamma_3^2+1-8p^2)-2p^2\Delta u_\alpha\gamma_\alpha(6\gamma_3^2-1)](\xi, t-r/c_L)$$
$$-\frac{1}{4\pi c_T}\int_\Gamma dS(\xi)\frac{1}{r}[2\Delta\dot{u}_3\gamma_3(\gamma_3^2-1)+\Delta\dot{u}_\alpha\gamma_\alpha(2\gamma_3^2-1)](\xi, t-r/c_T)$$
$$-\frac{1}{4\pi c_T}p\int_\Gamma dS(\xi)\frac{1}{r}[-\Delta\dot{u}_3\gamma_3(2p^2\gamma_3^2+1-2p^2)-2p^2\Delta\dot{u}_\alpha\gamma_\alpha\gamma_3^2](\xi, t-r/c_L) \ . \quad (63)$$

The component u_2 may be obtained by considering the symmetry between the coordinates x_1 and x_2.

Static case

The elastostatic counterpart of Eq. (60) may be obtained by simply dropping the time dependence:

$$u_k(\mathbf{s}) = \frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[3(1-p^2)W_{13k}+p^2U_{11k} - p^2U_{12k}+3(1-p^2)W_{15k}+p^2U_{14k}](\xi) \ . \quad (64)$$

The special form of Eq. (64) for the planar 3-D crack lying on the x_1x_2-plane is:

$$u_1(\mathbf{s}) = \frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}[3(1-p^2)(\Delta u_3\gamma_3+\Delta u_\alpha\gamma_\alpha)\gamma_1\gamma_3 + p^2(\Delta u_1\gamma_3-\Delta u_3\gamma_1)](\xi) \quad (65)$$

$$u_3(\mathbf{s}) = \frac{1}{4\pi}\int_\Gamma dS(\xi)\frac{1}{r^2}(\Delta u_3\gamma_3+\Delta u_\alpha\gamma_\alpha)(\xi) \times[3(1-p^2)\gamma_3^2+p^2] \ . \quad (66)$$

9
Discussion and conclusion

In the present article, we have enlarged Fukuyama and Madariaga's (1998) integration by parts technique and combined it with Tada and Yamashita's (1997) local Cartesian coordinate system, to derive, for the first time, a non-hypersingular time-domain BIE for the 3-D non-planar crack in a fully explicit form. We have given not only the BIE describing the traction-slip relation on the crack, but also similar expressions for the stress and displacement fields off the fault. Although we are not going to detail it in this article, we have confirmed that, in the special case in which the crack configuration is independent of one coordinate, all the equations that we have derived reduce to corresponding equations given in the Tada and Yamashita (1997) paper on the 2-D non-planar crack theory. This confirms the correctness of our algebra.

The present study completes the set of non-hypersingular BIEM theory for crack dynamics, that is based on the displacement discontinuity method, the time-domain representation and the integration by parts technique. In practical implementation to non-planar crack analysis, however, direct use of the non-planar crack equation is expected to face many obstacles, not least the difficulty of meshing the curved surface and the extremely complex expression of the equation. This fact favors the approach of Aochi, Fukuyama and Matsu'ura (1999a, b), who propose to numerically model 3-D non-planar crack dynamics problems by approximating the curved crack surface by a patchwork of small planar elements. In their approach, the influence of slip on one planar element on the traction on another element is to be evaluated by way of simpler equations (51)–(54), which are special cases of the more complicated equation (46) or (50).

In approximating a curved crack surface by a patchwork of small planar elements, however, there is a point that requires attention. The use of piecewise-constant interpolation is fairly common in the discretization of the slip distribution on the crack surface. Tada and Yamashita (1996) pointed out that, when a curved crack is modeled as a series of smaller planar elements and when piecewise-constant interpolation is applied to slip, slip "gets stuck" at spurious joints of differently oriented elements and results in a smaller expected slip than if the crack is modeled as a series of curved elements that are smoothly joined. Seelig and Gross' (1997) modeling method, which they put to use in the 2-D case, hints at one way to prevent spurious suppression of slip in such a case. They numerically modeled shear slip on a non-planar crack, in the same way as they would have modeled a traction-free open crack that was allowed to slip both in the shear and opening modes. They then numerically penalized the negative opening component of the slip, which would have meant material penetration. If we follow their method and penalize both positive and negative opening components of slip, this approach can be used to model pure shear cracks, free from the spurious suppression of slip at element joints.

Our present theoretical study thus marks a major step toward a more realistic 3-D numerical analysis of earthquake fault dynamics. With a view to future numerical modeling, Aochi, Fukuyama and Matsu'ura (1999a, b) derived a set of discretization kernels for Eqs. (51)–(54), or the stress components that would be expected in response to a uniform slip of a unit slip-rate taking place during a unit length of time on a quandrangular fault patch of a unit area. These discrete kernels will be used when one numerically analyzes the behavior of a 3-D non-planar crack, approximating the crack by a patchwork of small planar elements and based on the piecewise-constant approximation for the slip-rate. This or other sorts of practical application of the 3-D non-planar crack dynamics theory are much awaited, for a better understanding of the seismic rupture phenomena on faults with complex geometry.

Appendix A: 3-D Green's functions

The explicit forms of the displacement Green's functions of 3-D elasticity theory G_{ij} and its first-order spatial derivatives are:

$$G_{ij}(\mathbf{x},t;\boldsymbol{\xi},\tau) = \frac{1}{4\pi\mu r}\{[I+\delta(t-\tau-r/c_T)]\delta_{ij} + [-3I-\delta(t-\tau-r/c_T) + p^2\delta(t-\tau-r/c_L)]\gamma_i\gamma_j\} \tag{A1}$$

with

$$I \equiv \int_1^p \mathrm{d}\nu\cdot\nu\delta(t-\tau-\nu r/c_T) = -c_T^2\int_{1/c_L}^{1/c_T}\mathrm{d}\lambda\cdot\lambda\delta(t-\tau-\lambda r) = -\frac{c_T^2(t-\tau)}{r^2}[-H(t-\tau-r/c_T)+H(t-\tau-r/c_L)] \tag{A2}$$

and

$$\frac{\partial}{\partial x_k}G_{ij} = \frac{1}{4\pi\mu r^2}\{[15I+6\delta(t-\tau-r/c_T) - 6p^2\delta(t-\tau-r/c_L)+\frac{r}{c_T}\dot{\delta}(t-\tau-r/c_T) - p^2\frac{r}{c_L}\dot{\delta}(t-\tau-r/c_L)]\gamma_i\gamma_j\gamma_k + [-3I-2\delta(t-\tau-r/c_T)+p^2\delta(t-\tau-r/c_L) - \frac{r}{c_T}\dot{\delta}(t-\tau-r/c_T)]\gamma_k\delta_{ij} + [-3I-\delta(t-\tau-r/c_T)+p^2\delta(t-\tau-r/c_L)] \times(\gamma_i\delta_{jk}+\gamma_j\delta_{ik})\} \tag{A3}$$

$$\frac{\partial}{\partial x_i}G_T = \frac{1}{4\pi\mu r^2}\left[-\delta(t-\tau-r/c_T) - \frac{r}{c_T}\dot{\delta}(t-\tau-r/c_T)\right]\gamma_i \tag{A4}$$

$$\frac{\partial}{\partial x_i}G_L = \frac{1}{4\pi\mu r^2}p^2\left[-\delta(t-\tau-r/c_L) - \frac{r}{c_L}\dot{\delta}(t-\tau-r/c_L)\right]\gamma_i \tag{A5}$$

for the elastodynamic case. In the elastostatic case, they are

$$G_{ij}(\mathbf{x};\boldsymbol{\xi}) = \frac{1}{8\pi\mu r}[(1-p^2)\gamma_i\gamma_j+(1+p^2)\delta_{ij}] \tag{A6}$$

and

$$\frac{\partial}{\partial x_k}G_{ij} = \frac{1}{8\pi\mu r^2}\left[-3(1-p^2)\gamma_i\gamma_j\gamma_k-(1+p^2)\gamma_k\delta_{ij} +(1-p^2)(\gamma_i\delta_{jk}+\gamma_j\delta_{ik})\right] \tag{A7}$$

$$\frac{\partial}{\partial x_i}G_T = \frac{1}{4\pi\mu r^2}(-\gamma_i) \tag{A8}$$

$$\frac{\partial}{\partial x_i}G_L = \frac{1}{4\pi\mu r^2}(-p^2\gamma_i)\ . \tag{A9}$$

Appendix B: Proof to Eq. (30)

Fukuyama and Madariaga (1995) derived their Eq. (D1), or (30) of the present article, in the following way. Define a 2-D polar coordinate system (r,φ) on the surface of a plane crack, with the origin at $\mathbf{x}$ and the moving radius $\boldsymbol{\xi}-\mathbf{x}$. From the relation

$$\left(\frac{\partial}{\partial r}\right)_{\varphi,t}\Delta\dot{u}_i(\boldsymbol{\xi},t-r/c) = -\gamma_\alpha\left(\frac{\partial}{\partial\xi_\alpha}\right)_{t-r/c}\Delta\dot{u}_i(\boldsymbol{\xi},t-r/c) - \frac{1}{c}\Delta\ddot{u}_i(\boldsymbol{\xi},t-r/c) \tag{B1}$$

it follows that

$$\int_\Gamma \mathrm{d}S(\boldsymbol{\xi})\frac{1}{r}\Delta\ddot{u}_i(\boldsymbol{\xi},t-r/c) = \int_0^{2\pi}\mathrm{d}\varphi\int_0^\infty \mathrm{d}r\Delta\ddot{u}_i(\boldsymbol{\xi},t-r/c) = -c\int_0^{2\pi}\mathrm{d}\varphi[\Delta\dot{u}_i(\boldsymbol{\xi},t-r/c)]_{r=0}^\infty - c\int_0^{2\pi}\mathrm{d}\varphi\int_0^\infty \mathrm{d}r\gamma_\alpha\frac{\partial}{\partial\xi_\alpha}\Delta\dot{u}_i(\boldsymbol{\xi},t-r/c) = 2\pi c\Delta\dot{u}_i(\mathbf{x},t) - c\int_\Gamma \mathrm{d}S(\boldsymbol{\xi})\frac{\partial}{\partial\xi_\alpha}\Delta\dot{u}_i(\boldsymbol{\xi},t-r/c)\frac{\gamma_\alpha}{r}\ , \tag{B2}$$

which was to be demonstrated.

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