NON-LINEAR MODEL PREDICTIVE CONTROL: A PERSONAL RETROSPECTIVE†

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An overview of non-linear model predictive control (NMPC) is presented, with an extreme bias towards the author's experiences and published results. Challenges include multiple solutions (from non-convex optimization problems), and divergence of the model and plant outputs when the constant additive output disturbance (the approach of dynamic matrix control, DMC) is used. Experiences with the use of fundamental models, multiple linear models (MMPC), and neural networks are reviewed. Ongoing work in unmeasured disturbance estimation, prediction and rejection is also discussed.

On présente un aperçu général du contrôle prédictif par modèles non linéaires (NMPC), en mettant l'accent en particulier sur les expériences des auteurs et les résultats publiés. Les défis incluent des solutions multiples (à partir des problèmes d'optimisation non convexes), ainsi que la divergence entre les sorties de modèle et d'installation lorsque la perturbation de sortie additive constante (la méthode du contrôle de matrice dynamique, DMC) est utilisée. Les expériences avec les modèles fondamentaux, les modèles linéaires multiples (MMPC) et les réseaux neuronaux sont examinées. Le travail actuellement mené sur l'estimation, la prédiction et le rejet des perturbations non mesurées est également examiné.

Keywords: process control, non-linear control, state estimation, optimization, process models

INTRODUCTION

Model predictive control (MPC) has been the most successful advanced control technique applied in the process industries. The formulation naturally handles time-delays, multivariable interactions and constraints. Particularly in the petrochemical industry, MPC has often been tuned for robustness rather than a high level of dynamic performance. In addition to conservative tuning, performance has been limited by the use of linear models and the standard "additive output disturbance" assumption to compensate for plant-model mismatch.

There is a wealth of articles on MPC in general, with many different formulations and applications of non-linear MPC in the literature; my goal is not to review these articles. Rather, my goal is to provide a personal perspective of NMPC. My primary audience is graduate students and others just beginning their foray into model predictive control research and application. The MPC literature can be quite challenging to read and it is not always clear why certain implementation decisions are made. Rarely are the particular pitfalls fully presented. One of my goals is to point out some of the pitfalls of using fairly intuitive approaches to implementing MPC and NMPC. While I certainly do not do justice to the many contributions of other researchers involved in MPC, through my limited citations, I hope that the reader obtains a clearer perspective on some of the issues of critical importance. While much of the article is focused on non-linear MPC, I close the article with a discussion of disturbance rejection, using some simple linear examples.

For general reviews of model predictive control, see Bequette (1991a), Henson (1998), Morari and Lee (1999), Mayne et al. (2000) and Qin and Badgwell (2003). A tutorial introduction to dynamic matrix control (DMC) is presented in my process control textbook (Bequette, 2003), and the paper by Shah (1995); also, the books Bitmead et al. (1990), Camacho and Bordons (2004), Maciejowski (2002) and Rossiter (2003) derive various MPC algorithms, generally with a focus on linear systems.

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† *This article is an extension of a keynote talk given at the CSChE Conference in Sherbrooke, QC, October 2006.*

MOTIVATING NON-LINEAR BEHAVIOUR

Examples of challenging problems that may justify NMPC are shown in Figure 1. Systems with *input multiplicity*, where a single output value may be obtained from at least two different input values, have a region where the process gain changes sign. No fixed parameter linear controller with integral action can be designed for closed-loop stable behaviour at operating points on different “sides” of this peak. An additional problem is that systems with input multiplicity often have a non-minimum phase zero (resulting in “inverse response” or “wrong-way” behaviour) region on one side of the peak, as shown by Sistu and Bequette (1995). This results in an additional complication even for a non-linear strategy that accounts for the change in the sign of the gain of a system with input multiplicity. In the region with the NMP zero there is an inherent closed-loop performance limitation that may not exist in the other region. Thus, even a non-linear controller should be designed for different closed-loop performance depending on the operating region.

Systems with multiple steady-states (or *output multiplicity*), such as the example shown in Figure 2, can sometimes be stabilized with a single fixed parameter linear controller with integral action, but the closed-loop performance usually degrades substantially as the operating point is changed.

A typical cascade control strategy to regulate temperature in a continuously stirred tank reactor (CSTR) is shown in Figure 3. Here, the output of the reactor temperature controller (primary) is a set point for the jacket temperature controller (secondary), which manipulates the make-up jacket flow rate. In many control studies the effect of the secondary loop is neglected and it is assumed that the jacket temperature is directly manipulated to control the reactor temperature; this results in a two-state model, where concentration and reactor temperature are the states. An example steady-state input-output curve for reactor temperature as a function of jacket temperature is shown in Figure 4 (top). While this relationship is monotonic (no multiplicity), and the two-state model is often open-loop stable throughout the entire range, the actual relationship between jacket flow rate and reactor temperature, as obtained from a three-state model, may have multiple steady-state behaviour with open-loop unstable regions, as shown in Figure 4 (bottom). Here, by using a two-state model, one is inherently assuming a stabilizing secondary controller. This has important safety ramifications, because, if the secondary controller saturates or the loop is opened, the system can go unstable, resulting in either ignition (high temperature steady-state) or extinction (low temperature steady-state). My main point is that it is extremely important to understand particular problems that may arise when simplifying control strategies. Russo and Bequette (1997) provide a more detailed discussion on this topic, while Russo and Bequette (1995, 1998) present comprehensive studies of the impact of reactor design on operability.

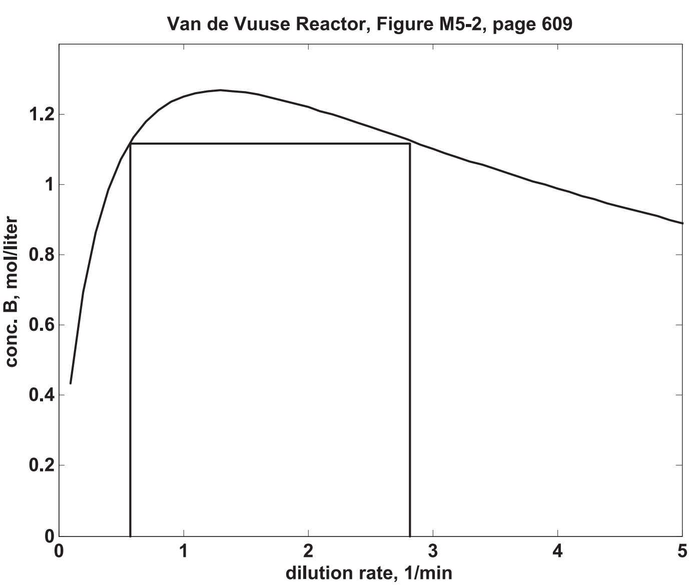

Figure 1. An example of input multiplicity, where a single desired steady-state output value may result from two different input values. For this particular example, the steady-state on the left-hand side also has a non-minimum phase zero (unstable inverse); Sistu and Bequette (1995).

OVERVIEW OF MODEL PREDICTIVE CONTROL

The basic concept of model predictive control is illustrated in Figure 5. A time step k, a sequence of M control moves (to be

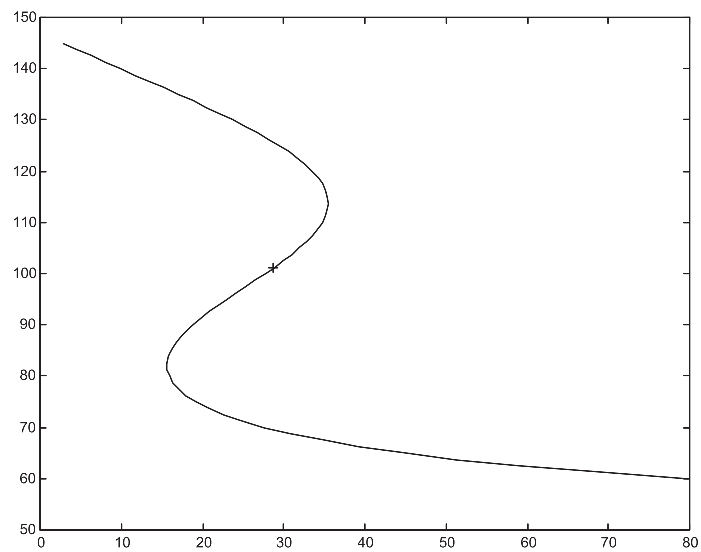

Figure 2. Output multiplicity (multiple steady-states) where there is a region where a single input value can result in several different output values; in this case, the intermediate output value is always unstable, while the lower or upper steady-state values are often open-loop stable

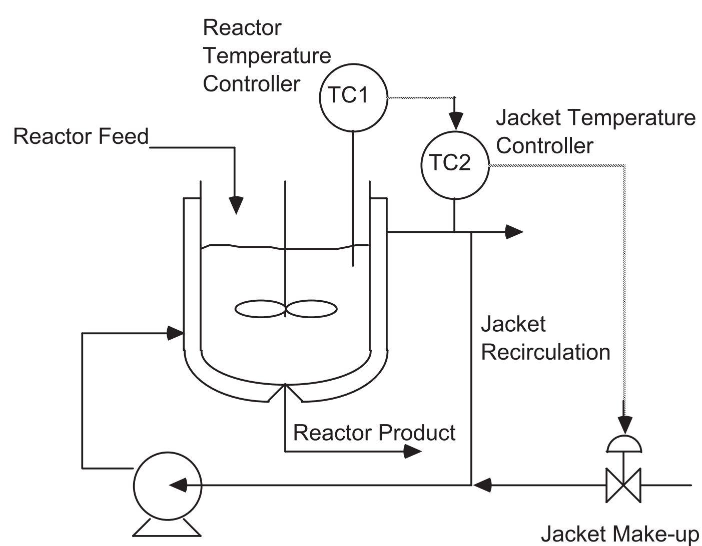

Figure 3. Typical control diagram for a continuously stirred tank reactor

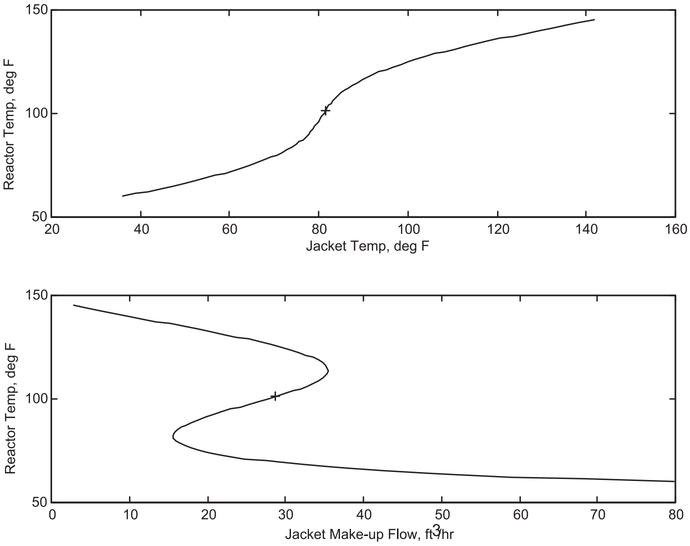

Figure 4. Reactor temperature as a function of the jacket temperature (top) and reactor temperature as a function of the jacket make-up flow rate (bottom)

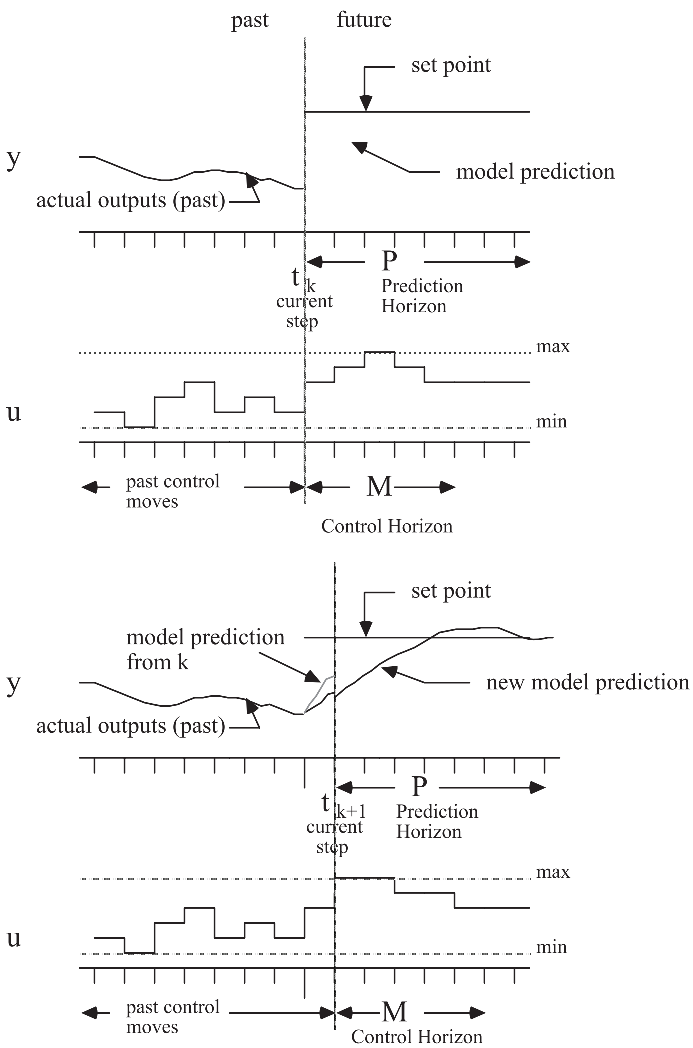

Figure 5. The basic MPC approach includes the optimization of M future control moves ("control horizon"), based on minimizing an objective function composed of model predictions over P time steps ("prediction horizon"), subject to constraints. The first control move is implemented, compensation for plant-model mismatch is performed, and the optimization problem is solved again.

applied at step k, k + 1,..., k + M-1) are adjusted to minimize an open-loop objective function over a prediction horizon of P steps (k + 1, ..., k + P). Only the first control move (time step k) is actually implemented, and a new output measurement is obtained at the next time step (k + 1). Rarely is the output prediction from step k to k + 1 perfect, so some type of model update is performed, and a new optimization problem is solved at step k + 1. For this reason, MPC is often called receding horizon control. While Figure 5 is used as an intuitive explanation of MPC, it should be clear that multiple inputs can be adjusted and that multiple outputs can be controlled, if properly handled in the objective function. Also, one of the many reasons for the success of MPC is the ability to explicitly enforce constraints on the manipulated inputs.

This description of MPC should raise many questions, such as:

- What type of model is used to predict the effect of control moves on outputs?
- What type of objective function is used?
- What type of optimization algorithm is used?
- What plant-model "mismatch" compensation procedure is used before a new optimization is performed? A related question is: what initial conditions are used to solve the model at each time step?

Perhaps the easiest question to address is the objective function. Most often a quadratic objective function is used, for two basic reasons: (1) it results in an analytical solution when a linear model is used and the problem is unconstrained; and (2) it seems natural to provide a greater penalty to larger deviations from the desired set point. Also, when a linear model is used, and the constrained problem is solved, this results in a so-called quadratic program (QP) and there exist efficient, robust and reliable QP codes. While linear programming problems are often used in large scale supply chain optimization problems, the solution is always at a constraint. For MPC problems, an LP formulation can often result in manipulated inputs that frequently hop from a minimum to maximum constraint.

In the sections that follow these questions are addressed for several different approaches that (we) have used to handle non-linear systems: (1) Non-linear Model Predictive Control (NMPC) using fundamental models; (2) EKF-based NMPC using successive linearization; (3) Multiple Model Predictive Control (MMPC); and (4) Artificial Neural Network-based MPC.

The initial conditions used to solve the model at each optimization time can simply consist of the values obtained by integrating the model from the previous time step, and compensating for the error in the plant output by using the so-called additive output disturbance assumption, as presented in the next section.

MODEL PREDICTIVE CONTROL USING FUNDAMENTAL MODELS

A foray into NMPC was based on the use of first-principles or fundamental ordinary differential equation models with the following form:

$$\dot{x} = f(x,u) \qquad y = g(x) \tag{1}$$

Where x is a vector of states, u is the vector of manipulated inputs, and y is a vector of outputs. It is implicitly assumed that there are vectors of parameters and disturbances. Integration of the model from time step k-1 to current time step k is represented by:

$$\hat{x}_k = F_{t_s}\left(\hat{x}_{k-1}, u_{k-1}\right)$$
$$\hat{y}_{k|k-1} = g\left(\hat{x}_k\right) \tag{2}$$

Where t_s is the sample time; the k:k-1 subscript notation is used to indicate the prediction at step k based on measurements at step k-1. At step k, an output measurement is available:

$$y_k$$

and the model error is calculated:

$$d_k = y_k - \hat{y}_{k|k-1} \tag{3}$$

A hypothetical set of current and future control moves:

$$u_k, u_{k+1}, \cdots, u_{k+P-1}$$

is chosen to minimize an objective function over a prediction horizon of P steps, and the model is integrated from time step k to k + P, based on the hypothetical control moves

$$\hat{x}_{k+1} = F_{t_s}\left(\hat{x}_k, u_k\right)$$
$$\hat{y}_{k+1|k} = g\left(\hat{x}_{k+1}\right) + d_k \tag{4}$$

The objective function is evaluated and the selection of control moves repeated until the optimum is obtained. Note that the equations above assume that the plant-model mismatch is performed using the standard additive output disturbance assumption, similar to what is commonly assumed in dynamic matrix control (DMC) (Cutler and Ramaker, 1980).

There are many ways to integrate or solve the modelling equations and the optimization problem. The most straightforward, and computationally expensive, approach is to use an optimization strategy, such as SQP (Biegler et al., 1997) as the "outer-loop" and a numerical integrator (based on Runge-Kutta or related techniques) as an "inner-loop" to evaluate the objective function at each iteration of the optimizer. This is referred to as a sequential strategy. As an alternative, orthogonal collocation on finite elements (OCFE) can be used to discretize the ordinary differential equations and solve them as a large set of algebraic equations. Again, a sequential approach can be used with the optimizer as the outer loop and the orthogonal collocation procedure as the inner loop. A simultaneous optimization and model solution strategy embeds the algebraic equations obtained from the OCFE method as equality constraints in the optimization problem. A major advantage to this is that it is straightforward to enforce state constraints, at least at the collocation points (Cuthrell and Biegler, 1987).

A non-linear quadratic matrix control (NLQDMC) strategy, proposed by Garcia (1984), successively linearizes the model at each time step and solves the optimization problem for the linear system of equations. Here, the non-linear model is used to "follow" the plant by integrating the non-linear equations from step k-1 to step k. The output is updated using the additive correction term (3). Based on an initial set of current and future control moves (usually just the most recent control action used from k-1 to k), the non-linear equations are then integrated from step k to step k + P. This can be viewed as the "free response," similar to the way that linear MPC is decoupled into free and forced response contributions for future predictions. The optimizer then uses the linearized model, evaluated at step k, to solve the optimization problem using inputs that are perturbed from the ones used for the integration of the non-linear model from k to k + P, and assuming a constant additive output disturbance for model output predictions. The main drawbacks are that this approach can only be used for open-loop stable systems, and cannot be used for inferential control, since the states are not corrected with feedback measurements. A detailed comparison of the computation times for the various implementation strategies, including different optimization codes and numerical integrators, is performed by Sistu et al. (1993). There are significant computational savings to the NLQDMC approach of Garcia (1984), particularly if analytical derivatives are used to in the model linearization. For the example problem the closed-loop performance of NLQDMC was nearly identical to that of the methods solving the full non-linear problem.

A major challenge to solving this open-loop optimal control problem is that it is, in general, non-convex. This can lead to multiple minima in the objective function, as shown in Figures 6 and 7 for an example problem with input multiplicity (Sistu and

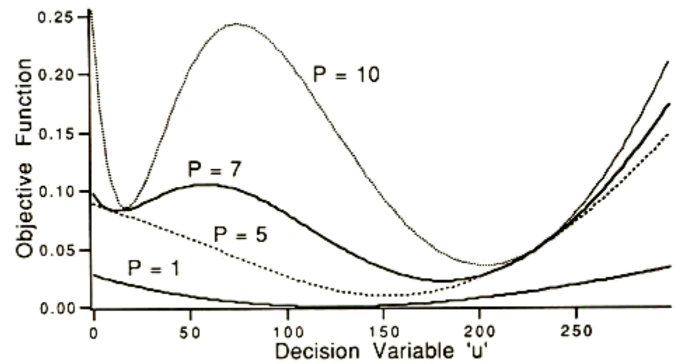

Figure 6. Illustration of a non-convex system. Here, the objective function has multiple minima for prediction horizons greater than 5, for a control horizon of 1 (Sistu and Bequette (1992)).

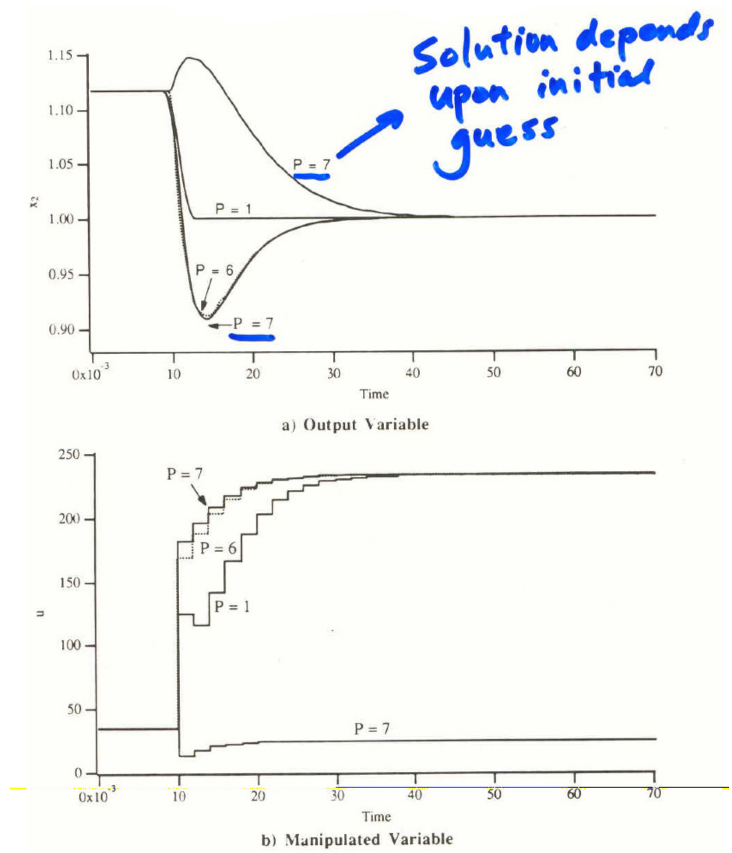

Figure 7. Illustration of the effect of multiple solutions. Depending on the optimization initialization, a prediction horizon of 7 can lead to two different steady-state solutions (Sistu and Bequette (1992)).

Bequette, 1992). Notice that a prediction horizon of 7 can lead to solutions for the manipulated input that are either on the "left" (low flows) or "right" (high flows) hand side of the peak in the steady-state input/output plot (Figure 1, with different flow rate units), depending upon the initial seed for the optimizer. This is one reason that many NMPC formulations have an objective function that includes all of the states, rather than just the controlled output as used in our simulation studies.

The additive output disturbance assumption is known to lead to poor performance when unmeasured disturbances occur at the process inputs. Also, simple output compensation cannot be used for linear unstable systems, because the additive disturbance would grow unbounded over time. One of the first interesting results was that a non-linear system may be controlled at an open-loop unstable operating point, using a non-linear model and the additive output assumption. This is possible because the model outputs may evolve to a stable steady-state, even while the plant output is being controlled to an unstable steady-state. The difference between the two states is the additive output term; an example is shown in Figure 8 (from Sistu and Bequette, 1991). While it is possible to stabilize the plant at an unstable point with the model output at a stable operating point,

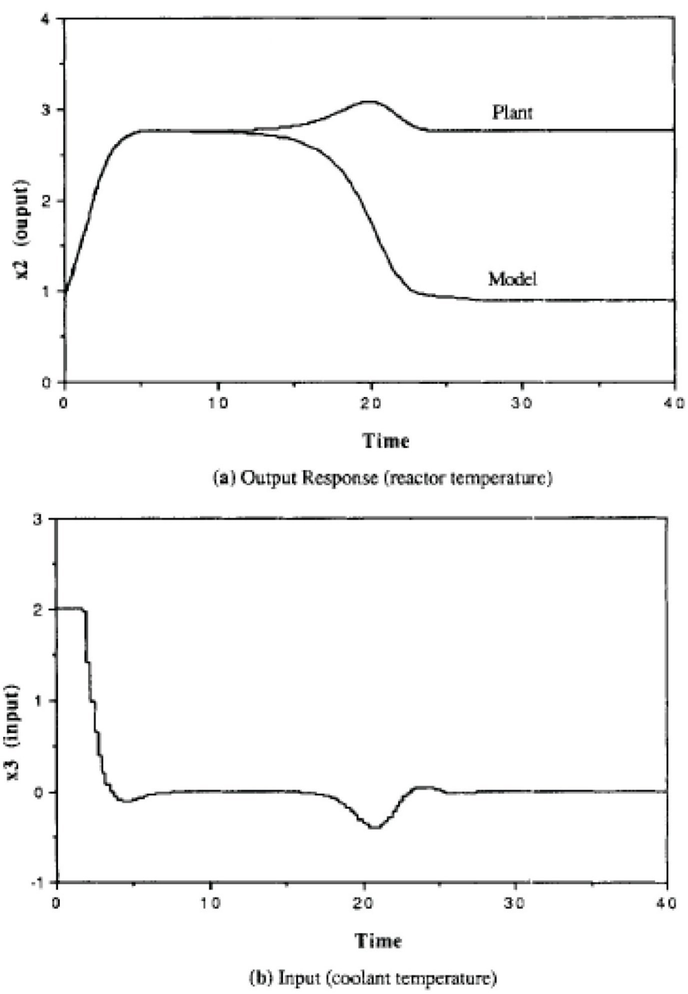

Figure 8. Set-point change to an open-loop unstable operating point. Notice that the plant can be controlled at the unstable point, while the model is attracted to a stable operating point. The difference between the two is compensated by the additive disturbance term (Sistu and Bequette (1991)).

closed-loop performance will generally be poor. Also, inferential control based on unmeasured states cannot be performed, since the unmeasured model state values will be different than the plant state values; note that this additive output disturbance assumption can be classified as an "open-loop observer." Clearly, a closed-loop state estimation or observer technique is needed for satisfactory closed-loop performance, particularly if inferential control is important. Here a number of closed-loop observer techniques, including solving a moving horizon problem by minimizing plant-model mismatch over a past horizon using constrained optimization (Sistu and Bequette, 1990; Bequette, 1991b; Sistu and Bequette, 1991; Ramamurthi et al., 1993) have been used.

Moving horizon estimation is the dual of model predictive (or receding horizon) control, and thus similar optimization and model solution approaches can be used. Decision variables can include the initial conditions of the states at the beginning of the horizon, disturbances and parameters. Usually, it is assumed that an estimated parameter is constant over the estimation horizon, but it may be desirable to "block" disturbances and allow them to change at frequent intervals (not necessarily every sample time) during the horizon. Sistu and Bequette (1990) extend the notion of an "activation threshold", an approach sometimes used in adaptive control, where the horizon estimator is only activated when plan-model mismatch is above a certain threshold; otherwise a simple additive disturbance assumption is used. Bequette (1991b) shows multirate results where output measurements and control actions are all available at different rates.

EKF-BASED NMPC

A major limitation to the NLQDMC approach of Garcia (1984) is that it uses an open-loop observer, limiting it to open-loop stable systems. Lee and Ricker (1994) extended the basic idea by adding a closed-loop observer in the form of an extended Kalman filter, resulting in EKF-based NMPC. The disturbance states using the EKF can be simple terms added to the state equations, or can be physical parameters, such as rate constants. The approach of Sandink et al. (2001) can be used to select appropriate parameters for real-time estimation. In addition, Kozub and MacGregor (1992a, b) provide detailed discussions on the design and implementation of state estimators using extended Kalman filters.

EKF-based NMPC was applied to chemical processes where fundamental models are available, but parameters are uncertain and there are limited state variable measurements. Particularly important is the ability to characterize disturbances that do not fit the traditional "additive output disturbance" characterization, which simply involves a shifting of the predicted process output by the most recent plant-model mismatch. In Prasad et al. (2002) a styrene polymerization reactor, with multirate sampling and delayed results from laboratory measurements was studied. A multi-level Kalman filter is used, where an updating KF is implemented when the delayed laboratory measurement value is received.

MULTIPLE MODEL PREDICTIVE CONTROL

Multiple Model Predictive Control (MMPC) uses a bank of models, where the models are selected to span the expected dynamic behaviour of the process. A weighting function selects the best model (or combination of models) that represents the current input-output behaviour. The weighted model is then used for the predictions in the optimal control move calculation. MMPC is particularly useful for processes where it would be hard to obtain fundamental models that realistically describe behaviour over a wide range of operating conditions. Classical adaptive control techniques will often result in estimated model parameters that are not physically consistent; also, there have been few successful multivariable adaptive control applications. One of the main application areas of our MMPC strategy is in the field of biomedical control systems. MMPC was used to regulate the blood pressure and cardiac output of patients in a surgical environment. Our approach does not require a period of open-loop process identification, so closed-loop control can begin immediately with the sedation of the patient. Yu et al. (1992) solved an unconstrained optimization problem. A constrained version is presented by Rao et al. (2003), while implementation issues are discussed by Aufderheide and Bequette (2003).

The primary limitation to our MMPC approach is the classic additive disturbance assumption of DMC. Our current effort extends the MMPC approach to include disturbances in each model in the bank as appended states, which are estimated using a Kalman filter.

ARTIFICIAL NEURAL NETWORK-BASED MPC

Similar to MMPC, an artificial neural network based approach to MPC may be desirable when it is too time consuming to develop a fundamental model, or when a fundamental model is difficult to formulate (such as many biomedical or physiological problems). Also, many engineers in industry are comfortable with using neural networks, particularly since there are a number of readily available software packages to use. A major disadvantage is that there tends to be a "black box" treatment and lack of understanding of what is happening in that "black box."

Prasad and Bequette (2003) presented a method of reducing the number of intermediate nodes in ANN models. Kuure-Kinsey et al. (2006) developed a novel formulation of an ANN MPC strategy that results in a time-varying linear term, enabling an analytical solution to an unconstrained problem, and a quadratic program (QP) for constraints. Significant computational savings for the linearized ANN approach are shown compared to the "full blown" ANN optimization problem solution. Also, while most ANN-based MPC approaches presented in the literature use additive output compensation, Kuure-Kinsey et al. (2006) developed a KF-based procedure to estimate input disturbances as appended states.

DISTURBANCE REJECTION

It was noted several times the limitation to the additive output step disturbance of DMC and early implementations of NMPC. Muske and Badgwell (2002) provide a nice analysis of disturbance rejection in linear MPC, by appending disturbance states that are estimated using a Kalman Filter. They proved that the number of disturbances that can be estimated without bias (obviously in the limit of a perfect model) is equal to the number of measurements. They focus on step input disturbances, but similar formulations result for other types of disturbances, such as ramps and periodic functions.

An additive output disturbance assumption, results in an augmented state space model with the following form:

$$\begin{aligned} \underbrace{\begin{bmatrix} x_{k+1} \\ p_{k+1} \end{bmatrix}}_{x^a_{k+1}} &= \underbrace{\begin{bmatrix} \Phi & 0 \\ 0 & I \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ p_k \end{bmatrix}}_{x^a_k} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k \\ y_k &= \underbrace{\begin{bmatrix} C & I \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ p_k \end{bmatrix}}_{x^a_k} \end{aligned} \tag{5}$$

Augmenting input disturbances as additional states yields the formulation:

$$\begin{aligned} \underbrace{\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix}}_{x^a_{k+1}} &= \underbrace{\begin{bmatrix} \Phi & \Gamma^d \\ 0 & I \end{bmatrix}}_{\Phi^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x^a_k} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\Gamma^a} u_k \\ y_k &= \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C^a} \underbrace{\begin{bmatrix} x_k \\ d_k \end{bmatrix}}_{x^a_k} \end{aligned} \tag{6}$$

and a Kalman filter with the following form:

$$\begin{aligned} \hat{x}^a_{k|k-1} &= \Phi^a \hat{x}^a_{k-1|k-1} + \Gamma^a u_k \\ \hat{x}^a_{k|k} &= \hat{x}^a_{k|k-1} + L_k \left(y_k - C^a \hat{x}^a_{k|k-1} \right) \end{aligned} \tag{7}$$

is used for state estimation for both output and input disturbances. When the disturbance really enters as a step at the input, the performance of the KF-based method can be significantly better than DMC (additive output disturbance), as shown in Figure 9 for a first-order example. It should be noted, however, that Tenney et al. (2004) provide an example where a linear MPC with output compensation stabilizes the non-linear process, while an input disturbance formulation cannot.

While it is satisfactory for many systems to assume step disturbances, where the most recent disturbance estimate is assumed to remain constant for predictions, we are motivating by problems where disturbances really occur as other forms, such as ramps or sine waves (periodic behaviour).

van de Vusse Reaction Example

Linearization of the classic van de Vusse reactor model (Bequette, 2003) results in the following state space model, where the first input is manipulated and the second input is a disturbance.

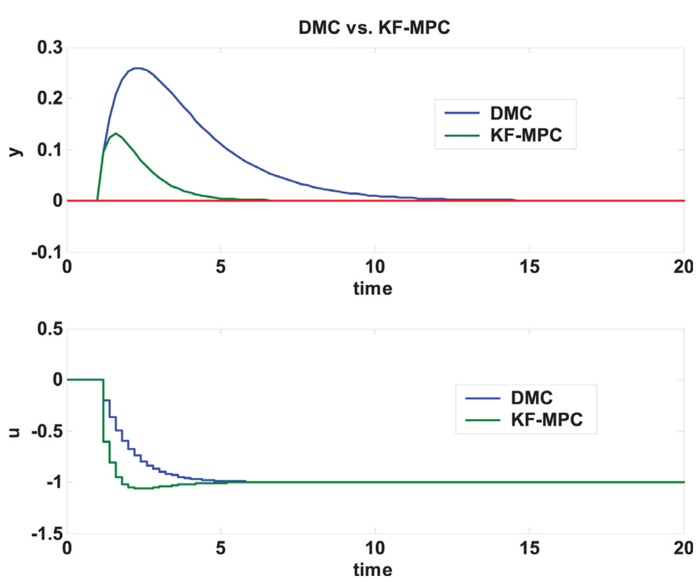

Figure 9. Comparison of performance of DMC vs. a KF-MPC approach that assumes a plant input disturbance, for a simple first-order system

$$\begin{bmatrix} \dot{C}_A \\ \dot{C}_B \end{bmatrix} = \begin{bmatrix} -2.4048 & 0 \\ 0.8333 & -2.2381 \end{bmatrix} \begin{bmatrix} C_A \\ C_B \end{bmatrix} + \begin{bmatrix} 7 & 0.5714 \\ -1.117 & 0 \end{bmatrix} \begin{bmatrix} F/V \\ C_{Af} \end{bmatrix} \quad (8)$$

First, consider a ramp disturbance. The performance of a KF-based MPC strategy with a ramp input disturbance estimate performs much better than an additive output disturbance formulation, as shown in Figure 10.

The disturbance model for a periodic disturbance is

$$\begin{bmatrix} \dot{C}_{Af} \\ \ddot{C}_{Af} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} C_{Af} \\ \dot{C}_{Af} \end{bmatrix} + \begin{bmatrix} 0 \\ d \end{bmatrix} \quad (9)$$

The performance if a KF-based MPC strategy with a periodic input disturbance estimate performs much better than an additive output disturbance formulation, as shown in Figure 11. Of course, this assumes that the period (or equivalently, the frequency) is known. If the period is estimated, then an extended Kalman filter formulation is required, where the period is an estimated state. Alternatively, a multiple model estimate strategy can be used, where each disturbance model corresponds to a different frequency disturbance.

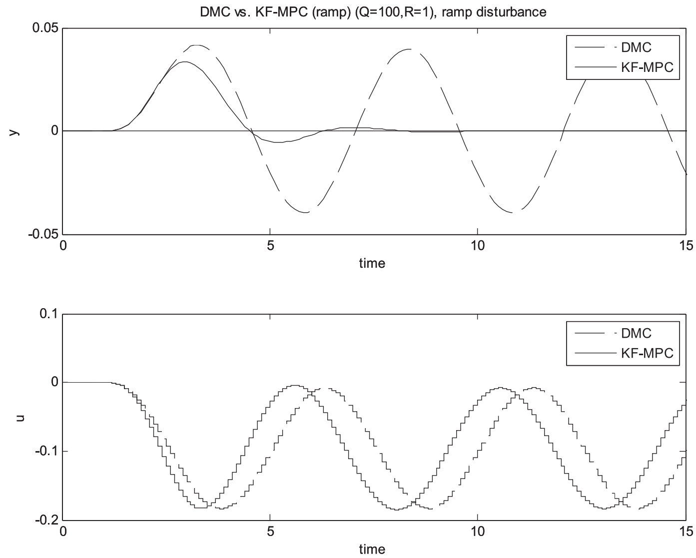

Figure 11. Performance for a Sin Disturbance. Comparison of KF-MPC assuming a sinusoidal disturbance with that of an additive output disturbance.

SUMMARIZING COMMENTS PERSPECTIVE ON CONTINUING CHALLENGES

In this article I have provided a concise overview of some of the challenges of developing and implementing non-linear predictive control algorithms. In general, the optimization problem is non-convex and may result in solutions with multiple minima. Also, the intuitive approach of compensating for plant-model mismatch with an additive output term, can result in poor performance when actual disturbances occur at the inputs. It is particularly important for open-loop unstable systems to use a closed-loop observer that compensates for the state estimates.

Since most process systems are open-loop stable, and input-output testing is often done to develop models, the issue of disturbance rejection is often more important than explicit state estimation. I encourage those of you developing MPC-based techniques to give more attention to the problem of disturbance estimation, prediction and compensation. Also, model development remains an ongoing challenge and it is usually not clear whether the additional effort of developing a non-linear model, instead of a linear model, will result in an economic benefit. While many investigators have developed non-linearity measures, these are not easy to apply in practice and usually require the *a priori* development of a non-linear model.

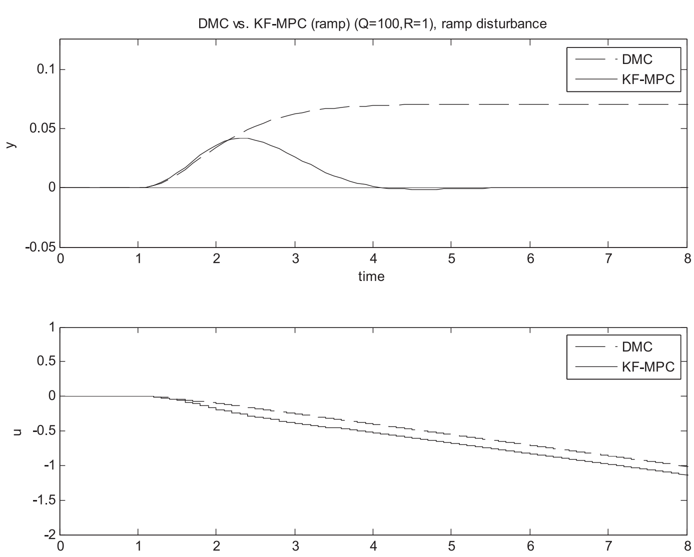

Figure 10. Performance for a Ramp Disturbance. Comparison of KF-MPC assuming a ramp disturbance with that of an additive output disturbance.

Understanding how to develop an appropriate closed-loop performance criterion for non-linear processes remains a challenge. Take the van de Vuuse reactor problem, for example. Operating points in the minimum-phase region (right-hand side of the "peak") are easy to control, and deadbeat control is possible. On the other hand, operation in the non-minimum phase region (right-hand side of the "peak") is much more difficult, requiring long prediction horizons and limited closed-loop performance. For non-linear systems, then, it is important to develop performance criteria based on knowledge of the inherent performance limitations at a particular operating point. Having a "variable" objective function certainly adds an additional degree of difficulty; as handled in the context of MMPC (Aufderheide and Bequette, 2003) by changing the objective function based on the dynamics of the current weighted model.

Finally, there has been limited discussion about real-time adaptation in the industrial applications literature. Most of the non-linear MPC strategies have been based on the use of artificial neural networks or gain-scheduling, neither of which are typically adapted in real-time. Practitioners are encouraged to provide more detailed discussions of what adaptive methods have and have not worked in MPC applications.

ACKNOWLEDGEMENTS

I wish to recognize the many contributions of my current and former students; I have not only learned a lot in the process, but have had an enjoyable time doing so (I hope that they can say the same). Also, I gratefully acknowledge the National Science Foundation (BES-0411693 and DGE-0504361) for recent support of this work.

REFERENCES

Aufderheide, B. and B. W. Bequette, "Extension of Dynamic Matrix Control to Multiple Models," Comp. Chem. Eng. **27**, 1079–1096 (2003).

Bequette, B. W., "Nonlinear Control of Chemical Processes: A Review," Ind. Eng. Chem. Res. **30**(7), 1391–1413 (1991a).

Bequette, B. W., "Nonlinear Predictive Control Using Multi-Rate Sampling," Can. J. Chem. Eng. **69**(1), 136–143 (1991b).

Bequette, B. W., "Process Control: Modeling, Design and Simulation," Prentice Hall, Upper Saddle River, NJ (2003).

Biegler, L. T., I. E. Grossmann and A. W. Westerberg, "Systematic Methods of Chemical Process Design," Prentice Hall, Upper Saddle River, NJ (1997).

Bitmead, R. R., M. Gevers and V. Wertz, "Adaptive Optimal Control—The Thinking Man's GPC," Prentice Hall, Englewood Cliffs (1990).

Camacho, E. F. and C. Bordons, "Model Predictive Control," 2nd ed., Springer, London (2004).

Cuthrell, J. E. and L. T. Biegler, "On the Optimization of Differential-Algebraic Process Systems," AIChE J., **33**, 1257–1270 (1987).

Cutler, C. R. and B. L. Ramaker, "Dynamic Matrix Control—A Computer Control Algorithm," in "Proc. Joint Automatic Control Conference," San Francisco, CA, Paper WP5–B (1980).

Garcia, C. E., "Quadratic Dynamic Matrix Control of Nonlinear Processes," Presented at the AIChE Annual Meeting, San Francisco, CA (1984).

Henson, M. A., "Nonlinear Model Predictive Control: Current Status and Future Directions," Comp. Chem. Eng. **23**, 187–202 (1998).

Kozub, D. J. and J. F MacGregor, "Feedback Control of Polymer Quality in Semi-Batch Copolymerization Reactors," Chem. Eng. Sci. **47**(4), 929–942 (1992a).

Kozub, D. J. and J. F MacGregor, "State Estimation for Semi-Batch Polymerization Reactors," Chem. Eng. Sci. **47**(5), 1047–1062 (1992b).

Kuure-Kinsey, M., R. Cutright and B. W. Bequette, "Computationally Efficient Neural Predictive Control based on a Feedforward Architecture," Ind. Eng. Chem. Res. **45**(25), 8575–8582 (2006).

Lee, J. H. and N. L. Ricker, "Extended Kalman Filter Based Nonlinear Model Predictive Control," Ind. Eng. Chem. Res. **33**(6), 1530–1541 (1994).

Maciejowski, J. M., "Predictive Control: With Constraints," Prentice Hall (2002).

Mayne, D. Q., J. B. Rawlings, C. V. Rao and P. O. M. Scokaert, "Constrained Model Predictive Control: Stability and Optimality," Automatica **36**, 789–814 (2000).

Morari, M. and J. H. Lee, "Model Predictive Control: Past, Present and Future," Comp. Chem. Eng. **23**, 667–682 (1999).

Muske, K. R. and T. A. Badgwell, "Disturbance Modeling for Offset–Free Linear Model Predictive Control," J. Process Control **12**, 617–632 (2002).

Prasad, V., M. Schley, L. P. Russo and B. W. Bequette, "Product Property and Production Rate Control of Styrene Polymerization," J. Process Control **12**(3), 353–372 (2002).

Prasad, V. and B. W. Bequette, "Nonlinear System Identification and Model Reduction Using Artificial Neural Networks," Comp. Chem. Eng. **27**, 1741–1754 (2003).

Qin, S. J. and T. A. Badgwell, "A Survey of Industrial Model Predictive Control Technology," Control Eng. Practice **11**, 733–764 (2003).

Ramamurthi, Y., P. B. Sistu and B. W. Bequette, "Control-Relevant Data Reconciliation and Parameter Estimation," Comp. Chem. Eng. **17**(1), 41–59 (1993).

Rao, R. R., B. Aufderheide and B. W. Bequette, "Experimental Studies on Multiple-Model Predictive Control for Automated Regulation of Hemodynamic Variables," IEEE Trans. Biomed. Eng. **50**(3), 277–288 (2003).

Rossiter, J. A., "Model-Based Predictive Control: A Practical Approach," CRC Press, New York (2003).

Russo, L. P. and B. W. Bequette, "Impact of Process Design on the Multiplicity Behavior of a Jacketed Exothermic CSTR," AIChE J. **41**(1), 135–147 (1995).

Russo, L. P. and B. W. Bequette, "State Space vs. Input/Output Representations for Cascade Control of Unstable Systems," Ind. Eng. Chem. Res. **36**(6), 2271–2278 (1997).

Russo, L. P. and B. W. Bequette, "Operability of Chemical Reactors. Multiplicity Behavior of a Jacketed Styrene Polymerization Reactor," Chem. Eng. Sci. **53**(1), 27–45 (1998).

Sandink, C., K. B. McAuley and P. J. McLellan, "Selection of Parameters for Updating in On-Line Models," Ind. Eng. Chem. Res. **40**, 3936–3950, (2001).

Shah, S. L., "A Tutorial Introduction to Constrained Long Range Predictive Control," Pulp Paper Canada **96**(4), 57–63 (1995).

Sistu, P. B. and B. W. Bequette, "Process Identification using Nonlinear Programming Techniques," in "Proc. of the American Control Conference," San Diego, CA (1990), pp. 1534–1539.

Sistu, P. B. and B. W. Bequette, "Nonlinear Predictive Control of Uncertain Processes: Application to a CSTR," AIChE J. **37**(11), 1711–1723 (1991).

Sistu, P. B. and B. W. Bequette, "Control of Nonlinear Nonminimum-Phase Processes," in "Proc. of the American Control Conference," Chicago, IL (1992), pp. 2602–2606.

Sistu, P. B., R. S. Gopinath and B. W. Bequette, "Computational Issues in Nonlinear Predictive Control," Comp. Chem. Eng. **17**(4), 361–366 (1993).

Sistu, P. B. and B. W. Bequette, "Model Predictive Control of Processes with Input Multiplicities," Chem. Eng. Sci. **50**(6), 921–936 (1995).

Tenney, M. J., J. B. Rawlings and S. J. Wright, "Closed-Loop Behavior of Nonlinear Model Predictive Control," AIChE J. **50**(9), 2142–2154 (2004).

Yu, C., R. J. Roy, H. Kaufman and B. W. Bequette, "Multiple-Model Adaptive Predictive Control of Mean Arterial Pressure and Cardiac Output," IEEE Trans. Biomed. Eng. **39**(8), 765–778 (1992).

Manuscript received March 19, 2007; revised manuscript received May 9, 2007; accepted for publication May 24, 2007.