

Non-uniform 2-slot Constellations for Bidirectional Relaying in Fading Channels

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Abstract—In this paper we introduce the constellation alphabets suitable for bidirectional relaying in *parametric wireless channels*. Based on the analysis of hierarchical minimum distance, we present a simple design algorithm for the *non-uniform 2-slot* constellation alphabets. These novel constellation alphabets outperform traditional linear modulation schemes in Hierarchical-Decode-and-Forward and Denoise-and-Forward relaying strategies in fading channels without sacrificing the overall system throughput.

Index Terms—Bidirectional relaying, hierarchical-decode-and-forward, wireless network coding.

I. INTRODUCTION

BIDIRECTIONAL relaying strategies based on a wireless-domain extension of the traditional Network Coding (NC) [1] principles offer a great potential to improve communication in 2-Way Relay Channels (2-WRC). Significant performance benefits were observed mainly for the Denoise and Forward (DNF) [2] and Hierarchical Decode and Forward (HDF) [3] strategies (Fig. 1).

While the HDF/DNF strategies are mature in the traditional AWGN channel, their performance in fading channels could be seriously degraded due to the inherent wireless channel *parameterization* (e.g. complex channel gain) [2], [3]. This performance degradation could be avoided by phase pre-rotation of both source node transmissions [2], [4] or by adapting the relay eXclusive output symbol mapping [3] to the actual channel conditions [2]. However, both these approaches have several drawbacks, including a practical infeasibility of (synchronized) multi-node transmission phase pre-rotation (in fading channels) or a sensitivity to channel estimation errors of adaptive solutions (inaccurate channel estimate results in an improper choice of relay output symbol mapper). Although the novel \mathbb{C}^2 constellation alphabets (introduced in [5]) are quite robust to channel parameterization, this robustness is redeemed by a reduction of the potential throughput.

In this paper we target a constellation alphabet parametric-design problem from a different way. The scope is to design the alphabet robust to channel parameterization effects while avoiding the requirement of phase pre-rotation (or adaptive

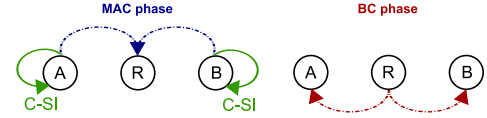


Figure 1. HDF/DNF signal flow in 2-WRC. The relay decodes only the superimposed (*hierarchical*) symbol in the Multiple-Access (MAC) phase of the communication (instead of a joint decoding of the two separate data streams). In the following Broadcast phase (BC) the hierarchical symbol is broadcasted to both destinations, which are then able to decode the desired information by using the Complementary Side Information (C-SI) about their own (previously sent) data [2], [3].

processing) but still preserving the \mathbb{C}^1 (per symbol slot) dimensionality constraint (to avoid the throughput reduction). Based on the analysis of the hierarchical (Euclidean) distance [6], we introduce the non-uniform 2-slot constellations and compare their Symbol Error Rate (SER) performance to that of the traditional linear modulation constellations.

II. 2-WRC WITH HDF STRATEGY

A parametric wireless 2-WRC system (Fig. 1) with 3 physically separated nodes (sources A, B and relay R) is considered in this paper. Data A source is co-located with the destination for data B (and vice versa). A signal space representation (with an *orthonormal* basis) of the transmitted channel symbols is $s_A(c_A), s_B(c_B)$ ($s_A, s_B \in \mathcal{A}_s \subset \mathbb{C}^N, |\mathcal{A}_s| = M$), where c_A, c_B are source node code symbols, $\mathcal{A}_s(\cdot)$ is a channel symbol memoryless mapper and M is the alphabet cardinality.

The constellation space signal received at the relay in MAC phase is

$$x = h_A s_A + h_B s_B + w \quad (1)$$

where w is the circularly symmetric complex Gaussian noise (variance σ_w^2 per complex dimension) and h_A, h_B are scalar complex channel coefficients (constant during the observation and known at the relay). The useful signal ($h_A s_A + h_B s_B$) can be equivalently expressed (after a rescaling by $1/h_A$) as

$$u(s_A, s_B) = s_A + h s_B \quad (2)$$

where $h = h_B/h_A, h_A, h_B, h \in \mathbb{C}^1$ [5]. As shown in [7], it is sufficient to consider only $|h| \leq 1$ for further analysis, since the problem is symmetrical around $|h| = 1$ (for $|h_B| > |h_A|$ the useful signal can be equivalently rescaled by $1/h_B$ to obtain $u' = h' s_A + s_B$, where $|h'| \leq 1$).

Signal $u(s_A, s_B)$ is a superposition of symbols from A, B , and hence it will be called a *compound symbol*. After receiving the signal $x = h_A u + w$, the relay performs an eXclusive mapping operation (see [3] for details) to map the received *compound symbol* to the relay output symbol $s_R(c_{AB})$, where

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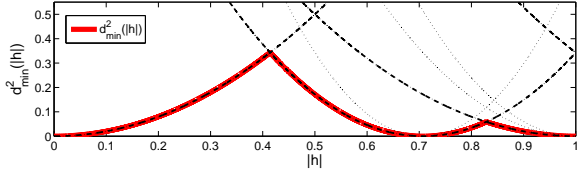


Figure 2. Set \mathcal{S}_p of min-distance parabolas for the QPSK alphabet (dashed lines) and the non-uniform 2-slot alphabet with $\mathcal{A}_s = \text{QPSK}$ and $s_f = 1$ (dotted lines). The hierarchical min-distance $d_{\min}^2(|h|)$ of both alphabets coincides.

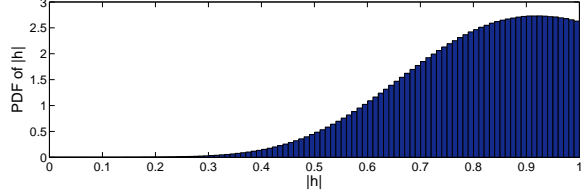


Figure 3. Probability distribution of channel parameter $|h| \leq 1$ (Rician fading channels $|h_A|, |h_B|$) with a Rician factor $K = 10$ dB.

$c_{AB} = \mathcal{X}(c_A^i, c_B^j)$. The eXclusive mapping operator $\mathcal{X}(\cdot)$ guarantees a fulfillment of the eXclusive law [3], i.e. the invertibility of the eXclusive mapping at the destinations (provided the C-SI is known). In the rest of the paper we will use a slightly relaxed notation: $\mathcal{X}(i, j) = \mathcal{X}(c_A^i, c_B^j)$.

A. Hierarchical Minimum Distance Analysis

The (squared) *hierarchical minimum distance* represents an approximation of the hierarchical decoder exact metric ([2], [5]). The analysis of the constellation space (Euclidean) distance properties of alphabets in the 2-WRC with HDF strategy was presented in [6]. The (squared) Euclidean distance ($d_{u^{i,j}, u^{i',j'}}^2(h)$) of a general pair of compound symbols ($u^{i,j} = s_A^i + h s_B^j$ and $u^{i',j'} = s_A^{i'} + h s_B^{j'}$) is defined therein as

$$d_{u^{i,j}, u^{i',j'}}^2(h) = \|\Delta s_A\|^2 + |h|^2 \|\Delta s_B\|^2 + 2\Re\{h^* z\} \quad (3)$$

where $z = \langle \Delta s_A; \Delta s_B \rangle$, $\Delta s_A = s^i - s^{i'}$, $\Delta s_B = s^j - s^{j'}$ and $i, i', j, j' \in \{1, 2, \dots, M\}$.

As shown in [6], the hierarchical minimum distance

$$d_{\min}^2(h) = \min_{(i,j,i',j'):\mathcal{X}(i,j) \neq \mathcal{X}(i',j')} d_{u^{i,j}, u^{i',j'}}^2(h) \quad (4)$$

is (for the worst case phase $\angle h$) lower bounded by a *set of parabolas* $\mathcal{S}_p = \{p_{i,j,i',j'}\}_{\mathcal{X}(i,j) \neq \mathcal{X}(i',j')}$, where each particular parabola is given by

$$p_{i,j,i',j'}(|h|) = \min_{\angle h} d_{u^{i,j}, u^{i',j'}}^2(h) = |h|^2 \|\Delta s_B\|^2 - 2|h||z| + \|\Delta s_A\|^2. \quad (5)$$

The lower bound defined by \mathcal{S}_p is always achieved for some specific $\angle h$ [6] and hence

$$d_{\min}^2(|h|) = \min_{(i,j,i',j'):\mathcal{X}(i,j) \neq \mathcal{X}(i',j')} p_{i,j,i',j'}(|h|). \quad (6)$$

This *parabolic behaviour* of the hierarchical min-distance (see an example in Fig. 2) results necessarily in eXclusive law failures ($d_{\min}^2(h) \rightarrow 0$), and consequently in destination decoding errors, since the relay cannot unambiguously determine the output symbol [6].

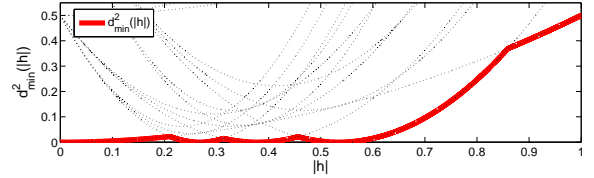


Figure 4. Hierarchical min-distance $d_{\min}^2(|h|)$ and the set \mathcal{S}_p of min-distance parabolas for the non-uniform 2-slot alphabet with $\mathcal{A}_s = \text{QPSK}$ and $s_f = 0.25$ (compare to Fig. 2 and distribution of $|h|$ in Fig. 3).

III. NON-UNIFORM 2-SLOT ALPHABETS

As proved in [6], the parabolic behaviour of hierarchical min-distance cannot be fully avoided for traditional linear modulation constellations in \mathbb{C}^1 (excepting the binary alphabets). However, as we will show in the following section, in case of *Rician fading channels* it is possible to suppress this harmful behaviour by a suitable design of 2-slot constellation alphabets $\mathcal{A}_s^A, \mathcal{A}_s^B$.

A. Parabolic Behaviour Analysis

It can be shown that for Rician source-relay channels ($|h_A|, |h_B|$), the probability distribution of channel parameter $|h| = |h_B|/|h_A|$ is diminishing as $|h| \rightarrow 0$ (see Fig. 3). Considering the hierarchical min-distance $d_{\min}^2(|h|)$ of QPSK (Fig. 2) along with the probability distribution of $|h|$ (Fig. 3) it is obvious that the performance of HDF/DNF system with QPSK alphabets will be poor. Fortunately, it is possible to decrease the negative impact of this parabolic min-distance behaviour by shifting the min-distance parabolas (5) towards the less probable values of $|h|$.

A position of each particular min-distance parabola (5) vertex (minimum) is given by

$$|h'_{\min}| = \arg \min_{|h|} p_{i,j,i',j'}(|h|) = \frac{|\langle \Delta s_A; \Delta s_B \rangle|}{\|\Delta s_B\|^2}, \quad (7)$$

which can be (in case of the constellation alphabets in \mathbb{C}^1) further simplified to

$$|h'_{\min}| = \frac{\|\Delta s_A\| \cdot \|\Delta s_B\|}{\|\Delta s_B\|^2} = \frac{\|\Delta s_A\|}{\|\Delta s_B\|}, \quad (8)$$

since any pair of vectors in \mathbb{C}^1 is always linearly dependent, which gives us the equality in the general *Cauchy-Schwartz inequality*.

B. Alphabet Design Algorithm

As it is obvious from (8), a position of the minimum of each particular min-distance parabola $p_{i,j,i',j'}(|h|)$ is given by the ratio of $\|\Delta s_A^{i,i'}\|$ and $\|\Delta s_B^{j,j'}\|$, i.e. by the corresponding min-distances of individual source alphabets $\mathcal{A}_s^A, \mathcal{A}_s^B$. Now it seems quite straightforward that it is possible to shift the particular min-distance parabola minimum towards $|h| \rightarrow 0$ by increasing $\|\Delta s_A^{i,i'}\|$ relatively to $\|\Delta s_B^{j,j'}\|$, which can be achieved by a suitable allocation of output power at both sources.

Since the average power constraint must be taken into account, it is not feasible to purely increase the output power

Algorithm 1 Non-uniform 2-slot constellation alphabet.

- 1) Pick a linear modulation constellation \mathcal{A}_s .
 - 2) Choose a power scaling factor $s_f \in (0, 2)$.
 - 3) Source A alphabet: $\mathcal{A}_s^A = [\sqrt{s_f}\mathcal{A}_s, \sqrt{2-s_f}\mathcal{A}_s]$.
 - 4) Source B alphabet: $\mathcal{A}_s^B = [\sqrt{2-s_f}\mathcal{A}_s, \sqrt{s_f}\mathcal{A}_s]$.
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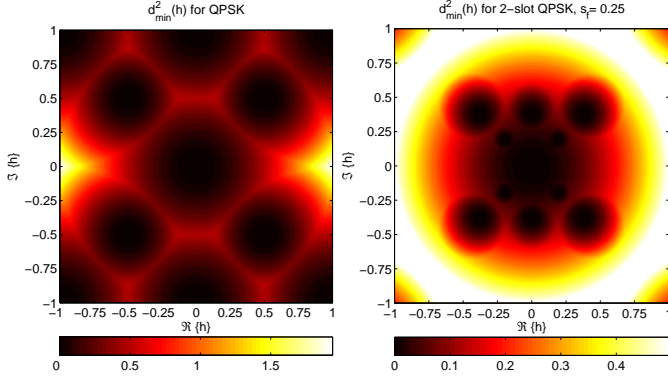


Figure 5. Hierarchical min-distance $d_{\min}^2(h)$ of the QPSK and the 2-slot alphabet with $\mathcal{A}_s = \text{QPSK}$, $s_f = 0.25$ as a function of channel parameter $h \in \mathbb{C}$. In case of the 2-slot alphabet the events corresponding to the eXclusive law failures ($d_{\min}^2(h) \rightarrow 0$) are shifted towards the less probable values of channel parameter h (see Fig. 3).

of one source relatively to the other one. We propose a design of novel 2-slot alphabets, where each 2-slot *super-symbol* is formed by pairing of two subsequent source symbols. This gives us an additional degree of freedom, since it is possible to arbitrarily re-distribute the available power among the two slots of the super-symbol. If we denote this power as $P_{2\text{slot}} = 2P_{1\text{slot}}$, it is obvious that the only restriction is that the power scaling coefficients for both slots in the super-symbol must sum up to 2, which gives us $P_{2\text{slot}} = s_f P_{1\text{slot}} + (2 - s_f) P_{1\text{slot}}$, where $s_f \in (0, 2)$ defines the *power scaling factor*.

Based on this observation we propose a design algorithm for the *non-uniform 2-slot* constellation alphabets (Algorithm 1). The power is re-allocated *non-uniformly* among the 2-slots according to s_f . Note that for $s_f = 1$ we get a pure 2-slot extension of a traditional linear modulation constellation with identical hierarchical min-distance properties (see Fig. 2). The hierarchical min-distance of a 2-slot alphabet with $s_f = 0.25$ is in Fig. 4. A comparison of the overall hierarchical min-distance $d_{\min}^2(h)$ properties (i.e. as a function of $h \in \mathbb{C}$) of the QPSK and the 2-slot alphabet with $\mathcal{A}_s = \text{QPSK}$, $s_f = 0.25$ is in Fig. 5.

IV. CONCLUDING REMARKS

Performance comparison of the proposed non-uniform 2-slot alphabets with $\mathcal{A}_s = \text{QPSK}$ (with various s_f) and the traditional linear modulation (QPSK) is in Fig. 6. Since the source alphabets $\mathcal{A}_s^A, \mathcal{A}_s^B$ are used only in the MAC phase, we analyze only the SER of Hierarchical (compound) symbols (H-SER) received by the relay.

Remarkable SNR gains can be observed for the proposed non-uniform 2-slot alphabets ($\sim 10 - 15\text{dB}$ in Fig. 6) for moderately high SNR. It is important to note that the overall system throughput is not sacrificed, since the cardinality of

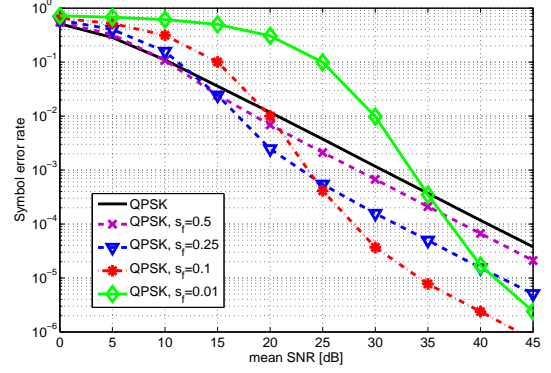


Figure 6. H-SER of the QPSK and the non-uniform 2-slot alphabets with $\mathcal{A}_s = \text{QPSK}$ (with various s_f). The decoder of the 2-slot alphabets decodes the compound symbols on a per-slot basis (for a better comparison with traditional linear modulation constellations). Rician fading channels h_A, h_B with a Rician factor $K = 10\text{dB}$ are assumed. The average SNR is defined as $\frac{1}{2\sigma_n^2} \mathbb{E} [|h_A|^2 + |h_B|^2]$. For simplicity reasons we do not use error-correcting codes at the relay. It is obvious that a crucial part of the alphabet design (Algorithm 1) is a choice of the scaling factor s_f (for a given \mathcal{A}_s). A suitable selection of s_f allows to trade-off the vulnerability to eXclusive law failures with the alphabet distance properties, resulting in an improved performance in the medium to high SNR region.

the 2-slot alphabet is $|\mathcal{A}_s^A| = |\mathcal{A}_s^B| = M^2$ for $|\mathcal{A}_s| = M$ (see Algorithm 1) and hence the promising parametric performance of the proposed 2-slot alphabets is not redeemed by a reduction of the potential throughput (as in [5]).

The non-uniform 2-slot constellation alphabet design could be generally characterized as an alphabet-diversity technique regarding the hierarchical min-distance. A suitable selection of the scaling factor s_f (in Algorithm 1) is evidently critical for alphabet performance. As it is obvious from Fig. 6, it is not appropriate to purely allocate most of the available source power to a one slot of the 2-slot alphabet ($s_f \ll 0.1$ for $\mathcal{A}_s = \text{QPSK}$), since the H-SER performance of such an alphabet will be poor (even for a reasonably high SNR). A suitable selection of the scaling factor s_f (for particular constellation alphabets) along with an analysis of general non-uniform N-slot constellation alphabets are scope of the future work.

REFERENCES

- [1] R. W. Yeung, S.-Y. R. Li, N. Cai, and Z. Zhang, *Network Coding Theory*. now Publishers, 2006.
- [2] T. Koike-Akino, P. Popovski, and V. Tarokh, "Optimized constellations for two-way wireless relaying with physical network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, pp. 773–787, June 2009.
- [3] J. Sykora and A. Burr, "Layered design of hierarchical exclusive codebook and its capacity regions for HDF strategy in parametric wireless 2-WRC," *IEEE Trans. Veh. Technol.*, 2011. Accepted for publication.
- [4] S. Zhang and S.-C. Liew, "Applying physical-layer network coding in wireless networks," *EURASIP J. on Wireless Comm. and Netw.*, vol. 2010, pp. 1–12, 2010. doi:10.1155/2010/870268.
- [5] T. Uricar and J. Sykora, "Design criteria for hierarchical exclusive code with parameter-invariant decision regions for wireless 2-way relay channel," *EURASIP J. on Wireless Comm. and Netw.*, vol. 2010, pp. 1–13, 2010. doi:10.1155/2010/921427.
- [6] T. Uricar and J. Sykora, "Hierarchical eXclusive alphabet in parametric 2-WRC - Euclidean distance analysis and alphabet construction algorithm," in *COST 2100 MCM*, (Aalborg, Denmark), pp. 1–9, June 2010. TD-10-11051.
- [7] T. Uricar, "Parameter-invariant hierarchical eXclusive alphabet design for 2-WRC with HDF strategy," *Acta Polytechnica*, vol. 50, no. 4, pp. 79–86, 2010.