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Brief note

NON-PERTURBATIVE SOLUTION FOR HYDROMAGNETIC FLOW OVER A LINEARLY STRETCHING SHEET

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In this paper, the Adomian decomposition method with Padé approximants are integrated to study the boundary layer flow of a conducting fluid past a linearly stretching sheet under the action of a transversely imposed magnetic field. A closed form power series solution based on Adomian polynomials is obtained for the similarity nonlinear ordinary differential equation modelling the problem. In order to satisfy the farfield condition, the Adomian power series is converted to diagonal Padé approximants and evaluated. The results obtained using ADM-Padé are remarkably accurate compared with the numerical results. The proposed technique can be easily employed to solve a wide range of nonlinear boundary value problems.

Key words: stretching sheet, hydromagnetic flow, ADM-Padé, numerical solution.

1. Introduction

Convective fluid flow due to a stretching surface is an important type of flow occurring in several engineering processes. The flow situations are encountered in a great number of industrial applications such as the extrusion of a polymer sheet from a dye, the drawing of plastic films and heat-treated materials travelling between a feed roll and a wind-up roll or materials manufactured by extrusion, glass-fiber and paper production, cooling of metallic sheets, crystal growing and many others. In these cases, the final products of desired characteristics depend on the rate of cooling during the process of stretching of the sheet. During the manufacture of these sheets, the mixture issues from a slit and is subsequently stretched to achieve the desired thickness. Sakiadis (1961) has written the pioneering work on the flows developed over a moving flat plate. Crane (1961) studied this configuration for the stretching sheet. Crane's problem has been studied by researchers under various aspects. For instance Rajagopal *et al*. (1984), Sankara and Watson

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(1985) and Andersson *et al*. (1992) extended Crane's problem for second grade, micropolar and power-law fluid models, respectively. Recent contributions concerning the boundary layer flows over stretching surfaces have been mode by Pavlov (1974), Mahapatra *et al*. (2009), Nandeppanavar *et al*. (2010), Ahmed and Asghar (2011), Makinde and Charles (2010).

Moreover, in the beginning of the 1980s, a new method for exactly solving nonlinear functional equations was proposed by Adomian, the so called Adomian decomposition method (1994). Over these years, this method has been applied to solve a wide range of problems arising from physics, biology, engineering. This method uses a decomposition of the nonlinear operator as a series function (2008). Recently, the development of a semi-numerical hybrid of Adomian–Padé (Makinde and Sibanda, 2008; Wazwaz, 2006; Kechil and Hashim, 2007) scheme to enhance the accuracy and convergence of ADM solutions has proven itself as a powerful tool and a potential alternative to traditional numerical techniques in various applications in sciences and engineering.

The present work examines the application of ADM-Padé technique to tackle the nonlinear differential equation modelling the steady laminar hydromagnetic flow of a conducting fluid past a stretching sheet. These ADM-Padé solutions are validated by a numerical solution obtained by the fourth-fifth order Runge-Kutta-Fehlberg method using a computation software MAPLE 14. Graphical results are displayed for several values of embedding parameters. The dimensionless expression of the local skin friction coefficient is discussed through graphical result.

2. Mathematical model

Consider a steady two-dimensional boundary layer flow of an incompressible and electrically conducting isothermal Newtonian liquid over a linearly stretching sheet (see Fig.1).

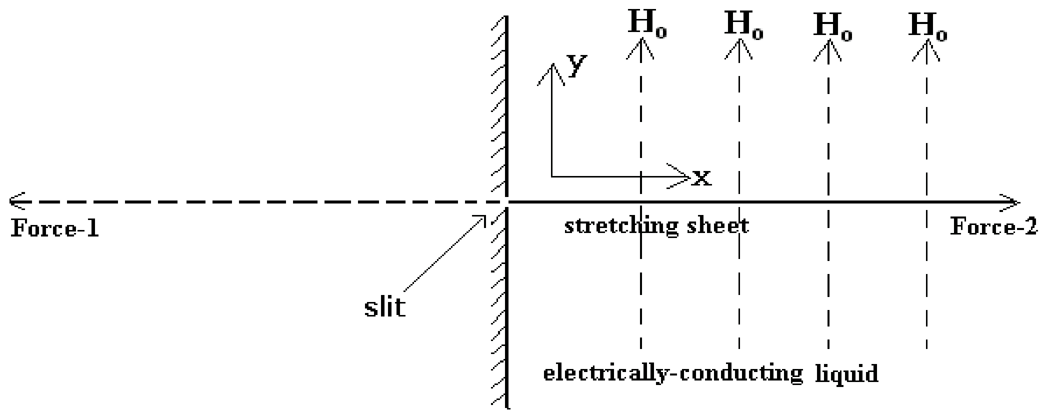

Fig.1. Schematic of the stretching sheet problem.

A uniform transverse magnetic field H_0 acts parallel to the y-axis and the conducting liquid in the half space $y > 0$ is considered to study the dynamics induced by the stretching sheet. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. A Hartmann formulation is done for the MHD problem. The conservation of mass and the momentum boundary layer equation for the quadratic stretching sheet problem are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma H_0^2}{\rho}u. \tag{2.2}$$

Equation (2.2) has been derived with the assumption that the contribution from the normal stress is of the same order of magnitude as the shear stress in addition to usual boundary layer approximations. The appropriate boundary conditions to the problem are

$$u(x,y)=cx, \qquad v=v_w, \qquad \text{at} \qquad y=0, \tag{2.3a}$$

$$u(x,y)=0 \qquad \text{as} \qquad y\to\infty. \tag{2.3b}$$

Here, u and v are the components of the liquid velocity in the x and y directions, respectively, $c>0$ is the stretching rate, ν is the kinematic viscosity, H_0 is the applied magnetic field and σ is the electrical conductivity. The constant v_w is the suction/injection parameter, where $v_w<0$ corresponds to the suction and $v_w>0$ to the injection of the fluid, moreover $v_w=0$.

We now introduce new dimensionless variables f and η such that

$$\psi=\sqrt{c\nu}\;\; x\,f(\eta), \tag{2.4}$$

and

$$\eta=y\sqrt{\frac{c}{\nu}}\;\; . \tag{2.5}$$

The velocity components u and υ are then related to the physical stream function ψ by

$$u=\frac{\partial\psi}{\partial y}\;, \qquad v=-\frac{\partial\psi}{\partial x}\;, \tag{2.6}$$

is introduced such that the continuity equation is automatically satisfied. Introducing the stream function $\psi(x,y)$ and the non-dimensional form of Eq.(2.3) in Eq.(2.6), we get

$$\nu\frac{\partial^3\psi}{\partial y^3}+\frac{\partial\left(\psi,\frac{\partial\psi}{\partial y}\right)}{\partial(x,y)}-\frac{\sigma H_0^2}{\rho}\frac{\partial\psi}{\partial y}=0\,, \tag{2.7}$$

$$\frac{\partial\psi}{\partial y}=cx, \qquad \frac{\partial\psi}{\partial x}=v_w \qquad \text{at} \qquad y=0, \tag{2.8a}$$

$$\frac{\partial\psi}{\partial y}=0\,, \qquad \text{as} \qquad y\to\infty. \tag{2.8b}$$

Substituting the similarity expression for the stream function in Eq.(2.4) into Eqs (2.7) and (2.8), we obtain

$$f''' + ff'' - f'^2 - Qf' = 0, \tag{2.9}$$

$$f(0) = V_C, \qquad f'(0) = 1, \qquad \text{at} \qquad \eta = 0, \tag{2.10}$$

$$f'(\infty) \to 0 \qquad \text{as} \qquad \eta \to \infty \tag{2.11}$$

where $Q = \dfrac{H_0^2 \sigma}{c\rho}$ is the Chandrasekhar number and $V_C = \dfrac{v_w}{\sqrt{c\rho}}$ suction/injection parameter. In this present work we assume $f(0) = V_C$ and $V_C > 0$ corresponds to suction (i.e., $v_c < 0$), $V_C < 0$ corresponds to blowing (i.e., $v_c > 0$) and $V_C = 0$ (i.e., $v_c = 0$) is the case when the surface is impermeable. The prime denotes differentiation with respect to η. For practical purposes, the functions $f(\eta)$ allow us to determine the skin friction coefficient given as $C_f = -f''(0)$.

3. ADM-Padé technique

The decomposition method was presented by Adomian (1994). We shall demonstrate the application of the ADM to obtain the analytical solution of Eqs (2.9)-(2.11), if we write in operator form as

$$L_1(f) = \left(\frac{df}{d\eta}\right)^2 - f\frac{df^2}{d\eta^2} + Q\frac{df}{d\eta} \tag{3.1}$$

where $L_1 = \dfrac{d^3}{d\eta^3}$ is a third order differential operator with the inverse operator $L_1^{-1} = \int_0^\eta \int_0^\eta \int_0^\eta (\text{Function})\, dtdtdt$.

Operating with L_1^{-1} on both sides of Eq.(3.1) and making use of the boundary condition in Eq.(2.11), we obtain

$$f(\eta) = V_C + \frac{1}{2}\alpha\eta^2 + L_1^{-1}\left(\left(\frac{df}{d\eta}\right)^2 - f\frac{df^2}{d\eta^2} + Q\frac{df}{d\eta}\right) \tag{3.2}$$

where and $\alpha = f''(0)$ is to be determined. In Adomian (1994), $f(\eta)$ may be expressed as

$$f(\eta) = V_c + \sum_{k=0}^{\infty} f_k(\eta), \tag{3.3}$$

in which the components $f_k(\eta)$, $k \geq 0$, are determined recursively. From Eqs (3.2) and (3.3), we have

$$\sum_{k=0}^{\infty} f_k(\eta) = V_C + \frac{1}{2}\alpha\eta^2 + L_1^{-1}\left(\sum_{k=0}^{\infty}\alpha_k - \sum_{k=0}^{\infty}\beta_k + Q\frac{df}{d\eta}\right). \tag{3.4}$$

In Eq.(3.4) α_k and β_k are the so-called Adomian polynomials (1994) representing the nonlinear terms. It can be show that

$$\sum_{k=0}^{\infty}\alpha_k=\left(\frac{df}{d\eta}\right)^2 \qquad \text{and} \qquad \sum_{k=0}^{\infty}\beta_k=f\frac{df^2}{d\eta^2} \tag{3.5}$$

where α_k and β_k are Adomian polynomials given by

$$\alpha_k=\frac{1}{j!}\frac{d}{dh^j}\left[\left(\sum_{i=0}^{\infty}f_{\eta_i}h^i\right)^2\right]_{h=0}, \qquad \beta_k=\frac{1}{j!}\frac{d}{dh^j}\left[\left(\sum_{i=0}^{\infty}f_i h^i\right)\left(\sum_{i=0}^{\infty}f_{\eta\eta_i}h^i\right)\right]_{h=0}. \tag{3.6}$$

Hence, we have the recursive Adomian algorithm for generating the series solutions expressed as follows

$$f_0=V_c+\eta+\frac{1}{2}\alpha\eta^2, \tag{3.7}$$

$$f_1=\frac{1}{6}(1+Q-\alpha V_c)\eta^3+\frac{1}{24}(1+Q)\alpha\eta^4+\frac{1}{120}\alpha^2\eta^5, \tag{3.8}$$

$$\begin{aligned}f_2=&\frac{1}{24}V_c(\alpha V_c-1-Q)\eta^4+\frac{1}{120}\left(Q+Q^2-V_c\alpha-2QV_c\alpha\right)\eta^5+\\&-\frac{1}{720}\alpha\left(3V_c\alpha-1-2Q-Q^2\right)\eta^6+\frac{1}{5040}(2Q-1)\alpha^2\eta^7-\frac{1}{40320}\alpha^3\eta^8.\end{aligned} \tag{3.9}$$

The series solution is given

$$\begin{aligned}f(\eta)=&\sum_{k=0}^{\infty}f_k(\eta)=V_c+\eta+\frac{1}{2}\alpha\eta^2+\frac{1}{6}(1+Q-\alpha V_c)\eta^3+\\&+\frac{1}{24}\big((1+Q)\alpha+V_c(\alpha V_c-1-Q)\big)\eta^4+\frac{1}{120}\left(\alpha^2+Q+Q^2-V_c\alpha-2QV_c\alpha\right)\eta^5+\\&-\frac{1}{720}\alpha\left(3V_c\alpha-1-2Q-Q^2\right)\eta^6+\frac{1}{5040}(2Q-1)\alpha^2\eta^7-\frac{1}{40320}\alpha^3\eta^8.\end{aligned} \tag{3.10}$$

Our aim in Eq.(3.10) mainly concerns the mathematical behavior of the solution $f(\eta)$. It is a well known fact that Padé approximants will converge on the entire real axis if $f(\eta)$ is free of singularities on the entire real axis. More importantly, the diagonal approximants provide most accurate results; therefore we will construct only diagonal approximants. The undetermined value of $\alpha=f''(0)$ is calculated from the boundary condition at infinity in Eq.(2.11). This is achieved by constructing a diagonal Padé approximant [*M*/*M*] for the partial sum of $f'(\eta)$, i.e.,

$$f'(\eta)=\frac{\sum_{i=0}^{M}a_i\eta^i}{\sum_{i=0}^{M}b_i\eta^i}+O\left(\eta^{2M+1}\right), \tag{3.11}$$

and in order to evaluate $f'(\infty) \to 0$, we obtain the limit of the partial sum in Eq.(3.11), i.e., $\lim_{\eta \to \infty} A = 0$, where

$$A = \frac{\sum_{i=0}^{M} a_i \eta^i}{\sum_{i=0}^{M} b_i \eta^i} . \tag{3.12}$$

The resulting polynomial in α from Eq.(3.12) is solved numerically to determine the approximate value of α.

4. Results and discussion

In order to validate the results obtained using the Adomian – Padé technique, we solved the entire problem purely numerically using the fourth-fifth order Runge-Kutta-Fehlberg method implemented on MAPLE 14 and compared the result obtained with that of [*10/10*] diagonal Pade approximant as illustrated in Tab.1. It is clearly observed from Tab.1 that the present results are in good agreement with the numerical solution. This favourable comparison lends confidence in the ADM – Padé results reported subsequently. Figure 2 shows the effect of increasing the magnetic field strength on the momentum boundary-layer thickness. It is now a well established fact that the magnetic field presents a damping effect on the velocity field by creating a drag force that opposes the fluid motion, causing the velocity to decrease. An increase in the Chandrasekhar number slows down the motion of the fluid and decreases the momentum boundary layer thickness. A similar trend is observed in Fig.3 with increasing suction ($V_c>0$) while an injection ($V_c<0$) causes the momentum boundary layer to thicken by increasing the fluid velocity. In Fig.4, it is interesting to note that the local skin friction generally increases with an increase in the intensity of the magnetic field. As the intensity of fluid suction increases, a further increase in the local skin friction is observed. An injection causes a decrease in the local skin friction at the sheet surface.

Tab.1. Comparison of results obtained by Adomian-Padé and numerical method.

Q	V_c	$\alpha = f''(0)$ ADM-Padé [*10/10*]	$\alpha = f''(0)$ Numerical Method
0	*0*	*-1.00000*	*-1.00000*
0	*0.5*	*-1.280777*	*-1.280777*
0.5	*0.5*	*-1.500000*	*-1.500000*
1	*0.5*	*-1.686140*	*-1.686140*
0.5	*-0.5*	*-1.00000*	*-1.000000*
0.5	*-1*	*-0.8228757*	*-0.8228757*

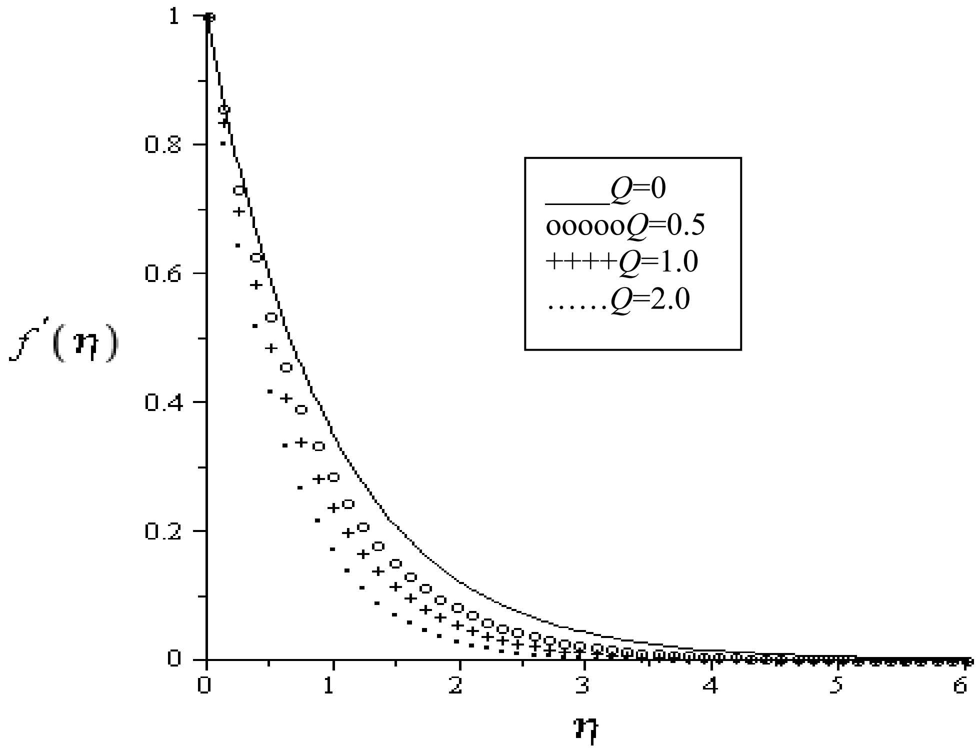

Fig.2. Effect of increasing magnetic field intensity on velocity profiles for $V_c = 0.1$.

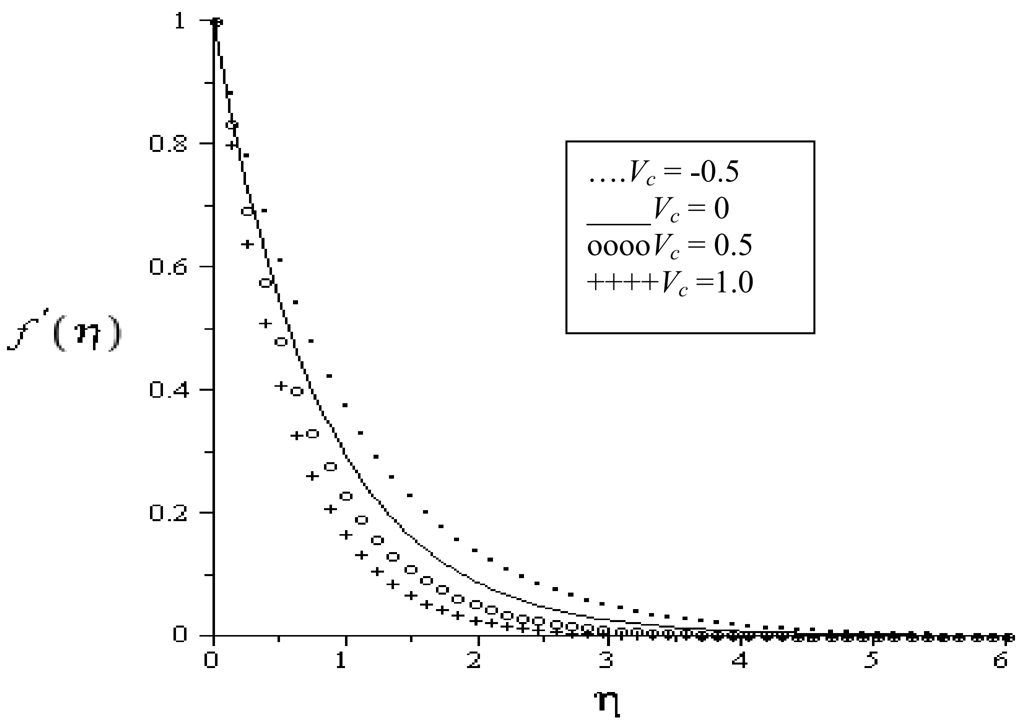

Fig.3. Effect of suction/injection on velocity profiles for Q= *0.5*.

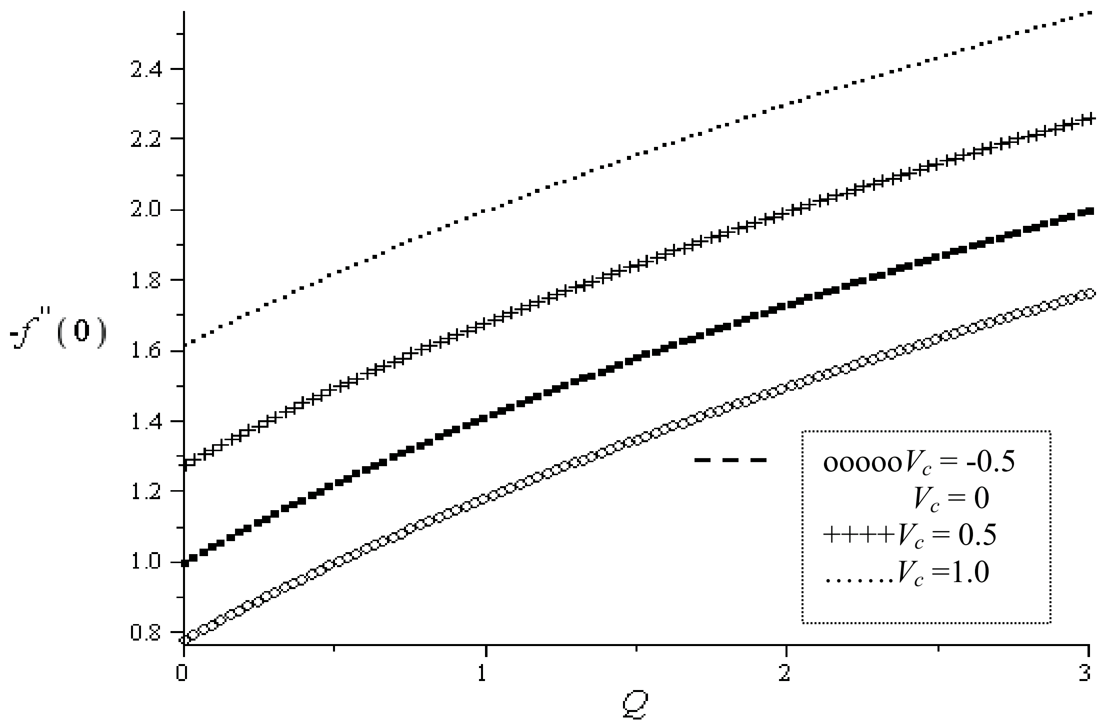

Fig.4. Effect of parameter variation on skin friction coefficients.

5. Conclusion

In this paper, we proposed and implemented an efficient technique based on ADM-Padé for solving a nonlinear ordinary differential equation modelling hydromagnetic flow over a stretching sheet. A comparison with the Runge-Kutta-Fehlberg numerical method was also made. The solution is very rapidly convergent by utilizing the ADM-Padé technique. This approach provides a potential alternative to traditional numerical techniques in various applications in sciences and engineering.

Nomenclature

c – constant rate of stretching [s^{-1}]
f – similarity function
H_0 – strength of the magnetic field [$w\ m\text{-}2$]
k_1 – $k_1 = \dfrac{k_0 c}{\mu}$ viscoelastic parameter
l – characteristic length [m]
M – $\left(H_0\sqrt{\dfrac{\sigma}{c\rho}}\right)$ Hartmann number
Q – M^2 (Chandrasekhar hydromagnetic number)
u – velocity component along the sheet [$m\ s\text{-}1$]
v – velocity component normal to the sheet [$m\ s\text{-}1$]
v_w – suction/injection
x – coordinate along the sheet [m]
y – coordinate normal to the sheet [m]
η – similarity variable

μ – dynamic viscosity [$kg\ m^{-1}s^{-1}$]
υ – kinematic fluid viscosity [$m^2\ s^{-1}$]
ρ – density [$kg\ m^{-3}$]
ψ – stream function [$m^2\ s^{-1}$]
σ – electrical conductivity [$mho\ m^{-1}$]

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