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Non-Recursive Control Design for Nonlinear System With Backlash-Like Hysteresis and Disturbance

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ABSTRACT In this paper, we propose a non-recursive homogeneity-based robust control for a kind of nonlinear systems with backlash-like hysteresis and external disturbance, as an alternative approach to the well known recursive backstepping design which needs to compute a number of partial derivatives. The backlash-like hysteresis and external disturbance are also considered in our design which makes our method more practical in the application of control engineering. Global asymptotical tracking performance is guaranteed with proposed control scheme. Some simulation results are provided for illustrating our theoretical results.

INDEX TERMS Non-recursive, nonlinear systems, global asymptotical tracking, backlash-like hysteresis.

I. INTRODUCTION

As an important area of control theory, nonlinear system control has been studied for decades, and some excellent control strategies have been proposed, such as input/output linearization [1], backstepping mechanism [2], finite/fixed-time control [3]–[5] and model predictive control [6]. Backstepping method is a promising approach based on recursive design procedure for nonlinear system with strictly feedback forms, see [2], [7], [8] for examples. Due to its recursive design procedure, backstepping method involves a number of partial derivative terms in virtual control and final control at each step. This may lead to a complex control algorithm and make it difficult for implementation, especially for high-order or high relative degree systems. On the other hand, homogeneous systems, which cover a broad category of inherently nonlinear systems [9], have been studied [10]–[12] and applied in nonlinear system control programming, such as [12]–[15]. In [15], the authors propoesd a innovation for the output feedback stabilization of a kind of nonlinear system, using the concept of *homogeneous domination*. The practical tracking problem has also been investigated for the nonlinear system with homogeneous technique in [16]. Unfortunately, most homogeneous control design approaches still need recursive design process except for [14], in which a non-recursive authentication is given for a chain of integrators. Note that practical systems are not just modeled by a chain of integrators. Thus it is important to find a suitable control design methodology to design stable control for general systems. Recently in [17], a non-recursive innovation is presented for a kind of nonlinear systems,thus abundant partial derivative terms have been avoided. Note that practical factors such as hysteresis and disturbances are unavoidable and should be taken into account. In [7], an adaptive control is designed for systems with hysteresis based on recursive-backstepping process. An alternative non-recursive approach for nonlinear systems with the unknown hysteresis and disturbance is needed, which motivates us to propose a new design strategy in this paper.

Inspired by the property of the weighted homogeneity and superiority of homogeneous domination, we proposed a non-recursive tracking arithmetic for a kind of nonlinear system. Firstly, a series of coordinates transforms with the desired steady-states are proposed to covert the tacking problem to stabilization problem with the consideration of back-lash hysteresis and external disturbance. Secondly, a novel homogeneous control law is designed with a scaling gain. Furthermore, the combined effect caused by approximating hysteresis and disturbances is handled in a similar way to [7]. The unknown bound of the effect is estimated online

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with an adaptive law. The obtained estimate is incorporated in the control law. Thirdly, a guideline of choosing the alterable gain value is derived by analyzing stability of the closed-loop system, and the other control parameters can be easily designed as the coefficients of a Hurwitz polynomial. Moreover, both asymptotic and finite-time tracking consequences can be built under our method through changing the value of the homogeneous degree. It is also shown that the accurate tracking can be achieved in the absence of hysteresis and disturbance.

Paper organization: The problem statement and some useful lemmas are listed in Section 2. The controller design and stability analysis are presented in Section 3, followed by some illustrative simulation examples in Section 4. Finally, the conclusion is given in Section 5.

Notations:

- $\mathcal{R}^+_{odd}$ — the ratios of a set of two positive integers.
- $\mathbb{N}_{j:i}$ — $\mathbb{N}_{j:i} = \{j, j+1, \cdots, i\}$ with integers j and i satisfying $0 \le j \le i$.
- $\mathbb{C}^i$ — the set of all differentiable functions whose first ith time derivatives are continuous.
- $\lfloor\cdot\rceil^a$ — a $\mathbb{C}^0$ function, where $\lfloor\cdot\rceil^a = \text{sgn}(\cdot)|\cdot|^a$.

(Weighted Homogeneity):

- Δ^r — is a map:$\mathcal{R}^+ \times \mathcal{R}^n \to \mathcal{R}^n$ named a one-parameter family of dilation.
- $\Delta^r_\epsilon x$ — $\Delta^r_\epsilon x = (\epsilon^{r_1}x_1, \ldots, \epsilon^{r_n}x_n)$, where $x = (x_1, \ldots, x_n) \in \mathcal{R}^n$ is a fixed choice of coordinates and $r \triangleq (r_1, r_2, \cdots, r_n)$ are positive real numbers.
- $V \in \mathcal{H}^\tau_{\Delta^r}$ — a continuous function: $\mathcal{R}^n \to \mathcal{R}$ with a given dilation Δ^r and a real number τ called Δ^r−homogeneous of degree τ if $V \circ \Delta^r_\epsilon = \epsilon^\tau V$.
- $f_j \in \mathcal{H}^{\tau+r_j}_{\Delta^r}$ — a continuous vector field $f(x) = \sum f_j(x)(\frac{\partial}{\partial x_j})$ named Δ^r− homogeneous of degree τ with $j \in \mathbb{N}_{1:n}$.
- $\bar{x}^\tau_{i\Delta^r}$ — a vector $\bar{x}^\tau_{i\Delta^r} = (x_1^{\tau/r_1}, \cdots, x_i^{\tau/r_i})^T$, where $x^\tau_{\Delta^r} = \bar{x}^\tau_{n\Delta^r}$ and $\lfloor x\rceil^\tau_{\Delta^r} = (\lfloor x_1\rceil^{\tau/r_1}, \cdots, \lfloor x_n\rceil^{\tau/r_n})^T$.
- $\|x\|_{\Delta^r}$ — a homogeneous p−norm, where $\|x\|_{\Delta^r} = (\sum_{i=1}^n |x_i|^{p/r_i})^{1/p}$.
- $\|x\|$ — a conventional $L - p$ norm, where $\|x\| = (\sum_{i=1}^n |x_i|^p)^{1/p}$.

In this paper, r is given by $r_1 = 1, r_i = r_{i-1} + \tau,\ i \in \mathbb{N}_{2:n}$ with a degree τ. We denote $\kappa \ge \max\{r_i + \tau\}_{i \in \mathbb{N}_{1:n}}$ and choose $p = 2$ for the sake of simplicity.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

In this section, we present the system model, the design problem and some useful Lemmas.

A. PROBLEM FORMULATION

Consider a kind of SISO nonlinear system model described by

$$\begin{cases} \dot{x}_1 = G_1x_2 + F_1(x_1) \\ \dot{x}_2 = G_2x_3 + F_2(\bar{x}_2) \\ \cdots \\ \dot{x}_n = G_nW(u(t)) + d(t) + F_n(\bar{x}_n) \\ y(t) = x_1, \end{cases} \tag{1}$$

where $\bar{x}_i = (x_1, x_2, \cdots, x_i)^T$ and $x_i \in R, i \in \{1, 2, \ldots, n\}$ denote the states of the plant. $F_i, i \in \{1, 2, \ldots, n\}$ are known system functions and $G_i, i \in \{1, 2, \ldots, r\}$ are known parameters. $u(t)$ is the designed controller and $W(u(t))$ denotes the plant input with backlash-like hysteresis of $u(t)$. The desired trajectory is y_d.

Assumption 1: The desired trajectory y_d and its nth derivatives are piecewise continuous, known and bounded.

Assumption 2: There exist a known constant $\sigma > 0$ and a constant τ, such that

$$|F_i(x_i) - F_i(\hat{x}_i)| \le \sigma \sum_{j=1}^{i} |x_j - \hat{x}_j|^{\frac{r_i+\tau}{r_j}}, \quad i \in \mathbb{N}_{1:n}.$$

Remark 1: Assumption 2 can be seen as a generalized continuous condition satisfied by many nonlinear functions. When $\tau = 0$, Assumption 2 is the widely known global Lipschitz continuous condition. And Assumption 2 becomes a Hölder continuous condition when $\tau < 0$. This model can be considered as a special fractional order system, which has a wide range of applications, especially in electronics [18] and electromagnetism [19].

The purpose of this paper is to design a controller which can ensure global stability of the closed-loop system in the presence of backlash-like hysteresis and external disturbance. In addition, the system state x_1 will converge to the desired trajectory y_d asymptotically in the case of $\tau \ge 0$, or in finite time with $\tau < 0$.

B. BACKLASH-LIKE HYSTERESIS AND LEMMA INTRODUCTION

The property of backlash-like hysteresis and some useful lemmas are introduced in this part for the convenience of readers. Based on the analysis in [7], [20], the input backlash-like hysteresis of the system is described by

$$\frac{dW(t)}{dt} = \alpha\left|\frac{du(t)}{dt}\right|(cu(t) - W(t)) + B_1\frac{du(t)}{dt} \tag{2}$$

where α, c and B_1 are constants, c is the slope coefficient of the line satisfying $c > B_1$. The above equation can be solved as

$$W(u(t)) = cu(t) + d_1(u) \tag{3}$$

$$d_1(u) = [W(0) - cu(0)]e^{-\alpha(u(t)-u(0))sgn(\dot{u})} + e^{-\alpha(u(t))sgn(\dot{(u)})}\int_{u(0)}^{u(t)} (B_1 - c)e^{\alpha s sgn(\dot{u})}ds \tag{4}$$

where d_1 can be seen as the bounded approximation error of backlash-like hysteresis by the linear approximator $cu(t)$. The backlash-like hysteresis with parameters $B_1 = 0.345$ $\alpha = 1$ and $c = 3.1635$ is shown in Fig.1.

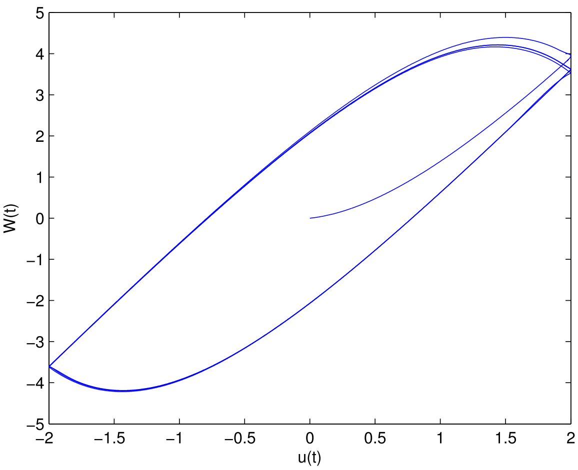

FIGURE 1. **Backlash-like hysteresis with $B_1 = 0.345$ $\alpha = 1$, $c = 3.1635$ and $u(t) = 2\sin(2.3t)$.**

Then we have the system model as

$$\begin{cases} \dot{x}_1 = G_1 x_2 + F_1(x_1) \\ \dot{x}_2 = G_2 x_3 + F_2(x_1, x_2) \\ \dots \\ \dot{x}_n = G_n cu(t) + D(t) + F_n(x_1, \dots, x_n) \\ y(t) = x_1, \end{cases} \tag{5}$$

where $D(t) = d(t) + G_n d_1(u)$ and $D(t) \leq D^*$ with D^* being an unknown positive constant. Then we give some lemmas which are used in next section:

Lemma 1 [14]: Let $V_1(x) \in \mathcal{H}_{\Delta^r}^{\tau_1}$ and $V_2(x) \in \mathcal{H}_{\Delta^r}^{\tau_2}$, respectively, then one has

i) $V_1(x)V_2(x) \in \mathcal{H}_{\Delta^r}^{\tau_1+\tau_2}$.

ii) $\frac{\partial V_1(x)}{\partial x_i} \in \mathcal{H}_{\Delta^r}^{\tau_1 - r_i}$, $i \in \mathbb{N}_{1:n}$.

iii) If $V_1(x) \geq 0$, then

$$\left(\min_{\{x:V_1(x)=1\}} V_2(x)\right) V_1^{\frac{\tau_2}{\tau_1}}(x) \leq V_2(x) \leq \left(\max_{\{x:V_1(x)=1\}} V_2(x)\right) V_1^{\frac{\tau_2}{\tau_1}}$$

Lemma 2 [10]: For a system $\dot{x} = f(x)$, $x \in \mathcal{R}^n$, where $f(x)$ is a continuous function with homogeneous of degree τ.The whole system is globally stable when the origin is locally asymptotically stable. The whole system is globally finite-time stable when $\tau \leq 0$.

Lemma 3 [21]: For a nonlinear system $\dot{x} = f(x, t)$, $f(0, t) = 0$. If there exists a positive-definite V and $\varpi > 0$, $\varpi \in R$ and $\iota \in (0, 1)$, $\varpi \in R$, and $\dot{V} + \varpi V^{\iota}$ is semi-negative definite, then $x = 0$ can be defined as a globally finite-time stable equilibrium and the whole system can reach stable with $T \leq \frac{V^{1-\iota}(x_0, t_0)}{\varpi(1-\iota)}$ for any $x_0 = x(t_0)$.

Lemma 4 [22]: Let $\mathcal{X}$ and $\mathcal{Y}$ be topological space, where $\mathcal{Y}$ is a compact set. Let $\mathbb{O}$ be an open set of $\mathcal{X} \times \mathcal{Y}$ and a slice $\{\mathbf{x}_0\} \times \mathcal{Y}$ of $\mathcal{X} \times \mathcal{Y}$ belongs to $\mathbb{O}$, then $\mathbb{O}$ also contain $\mathcal{W} \times \mathcal{Y}$ where $\mathcal{W}$ is a neighborhood of $\mathbf{x}_0$ in $\mathcal{X}$.

Lemma 5 [23]: With constants $m > 0, n > 0$. Given any positive smooth function $\Re(x, y)$, the following inequality holds

$$|x|^m |y|^d \leq \frac{m}{m+d} \Re(x, y) |x|^{m+d} + \frac{m}{d+m} \Re^{-m/d}(x, y) |y|^{m+d}.$$

III. DESIGN OF NON-RECURSIVE CONTROLLER AND STABILITY ANALYSIS

It is noticed that many existing results of homogeneous control design focus on the stabilization problems which are easier to design and analyze. We firstly use a coordinate transformation to reduce the tracking problem to the stabilization problem, then a homogeneous control is given, followed by the stability analysis in remaining part of this section.

A. SYSTEM ANALYSIS AND CONTROLLER DESIGN

To begin our design, we first define the auxiliary variables x_i^* as

$$\begin{cases} x_1^* = G_0^{-1} y_d \\ x_2^* = G_1^{-1}(\dot{x}_1^* - F_1(\bar{x}_1^*)) \\ \dots \\ x_n^* = G_{n-1}^{-1}(\dot{x}_{n-1}^* - F_{n-1}(\bar{x}_{n-1}^*)) \\ x_{n+1}^* = G_n^{-1}(\dot{x}_n^* - F_n(\bar{x}_n^*)) \end{cases} \tag{6}$$

where $\bar{x}_i^* = (x_1^*, \cdots, x_i^*)^T$ and $x_{i\in\mathbb{N}_{1:n}}^*$ can be seen as the system desired steady states. Then we define the coordinates with z_i as

$$\begin{cases} z_1 = \prod_{m=1}^{1} G_{m-1}(x_1 - x_1^*) \\ z_2 = \prod_{m=1}^{2} G_{m-1}(x_2 - x_2^*) \\ \dots \\ z_n = \prod_{m=1}^{n} G_{m-1}(x_n - x_n^*) \end{cases} \tag{7}$$

where $\prod_{m=1}^{j} G_{m-1} = G_0 G_1 \dots G_{j-1}$, and $G_0 = 1$. The system model (1) can be transferred to

$$\begin{cases} \dot{z}_1 = z_2 + \prod_{m=1}^{1} G_{m-1} \tilde{F}_1 \\ \dot{z}_2 = z_3 + \prod_{m=1}^{2} G_{m-1} \tilde{F}_2 \\ \dots \\ \dot{z}_n = \prod_{m=1}^{n+1} G_{m-1} c(u(t) - x_{n+1}^*) \\ \quad + \prod_{m=1}^{n} G_{m-1} \tilde{F}_n + \prod_{m=1}^{n} G_{m-1} D(t) \end{cases} \tag{8}$$

where $\tilde{F}_i = F_i((x_1, \dots, x_i)) - F_i((x_1^*, \dots, x_i^*))$ satisfies Assumption 2. Denote $z = (z_1, \cdots, z_n)^T$ and introduce a new coordinate ζ as

$$\zeta = \mathbb{L} z \tag{9}$$

$$\mathbb{L} = \mathrm{diag}\{1/L^0, 1/L, \dots, 1/L^{n-1}\} \tag{10}$$

where $\zeta = (\zeta_1, \cdots, \zeta_n)^T$. Then we have the following new system model

$$\begin{cases} \dot{\zeta}_1 = L\zeta_2 + \frac{\prod_{m=1}^{1} G_{m-1}}{L^0}\tilde{F}_1 \\ \dot{\zeta}_2 = L\zeta_3 + \frac{\prod_{m=1}^{2} G_{m-1}}{L^1}\tilde{F}_2 \\ \cdots \\ \dot{\zeta}_n = \frac{\prod_{m=1}^{n+1} G_{m-1}c}{L^{n-1}}(u(t) - x_{n+1}^*) \\ + \frac{\prod_{m=1}^{n} G_{m-1}}{L^{n-1}}\tilde{F}_r + \frac{\prod_{m=1}^{n} G_{m-1}}{L^{n-1}}D(t) \end{cases} \tag{11}$$

From Assumption 2, one has

$$\begin{aligned} \tilde{F}_i &\le \frac{\sigma\prod_{m=1}^{i} G_{m-1}}{L^{i-1}} \sum_{j=1}^{i}\left(\prod_{m=1}^{j} G_{m-1}\right)^{-\frac{r_i+\tau}{r_j}} |z_j|^{\frac{r_i+\tau}{r_j}} \\ &\le \sigma\prod_{m=1}^{i} G_{m-1}\Theta\left(\frac{\|L^0\|^{\frac{r_i+\tau}{r_1}}}{L^{i-1}}\|\zeta_1\|^{\frac{r_i+\tau}{r_1}} \right. \\ &\quad \left. + \ldots + \frac{\|L^{r-1}\|^{\frac{r_r+\tau}{r_1}}}{L^{i-1}}\|\zeta_i\|^{\frac{r_i+\tau}{r_i}}\right) \\ &= \sigma^*\prod_{m=1}^{i} G_{m-1}\left(L_0^*\|\zeta_1\|^{\frac{r_i+\tau}{r_1}} + \ldots + L_{i-1}^*\|\zeta_i\|^{\frac{r_i+\tau}{r_i}}\right) \end{aligned} \tag{12}$$

where $\Theta = \min\{\left(\prod_{m=1}^{j} G_{j-1}\right)^{-\frac{r_i+\tau}{r_j}}\}_{j\in N_{1:n}}$, $\sigma^* = \sigma\prod_{m=1}^{i} G_{m-1}\Theta$ and $L_j^* = L^{(j-1)\frac{r_i+\tau}{r_j}-(i-1)}$. Now we have

$$L_j^* = L^{(j-1)\frac{r_i+\tau}{r_j}-(i-1)} \le L^{1-\frac{1}{\max_{j\in N_{1:i}}\{r_j\}}} \tag{13}$$

So

$$\tilde{F}_i \le \sigma^* L^{1-\frac{1}{\max_{j\in N_{1:i}}\{r_j\}}}\bar{F}_i \tag{14}$$

where $\bar{F}_i = \left(\|\zeta_1\|^{\frac{r_i+\tau}{r_1}} + \ldots + \|\zeta_i\|^{\frac{r_i+\tau}{r_i}}\right)$. The controller can be designed as

$$\begin{aligned} u(t) = &-L^n\mathcal{K}\left(\prod_{m=1}^{n+1} G_{m-1}c\right)^{-1}\lfloor\zeta\rceil_{\Delta^r}^{r_n+\tau} \\ &+ G_n^{-1}(\dot{x}_n^* - F_n(x_1^*, \ldots, x_n^*)) - (G_n c)^{-1} sgn(z_n)\hat{D} \end{aligned} \tag{15}$$

where $\mathcal{K} = (k_1, \ldots, k_n)$ is the coefficient vector of a Hurwitz polynomial $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$, $\tau \ge 0$ is a homogeneous degree. $\hat{D}$ is the estimate of D^*. The update law of $\hat{D}$ is given as

$$\dot{\hat{D}} = \eta\sum_{i=1}^{n}|\zeta_i^{\frac{k-\frac{\tau}{2}}{r_i}}||\zeta_n^{\frac{k-\frac{\tau}{2}-r_n}{r_n}}| \tag{16}$$

B. STABILITY ANALYSIS

Now we are going to give the main achievement of this paper.

Theorem 1: Based on the closed system (1) satisfying Assumptions 1 and 2. With the application of controller (15) and the parameter update law (16), the closed-loop system obtain global stability. For any initial value $x(0)$ and given tolerance $\varepsilon > 0$, there exists a finite time $T^* > 0$, such that $\|\zeta\|\varepsilon, \forall t > T^*$.

Proof: With the controller (15), the system closed-loop system is obtained as

$$\begin{cases} \dot{\zeta}_1 = L\zeta_2 + \frac{\prod_{m=1}^{1} G_{m-1}}{L^0}\tilde{F}_1 \\ \dot{\zeta}_2 = L\zeta_3 + \frac{\prod_{m=1}^{2} G_{m-1}}{L^1}\tilde{F}_2 \\ \cdots \\ \dot{\zeta}_r = -L\mathcal{K}\lfloor\zeta\rceil_{\Delta^r}^{r_n+\tau} + \frac{\prod_{m=1}^{n} G_{m-1}}{L^{n-1}}\tilde{F}_r \\ -\frac{\prod_{m=1}^{n} G_{m-1}}{L^{n-1}} sgn(z_n)\hat{D} + \frac{\prod_{m=1}^{n} G_{m-1}}{L^{n-1}}D(t) \end{cases} \tag{17}$$

Now homogeneous Lyapunov function can be designed as

$$W(\zeta, \tilde{D}) = V + \frac{\Psi^*}{2\eta}\tilde{D}^2 \tag{18}$$

where $V = \frac{1}{2}\lfloor\zeta\rceil_{\Delta^r}^{\kappa-\frac{\tau}{2}T} P\lfloor\zeta\rceil_{\Delta^r}^{\kappa-\frac{\tau}{2}}$, $\tilde{D} = D^* - \hat{D}$ and $P = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \vdots & \ddots \\ p_{n,1} & \cdots & p_{n,n} \end{bmatrix}$ is a positive definite matrix that satisfies $A_0^T P + PA_0 = -I$, where I is the identity matrix. $A_\tau = \begin{bmatrix} 0 & a_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{n-1} \\ -k_1 b_1 & -k_2 b_2 & \cdots & -k_n b_n \end{bmatrix}$ with $a_j = \frac{\kappa-\frac{\tau}{2}}{r_j}|\zeta_j|^{\frac{\kappa-\tau-r_j}{r_j}}|\zeta_{j+1}|^{\frac{-\kappa+r_{j+1}}{r_{j+1}}}$ and $b_j = \frac{\kappa-\tau/2}{r_n}|\zeta_n|^{\frac{\kappa-\tau-r_n}{r_n}}|\zeta_j|^{\frac{-\kappa+r_n+\tau}{r_j}}$. Ψ^* is a positive constant defined in (22) and η is a designed positive constant. And $\kappa = \max\{r_n+1\}$, we have $V \in \mathbb{C}^1 \cap \mathcal{H}_{\Delta^r}^{2\kappa-\tau}$. The derivative of the Lyapunov function V can be obtained as

$$\begin{aligned} \dot{V} = &\sum_{i=1}^{n-1}\frac{\partial V}{\partial\zeta_i}L\zeta_{i+1} - \frac{\partial V}{\partial\zeta_n}L\mathcal{K}\lfloor\zeta\rceil_{\Delta^r}^{r_n+\tau} \\ &+ \sum_{i=1}^{n}\frac{\partial V}{\partial\zeta_i}\frac{\prod_{m=1}^{i} G_{m-1}}{L^{i-1}}\tilde{F}_i + \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{i} G_{m-1}}{L^{i-1}}D(t) \\ &- \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{n} G_{m-1}}{L^{n-1}} sgn(z_n)\hat{D} \\ = &L\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}(\theta^{-1}P\theta A + A^T\theta P\theta^{-1})\lfloor\zeta\rceil_{\Delta^r}^{\kappa} \\ &+ \sum_{i=1}^{n}\frac{\partial V}{\partial\zeta_i}\frac{\prod_{m=1}^{i} G_{m-1}}{L^{i-1}}\tilde{F}_i + \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{i} G_{m-1}}{L^{i-1}}D(t) \\ &- \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{n} G_{m-1}}{L^{n-1}} sgn(z_n)\hat{D} \end{aligned} \tag{19}$$

where $\theta = \begin{bmatrix} |\zeta_1|^{\frac{\tau/2}{r_1}} & 0 & \cdots & 0 \\ 0 & |\zeta_2|^{\frac{\tau/2}{r_2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & |\zeta_n|^{\frac{\tau/2}{r_n}} \end{bmatrix}$ and we define $\varphi_\tau = \theta^{-1}P\theta A + A^T\theta P\theta^{-1}$. When $\tau = 0$, it is easy to get that $\varphi_0 = A_0^TP + PA_0 = -I$ with $A_0 = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -k_1 & -k_2 & \cdots & -k_n \end{bmatrix}$ and $L\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}(A_0^TP + PA_0)\lfloor\zeta\rceil_{\Delta^r}^{\kappa} = -L\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa}$. With the Lemma.4, there exist two constants ς_1 and ς_2, such that $\varphi_\tau(-\varsigma_1, \varsigma_2) < 0$. Now we have

$$\dot V \le -LB_1\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} + \sum_{i=1}^{n}\frac{\partial V}{\partial\zeta_i}\frac{\prod_{m=1}^{i}G_{m-1}}{L^{i-1}}\tilde F_i + \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{i}G_{m-1}}{L^{i-1}}D(t) - \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{n}G_{m-1}}{L^{n-1}}sgn(z_n)\hat D \tag{20}$$

where B_1 is a positive constant. From (14), we have

$$\begin{aligned}\dot V &\le -LB_1\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} + \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{i}G_{m-1}}{L^{i-1}}D(t) \\ &\quad + \sum_{i=1}^{n}\left|\frac{\partial V}{\partial\zeta_i}\right|\sigma^*L^{1-\frac{1}{\max_{j\in N_{1:i}}\{r_j\}}}\bar F_i \\ &\quad - \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{n}G_{m-1}}{L^{n-1}}sgn(z_n)\hat D \\ &\le -LB_1\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} + \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{i}G_{m-1}}{L^{i-1}}D(t) \\ &\quad + \sigma^*L^{1-\frac{1}{\max_{j\in N_{1:n}}\{r_j\}}}\sum_{i=1}^{n}\left|\frac{\partial V}{\partial\zeta_i}\right|\bar F_i \\ &\quad - \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{n}G_{m-1}}{L^{n-1}}sgn(z_n)\hat D \end{aligned} \tag{21}$$

With $\frac{\partial V}{\partial\zeta_i} \in \mathcal{H}_{\Delta^\gamma}^{2\kappa-\tau-r_i}$, $\bar F_i \in \mathcal{H}_{\Delta^r}^{r_i+\tau}$ and $\frac{\partial V}{\partial\zeta_n} = (\sum_{i=1}^{n-1}\{(p_{i,n} + p_{n,i})\frac{k-\frac{\tau}{2}}{2\,r_n}\zeta_i^{\frac{k-\frac{\tau}{2}}{r_i}}\} + p_{n,n}\frac{2k-\tau}{2r_n}\zeta_n^{\frac{k-\frac{\tau}{2}}{r_n}})\zeta_n^{\frac{k-\frac{\tau}{2}-r_n}{r_n}}$, the following result can be obtained with Lemma.1

$$\begin{aligned}\dot V &\le -LB_1\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} + B_2\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} \\ &\quad + \frac{\partial V}{\partial\zeta_n}\frac{\prod_{m=1}^{i}G_{m-1}}{L^{i-1}}(-sgn(z_n)\hat D + D(t)) \\ &\le -C\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} \\ &\quad + (\sum_{i=1}^{n-1}\{(p_{i,n} + p_{n,i})\frac{k-\frac{\tau}{2}}{2r_n}\zeta_i^{\frac{k-\frac{\tau}{2}}{r_i}}\} + p_{n,n}\frac{2k-\tau}{2r_n}\zeta_n^{\frac{k-\frac{\tau}{2}}{r_n}}) \\ &\quad *\zeta_n^{\frac{k-\frac{\tau}{2}-r_n}{r_n}}\frac{\prod_{m=1}^{i}G_{m-1}}{L^{i-1}}(-sgn(\zeta_n)\hat D + D(t)) \\ &\le -C\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} \\ &\quad + \Psi\sum_{i=1}^{n}|\zeta_i^{\frac{k-\frac{\tau}{2}}{r_i}}||\zeta_n^{\frac{k-\frac{\tau}{2}-r_n}{r_n}}|\frac{\prod_{m=1}^{i}G_{m-1}}{L^{i-1}}(-\hat D + D^*) \\ &\le -C\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} + \Psi^*\sum_{i=1}^{n}|\zeta_i^{\frac{k-\frac{\tau}{2}}{r_i}}||\zeta_n^{\frac{k-\frac{\tau}{2}-r_n}{r_n}}||\tilde D| \end{aligned} \tag{22}$$

where $\Psi = \max\{|p_{1,n} + p_{n,1}|\frac{k-\frac{\tau}{2}}{r_n}, \ldots, |p_{n-1,n} + p_{n,n-1}|\frac{k-\frac{\tau}{2}}{r_n}, |p_{n,n}|\frac{2k-\tau}{2r_n}\}$, $\Psi^* = \Psi\frac{\prod_{m=1}^{i}G_{m-1}}{L^{i-1}}$ and $C = LB_1 - B_2$ is a positive constant. From (18), we have

$$\begin{aligned}\dot W &\le -C\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} + \frac{\Psi^*}{\eta}\tilde D(-\dot{\hat D}) \\ &\quad + \Psi^*\sum_{i=1}^{n}|\zeta_i^{\frac{k-\frac{\tau}{2}}{r_i}}||\zeta_n^{\frac{k-\frac{\tau}{2}-r_n}{r_n}}||\tilde D| \\ &\le -C\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} \\ &\quad + \frac{\Psi^*}{\eta}|\tilde D|(\eta\sum_{i=1}^{n}|\zeta_i^{\frac{k-\frac{\tau}{2}}{r_i}}||\zeta_n^{\frac{k-\frac{\tau}{2}-r_n}{r_n}}| - \dot{\hat D}) \end{aligned} \tag{23}$$

With the update law (16),

$$\dot W \le -C\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} \tag{24}$$

When $\tau \in [0, +\varsigma_2)$, from (18) and (24), we know $(\zeta, \tilde D)$ is global uniformly bounded for $t \in [0, +\infty)$, e.i. the whole system is asymptotically stable. When $\tau \in (\varsigma_1, 0]$, then there exist a positive constant $\mathbb{D}$ such that $|\tilde D| \le \mathbb{D}$. From (22),

$$\begin{aligned}\dot V &\le -C\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} \\ &\quad + \Psi\sum_{i=1}^{n}|\zeta_i^{\frac{k-\frac{\tau}{2}}{r_i}}||\zeta_n^{\frac{k-\frac{\tau}{2}-r_n}{r_n}}|\frac{\prod_{m=1}^{i}G_{m-1}}{L^{i-1}}(-\hat D + D^*) \\ &\le -\frac{C}{2}\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} - \frac{C}{2}|\zeta_n|^{\frac{2k}{r_n}} + \Theta(-\hat D + D^*) \end{aligned} \tag{25}$$

where $\Theta = \Psi\sum_{i=1}^{n}|\zeta_i^{\frac{k-\frac{\tau}{2}}{r_i}}||\zeta_n^{\frac{k-\frac{\tau}{2}-r_n}{r_n}}|\frac{\prod_{m=1}^{i}G_{m-1}}{L^{i-1}} \ge 0$. Now we define $V^*(\zeta_n, \hat D) = -\frac{C}{2}|\zeta_n|^{\frac{2k}{r_n}} + (-\hat D + D^*)\Psi(t)$ with $V^*(0, \hat D) = 0$. There must exist a constant $\lambda > 0$ and $\Omega = \{(\zeta_n, \hat D) : V(\zeta_n, \hat D) \le \lambda\}$, such that $V^* < 0$. Once $(\zeta_n, \hat D) \in \Omega$, it will be always in Ω as shown below.

- Case 1: When $(\zeta_n, \hat D) \in \Omega$, then we have

$$\dot V \le -\frac{C}{2}\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} \le -cV^{\frac{2k}{2k-\tau}} \tag{26}$$

- Case 2: When $(\zeta_n, \hat D) \notin \Omega$, we firstly prove that $(\zeta_n, \hat D)$ will converge to Ω with in a finite time T_1. Actually, when $(\zeta_n, \hat D)$ is outside Ω, we much have $V > \lambda$.

$$\begin{aligned} W(0) &\ge W(0) - W(\mu) \ge \int_0^\mu C\lfloor\zeta(s)\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta(s)\rceil_{\Delta^r}^{\kappa}ds \\ &\ge \int_0^\mu 2CV^{\frac{2k}{2k-\tau}}ds \ge 2C\lambda^{\frac{2k}{2k-\tau}}\mu \end{aligned} \tag{27}$$

then $(\zeta_n, \hat D)$ will approach to Ω within:

$$T_1 = \frac{W(0)}{2C\lambda^{\frac{2k}{2k-\tau}}} \tag{28}$$

Once $(\zeta_n, \hat D)$ enters Ω, we have

$$\dot V \le -\frac{C}{2}\lfloor\zeta\rceil_{\Delta^r}^{\kappa\,T}\lfloor\zeta\rceil_{\Delta^r}^{\kappa} \le -cV^{\frac{2k}{2k-\tau}} \tag{29}$$

The finite-time stabilization can be shown from Lemma3. Thus, the proof has completed.

IV. SIMULATION EXAMPLE

In this section, we consider the following system to illustrate the presented theory.

$$\begin{aligned}\dot{x}_1 &= G_1 x_2\\ \dot{x}_2 &= G_2 x_3 - B^{-1} N \sin x_1\\ \dot{x}_3 &= G_3 W(t) - M * ln(1 + x_2^2) + d(t)\end{aligned} \tag{30}$$

where $G_1 = 1, G_2 = 0.5$ and $G_3 = 2$. The initial states of the system are chosen to be $x(0) = [1, -2, 0]$. The parameters of the system are $B = 1, N = 10, M = 0.05$ for simulation purpose. The external disturbance has been added in $t = 4$ with $d(t) = 10 * sin(t)$ and $W(t)$ is the backlash-like input with system control $u(t)$. Define $F_2 = -B^{-1} N \sin x_1$ and $F_3 = -Mln(1 + x_2^2)$, then we have

$$|F_2(\bar{x}_i) - F_2(\hat{\bar{x}}_i)| \leq \sigma |x_1 - \hat{x}_1|^{\frac{1+\tau}{1}} \tag{31}$$

$$|F_i(\bar{x}_i) - F_i(\hat{\bar{x}}_i)| \leq \sigma |x_2 - \hat{x}_2|^{\frac{1+3\tau}{1+\tau}} \tag{32}$$

with $\tau < 0$. It is concluded that Assumption 2 is satisfied with $r_1 = 1$,$r_2 = 1 + \tau$,$r_3 = 1 + 2\tau$. Now we have the following system with (6).

$$\begin{cases} x_1^* = y_d\\ x_2^* = G_1^{-1} \dot{x}_1^*\\ x_3^* = G_2^{-1}(\dot{x}_2^* - F_2(x_1^*, x_2^*))\\ x_4^* = G_3^{-1}(\dot{x}_3^* - F_3(x_1^*, x_2^*, x_3^*)) \end{cases} \tag{33}$$

we assume that y_d is a bounded smooth reference signal, where $y_d = sin(\pi t)$. Based on the coordinate (7), we get

$$\begin{cases} \dot{z}_1 = z_2\\ \dot{z}_2 = z_3 + G_1^{-1} \tilde{F}_2\\ \dot{z}_3 = \prod_{m=1}^{4} G_{m-1} c(u(t) - x_4^*)\\ \quad + \prod_{m=1}^{3} G_{m-1} \tilde{F}_3 + \prod_{m=1}^{3} G_{m-1} D(t) \end{cases} \tag{34}$$

Then the system is simplified as follows:

$$\begin{cases} \dot{\zeta}_1 = L\zeta_2 + \dfrac{\prod_{m=1}^{1} G_{m-1}}{L^0} \tilde{F}_1\\ \dot{\zeta}_2 = L\zeta_3 + \dfrac{\prod_{m=1}^{2} G_{m-1}}{L^1} \tilde{F}_2\\ \dot{\zeta}_3 = \dfrac{\prod_{m=1}^{4} G_{m-1} c}{L^2}(u(t) - x_4^*)\\ + \dfrac{\prod_{m=1}^{3} G_{m-1}}{L^2} \tilde{F}_r + \dfrac{\prod_{m=1}^{3} G_{m-1}}{L^2} D(t) \end{cases} \tag{35}$$

Now we design the following controller

$$u(t) = -L^3 K(\lfloor z_1 \rceil^{\frac{r_3+\tau}{r_1}}, \lfloor z_2/L \rceil^{\frac{r_3+\tau}{r_2}}, \lfloor z_3/L^2 \rceil^{\frac{r_3+\tau}{r_3}})^T + x_4^* - (G_3 c)^{-1} sgn(z_3)\hat{D} \tag{36}$$

where $K = [6, 11, 6]$ with Hurwitz property and $L = 3$, $\tau = -2/15$. As shown in Fig.2, the tracking problem is achieved by the proposed control method. Even though the disturbance exists in the system, the perfect tracking performance can be promised in our simulation.

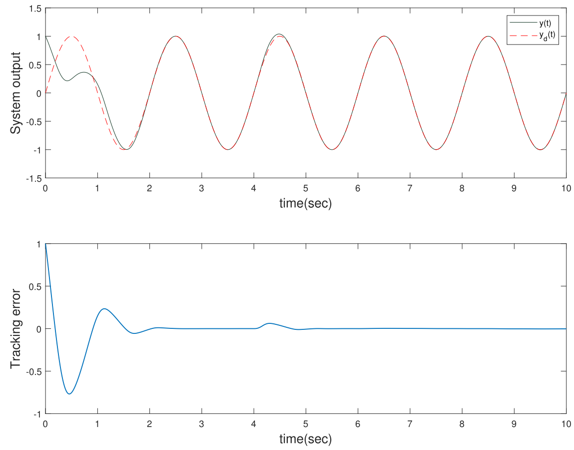

FIGURE 2. System tracking error.

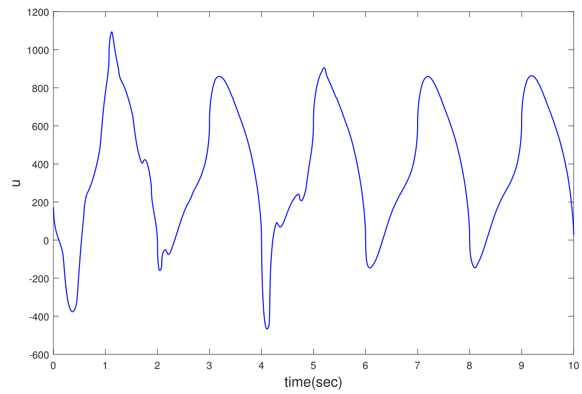

FIGURE 3. Time history of control input.

V. CONCLUSION

In this paper, a homogeneous non-recursive controller has been desingned for the tracking problem of a class of non-linear system with backlash-like hysteresis and external disturbances. The tracking controller is much simpler in the non-recursive process than the recursive design. Practical tracking control can be unified with external disturbances and backlash-like hysteresis. Both asymptotical and finite-time convergence have been achieved in the proposed method with the proper gain value and homogeneous degree. The feasibility of our schemes has been illustrated via some simulation results.

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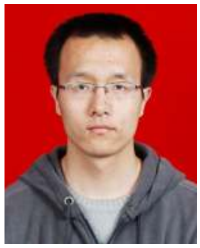

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