

8-1-2007

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Non-linear Dynamics and Leadership Emergence

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Abstract: The process by which leaders emerge from leaderless groups is well-documented, but not nearly as well understood. This article describes how non-linear dynamical systems concepts of attractors, bifurcations, and self-organization culminate in a swallowtail catastrophe model for the leadership emergence process, and presents the experimental results that the model has produced thus far for creative problem solving, production, and coordination-intensive groups. Several control variables have been identified that vary in their function depending on what type of group is involved, e.g. creative problem solving, production, and coordination-intensive groups. The exposition includes the relevant statistical strategies that are based on non-linear regression along with some directions for new research questions that can be explored through this non-linear model.

Keywords: Leaderless group; Emergent leadership; Catastrophe model; Creative problem solving; Coordination; Self-organization

The emergence of leaders from leaderless groups is a well-documented phenomenon (e.g. Ansbacher, 1951; Bass, 1949, 1954; Cattell & Stice, 1954). Leaderless group exercises have become a staple in assessment centers for leadership identification. In the traditional research paradigm, group participants might be measured on a number of traits that could possibly be related to leadership behaviors. Members of the group then interact while carrying out a task. Then magic happens, and a leader emerges from the group at the end of the discussion period. The leaders are typically determined by a vote or by questionnaire items that have essentially the same purpose. Cattell & Stice (1954) found that not only did leaders emerge from leaderless discussion groups, but they also found two types of leaders: those who were regarded as the leaders overall by the group members and the technical leaders. Each type of leader displayed a distinctive set of personality traits. As expected, most group members were not identified as leaders.

The process of emergence remained a black box, however, until recently. The non-linear dynamical systems (NDS) concepts of self-organization (Bak, 1996; Haken, 1984; Holland, 1995; Kauffman, 1993, 1995), phase shifts, and catastrophe models for discontinuous changes in events (Thom, 1975; Zeeman, 1977) have unraveled the part of the process where the magic happens in leadership emergence and other social phenomena (Guastello, 1995a, 2002). This article recounts the recent theoretical and empirical studies that have resulted in a generalizable non-linear model for the emergence of leaders. The general model contains some variations depending on whether the group is involved in creative problem solving, production, or coordination-intensive tasks.

1. NDS and catastrophe models for discontinuous change

NDS theory is not simply a group of methods for non-linear data analysis. It is a set of concepts that describe the various ways by which a system can change over time (Abraham & Shaw, 1992; Sprott, 2003). When NDS concepts are applied in psychology, the goal is to build a theory that elucidates how the dynamical concepts of changes in systems occur in a situation, and how psychological

constructs are involved either as order parameters or control parameters.

Order parameters are essentially dependent measures in the social scientist's worldview. There may be more than one order parameter in some complex dynamical systems, however. Order parameters within a system might be completely independent of each other, or they might interact with each other as they evolve over time. *Control parameters* are essentially independent variables, with the important difference that they can act in ways that are more interesting than the simple additive relationships that are found in conventional research designs. Three distinct types of control parameters — asymmetry, bifurcation, and bias — are involved in leadership emergence dynamics, as described in a later section of this article.

The catastrophe models for discontinuous changes in events (Thom, 1975; Zeeman, 1977) involve combinations of attractors and bifurcations. An *attractor* is a stable state of behavior. Elements of a system (objects, people) gravitate toward these stable states and tend to remain there unless a powerful force is applied. A *bifurcation* is a pattern of instability; in the cases considered here, the bifurcations involve the differentiation of a system into multiple stable and unstable states.

According to the classification theorem (Thom, 1975) given a maximum of four control parameters, all discontinuous changes of events can be modeled by one of seven elementary topological models (with qualifications). The models describe change between (or among) qualitatively distinct forms for behavior. The elementary catastrophe models are hierarchical and vary in the complexity of the behavior spectra they encompass. Change in behavior is described by differential equations that represent the structure of the behavior spectrum, or *response surface*. The cusp model that is shown in Fig. 1 is one of the simpler catastrophe models, and it is one that is most frequently used. The *cusp* response surface is 3-dimensional and describes changes between two stable states of behavior (attractors). The two attractors are separated by a bifurcation structure (manifold). The shaded region of the response surface represents a region where

very few points, which represent behaviors (e.g. of people) within the system, are likely to fall.

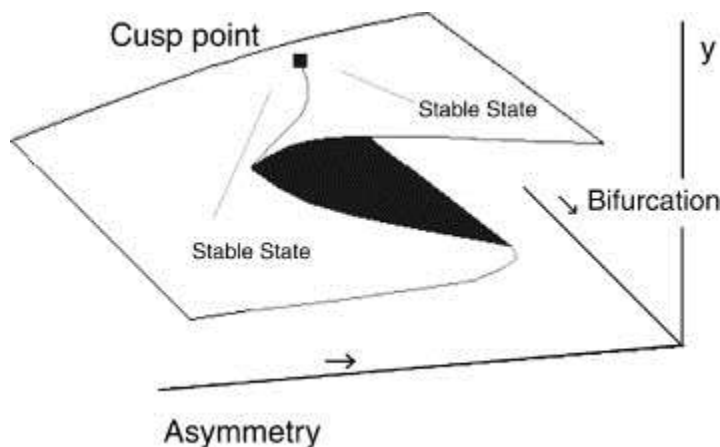


Fig. 1. The cusp catastrophe model.

Movement of points within the system around its response surface is governed by two control parameters. The asymmetry parameter governs how close the system is to discontinuous change in behavior. Imagine that the behavior of the system begins at the lower stable state of the response surface. If the asymmetry variable changes, no change in the behavior of the system is observed until a critical point is reached, where behavior changes suddenly. Behavior can change in the reverse direction, and again no change in the behavior of the system is observed while the asymmetry variable is changing until once again, a critical point is reached. Note that the critical points for moving in the "upward" direction are different from those associated with movement in the "downward" direction.

The bifurcation parameter governs how large the change will be. For large values of the bifurcation variable, change is discontinuous and rather dramatic as the system changes from one stable state to another. For low values of the bifurcation variable, change is gradual and the resulting behavioral states are not stable. The cusp point (shown in Fig. 1) represents the behavioral region of greatest instability and indeterminism. Anything can happen at the cusp point: The system can move to one or the other stable states, with just a little deflection from the control parameters, or it could remain in the unstable area and display small but unstable changes in either

direction. The equation of the cusp response surface for a process that changes over time is:

$$d y / d t = y^3 - b y - a \quad (1)$$

where y is the dependent measure, b is the bifurcation variable, and a is the asymmetry variable.

There is a connection between the self-organizing processes and catastrophe models that has been known for some time. A process that is akin to a phase transition takes place when a system self-organizes from one (resting) state to another (Gilmore, 1981; Kelso, 1995; Puu, 2001; Thompson, 1982; Zhang, 2002). Catastrophe models can be used to model phase transitions at several possible levels of complexity, and their mathematical properties can and should be taken literally.

The research program for leadership emergence does in fact adopt a literal catastrophe model. Leadership emergence requires the use of the swallowtail catastrophe, however, which is the next more complex catastrophe model in Thom's series. The swallowtail model involves three control parameters, two stable states, and an unstable state. The dynamics of the swallowtail model are described in a subsequent section of this article.

1.1. Self-organization and leadership emergence

The non-linear theory behind leadership emergence (Guastello, 1998) was grounded in the rugged landscape model of self-organization (Kauffman, 1993, 1995). The rugged landscape model culminates in the $NK[C]$ function as follows. K represents the number of traits that organisms must possess to survive in their particular ecological niche. N refers to the number of organisms that have adapted to a niche by virtue of a particular number of traits; one-trait organisms would be the most common, two-trait organisms would be less common, and not so many would survive in niches that require five (perhaps) traits to adapt there. C refers to the complexity of interactions within a niche. If C is large, the landscape is said to be

rugged, meaning that it is more challenging to join that niche or, once there, leave it for another one.

As leaderless groups interact while performing a task, their members become differentiated into primary leaders, secondary leaders, and the majority of the group who remain non-leaders after the differentiation process has occurred. The resulting frequency distribution would take the form that is shown in Fig. 2. The horizontal axis corresponds to K in the Kauffman's $NK[C]$ function, and represents the number of traits that are associated with the social niche that a person occupies when the group self-organizes. Primary leaders are in the mode furthest to the right in the diagram. Secondary leaders of various sorts occupy the statistical mode in the middle. The vast majority occupies the large mode at the left. The large mode is technically unstable, meaning that members of this subgroup could wander into a leadership mode if the values of control parameters were conducive. The vertical axis is N , the number of cases associated with a value of K , in both the $NK[C]$ function and in most any general frequency distribution.

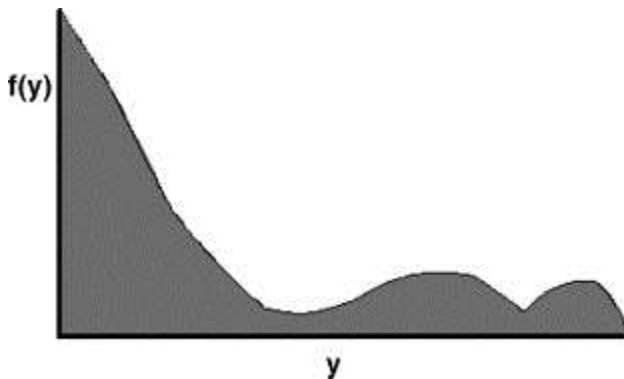


Fig. 2. An interesting slice of the swallowtail catastrophe distribution showing the presence of stable states, the unstable state, and separations among the states.

The K traits can be defined psychologically as personality traits, insofar as personality traits play out in actual social behaviors. More proximally to the present model, however, the K traits can be defined as social contributions or schemata in work-related conversations, such as asking questions, giving answers, initiating a new path of discussion, facilitating the expression of ideas by others, following a line of reasoning started by someone else, and so on (Bottger, 1984; Guastello, 1995a,b). Leaders tend to have wider repertoires of

conversational behavior than non-leaders. According to Graen & Uhl-Bien (1995), the building block of leadership is the dyadic relationship between the leader and each of the members, and the quality of the social exchange and reciprocity among them. A high-quality interaction would be characterized by four principles — loyalty, respect, contribution, and positive affect — that comprise a single indicator of LMX (Liden & Maslyn, 1998). High ratings on LMX have been associated with work outcomes such as individuals' work performance, job satisfaction, satisfaction with supervision in particular, increased role clarity and reduced role conflict, and leader–member agreement (Gerstner & Day, 1997). When enough interactions have occurred, some people will attract more interactions than others will, hence leaders and other roles will emerge from the group (Graen & Uhl-Bien, 1995). Thus local interactions give rise to global phenomena (Zaror & Guastello, 2000).

Research on leadership emergence indicates, however, that the constellation of K traits is just one of three control parameters that are apparently involved in the process (Guastello, 1998; Guastello, Craven, Zygowicz, & Bock, 2005; Zaror & Guastello, 2000). Group interactions can range from light socialization to task-specific insights and problem solving. According to Kauffman's model, the C factor signifies the complexity of interaction of agents within a (virtual) ecological niche. The complexity of interaction is observable as a variety and quantity of conversational behaviors such as asking questions, offering creative ideas, expanding on the ideas of others, facilitating the expression of others, and so forth. In this context, ruggedness would take the form of distinct role separations among the participants in a group from which leaders emerge, and thus in the distinctiveness of the modes of density in Fig 2.

Asymmetries in members' interaction patterns eventually occur whereby some group members become more central to the group's interaction pattern than do other members. When this asymmetry occurs, group members will have self-organized into roles that exhibit broad leadership or secondary leadership (Guastello, 1998; Guastello et al., 2005; Zaror & Guastello, 2000). This principle is consistent with leader–member exchange theory (Graen & Uhl-Bien, 1995), which characterized the dyadic communications between a would-be leader and individual members of the group as the basic building blocks of

the leadership process. It appears from several viewpoints that the quality of the communications between the would-be leader and members are more important than the sheer quantity of communications (Bass, 1990; Bonito & Hollingshead, 1997; Bottger, 1984; Fisher, 1986; Graen & Uhl-Bien, 1995; Guastello, 1995a; Guastello et al., 2005).

Secondary leadership might reflect particular social contributions such as technical contributions or conflict resolution (Cattell & Stice, 1954). Bales (1950) distinguished between task leaders and process leaders in the early stages of his group process theory. Bales (1999) explicated later that many possible group structures form, dissolve and reorganize in addition to leadership emergence.

The particular leaders that emerge in a given situation will be predicated on the task complexity, information requirements, performance verifiability (Hirokawa, 1990), and group's preferences for dominant, considerate, or radical thinking on the part of their leaders (Bales, 1999). It is also apparent that the type of task governs what traits or behaviors are most relevant for leadership emergence (Barge, 1996; Guastello et al., 2005; House & Mitchell, 1975; Kolb, 1992; Neuman & Wright, 1999; Zander, 1994). Thus the research program described here includes different types of tasks where the dynamics of leadership emergence could play out differently, even though the fundamental dynamic process could be the same. The core dynamics are described next, followed by an elaboration of some different types of tasks and how they might impact on the leadership emergence process.

1.2. Swallowtail catastrophe model

The presence of self-organizing processes might suggest an inverse power law (Bak, 1996) or a catastrophe model (Guastello, 2002; Puu, 2001; Zhang, 2002) as a descriptive probability density function. Empirical studies illustrate, however, that the swallowtail catastrophe model is an excellent fit for leadership emergence data (Guastello, 1998; Guastello & Bond, 2007a; Guastello et al., 2005; Zaror & Guastello, 2000), and better suited than an inverse power law (Guastello, 2005a). In fact, the peculiar distribution shown in Fig. 2 is

actually unique to the swallowtail catastrophe model. The visual similarity between the distribution in Fig. 2 and the plot of N vs. K (Kauffman, 1993, p. 130, and reprinted in Guastello, 1998, p. 307) should be readily apparent.

The swallowtail response surface is shown in Fig 3. The swallowtail model distinguishes two stable states and a large unstable state. The unstable state is separated from the two stable states by a substantial antimode. The separation areas are created by an underlying bifurcation structure that is a critical part of the model. Once again, the shaded regions represent areas where points, which represent the behavior of the system (or people within it), are unlikely to fall. Because the response surface is four-dimensional, it must be presented in two three-dimensional sections. The equation for the response surface is shown in Eq. (2):

$$df(y)/dy = y^4 - cy^2 - by - a \tag{2}$$

where y is the dependent measure, i.e., an index of leadership, and a , b , and c are control parameters.

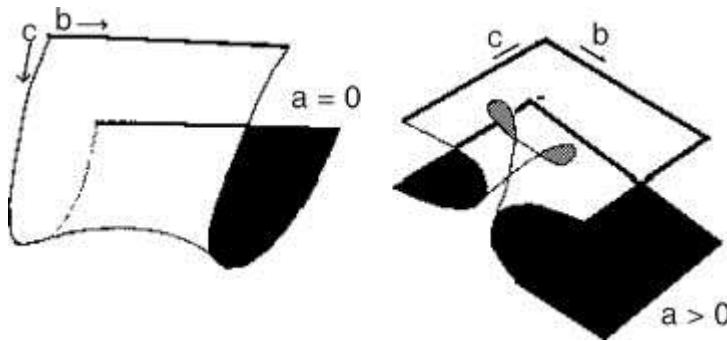


Fig. 3. The swallowtail catastrophe response surface, shown as two three-dimensional sections.

Control parameter a , also known as the asymmetry parameter, governs the broad distinction between two subsections of the swallowtail response surface. For low values of a , data points, which represent people in the system, will either fall within a single mode associated with non-leaders, or be sent to an undefined place somewhere else. At higher values of a , "somewhere else" is the other subsection of the surface where it is possible to see two possible stable

states (attractors), and points can move around the surface between the stable and unstable states. The separation among the stable and unstable states is produced by a bifurcation mechanism that gives rise to the three control parameters.

Control parameters b and c determine whether a point will fall into one of the two distinct stable states, or in the shaded and unstable region between them. Control b is also known as the bifurcation parameter, and it denotes the extent to which points move from the ambiguous area of the surface (rear) to the stable states that signify leadership roles (unfolded portion, front).

Control c is also known as the *bias* or *swallowtail* parameter. The bias parameter c , is responsible for distinguishing between the primary and secondary leadership roles. Points can also move between the two stable states so long as the asymmetry parameter remains high. If a drops too far in value, however, the point makes a discontinuous shift back to the unstable state on the left-hand (in Fig. 3) portion of the surface.

2. Research design

The research design for studying leadership emergence within the swallowtail catastrophe paradigm is described below in generic form. Several parts of the process are not especially different from what would be used in a conventional research design that involves leaderless groups. The analysis, however, is unique to the swallowtail model.

2.1. Participants and procedure

Participants in the studies conducted thus far have involved college students who were recruited from the usual sources and assigned by the experimenters to groups with average sizes of 4 to 14 people. The matter of group size is interesting in its own right, and it will be discussed separately later on.

Three experimental tasks have been utilized thus far. One was a creative problem solving task ("Island Commission" by Gillan, 1979, 2002) that involved average group sizes of 8 or 14 people (Guastello,

1998; Guastello et al., 2005; Zaror & Guastello, 2000). One was a simplistic production task that involved negligible prior skill and an average group size of 7 people (Guastello et al., 2005). The third was a coordination-intensive task that involved a card game and group sizes of 4 people (Guastello & Bond, 2007a).

3. Measurements

In each type of task situation, participants completed a brief questionnaire at the end of the game that asked who was most like the leader and second most like the leader. The specific instruction was: "Mark a '1' in the space to the left of the name of the person who acted *most* like the leader of the group. If no member of the group acted like the leader, you can check the response, 'No member of the group behaved in this fashion.' Mark a '2' in the space to the left of the name of the person who acted *second most* like the leader of the group. You only need to mark the top two people."

The study of coordination-intensive groups involved an experimental manipulation wherein the participants were not allowed to talk. In that case the instructions continued: "If there was no such second group member, you can mark a '2' in the space to the left of the response, 'No member of the group behaved in this fashion.'"

For the data analysis, persons who were designated as most like the leader by a particular participant received a score of 2, and those who were designated as second most like the leader were given a score of 1. All others received a score of 0. The ratings that were given to each person were summed over the ratings that were received from all the participants in the group. Thus for a group of 8 people the scores ranged from 0 to 16.

This strategy for obtaining leadership was validated against videotapes that were coded by independent observers for a variety of conversational behaviors (Guastello, 1995a,b). The ratings of leadership were significantly correlated with the observed frequencies of occurrence of information seeking, information giving, tension reduction, clarifying responses and ideas, gatekeeping, initiating a stream of discussion, and following the ideas of others. This core set of conversational contributions originated with the work of Benne &

Sheats (1948); the questionnaires also contained questions about various aspects of leadership style that had become salient in the leadership literature since that time. The original questionnaires appear in the original articles (Guastello & Bond, 2007a; Guastello et al., 2005), and summaries of the concepts that turned out to be relevant appear in Table 1.

Table 1. Summary of results from leadership emergence studies with the swallowtail catastrophe model^a

Indicator	Creative problem solving groups	Production groups	Coordination-intensive groups
Asymmetry	General participation and control of the conversation incl. gatekeeping, initiating following, harmonizing facilitating the ideas of others, task orientation consideration of other players' interests, concern for solution quality.	Tension reduction incl. harmonizing, giving information, goal realism.	General participation and control of the conversation; incl. gatekeeping, initiating, following, creative ideas, facilitating the ideas of others.
Bifurcation	Giving information, creative ideas, competitive behavior concern for solution quality.	Creative and task control, controlling the conversation	Verbal vs. non-verbal working conditions
Bias	Unknown	Unknown	Task control
R^2 pdf	> .99	> .99	> .99
R^2 with control variables	.74	.62	.61
R^2 linear comparison	.80	.75	.58

^aSummarized from Guastello et al. (2005) and Guastello and Bond (2007a).

4. Analyses

Although there were some opportunities for conventional statistical analyses, only those germane to the swallowtail hypothesis are described here. An important choice has to be made here. There are two possible approaches to the analysis of catastrophe data. One requires that the dependent variable be measured at two points in time, and it can be accomplished through polynomial regression within the general linear model (Guastello, 1985, 1992, 1995a, 2002,

2005b,c). The other approach only requires that the dependent measure be observed at one point in time, but it requires analysis through non-linear regression. The latter choice was preferred for the leadership emergence studies for two reasons. First, there was a concern that any attempt to measure leadership before the group session was completed could contaminate the final ratings of leadership or other variables. Second, strictly speaking, the true Time 1 measurement of leadership for all participants in a leaderless group is 0.0, and thus there is no variation among the participants.

The swallowtail model was tested in two phases of analysis. One involved a test of the swallowtail model structure without testing for specific control variables. The second included hypotheses concerning the control variables. Potential control variables resulted from a factor analysis of the questionnaire (without the leadership variable) and from experimental manipulations that were also introduced.

The swallowtail pdf is shown in Eq. (3) and Fig. 2:

$$\text{pdf}(z) = \xi \exp \left[-z^5/5 + z^4/4 + cz^3/3 + bz^2/2 + az \right], \quad (3)$$

where z is the leadership measurement and a , b , and c are the control parameters.

The variable y in Eq. (2) is transformed into z in Eq. (3) with respect to *location* (λ) and *scale* (σ_s), as defined in Eq. (4):

$$z = (y - \lambda) / \sigma_s \quad (4)$$

In most discussions of probability functions, "location" refers to the mean of the function. The pdf for a non-linear dynamical process is a member of an exponential family of distributions and is asymmetrical, unlike the so-called normal distribution (Cobb, 1981; Guastello, 2005a). Thus the location parameter for Eq. (3) is (usually) the lower limit of the distribution, which is the lowest observed value in the series. The transformation in Eq. (3) has the added advantage of fixing a zero point and thus transforming measurements with interval scales (common in the social sciences) into ratio scales. A

fixed location point defines where the non-linear function is going to start. The same transformation is made on variables that are scheduled to be used as potential control variables a , b , or c .

The scale parameter in common discussions of pdfs is (usually) the standard deviation of the distribution. The standard deviation is used here also. The use of the scale parameter later on while testing structural equations serves the purpose of eliminating bias between two or more variables that are multiplied together. Although the results of linear regression are not affected by values of location and scale when the independent variables are simply additive, non-linear models are clearly affected by the transformation.

The pdf is tested as a non-linear regression model in Eq. (5):

$$\text{pdf}(z) = \xi \exp[\theta_1 z^5 + \theta_2 z^4 + \theta_3 cz^3 + \theta_4 bz^2 + \theta_5 az] \quad (5)$$

Note where the regression weights θ_i are inserted in Eq. (5). ξ is also treated as a regression weight. Pdf(z) is the cumulative probability of z within the distribution, i.e., a probit transformation of z .

If the control parameters are not known yet, variables a , b , and c , in Eq. (5) can be ignored. This was the starting point in evaluating a swallowtail hypothesis. In the initial research (Guastello, 1998; Zaror & Guastello, 2000), the term $\theta_2 z^4$ was dropped from the model in order to register significant weights for the remaining terms. $\theta_2 z^4$ is not formally a part of the swallowtail function; rather it is introduced as an additional correction for location (Guastello, 2002). $\theta_2 z^4$ might have displayed statistical significance if one part of the response surface was more strongly represented by the data than another part of the surface; this actually happened in the analysis for coordination-intensive groups (Guastello & Bond, 2007a). Further elaboration on the statistical theory that is pertinent to non-linear dynamics appears elsewhere (Guastello, 1995a, 2002, 2005a,b,c).

Non-linear regression is available on the more comprehensive statistical packages. A few procedural steps are outlined here to help

orient first-time users of this medium of analysis, which is actually over 40 years old. The first step is to define a model and to specify where the regression weights should go, including any constants. The second step is to specify initial values for the regression weights. If there is no reason a priori for picking one value over another, one might start by using a small value such as + 0.5 for each parameter. The third step is to choose an error function; least squares is usually the default choice.

The non-linear regression algorithm then proceeds through an iterative process (Newton–Raphson search) of fitting the data to the model, fitting the data to a derivative of the model, adjusting the parameters, and fitting again. The process continues until any further adjustments produce negligible differences in parameter estimates. The calculation of results concludes with an ANOVA table, overall R^2 , and confidence intervals on each of the non-linear regression parameters. Detailed elaborations on the theory behind these computational steps can be found elsewhere (e.g. Ratkowski, 1983; Seber & Wild, 2003; Stortelder, 1998).

Non-linear regression offers hypothesis testing of the standard Neyman–Pearson variety on each of the regression weights. In optimal circumstances, all weights would attain significance at the .05 level. If this is not the case, one should consider dropping the parameter of least theoretical value, such as θ_{2z^4} in Eq. (5).

The next step is to compare R^2 for the non-linear model against a linear model that is composed of the same dependent measure and the same independent variables. Ideally the R^2 for the non-linear model should exceed the R^2 for the linear counterpart. There is no significance test here because, logically, the non-linear model should be at least as good as the linear alternative. Historically, in the cases where a comparison of R^2 was available and the conclusion favored the non-linear interpretation or theory, the average R^2 coefficients favored the non-linear conclusion by a ratio of 2:1 (Guastello, 1995a, 2002).

4.1. What eventually emerged

In the big picture of NDS and organizational behavior, the fundamental notion of the nature of the organization has evolved from

a mechanistic bureaucracy, to a humanistic enterprise, to an organic living system, to a complex adaptive system (Anderson, 1999; Dooley, 1997; Guastello, 2001, 2002; Kiel, 1996; Osborn, Hunt, & Jauch, 2002; Yuan & McKelvey, 2004). The complex adaptive system places a strong emphasis on an organization's ability to enact successful creative problem solving as matter of routine. Coordination is another special capability of a complex adaptive system. Routine production is not as glamorous, but one would hope that the organizations of the near future would still have something to produce that is measurable in quantities.

4.2. Creative problem solving groups

This notion of a complex adaptive system took form several years after two other important lines of thought concerning leadership congealed. In one, Bass (1985) introduced the concept of the transformational leaders who encouraged their constituencies to think differently, which often meant creatively, and who also made noticeable intellectual contributions to their groups' work. In another, Simonton (1988) pointed out the close connection between leadership and creative excellence, noting that it is an act of leadership to encourage people to think differently, as a good many creative scientists and artists have done (Guastello, 1995b). Thus it seemed reasonable to start the plan of study of NDS and leadership emergence with creative problem solving groups.

In the swallowtail model for creative problem solving groups (Guastello et al., 2005), the asymmetry parameter was defined by a large group of social contributions, not unlike what one would expect from Kauffman's *K* parameter: clarifying responses and ideas, gatekeeping, initiating, following, harmonizing, facilitating the ideas of others, controlling the conversation, task orientation, consideration of the other players' interest (as defined by their roles in the game), and concern for the quality of the game outcomes. This group of behaviors was defined collectively as *controlling the conversation* (Guastello et al., 2005).

The bifurcation parameter was composed of four variables, which were collectively defined as *creativity*: giving information, creative ideas, competitive behavior, and concern for the game

outcomes. Here the latter two variables indicated a strong immersion in the creative role-play. Although another variable, *tension reduction*, was identified in the questionnaire data, it had no effect when tested as any of the control parameters. The bias factor for creative problem solving groups remains unknown.

4.3. Production groups

The study of production groups involved a game where the group members had to plan some work then produce two, three, or four dozen units of work in a fixed amount of time (Guastello et al., 2005). The "work" was simplistic and repetitive. It is noteworthy that the groups that were assigned the goal of producing two dozen units actually produced more work than the groups that we assigned three or four dozen units. Hence the variable *goal realism* entered the study, where the two-dozen condition was scored as high realism and the other two conditions were scored as low-realism.

The results of the study showed that goal realism and tension reduction both contributed to the asymmetry parameter for leadership emergence in production groups. Two variables contributed to the bifurcation parameter, which contributes to the strength of the separation of the two modes for primary and secondary leaders: *task control* and *creative control*. Creative control was a combination of creative ideas and controlling the conversation during the first phase of the game. Task control was a combination of task orientation, clarifying responses and ideas, gatekeeping, following, and controlling the conversation. The bias parameter, which would sort participants into primary or secondary leaders, also remains unknown for production groups.

4.4. Coordination-intensive groups

Coordination in a work group occurs when group members perform either the same task or reciprocal tasks at the appropriate time to facilitate a group performance objective. Here the work of sports teams, theatrical performers, hospital emergency room personnel, and some military and industrial operations come to mind. According to the game theory (von Neumann & Morgenstern, 1953),

there are a few different forms of coordination, not just one as depicted in conventional group dynamics research. Prisoners' Dilemma is perhaps the best known coordination game, where the two players must select the cooperation option, instead of the competition option, at the same time in order to benefit from the option. Three others are strictly cooperative, and do not include competitive options: Bandwagon (Guastello & Philippe, 1997), Stag Hunt (Guastello & Bond, 2004), and Intersection (Guastello & Bond, 2007a; Guastello & Guastello, 1998).

The leadership emergence studies for coordination-intensive groups (Guastello & Bond, 2007a,b) addressed the type of coordination that occurs at 4-way stop intersections where the drivers who approach the intersection must figure out what turn-taking rules are in play and when it is their turn to proceed through the intersection. For pragmatic reasons the logic of the Intersection game was converted to a card-playing game.

The coordination studies also utilized an experimental manipulation whereby the groups were allowed to talk in some conditions, but not allowed to talk in other conditions. This manipulation was suggested from other NDS studies in coordination where it was shown that leaders were not necessary for coordination to occur. Of course, leadership theorists would expect leaders to emerge nonetheless; indeed they did, but to no greater extent in verbalizing groups than in non-verbal groups when the analysis was confined to simple comparisons of mean ratings on the leadership variable (Guastello & Bond, 2007b).

The results also indicated that leaders did emerge in a manner that was consistent with the swallowtail function (Guastello and Bond, 2007a). All three control parameters were successfully identified for leadership emergence in coordination-intensive groups. Parameter *a* was a *general participation* factor that was about as broad in scope as the *a* that was obtained for creative problem solving groups. Parameter *b* was whether the group worked *verbally or non-verbally*; people in verbalizing groups were more often distinctively associated with the primary leadership mode of the response surface. Parameter *c* was task control, which consisted of asking questions, controlling the card play, task orientation, and competitive behavior.

Table 1 summarizes the findings from the experiments described above. In all cases the R^2 for the swallowtail pdf was greater than .99, indicating that the data corresponded very well to the swallowtail catastrophe dynamics as hypothesized. Similar results were reported previously as well (Guastello, 1998; Zaror & Guastello, 2000).

When the control variables were entered into the model, the R^2 remained high, but did drop somewhat. This is not unusual. Unlike ordinary linear regression, non-linear regression does not produce an increase in R^2 simply because a new variable is added. Additional variables could lower the R^2 because new chances for error are introduced.

The R^2 for the linear comparison models was also very high, and higher than that obtained for the swallowtail model in two out of three cases. Ideally, the R^2 for the non-linear model should exceed the R^2 for the linear model, but the results are encouraging nonetheless, because we are still looking for parameter c for two of the models. A third possible control variable was identified in the factor analysis of questionnaire items in both the experiments for creative problem solving and production groups, but it did not result in a significant regression weight and were thus discarded in each case. The search continues for better candidates for parameter c , and it is probable that the R^2 for the completed model would supersede that of the linear model when the last control variable is put into place.

Researchers in this area should be aware of another quirk in the data. All the questionnaire items displayed an exponential distribution, which would help to increase *some* of the linear correlations with leadership variable (others were rather low). The questionnaire items probably would not have been constructed to display exponential distributions if it were not for the presumption of a potential non-linear dynamical process. The participants rated each other as (for instance) most like the leader in the group (score 2 points), and second-most like the leader (score 1 point); no other points were assigned to any other rankings. The odds of being most like the leader in the mind of one person are .125 for a group of 8 people. When the ratings are added together over all people in a group for each person in the group, an exponential distribution results; this distribution will occur by virtue

of adding together scores on several dichotomous variables where membership in one of the categories is rare, no matter what the underlying frequency distribution of the items happens to be. The exponential structure was "complexified" a bit by allowing for the option of identifying the person who was second-most like the leader, thus producing the possibilities of the statistical mode for secondary leaders. The same statistical process was occurring in the definition of control variables, but with more responses added together in most cases. Thus linear correlations between exponentially distributed variables would be unusually large by virtue of the values indicating non-leaders being consistent for most people in the study. In short, the linear models capitalized on a scaling attribute that probably would not have been introduced under the assumptions of simple linear models and normally distributed variables. It should be mentioned in addition, however, that many of the catastrophe studies on record involved normally distributed control variables along with catastrophically distributed behavioral outcome variables. This mismatch in the shapes of frequency distributions would produce a challenge for the linear model and its assumptions, but would be very friendly to the non-linear models.

A related issue was that there was a high correlation between the leadership variable and another questionnaire item, control of the conversation, which even displayed a swallowtail distribution itself on one occasion (see Guastello et al., 2005 for details). This observation suggested that the group participants were equating leadership with control issues. Perhaps that is not the concept of leadership that leadership theorists would want to encourage in all circumstances.

5. Discussion and future research directions

In light of the known relationships between catastrophe models, phase shifts, and self-organizing phenomena, the swallowtail functions for the emergence of leadership from a leaderless group indicate both the conceptual and empirical signs of a self-organizing process. The same structural model generalizes for three different types of work environments. This point by itself tells us something about the proverbial magic that we did not know beforehand. The swallowtail model describes a bifurcation mechanism with a set of control

variables that explain what the control variables in the process actually do, and there are three specific functions. These functions would not have been made obvious by any known form of linear analysis. Knowledge of the mechanism also tells us that if we have succeeded in finding two out of three control variables, there is a third lurking in the system somewhere. In the cases of creative problem solving and production groups, we might not know what parameter *c* is, but we do know what it should be doing when we find it. Verbalizing vs. non-verbalizing groups was identified as parameter *c* for coordination-intensive groups, and this finding might suggest some clues as to where to look to complete the models for the other types of groups.

First impressions indicate that different psychological variables were operating as control parameters in each type of situation. One recurrent theme, nonetheless, was that a thick constellation of characteristics or behaviors contributed to parameter *a* in two out of three cases. In production groups, however, parameter *a* was composed of tension reduction and goal realism. It would appear that leaders of the usual sort would not become apparent under conditions where the goal was felt to be impossible and success was not likely. Tension reduction was a factor emanating from the questionnaire in all three applications, but it only worked as a control variable in the production problem — in precisely the location where other traits would have appeared. The production situation apparently called for someone to keep the spirits up in the face of absurdity. Task and creative control were involved in all three types of situations, although the role of these variables as control parameters shifted from one situation to another.

The research on emergent leadership that is described here should be regarded as a beginning of a line of inquiry, rather than an endpoint. For instance, only one type of creative problem solving, production, or coordination-intensive group has been tested thus far, and it would be reasonable to determine whether similar results would be obtained for other examples of the tasks, especially where the control parameters are concerned. Not to make matters any less complicated, coordination is actually a group of related phenomena, not a singular one (Guastello, 2002), and research on leadership emergence in other types of coordination tasks is now in progress.

Parameter c was identified in only one of the three cases. It was speculated (Guastello et al., 2005) that c might be related to group homogeneity or heterogeneity. The tasks were performed to date by college student populations who did not have a great personal investment in the game, and could be considered relatively homogenous overall. Real-world groups with real-world tasks could show more colorful results, especially when real differences in expertise and perspectives were in evidence.

Another avenue of pursuit might invoke some variables and measurements that could be used as predictors in a personnel selection and development context. The present examples relied on post-game assessments of behaviors that were exhibited during the work and conversations. What psychological constructs, abilities or traits correspond to those behavior clusters? It is probably the case that personality traits have been overworked in the conventional leadership literature. It would be at least as interesting, however, to associate personality and other person characteristics with particular things that leaders do, and thus make more specific associations between traits and control parameters.

Group size is another aspect of leadership emergence that needs to be investigated systematically. Creative problem solving groups perform better when there is a critical mass of people and ideas to get the job done (Dennis & Valacich, 1993). The optimal size of production groups is probably going to vary with the nature and complexity of the work in question, and some very curious non-linear dynamics have been recorded for (hierarchical) production groups under conditions of downsizing (Guastello & Johnson, 1999). There are a few precedent pieces of research on the matter concerning leadership and group size that are separated in time. Bass and Norton (1951) studied leaderless groups of sizes ranging from 2 to 12 people and found that the maximum stratification among participants occurred at group sizes of 6. Bonito and Hollingshead (1997) reported that the leader's share of talking time increased as the group size increased from 4 to 10 participants. The swallowtail model, however, seems to hold up consistently for group sizes in the range from 6 to 18 people. The model became a bit degenerated in the coordination groups, however, in that the left and right portions of Fig. 3 were not clearly separated (Guastello & Bond, 2007a); thus secondary leaders

were not differentiated very well from the non-leaders. A possible explanation is that the coordination task required equal participation from each participant in some aspects of the work, and this requirement was not built into the other types of tasks.

Hierarchies pose another class of research questions. The first vestiges of a hierarchy in a group form when primary and secondary leaders emerge. It is unknown whether the rules for further leadership emergence change once a particular hierarchy has solidified. One piece of evidence suggests that they might indeed change: Bass and Wurster (1953) studied a group of industrial supervisors with different company ranks, and their findings indicated that prior rank distorted the perceptions of leadership when those people were placed in experimentally designed leaderless group discussion.

Thus as the old saying goes, "more research is needed." So far we know that the swallowtail catastrophe model does a decent job of capturing the dynamics leading up to the emergence of leaders from leaderless groups in three distinctive types of task. As such it captures the self-organization dynamics in the social process. The nature of the control variables changes from type of task to another. Two out of three control parameters have been identified for two types of task, and all three parameters were identified for a third type of task. The possible interactions between group size and task type could produce some interesting variations in the dynamics, and these possibilities need to be explored further. The full range of task possibilities has not been explored, nor have the contrasts that could exist between groups and tasks that are contrived in a laboratory and those that exist in the real-world, where people have vested personal interests in the group's outcomes. Here the possible influences of hierarchies and past relationships among the participants have not been explored.

To summarize, leadership emergence is a self-organizing process that starts with bilateral interactions among group members. Eventually a phase shift occurs wherein a group structure emerges with a primary leader, secondary leader, and non-leaders. The phase shift is aptly described by the swallowtail catastrophe model, which contains three control parameters. Two of the three control parameters have been identified for creative problem solving and production groups; the search continues for the third control

parameter. All three control parameters were identified for coordination-intensive groups of one particular variety of coordination. Additional questions remain unanswered concerning the nature of control parameters for other types of groups, and the impact of group size and pre-existing hierarchical structures on the dynamics of leadership emergence.

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