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REVIEW ARTICLE

‘Nonclassical’ states in quantum optics: a ‘squeezed’ review of the first 75 years

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Online at stacks.iop.org/JOptB/4/R1**Abstract**

Seventy five years ago, three remarkable papers by Schrödinger, Kennard and Darwin were published. They were devoted to the evolution of Gaussian wave packets for an oscillator, a free particle and a particle moving in uniform constant electric and magnetic fields. From the contemporary point of view, these packets can be considered as prototypes of the coherent and squeezed states, which are, in a sense, the cornerstones of modern quantum optics. Moreover, these states are frequently used in many other areas, from solid state physics to cosmology. This paper gives a review of studies performed in the field of so-called ‘nonclassical states’ (squeezed states are their simplest representatives) over the past seventy five years, both in quantum optics and in other branches of quantum physics.

My starting point is to elucidate who introduced different concepts, notions and terms, when, and what were the initial motivations of the authors. Many new references have been found which enlarge the ‘standard citation package’ used by some authors, recovering many undeservedly forgotten (or unnoticed) papers and names. Since it is practically impossible to cite several thousand publications, I have tried to include mainly references to papers introducing new types of quantum states and studying their properties, omitting many publications devoted to applications and to the methods of generation and experimental schemes, which can be found in other well known reviews. I also mainly concentrate on the initial period, which terminated approximately at the border between the end of the 1980s and the beginning of the 1990s, when several fundamental experiments on the generation of squeezed states were performed and the first conferences devoted to squeezed and ‘nonclassical’ states commenced. The 1990s are described in a more ‘squeezed’ manner: I have confined myself to references to papers where some new concepts have been introduced, and to the most recent reviews or papers with extensive bibliographical lists.

Keywords: Nonclassical states, squeezed states, coherent states, even and odd coherent states, quantum superpositions, minimum uncertainty states, intelligent states, Gaussian packets, non-Gaussian coherent states, phase states, group and algebraic coherent states, coherent states for general potentials, relativistic oscillator coherent states, supersymmetric states, para-coherent states, q -coherent states, binomial states, photon-added states, multiphoton states, circular states, nonlinear coherent states

1. Introduction

The terms ‘coherent states’, ‘squeezed states’ and ‘nonclassical states’ can be encountered in almost every modern paper on

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quantum optics. Moreover, they are frequently used in many other areas, from solid state physics to cosmology. In 2001 and 2002 it will be seventy five years since the publication of three papers by Schrödinger [1], Kennard [2] and Darwin [3], in which the evolutions of Gaussian wavepackets for an oscillator, a free particle and a particle moving in uniform constant

electric and magnetic fields were considered. From the contemporary point of view, these packets can be considered as prototypes of the coherent and squeezed states, which are, in a sense, the cornerstones of modern quantum optics. Since the squeezed states are the simplest representatives of a wide family of ‘nonclassical states’ in quantum optics, it seems appropriate, bearing in mind the jubilee date, to give an updated review of studies performed in the field of nonclassical states over the past seventy five years.

Although extensive lists of publications can be found in many review papers and monographs (see, e.g., [4–7]), the subject has not been exhausted, because each of them highlighted the topic from its own specific angle. My starting point was to elucidate who introduced different concepts, notions and terms, when, and what were the initial motivations of the authors. In particular, while performing the search, I discovered that many papers and names have been undeservedly forgotten (or have gone unnoticed for a long time), so that ‘the standard citation package’ used by many authors presents a sometimes rather distorted historical picture. I hope that the present review will help many researchers, especially the young, to obtain a less deformed vision of the subject.

One of the most complicated problems for any author writing a review is an inability to supply the complete list of all publications in the area concerned, due to their immense number. In order to diminish the length of the present review, I tried to include only references to papers introducing *new types of quantum states* and studying their properties, omitting many publications devoted to *applications* and to the methods of generation and experimental schemes. The corresponding references can be found, e.g., in other reviews [4–6]. I also concentrate mainly on the initial period, which terminated approximately at the border between the end of the 1980s and the beginning of the 1990s, when several fundamental experiments on the generation of squeezed states were performed and the first conferences devoted to squeezed and ‘nonclassical’ states commenced. The 1990s are described in a more ‘squeezed’ manner, because the recent history will be familiar to the readers. For this reason, describing that period, I have confined myself to references to papers where some new concepts have been introduced, and to the most recent reviews or papers with extensive bibliographical lists, bearing in mind that in the Internet era it is easier to find recent publications (contrary to the case of forgotten or little known old publications).

2. Coherent states

It is well known that it was Schrödinger [1] who constructed for the first time in 1926 the ‘nonspreading wavepackets’ of the harmonic oscillator. In modern notation, these packets can be written as (in the units $\hbar = m = \omega = 1$):

$$\langle x|\alpha\rangle = \pi^{-1/4} \exp(-\frac{1}{2}x^2 + \sqrt{2}x\alpha - \frac{1}{2}\alpha^2 - \frac{1}{2}|\alpha|^2). \quad (1)$$

A complex parameter α determines the mean values of the coordinate and momentum according to the relations:

$$\langle \hat{x} \rangle = \sqrt{2} \operatorname{Re} \alpha, \quad \langle \hat{p} \rangle = \sqrt{2} \operatorname{Im} \alpha.$$

The variances of the coordinate and momentum operators, $\sigma_x = \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2$ and $\sigma_p = \langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2$, have equal values, $\sigma_x = \sigma_p = 1/2$, so their product assumes the minimal value permitted by the Heisenberg uncertainty relation,

$$(\sigma_x \sigma_p)_{\min} = 1/4$$

(in turn, this relation, which was formulated by Heisenberg [8] as an approximate inequality, was strictly proven by Kennard [2] and Weyl [9]).

The simplest way to arrive at formula (1) is to look for the eigenstates of the non-Hermitian annihilation operator

$$\hat{a} = (\hat{x} + i\hat{p})/\sqrt{2} \quad (2)$$

satisfying the commutation relation

$$[\hat{a}, \hat{a}^\dagger] = 1, \quad (3)$$

so that:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle. \quad (4)$$

For example, this was done by Glauber in [10], where the name ‘coherent states’ appeared in the text for the first time. However, several authors did similar things before him. The annihilation operator possessing property (3) was introduced by Fock [11], together with the eigenstates $|n\rangle$ of the number operator $\hat{n} = \hat{a}^\dagger \hat{a}$, known nowadays as the ‘Fock states’ (the known eigenfunctions of the harmonic oscillator in the coordinate representation in terms of the Hermite polynomials were obtained by Schrödinger in [12]). And it was Iwata who considered for the first time in 1951 [13] the eigenstates of the non-Hermitian annihilation operator \hat{a} , having derived formula (1) and the now well known expansion over the Fock basis:

$$|\alpha\rangle = \exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (5)$$

The states defined by means of equations (4) and (5) were used as some auxiliary states, permitting to simplify calculations, by Schwinger [14] in 1953. Later, their mathematical properties were studied independently by Rashevskiy [15], Klauder [16] and Bargmann [17], and these states were discussed briefly by Henley and Thirring [18].

The coherent state (5) can be obtained from the vacuum state $|0\rangle$ by means of the unitary *displacement operator*:

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle, \quad \hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a}) \quad (6)$$

which was actually used by Feynman and Glauber as far back as 1951 [19,20] in their studies of quantum transitions caused by the classical currents (which are reduced to the problem of the forced harmonic oscillator, studied, in turn, in the 1940s–1950s by Bartlett and Moyal [21], Feynman [22], Ludwig [23], Husimi [24] and Kerner [25]).

However, only after the works by Glauber [10] and Sudarshan [26] (and especially Glauber’s work [27]), did the coherent states become widely known and intensively used by many physicists. The first papers published in magazines, which had the combination of words ‘coherent states’ in their titles, were [27,28].

Coherent states have always been considered as ‘the most classical’ ones (among the *pure* quantum states, of

course): see, e.g., [29]. Moreover, they can serve [27] as a starting point to define the 'nonclassical states' in terms of the Glauber–Sudarshan P -function, which was introduced in [10] to represent thermal states and in [26] for arbitrary density matrices:

$$\hat{\rho} = \int P(\alpha) |\alpha\rangle\langle\alpha| d \operatorname{Re} \alpha d \operatorname{Im} \alpha, \quad (7)$$

$$\int P(\alpha) d \operatorname{Re} \alpha d \operatorname{Im} \alpha = 1.$$

Indeed, the following definition was given in [27]: 'If the singularities of $P(\alpha)$ are of types stronger than those of delta function, e.g., derivatives of delta function, the field represented will have no classical analog'.

In this sense, the thermal (chaotic) fields are classical, since their P -functions are usual positive probability distributions. The states of such fields, being quantum mixtures, are described in terms of the density matrices (introduced by Landau [30]). Also, the coherent states are 'classical', because the P -distribution for the state $|\beta\rangle$ is, obviously, the delta function, $P_\beta(\alpha) = \delta(\alpha - \beta)$. On the other hand, these states lie 'on the border' of the set of the 'classical' states, because the delta function is the most singular distribution admissible in the classical theory. Consequently, it is enough to make slight modifications in each of the definitions (1), (4), (5), or (6), to arrive at various families of states, which will be 'nonclassical'. For this reason, many classes of states which are labelled nowadays as 'nonclassical', appeared in the literature as some kinds of 'generalized coherent states'. At least four different kinds of generalizations exist.

- (I) One can generalize equations (1) or (5), choosing different sets of coefficients $\{c_n\}$ in the expansion

$$|\psi\rangle = \sum_n c_n |n\rangle \quad (8)$$

or writing different explicit wavefunctions in other (continuous) representations (coordinate, momentum, Wigner–Weyl, etc).

- (II) One can replace the exponential form of the operator $\hat{D}(\alpha)$ in equation (6) by some other operator function, also using some other set of operators \hat{A}_k instead of operator \hat{a} (e.g., making some kinds of 'deformations' of the fundamental commutation relations like (3)), and taking the 'initial' state $|\varphi\rangle$ other than the vacuum state. This line leads to states of the form

$$|\psi_\lambda\rangle = f(\hat{A}_k, \lambda) |\varphi\rangle, \quad (9)$$

where λ is some continuous parameter.

- (III) One can look for the 'continuous' families of eigenstates of the new operators \hat{A}_k , trying to find solutions to the equation:

$$\hat{A}_k |\psi_\mu\rangle = \mu |\psi_\mu\rangle. \quad (10)$$

- (IV) One can try to 'minimize' the generalized Heisenberg uncertainty relation for Hermitian operators \hat{A} and \hat{B} different from \hat{x} and \hat{p} ,

$$\Delta A \Delta B \geq \frac{1}{2} |[\hat{A}, \hat{B}]|, \quad (11)$$

looking, for instance, for the states for which (11) becomes the equality.

Actually, all these approaches have been used for several decades of studies of 'generalized coherent states'. Some concrete families of states found in the framework of each method will be described in the following sections.

The 'nonclassicality' of the Fock states and their finite superpositions was mentioned in [31]. A simple criterium of 'nonclassicality' can be established, if one considers a generic Gaussian wavepacket with *unequal* variances of two quadratures, whose P -function reads [32] (in the special case of the statistically uncorrelated quadrature components)

$$P_G(\alpha) = \mathcal{N} \exp \left[-\frac{(\operatorname{Re} \alpha - a)^2}{\sigma_x - 1/2} - \frac{(\operatorname{Im} \alpha - b)^2}{\sigma_p - 1/2} \right] \quad (12)$$

(a and b give the position of the centre of the distribution in the α -plane; the symbol \mathcal{N} hereafter is used for the normalization factor). Since function (12) exists as a normalizable distribution only for $\sigma_x \geq 1/2$ and $\sigma_p \geq 1/2$, the states possessing one of the quadrature component variances less than $1/2$ are *nonclassical*. This statement holds for any (not only Gaussian) state. Indeed, one can easily express the quadrature variance in terms of the P -function:

$$\sigma_x = \frac{1}{2} \int P(\alpha) [(\alpha + \alpha^* - \langle \hat{a} + \hat{a}^\dagger \rangle)^2 + 1] d \operatorname{Re} \alpha d \operatorname{Im} \alpha.$$

If $\sigma_x < 1/2$, then function $P(\alpha)$ must assume negative values, thus it cannot be interpreted as a classical probability [32].

Another example of a 'nonclassical' state is a superposition of two different coherent states [33]. As a matter of fact, *all pure states*, excepting the coherent states, are 'nonclassical', both from the point of view of their physical contents [34] and the formal definition in terms of the P -function given above [35]. However, speaking of 'nonclassical' states, people usually have in mind not an arbitrary pure state, but members of some families of quantum states possessing more or less useful or distinctive properties. One of the aims of this paper is to provide a brief review of the known families which have been introduced whilst trying to follow the historical order of their appearance.

According to the *Web of Science* electronic database of journal articles, the combination of words 'nonclassical states' appeared for the first time in the titles of the papers by Helstrom, Hillery and Mandel [36]. The first three papers containing the combination of words 'nonclassical effects' were published by Loudon [37], Zubairy, and Lugiato and Strini [38]. 'Nonclassical light' was the subject of the first three studies by Schubert, Janszky *et al* and Gea-Banacloche [39].

3. Squeezed states

3.1. Generic Gaussian wavepackets

Historically, the first example of the nonclassical (squeezed) states was presented as far back as in 1927 by Kennard [2] (see the story in [40]), who considered, in particular, the evolution in time of the *generic Gaussian wavepacket*

$$\psi(x) = \exp(-ax^2 + bx + c) \quad (13)$$

of the harmonic oscillator. In this case, the quadrature variances may be arbitrary (they are determined by the real

and imaginary parts of the parameter a), but they satisfy the Heisenberg inequality $\sigma_x \sigma_p \geq 1/4$. Within twenty five years, Husimi [41] found the following family of solutions to the time-dependent Schrödinger equation for the harmonic oscillator (we assume here $\omega = m = \hbar = 1$)

$$\begin{aligned} \psi(x, t) = & [2\pi \sinh(\beta + it)]^{-1/2} \\ & \times \exp\{-\frac{1}{4}[\tanh[\frac{1}{2}(\beta + it)](x + x')^2 \\ & + \coth[\frac{1}{2}(\beta + it)](x - x')^2]\}, \end{aligned} \quad (14)$$

where x' and $\beta > 0$ are arbitrary real parameters. He showed that the quadrature variances oscillate with twice the oscillator frequency between the extreme values $\frac{1}{2} \tanh \beta$ and $\frac{1}{2} \coth \beta$. A detailed study of the Gaussian states of the harmonic oscillator was performed by Takahasi [42].

3.2. The first appearance of the squeezing operator

An important contribution to the theory of squeezed states was made by Infeld and Plebański [43–45], whose results were summarized in a short article [46]. Plebański introduced the following family of states

$$|\tilde{\psi}\rangle = \exp[i(\eta\hat{x} - \xi\hat{p})] \exp\left[\frac{i}{2} \log a (\hat{x}\hat{p} + \hat{p}\hat{x})\right] |\psi\rangle, \quad (15)$$

where ξ , η and $a > 0$ are real parameters, and $|\psi\rangle$ is an *arbitrary* initial state. Evidently, the first exponential in the right-hand side of (15) is nothing but the *displacement operator* (6) written in terms of the Hermitian quadrature operators. Its properties were studied in the first article [43]. The second exponential is the special case of the *squeezing operator* (see equation (21)). For the initial *vacuum* state, $|\psi_0\rangle = |0\rangle$, the state $|\tilde{\psi}_0\rangle$ (15) is exactly the squeezed state in modern terminology, whereas by choosing other initial states one can obtain various *generalized squeezed states*. In particular, the choice $|\psi_n\rangle = |n\rangle$ results in the family of the *squeezed number states*, which were considered in [45, 46]. In the case $a = 1$ (considered in [43]) we arrive at the states known nowadays by the name *displaced number states*. Plebański gave the explicit expressions describing the time evolution of the state (15) for the harmonic oscillator with a constant frequency and proved the completeness of the set of ‘displaced’ number states. Infeld and Plebański [44] performed a detailed study of the properties of the unitary operator $\exp(i\hat{T})$, where \hat{T} is a generic inhomogeneous quadratic form of the canonical operators \hat{x} and \hat{p} with constant c -number coefficients, giving some classification and analysing various special cases (some special cases of this operator were discussed briefly by Bargmann [17]). Unfortunately, the publications cited appeared to be practically unknown or forgotten for many years.

3.3. From ‘characteristic’ to ‘minimum uncertainty’ states

In 1966, Miller and Mishkin [47] deformed the defining equation (4), introducing the ‘characteristic states’ as the eigenstates of the operator

$$\hat{b} = u\hat{a} + v\hat{a}^\dagger \quad (16)$$

(for real u and v). In order to preserve the canonical commutation relation $[\hat{b}, \hat{b}^\dagger] = 1$ one should impose the constraint $|u|^2 - |v|^2 = 1$.

Similar states were considered by Lu [48], who called them ‘new coherent states,’ and by Bialynicki-Birula [49]. The general structure of the wavefunctions of these states in the coordinate representation is

$$\langle x|\beta\rangle = (\pi w_-^2)^{-1/4} \exp\left[\frac{\sqrt{2}\beta x}{w_-} - \frac{w_+ x^2}{2w_-} - \frac{w_-^* \beta^2}{2w_-} - \frac{|\beta|^2}{2}\right], \quad (17)$$

where $w_\pm = u \pm v$ and $\hat{b}|\beta\rangle = \beta|\beta\rangle$.

Wavefunction (17) also arises if one looks for states in which the uncertainty product $\sigma_x \sigma_p$ assumes the minimal possible value $1/4$. This approach was used in [29, 50–52]. The variances in the state (17) are given by

$$\sigma_x = \frac{1}{2}|u - v|^2, \quad \sigma_p = \frac{1}{2}|u + v|^2 \quad (18)$$

so that for *real* parameters u and v one has $\sigma_x \sigma_p = 1/4$ due to the constraint $|u|^2 - |v|^2 = 1$. In this case one can express the ‘transformed’ operator \hat{b} in (16) in terms of the quadrature operators

$$\hat{x} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}, \quad \hat{p} = (\hat{a} - \hat{a}^\dagger)/(i\sqrt{2}), \quad (19)$$

as:

$$\hat{b} = (\lambda^{-1}\hat{x} + \lambda\hat{p})/\sqrt{2}, \quad \lambda^{-1} = u + v, \quad \lambda = u - v. \quad (20)$$

The term ‘minimum uncertainty states’ (MUS) seems to be used for the first time in the paper by Mollow and Glauber [32], where it was applied to the special case of the Gaussian states (12) with $\sigma_x \sigma_p = 1/4$. Stoler [51] showed that the MUS can be obtained from the oscillator ground state by means of the unitary operator:

$$\hat{S}(z) = \exp[\frac{1}{2}(z\hat{a}^2 - z^*\hat{a}^{\dagger 2})]. \quad (21)$$

In the second paper of [51] the operator $\hat{S}(z)$ was written for real z in terms of the quadrature operators as $\exp[ir(\hat{x}\hat{p} + \hat{p}\hat{x})]$, which is exactly the form given by Plebański [46]. The conditions under which the MUS preserve their forms were studied in detail in [51, 53].

3.4. Coherent states of nonstationary oscillators and Gaussian packets

The operators like (16) and the state (17) arise naturally in the process of the dynamical evolution governed by the Hamiltonian

$$\hat{H} = \omega\hat{a}^\dagger\hat{a} + \kappa\hat{a}^{\dagger 2}e^{-2i\omega t - i\varphi} + \text{h.c.}, \quad (22)$$

which describes the *degenerate parametric amplifier*. This problem was studied in detail by Takahasi in 1965 [42]. Similar *two-mode* states were considered implicitly in 1967 by Mollow and Glauber [32], who developed the quantum theory of the *nondegenerate parametric amplifier*, described by the interaction Hamiltonian between two modes of the form $\hat{H}_{\text{int}} = \hat{a}^\dagger \hat{b}^\dagger e^{-i\omega t} + \text{h.c.}$

In the case of an oscillator with arbitrary time-dependent frequency $\omega(t)$, the evolution of initially coherent states was considered in [54], where the operator $\exp[(\hat{a}^{\dagger 2} + \hat{a}^2)s]$ naturally appeared. The generalizations of the coherent

state (4) as the eigenstates of the *linear time-dependent integral of motion operator*

$$\hat{A} = [\varepsilon(t)\hat{p} - \dot{\varepsilon}(t)\hat{x}]/\sqrt{2}, \quad [\hat{A}, \hat{A}^\dagger] = 1, \quad (23)$$

where function $\varepsilon(t)$ is a specific complex solution of the *classical* equation of motion

$$\ddot{\varepsilon} + \omega^2(t)\varepsilon = 0, \quad \text{Im} = 1(\dot{\varepsilon}\varepsilon^*), \quad (24)$$

have been introduced by Malkin and Man'ko [55]. The integral of motion (23) satisfies the equation

$$i\partial\hat{A}/\partial t = [\hat{H}, \hat{A}]$$

(where \hat{H} is the Hamiltonian) and it has the structure (16), with *complex* coefficients $u(t)$ and $v(t)$, which are certain combinations of the functions $\varepsilon(t)$ and $\dot{\varepsilon}(t)$. The eigenstates of \hat{A} have the form (17). They are coherent with respect to the time-dependent operator \hat{A} (equivalent to $\hat{b}(t)$ (16) and generalizing (20)), but they are *squeezed* with respect to the quadrature components of the 'initial' operator \hat{a} defined via (19). For the most general forms in the case of one degree of freedom see, e.g., [56] and the review [57].

Coherent states of a charged particle in a constant homogeneous magnetic field (generalizations of Darwin's packets [3]) have been introduced by Malkin and Man'ko in [58] (see also [59]). Generalizations of the nonstationary oscillator coherent states to systems with several degrees of freedom, such as a charged particle in nonstationary homogeneous magnetic and electric fields plus a harmonic potential, were studied in detail in [60–62]. Multidimensional time-dependent coherent states for arbitrary quadratic Hamiltonians have been introduced in [63]. Their wavefunctions were expressed in the form of generic Gaussian exponentials of N variables. From the modern point of view, all those states can be considered as squeezed states, since the variances of different canonically conjugated variables can assume values which are less than the ground state variances. Similar one-dimensional and multidimensional quantum Gaussian states were studied in connection with the problems of the theories of photodetection, measurements, and information transfer in [64, 65]. The method of linear time-dependent integrals of motion turned out to be very effective for treating both the problem of an oscillator with time-dependent frequency (other approaches are due to Fujiwara, Husimi and, especially, Lewis and Riesenfeld [24, 66]) and generic systems with multidimensional quadratic Hamiltonians. For detailed reviews see, e.g., [57, 67].

Approximate quasiclassical solutions to the Schrödinger equation with arbitrary potentials, in the form of the Gaussian packets whose centres move along the classical trajectories, have been extensively studied in many papers by Heller and his coauthors, beginning with [68]. The first coherent states for the *relativistic* particles obeying the Klein–Gordon or Dirac equations have been introduced in [69].

3.5. Possible applications and first experiments with 'two-photon' and 'squeezed' states

The first detailed review of the states defined by the relations (16) and (17) was given in 1976 by Yuen [70], who

proposed the name 'two-photon states'. Up to that time, it was recognized that such states can be useful for solving various fundamental physical and technological problems. In particular, in 1978, Yuen and Shapiro [71] proposed to use the two-photon states in order to improve optical communications by reducing the quantum fluctuations in one (signal) quadrature component of the field at the expense of the amplified fluctuations in another (unobservable) component. Also, the states with the reduced quantum noise in one of the quadrature components appeared to be very important for the problems of measurement of weak forces and signals, in particular, for the detectors of gravitational waves and interferometers [72–76]. With the course of time, the terms 'squeezing', 'squeezed states' and 'squeezed operator', introduced in the papers by Hollenhorst and Caves [72, 75], became generally accepted, especially after the article by Walls [77], and they have replaced other names proposed earlier for such states. It is interesting to notice in this connection, that exactly at the same time, the same term 'squeezing' was proposed (apparently completely independently) by Brosa and Gross [78] in connection with the problem of nuclear collisions (they considered a simple model of an oscillator with time-dependent *mass* and fixed elastic constant, whereas in the simplest quantum optical oscillator models squeezed states usually appear in the case of time-dependent *frequency* and fixed mass). The words 'squeezed states' were used for the first time in the titles of papers [79–82], whereas the word 'squeezing' appeared in the titles of studies (in the field of quantum optics) [83–85], and 'squeezed light' was discussed in [86].

At the end of the 1970s and the beginning of the 1980s, many theoretical and experimental studies were devoted to the phenomena of *antibunching* or *sub-Poissonian photon statistics*, which are unequivocal features of the quantum nature of light. The relations between antibunching and squeezing were discussed, e.g., in [87], and the first experiment was performed in 1977 [88] (for the detailed story see, e.g., [37, 89]). Many different schemes of generating squeezed states were proposed, such as the four-wave mixing [90], the resonance fluorescence [91, 92], the use of the free-electron laser [80, 93], the Josephson junction [80, 94], the harmonic generation [84, 95], two- and multiphoton absorption [83, 96] and parametric amplification [79, 83, 97], etc. In 1985 and 1986 the results of the first successful experiments on the generation and detection of squeezed states were reported in [98] (backward four-wave mixing), [99] (forward four-wave mixing) and [100] (parametric down conversion). Details and other references can be found, e.g., in [101].

3.6. Correlated, multimode and thermal states

The states (17) with *complex* parameters u, v do not minimize the Heisenberg uncertainty product. But they are MUS for the *Schrödinger–Robertson uncertainty relation* [102]

$$\sigma_x\sigma_p - \sigma_{px}^2 \geq \hbar^2/4, \quad (25)$$

which can also be written in the form

$$\sigma_p\sigma_x \geq \hbar^2/[4(1-r^2)], \quad (26)$$

where r is the *correlation coefficient* between the coordinate and momentum:

$$r = \sigma_{px} / \sqrt{\sigma_p \sigma_x} \quad \sigma_{px} = \frac{1}{2} \langle \hat{p}\hat{x} + \hat{x}\hat{p} \rangle - \langle \hat{p} \rangle \langle \hat{x} \rangle.$$

For arbitrary Hermitian operators, inequality (25) should be replaced by [102]:

$$\sigma_A \sigma_B - \sigma_{AB}^2 \geq | \langle [\hat{A}, \hat{B}] \rangle | / 4.$$

For further generalizations see, e.g., [103, 104].

Actually, the left-hand side of (26) takes on the minimal possible value $\hbar^2/4$ for *any pure* Gaussian state (this fact was known to Kennard [2]), which can be written as:

$$\psi(x) = \mathcal{N} \exp \left[-\frac{x^2}{4\sigma_{xx}} \left(1 - \frac{ir}{\sqrt{1-r^2}} \right) + bx \right]. \quad (27)$$

In order to emphasize the role of the correlation coefficient, state (27) was named the *correlated coherent state* [105]. It is unitary equivalent to the squeezed states with complex squeezing parameters, which were considered, e.g., in [106]. The dynamics of Gaussian wavepackets possessing a nonzero correlation coefficient were studied in [107]. Correlated states of the free particle were considered (under the name ‘contractive states’) in [108]. Gaussian correlated packets were applied to many physical problems: from neutron interferometry [109] to cosmology [110]. Reviews of properties of correlated states (including multidimensional systems, e.g., a charge in a magnetic field) can be found in [57, 111–113].

In the 1990s, various features of squeezed states (including phase properties and photon statistics) were studied and reviewed, e.g., in [114]. Different kinds of *two-mode squeezed states*, generated by the two-mode squeeze operator of the form $\exp(z\hat{a}\hat{b} - z^*\hat{a}^\dagger\hat{b}^\dagger + \dots)$, were studied (sometimes implicitly or under different names, such as ‘cranked oscillator states’ or ‘sheared states’) in [115]. *Multimode squeezed states* were considered in [116]. The properties of generic (pure and mixed) *Gaussian states* (in one and many dimensions) were studied in [117–121]. Their special cases, sometimes named *mixed squeezed states* or *squeezed thermal states*, were discussed in [122–124]. Mixed analogues of different families of coherent and squeezed states were studied in [125]. *Greybody states* were considered in [126]. ‘Squashed states’ have been introduced recently in [127].

Implicitly, the squeezed states of a charged particle moving in a homogeneous (stationary and nonstationary) magnetic field were considered in [60, 61]. In the explicit form they were introduced and studied in [111] and independently in [128]. For further studies see, e.g., [129–131]. In the case of an arbitrary (inhomogeneous) electromagnetic field or an arbitrary potential, the quasiclassical Gaussian packets centred on the classical trajectories (frequently called *trajectory-coherent states*) have been studied in [132].

The specific families of two- and multimode quantum states connected with the polarization degrees of freedom of the electromagnetic field (*biphotons, unpolarized light*) have been introduced by Karassiov [133]. For other studies see, e.g., [134].

3.7. Invariant squeezing

The instantaneous values of variances σ_x , σ_p and σ_{xp} cannot serve as true measures of squeezing in all cases, since they depend on time in the course of the free evolution of an oscillator. For example,

$$\sigma_x(t) = \sigma_x(0) \cos^2(t) + \sigma_p(0) \sin^2(t) + \sigma_{xp}(0) \sin(2t)$$

(in dimensionless units), and it can happen that both variances σ_x and σ_p are large, but nonetheless the state is highly squeezed due to large nonzero covariance σ_{xp} . It is reasonable to introduce some invariant characteristics which do not depend on time in the course of free evolution (or on phase angle in the definition of the field quadrature as $\hat{E}(\varphi) = [\hat{a} \exp(-i\varphi) + \hat{a}^\dagger \exp(i\varphi)] / \sqrt{2}$ [135]). They are related to the invariants of the total variance matrix or to the lengths of the principal axes of the ellipse of equal quasiprobabilities, which gives a graphical image of the squeezed state in the phase space [136] (this explains the name ‘principal squeezing’ used in [137]). The minimal σ_- and maximal σ_+ values of the variances σ_x or σ_p can be found by looking for extremal values of $\sigma_x(t)$ as a function of time [130]

$$\sigma_{\pm} = \mathcal{E} \pm \sqrt{\mathcal{E}^2 - d}, \quad (28)$$

where $\mathcal{E} = \frac{1}{2}(\sigma_x + \sigma_p) \equiv \frac{1}{2} + \langle \hat{a}^\dagger \hat{a} \rangle - |\langle \hat{a} \rangle|^2$ is the *energy of quantum fluctuations* (which is conserved in the process of free evolution), whereas parameter $d = \sigma_x \sigma_p - \sigma_{xp}^2$, coinciding with the left-hand side of the *Schrödinger–Robertson uncertainty relation* (25), determines (for Gaussian states) the quantum ‘purity’ [118] $\text{Tr} \hat{\rho}_{\text{Gauss}}^2 = 1/\sqrt{4d}$, being the simplest example of the so-called *universal quantum invariants* [138]. The expressions equivalent to (28) were obtained in [135–137]. Evidently, $\mathcal{E} \leq \frac{1}{2} + \bar{n}$, where \bar{n} is the mean photon number. Then one can easily derive from (28) the inequalities:

$$\sigma_- \geq \bar{n} + 1/2 - \sqrt{(\bar{n} + 1/2)^2 - d} > \frac{d}{1 + 2\bar{n}}. \quad (29)$$

For pure states ($d = 1/4$) these inequalities were found in [139]. For quantum mixtures, squeezing ($\sigma_- < 1/2$) is possible provided $\bar{n} \geq d^2 - 1/4$.

3.8. General concepts of squeezing

The first definition of squeezing for arbitrary Hermitian operators \hat{A} , \hat{B} was given by Walls and Zoller [91]. Taking into account the uncertainty relation (11), they said that fluctuations of the observable A are ‘reduced’ if:

$$(\Delta A)^2 < \frac{1}{2} | \langle [\hat{A}, \hat{B}] \rangle |. \quad (30)$$

This definition was extended to the case of several variables (whose operators are generators of some algebra) in [140] and specified in [141].

Hong and Mandel [142] introduced the concept of higher-order squeezing. The state $|\psi\rangle$ is squeezed to the $2n$ th order in some quadrature component, say \hat{x} , if the mean value $\langle \psi | (\Delta \hat{x})^{2n} | \psi \rangle$ is less than the mean value of $(\Delta \hat{x})^{2n}$ in the

coherent state. If \hat{x} is defined as in (19), then the condition of squeezing reads:

$$\langle (\Delta\hat{x})^{2n} \rangle < 2^{-n}(2n-1)!!.$$

In particular, for $n = 2$ we have the requirement $\langle (\Delta\hat{x})^4 \rangle < 3/4$. Hong and Mandel showed that the usual squeezed states are squeezed to any even order $2n$. The methods of producing such squeezing were proposed in [143].

Other definitions of the higher-order squeezing are usually based on the Walls–Zoller approach. Hillery [144] defined the second-order squeezing taking in (30):

$$\hat{A} = (\hat{a}^2 + \hat{a}^{\dagger 2})/2, \quad \hat{B} = (\hat{a}^2 - \hat{a}^{\dagger 2})/(2i).$$

A generalization to the k th-order squeezing, based on the operators $\hat{A} = (\hat{a}^k + \hat{a}^{\dagger k})/2$ and $\hat{B} = (\hat{a}^k - \hat{a}^{\dagger k})/(2i)$, was made in [145]. The uncertainty relations for higher-order moments were considered in [103]. For other studies on higher order squeezing see, e.g., [124, 146–149].

The concepts of the *sum squeezing* and *difference squeezing* were introduced by Hillery [150], who considered two-mode systems and used sums and differences of various bilinear combinations constructed from the creation and annihilation operators. These concepts were developed (including multimode generalizations) in [151]. A method of constructing squeezed states for general systems (different from the harmonic oscillator) was described in [152], where the eigenstates of the operator $\mu\hat{a}^2 + \nu\hat{a}^{\dagger 2}$ were considered as an example (see also [153]).

The concept of *amplitude squeezing* was introduced in [154]. It means the states possessing the property $\Delta n < \sqrt{\langle n \rangle}$ (for the coherent states, $\Delta n = \sqrt{\langle n \rangle}$, where n is the photon number). For further development see, e.g., [155]. Physical bounds on squeezing due to the finite energy of real systems were discussed in [156].

3.9. Oscillations of the distribution functions

While the photon distribution function $p_n = \langle n | \hat{\rho} | n \rangle$ is rather smooth for the 'classical' thermal and coherent states (being given by the Planck and Poisson distributions, respectively), it reveals strong oscillations for many 'nonclassical' states. The function p_n for a generic squeezed state was given in [70]

$$p_n = \frac{|v/(2u)|^n}{n!|u|} \exp \left[\operatorname{Re} \left(\frac{\beta^2 v^*}{u} \right) - |\beta|^2 \right] \left| H_n \left(\frac{\beta}{\sqrt{2uv}} \right) \right|^2,$$

where $H_n(z)$ is the Hermite polynomial. The graphical analysis of this distribution made in [157] showed that for certain relations between the squeezing and displacement parameters, the photon distribution function exhibits strong irregular oscillations, whereas for other values of the parameters it remains rather regular. Ten years later, these oscillations were rediscovered in [158–162], and since that time they have attracted the attention of many researchers, mainly due to their interpretation [163] as the manifestation of the *interference in phase space* (a method of calculating quasiclassical distributions, based on the areas of overlapping in the phase plane, was used earlier in [164]).

It is worth mentioning that as far back as 1970, Walls and Barakat [165] discovered strong oscillations of the photon

distribution functions, calculating eigenstates of the *trilinear* Hamiltonian $\hat{H}_{WB} = \sum_{k=1}^3 \omega_k \hat{a}_k^\dagger \hat{a}_k + \kappa (\hat{a}_1 \hat{a}_2^\dagger \hat{a}_3^\dagger + \text{h.c.})$. The parametric amplifier time-dependent Hamiltonian (22), which 'produces' squeezed states, can be considered as the semiclassical approximation to \hat{H}_{WB} . For recent studies on trilinear Hamiltonians and references to other publications see [166].

The photocount distributions and oscillations in the two-mode nonclassical states were studied in [167]. The influence of thermal noise was studied in [123, 159, 168, 169]. The cumulants and factorial moments of the squeezed state photon distribution function were considered in [162, 168]. They exhibit strong oscillations even in the case of slightly squeezed states [170]. For other studies see, e.g., [171].

4. Non-Gaussian oscillator states

4.1. Displaced and squeezed number states

These states are obtained by applying the displacement operator $\hat{D}(\alpha)$ (6) or the squeezing operator $\hat{S}(z)$ (21) to the states different from the vacuum oscillator state $|0\rangle$. As a matter of fact, the first examples were given by Plebański [43], who studied the properties of the state $|n, \alpha\rangle = \hat{D}(\alpha)|n\rangle$. His results were rediscovered in [172], where the name *semicoherent state* was used. The general construction $\hat{D}(\alpha)|\psi_0\rangle$ for an arbitrary *fiducial state* $|\psi_0\rangle$ was considered by Klauder in [16]. The special case of the states $|n, z\rangle = \hat{S}(z)|n\rangle$ (known now under the name *squeezed number states*) for real z was considered in [45, 46]. These states were also briefly discussed by Yuen [70]. The *displaced number states* were considered in connection with the time-energy uncertainty relation in [173]. They can be expressed as [174]:

$$|n, \alpha\rangle = \hat{D}(\alpha)|n\rangle = \mathcal{N}(\hat{a}^\dagger - \alpha^*)^n |\alpha\rangle. \quad (31)$$

The detailed studies of different properties of displaced and squeezed number states were performed in [123, 146, 175].

4.2. First finite superpositions of coherent states

Titulaer and Glauber [176] introduced the 'generalized coherent states', multiplying each term of expansion (5) by arbitrary phase factors:

$$|\alpha\rangle_g = \exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \exp(i\vartheta_n) |n\rangle. \quad (32)$$

These are the most general states satisfying Glauber's criterion of 'coherence' [27]. Their general properties and some special cases corresponding to the concrete dependences of the phases ϑ_n on n were studied in [177, 178].

In particular, Bialynicka-Birula [177] showed that in the *periodic case*, $\vartheta_{n+N} = \vartheta_n$ (with arbitrary values $\vartheta_0, \vartheta_1, \dots, \vartheta_{N-1}$), state (32) is the superposition of N Glauber's coherent states, whose labels are uniformly distributed along the circle $|\alpha| = \text{const}$:

$$|\phi\rangle = \sum_{k=1}^N c_k |\alpha_0 \exp(i\phi_k)\rangle, \quad \phi_k = 2\pi k/N. \quad (33)$$

The amplitudes c_k are determined from the system of N equations ($m = 0, 1, \dots, N - 1$)

$$\sum_{k=1}^N c_k \exp(im\phi_k) = \exp(i\vartheta_m), \quad (34)$$

and they are different in the generic case. Stoler [178] has noticed that any sum (33) is an eigenstate of the operator \hat{a}^N with the eigenvalue α_0^N , therefore it can be represented as a superposition of N orthogonal states, each one being a certain combination of N coherent states $|\alpha_0 \exp(i\phi_k)\rangle$.

4.3. Even and odd thermal and coherent states

An example of ‘nonclassical’ *mixed* states was given by Cahill and Glauber [33], who discussed in detail the *displaced thermal states* (such states, which are obviously ‘classical’, were also studied in [32, 179]) and compared them with the following quantum mixture:

$$\hat{\rho}_{ev-th} = \frac{2}{2+\bar{n}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{2+\bar{n}} \right)^n |2n\rangle\langle 2n|. \quad (35)$$

The *even and odd coherent states*

$$|\alpha\rangle_{\pm} = \mathcal{N}_{\pm} (|\alpha\rangle \pm |-\alpha\rangle), \quad (36)$$

where $\mathcal{N}_{\pm} = (2[1 \pm \exp(-2|\alpha|^2)])^{-1/2}$, have been introduced by Dodonov *et al* [180]. They are not reduced to the superpositions given in (33), since coefficients $c_1 = \pm c_2$ can never satisfy equations (34) for real phases ϑ_0, ϑ_1 . Besides, the photon statistics of the states (36) are quite different from the Poissonian statistics inherent to all states of the form (32). Even/odd states possess many remarkable properties: if $|\alpha| \gg 1$, they can be considered as the simplest examples of *macroscopic quantum superpositions* or ‘*Schrödinger cat states*’; being eigenstates of the operator \hat{a}^2 (cf equation (10)), the states $|\alpha\rangle_{\pm}$ are the simplest special cases of the *multiphoton states* (see later); they can be obtained from the vacuum by the action of *nonexponential* displacement operators

$$\hat{D}_+(\alpha) = \cosh(\alpha\hat{a}^\dagger - \alpha^*\hat{a}) \quad \hat{D}_-(\alpha) = \sinh(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$$

(cf equation (9)), etc. Moreover, from the modern point of view, the special cases of the time-dependent wavepacket solutions of the Schrödinger equation for the (singular) oscillator with a time-dependent frequency found in [180] are nothing but the *odd squeezed states*.

Parity-dependent squeezed states

$$[\hat{S}(z_1)\hat{\Pi}_+ + \hat{S}(z_2)\hat{\Pi}_-]|\alpha\rangle$$

where $\hat{\Pi}_{\pm}$ are the projectors to the even/odd subspaces of the Hilbert space of Fock states, were studied in [181]. The actions of the squeezing and displacement operators on the superpositions of the form $|\alpha, \tau, \varphi\rangle = \mathcal{N}(|\alpha\rangle + \tau e^{i\varphi}|-\alpha\rangle)$ (which contain as special cases even/odd, Yurke–Stoler, and coherent states) were studied in [182] (the special case $\tau = 1$ was considered in [183]). For other studies see, e.g., [6, 184–188]. Multidimensional generalizations have been studied in [189].

The name *shadowed states* was given in [190] to the mixed states whose statistical operators have the form $\hat{\rho}_{sh} = \sum p_n |2n\rangle\langle 2n|$ (i.e., generalizing the even thermal state (35)). Mixed analogues of the even and odd states were considered in [191].

5. Coherent phase states

The state of the classical oscillator can be described either in terms of its quadrature components x and p , or in terms of the amplitude and phase, so that $x + ip = A \exp(i\varphi)$. Moreover, in classical mechanics one can introduce the action and angle variables, which have the same Poisson brackets as the coordinate and momentum: $\{p, x\} = \{I, \varphi\} = 1$. However, in the quantum case we meet serious mathematical difficulties trying to define the phase operator in such a way that the commutation relation $[\hat{n}, \hat{\varphi}] = i$ would be fulfilled, if the photon number operator is defined as $\hat{n} = \hat{a}^\dagger \hat{a}$. These difficulties originate in the fact that the spectrum of operator \hat{n} is *bounded* from below.

The first solution to the problem was given by Susskind and Glogower [192], who introduced, instead of the phase operator itself, the *exponential phase operator*

$$\hat{E}_- \equiv \hat{C} + i\hat{S} \equiv \sum_{n=1}^{\infty} |n-1\rangle\langle n| = (\hat{a}\hat{a}^\dagger)^{-1/2}\hat{a}, \quad (37)$$

which can be considered, to a certain extent, as a quantum analogue of the classical phase $e^{i\varphi}$ [29, 193]. Operator \hat{E}_- and its Hermitian ‘cosine’ and ‘sine’ components satisfy the ‘classical’ commutation relations:

$$[\hat{C}, \hat{n}] = i\hat{S}, \quad [\hat{S}, \hat{n}] = -i\hat{C}, \quad [\hat{E}_-, \hat{n}] = \hat{E}_-. \quad (38)$$

However, operator \hat{E}_- is *nonunitary*, since the commutator with its Hermitically conjugated partner $\hat{E}_+ \equiv \hat{E}_-^\dagger$ is not equal to zero:

$$[\hat{E}_-, \hat{E}_+] = 1 - \hat{C}^2 - \hat{S}^2 = |0\rangle\langle 0|. \quad (39)$$

It is well known that the annihilation operator \hat{a} has no inverse operator in the full meaning of this term. Nonetheless, it possesses the *right inverse operator*

$$\hat{a}^{-1} = \sum_{n=0}^{\infty} \frac{|n+1\rangle\langle n|}{\sqrt{n+1}} = \hat{a}^\dagger (\hat{a}\hat{a}^\dagger)^{-1} = \hat{E}_+ (\hat{a}\hat{a}^\dagger)^{-1/2}, \quad (40)$$

which satisfies, among many others, the relations:

$$\hat{a}\hat{a}^{-1} = 1, \quad [\hat{a}, \hat{a}^{-1}] = |0\rangle\langle 0|, \quad [\hat{a}^\dagger, \hat{a}^{-1}] = \hat{a}^{-2}.$$

This operator was discussed briefly by Dirac [194], who noticed that Fock considered it long before. However, it has only found applications in quantum optics in the 1990s (see the paragraph on *photon-added states* later). Lerner [195] noticed that the commutation relations (38) do not determine the operators \hat{C}, \hat{S} uniquely. Earlier, the same observation was made by Wigner [196] with respect to the triple $\{\hat{n}, \hat{a}, \hat{a}^\dagger\}$ (see the next section). In the general case, besides the ‘polar decomposition’ (37), which is equivalent to the relations

$$\hat{E}_- |n\rangle = (1 - \delta_{n0}) |n-1\rangle, \quad \hat{E}_+ |n\rangle = |n+1\rangle, \quad (41)$$

one can define operator $\hat{U} = \hat{C} + i\hat{S}$ via the relation $\hat{U}|n\rangle = f(n)|n-1\rangle$, where function $f(n)$ may be arbitrary enough, being restricted by the requirement $f(0) = 0$ and certain other constraints which ensure that the spectra of the ‘cosine’ and ‘sine’ operators belong to the interval $(-1, 1)$. The properties of Lerner’s construction were studied in [197].

The states with the definite phase

$$|\varphi\rangle = \sum_{n=0}^{\infty} \exp(in\varphi)|n\rangle \quad (42)$$

were considered in [29, 192]. However, they are not normalizable (like the coordinate or momentum eigenstates). The normalizable *coherent phase states* were introduced in [198] as the eigenstates of the operator \hat{E}_- :

$$|\varepsilon\rangle = \sqrt{1 - \varepsilon\varepsilon^*} \sum_{n=0}^{\infty} \varepsilon^n |n\rangle, \quad \hat{E}_-|\varepsilon\rangle = \varepsilon|\varepsilon\rangle, \quad |\varepsilon| < 1. \quad (43)$$

The *pure* quantum state (43) has the same probability distribution $|\langle n|\varepsilon\rangle|^2$ as the mixed *thermal state* described by the statistical operator

$$\hat{\rho}_{th} = \frac{1}{1 + \bar{n}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{1 + \bar{n}} \right)^n |n\rangle\langle n|, \quad (44)$$

if one identifies the mean photon number \bar{n} with $|\varepsilon|^2/(1 - |\varepsilon|^2)$ [199].

The two-parameter set of states ($\kappa \neq -1$)

$$|z; \kappa\rangle = \mathcal{N} \sum_{n=0}^{\infty} \left[\frac{\Gamma(n + \kappa + 1)}{n! \Gamma(\kappa + 1)} \right]^{1/2} \left(\frac{z}{\sqrt{\kappa + 1}} \right)^n |n\rangle \quad (45)$$

has been introduced in [200] and studied in detail from different points of view in [180, 199, 201]. These states are eigenstates of the operator:

$$\hat{A}_\kappa = \hat{E}_- \left[\frac{(\kappa + 1)\hat{n}}{\kappa + \hat{n}} \right]^{1/2} \equiv \left[\frac{(\kappa + 1)}{\kappa + 1 + \hat{n}} \right]^{1/2} \hat{a}. \quad (46)$$

If $\kappa = 0$, $\hat{A}_0 = \hat{E}_-$, and the state $|z; 0\rangle$ coincides with (43). If $\kappa \rightarrow \infty$, then $\hat{A}_\infty = \hat{a}$, and the state (45) goes to the *coherent state* $|z\rangle$ (5). In the 1990s the state (45) appeared again under the name *negative binomial state* (see (67) in section 8.4). In the special case $\kappa = -p - 1$, $p = 1, 2, \dots$, the state (45) goes to the superposition of the first $p + 1$ Fock states

$$|z; p\rangle = \mathcal{N} \sum_{n=0}^p \left[\frac{p!}{n!(p-n)!} \right]^{1/2} \left(\frac{z}{\sqrt{p}} \right)^n |n\rangle, \quad (47)$$

which is nothing other than the *binomial state* introduced in 1985 (see section 8.4).

Sukumar [202] introduced the states ($\alpha = (r/m)e^{i\varphi}$, $\beta = \tanh r$, $m = 1, 2, \dots$)

$$\exp(\alpha \hat{T}_m^\dagger - \alpha^* \hat{T}_m)|0\rangle = \sqrt{1 - \beta^2} \sum_{n=0}^{\infty} (\beta e^{i\varphi})^n |mn\rangle,$$

where $\hat{T}_m = \hat{E}_m^\dagger \hat{n} \equiv (\hat{n} + m)\hat{E}_m^\dagger$, and operators \hat{T}_\pm act on the Fock states as follows:

$$\hat{T}_m |n\rangle = \begin{cases} n |n - m\rangle, & n \geq m \\ 0, & n < m, \end{cases}$$

$$\hat{T}_m^\dagger |n\rangle = (n + m) |n + m\rangle.$$

For $m = 1$ one arrives again at the state (43).

The phase coherent state (43) was rediscovered in the beginning of the 1990s in [203] as a pure analogue of the thermal state. It was also noticed that this state yields a strong squeezing effect. By analogy with the usual squeezed states, which are eigenstates of the linear combination of the operators \hat{a}, \hat{a}^\dagger (16), the *phase squeezed states* (PSS) were constructed in [204] as the eigenstates of the operator $\hat{B} = \mu \hat{E}_- + \nu \hat{E}_+$. The coefficients of the decomposition of PSS over the Fock basis are given by

$$c_n = \mathcal{N} (z_+^{n+1} - z_-^{n+1}), \quad z_\pm = \left(\beta \pm \sqrt{\beta^2 - 4\mu\nu} \right) / (2\mu),$$

where β is the complex eigenvalue of \hat{B} and \mathcal{N} is the normalization factor. It is worth noting, however, that PSS have actually been introduced as far back as 1974 by Mathews and Eswaran [205], who minimized the ratio $(\Delta C)^2 (\Delta S)^2 / |[\hat{C}, \hat{S}]|^2$, solving the equation (ν and λ are parameters):

$$[(1 + \nu)\hat{E}_- + (1 - \nu)\hat{E}_+]|\psi\rangle = \lambda|\psi\rangle. \quad (48)$$

The continuous representation of arbitrary quantum states by means of the phase coherent states (43) (an analogue of the Klauder–Glauber–Sudarshan coherent state representation) was considered in [206] (where it was called the *analytic representation in the unit disk*), [207] (under the name *harmonious state representation*), and [208]. Eigenstates of operator $\hat{Z}(\sigma) = \hat{E}_-(\hat{n} + \sigma)$,

$$|z; \sigma\rangle = \sum_{n=0}^{\infty} \frac{z^n |n\rangle}{\Gamma(n + \sigma + 1)}, \quad \hat{Z}(\sigma)|z; \sigma\rangle = z|z; \sigma\rangle, \quad (49)$$

were named as *philophase states* in [209]. If $\sigma = 0$, then (49) goes to the special case of the Barut–Girardello state (54) with $k = 1/2$.

The name 'pseudothermal state' was given to the state (43) in [210], where it was shown that this state arises naturally as an exact solution to certain nonlinear modifications of the Schrödinger equation. *Shifted thermal states*, which can be written as $\hat{\rho}_{th}^{(\text{shift})} = \hat{E}_+ \hat{\rho}_{th} \hat{E}_-$, have been considered in [211] (see also [212]).

For the most recent study on phase states see [213]. Comprehensive discussions of the problem of phase in quantum mechanics can be found, e.g., in [193, 214], and a detailed list of publications up to 1996 was given in the tutorial review [215].

6. Algebraic coherent states

6.1. Angular momentum and spin-coherent states

The first family of the coherent states for the angular momentum operators was constructed in [216], actually as some special two-dimensional oscillator coherent state, using the Schwinger representation of the angular momentum operators in terms of the auxiliary bosonic annihilation operators \hat{a}_+ and \hat{a}_- :

$$\hat{J}_+ = \hat{a}_+^\dagger \hat{a}_-, \quad \hat{J}_- = \hat{a}_-^\dagger \hat{a}_+, \quad \hat{J}_3 = \frac{1}{2} (\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-). \quad (50)$$

Such ‘oscillator-like’ angular momentum coherent states were studied in [217]. A possibility of constructing ‘continuous’ families of states using different modifications of the unitary displacement (6) and squeezing (21) operators was recognized in the beginning of the 1970s. The first explicit example is frequently related to the *coherent spin states* [218] (also named *atomic coherent states* [219] and *Bloch coherent states* [220])

$$|\vartheta, \varphi\rangle = \exp(\zeta \hat{J}_+ - \zeta^* \hat{J}_-) |j, -j\rangle \quad \zeta = (\vartheta/2) \exp(-i\varphi), \quad (51)$$

where \hat{J}_\pm are the standard raising and lowering spin (angular momentum) operators, $|j, m\rangle$ are the standard eigenstates of the operators \hat{J}^2 and \hat{J}_z , and ϑ, φ are the angles in the spherical coordinates. However, the special case of these states for spin- $\frac{1}{2}$ was considered much earlier by Klauder [16], who introduced the fermion coherent states $|\beta\rangle = \sqrt{1 - |\beta|^2} |0\rangle + \beta |1\rangle$, where β could be an arbitrary complex number satisfying the inequality $|\beta| \leq 1$. Also, Klauder studied generic states of the form (51) in [221]. A detailed discussion of the *spin-coherent states* was given in [222], whereas *spin-squeezed states* were discussed in [223]. For recent publications on the angular momentum coherent states see, e.g., [224].

6.2. Group coherent states

The operators \hat{J}_\pm, \hat{J}_z are the generators of the algebra $su(2)$. A general construction looks like

$$|\xi_1, \xi_2, \dots, \xi_n\rangle = \exp(\xi_1 \hat{A}_1 + \xi_2 \hat{A}_2 + \dots + \xi_n \hat{A}_n) |\psi\rangle, \quad (52)$$

where $\{\xi_k\}$ is a continuous set of complex or real parameters, \hat{A}_k are the generators of some algebra, and $|\psi\rangle$ is some ‘basic’ (‘fiducial’) state. This scheme was proposed by Klauder as far back as 1963 [221], and later it was rediscovered in [225]. A great amount of different families of ‘generalized coherent states’ can be obtained, choosing different algebras and basic states. One of the first examples was related to the $su(1, 1)$ algebra [225]

$$|\zeta; k\rangle \sim \exp(\zeta \hat{K}_+) |0\rangle \sim \sum_{n=0}^{\infty} \left[\frac{\Gamma(n+2k)}{n! \Gamma(2k)} \right]^{1/2} \zeta^n |n\rangle, \quad (53)$$

where k is the so-called Bargmann index labelling the concrete representation of the algebra, and \hat{K}_+ is the corresponding rising operator. Evidently, the states (45) and (53) are the same, although their interpretation may be different. Some particular realizations of the state (53) connected with the problem of quantum ‘singular oscillator’ (described by the Hamiltonian $H = p^2 + x^2 + gx^{-2}$) were considered in [180, 201]. The ‘generalized phase state’ (45) and its special case (47) can also be considered as coherent states for the groups $O(2, 1)$ or $O(3)$, with the ‘displacement operator’ of the form [199]:

$$\hat{D}_k(z) = \exp\left(z\sqrt{\hat{n}(\hat{n}+\kappa)} \hat{E}_+ - z^* \hat{E}_- \sqrt{\hat{n}(\hat{n}+\kappa)}\right).$$

A comparison of the coherent states for the Heisenberg–Weyl and $su(2)$ algebras was made in [226] (see also [227]). Coherent states for the group $SU(n)$ were studied in [220, 228], whereas the groups $E(n)$ and $SU(m, n)$ were considered in [229]. *Multilevel atomic coherent states* were introduced in [230].

The name *Barut–Girardello coherent states* is used in modern literature for the states which are eigenstates of some non-Hermitian *lowering operators*. The first example was given in [231], where the eigenstates of the lowering operator \hat{K}_- of the $su(1, 1)$ algebra were introduced in the form:

$$|z; k\rangle = \mathcal{N} \sum_{n=0}^{\infty} \frac{z^n |n\rangle}{\sqrt{n! \Gamma(n+2k)}}. \quad (54)$$

The corresponding wavepackets in the coordinate representation, related to the problem of nonstationary ‘singular oscillator’, were expressed in terms of Bessel functions in [180].

The first two-dimensional analogues of the Barut–Girardello states, namely, eigenstates of the product of commuting boson annihilation operators $\hat{a}\hat{b}$,

$$\hat{a}\hat{b}|\xi, q\rangle = \xi|\xi, q\rangle, \quad \hat{Q}|\xi, q\rangle = q|\xi, q\rangle, \quad (55)$$

$$\hat{Q} = \hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b},$$

were introduced by Horn and Silver [232] in connection with the problem of pions production. In the simplified variant these states have the form (actually Horn and Silver considered the infinite-dimensional case related to the quantum field theory):

$$|\xi, q\rangle = \mathcal{N} \sum_{n=0}^{\infty} \frac{\xi^n |n+q, n\rangle}{[n!(n+q)!]^{1/2}}. \quad (56)$$

Operator \hat{Q} can be interpreted as the ‘charge operator’; for this reason state (56) appeared in [233] under the name ‘charged boson coherent state.’ Joint eigenstates of the operators $\hat{a}_+ \hat{a}_+$ and $\hat{a}_+ \hat{a}_-$, which generate the angular momentum operators according to (50), were studied in [234]. The states (56) have found applications in quantum field theory [235]. Nonclassical properties of the states (56) (renamed as *pair coherent states*) were studied in [236]. An example of two-mode $SU(1, 1)$ coherent states was given in [237]. The generalization of the ‘charged boson’ coherent states (56) in the form of the eigenstates of the operator $\mu \hat{a}\hat{b} + \nu \hat{a}^\dagger \hat{b}^\dagger$, satisfying the constraint $(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b})|\psi\rangle = 0$, was studied in [238]. Nonclassical properties of the *even and odd charge coherent states* were studied in [239].

The first explicit treatments of the squeezed states as the $SU(1, 1)$ *coherent states* were given in studies [140, 240], whose authors considered, in particular, the realization of the $su(1, 1)$ algebra in terms of the operators:

$$\hat{K}_+ = \hat{a}^{\dagger 2}/2, \quad \hat{K}_- = \hat{a}^2/2, \quad \hat{K}_3 = (\hat{a}^\dagger \hat{a} + \hat{a}\hat{a}^\dagger)/4. \quad (57)$$

If $(\Delta K_1)^2 < |\langle \hat{K}_3 \rangle|/2$ or $(\Delta K_2)^2 < |\langle \hat{K}_3 \rangle|/2$ (where $\hat{K}_\pm = \hat{K}_1 \pm i\hat{K}_2$), then the state was named $SU(1, 1)$ squeezed [140] (in [144], similar states were named *amplitude-squared squeezed states*). The relations between the squeezed states and the Bogolyubov transformations were considered in [241]. ‘Maximally symmetric’ coherent states have been considered in [242]. $SU(2)$ and $SU(1, 1)$ phase states have been constructed in [243]. The algebraic approaches in studying the squeezing phenomenon have been used in [244]. Analytic representations based on the $SU(1, 1)$ group coherent states and Barut–Girardello states were compared in [245]. $SU(3)$ -coherent states were considered in [246]. Coherent states defined *on a circle* and *on a sphere* have been studied in [247]. For other studies on group and algebraic states and their applications see, e.g., [7, 248, 249].

7. 'Minimum uncertainty' and 'intelligent' states

For the operators \hat{A} and \hat{B} different from \hat{x} and \hat{p} , the right-hand side of the uncertainty relation (11) depends on the quantum state. The problem of finding the states for which (11) becomes the equality was discussed by Jackiw [50], who showed that it is reduced essentially to solving the equation:

$$(\hat{A} - \langle \hat{A} \rangle)|\psi\rangle = \lambda(\hat{B} - \langle \hat{B} \rangle)|\psi\rangle. \quad (58)$$

Jackiw found the explicit form of the states minimizing one of several 'phase-number' uncertainty relations, namely

$$(\Delta\hat{n})^2(\Delta\hat{C})^2/\langle\hat{S}\rangle^2 \geq 1/4, \quad (59)$$

in the form $|\psi\rangle = \mathcal{N} \sum_{n=0}^{\infty} (-i)^n I_{n-\lambda}(\gamma)|n\rangle$, where $I_\nu(\gamma)$ is the modified Bessel function, and λ, γ are some parameters.

Eswaran considered the $\hat{n}-\hat{C}$ pair of operators and solved the equation $(\hat{n} + i\gamma\hat{C})|\psi\rangle = \lambda|\psi\rangle$ [197]. If the product $\Delta n \Delta \varphi$ of the number and phase 'uncertainties' remains of the order of 1, then we have the *number-phase minimum uncertainty state* [250]. The methods of generating the MUS for the operators $\hat{N}, \hat{C}, \hat{S}$ (37) were discussed in [251]. For recent studies see, e.g., [252].

Multimode generalizations of the (Gaussian) MUS were discussed in [253], and their detailed study was given in [254]. *Noise minimum states*, which give the minimal value of the photon number operator fluctuation σ_n for the fixed values of the lower order moments, e.g., $\langle\hat{a}\rangle, \langle\hat{a}^2\rangle, \langle\hat{n}\rangle$, were considered in [255]. This study was continued in [256], where eigenstates of operator $\hat{a}^\dagger\hat{a} - \xi^*\hat{a} - \xi\hat{a}^\dagger$ were found in the form

$$|\xi; M\rangle = (\hat{a}^\dagger + \xi^*)^M |\xi\rangle_{\text{coh}} \quad (60)$$

(these states differ from (31), due to the opposite sign of the term ξ^*). Different families of MUS related to the powers of the bosonic operators were considered in [257]. The eigenstates of the most general linear combination of the operators $\hat{a}, \hat{a}^\dagger, \hat{a}^2, \hat{a}^{\dagger 2}$, and $\hat{a}^\dagger\hat{a}$ were studied in [182], where the general concept of *algebra eigenstates* was introduced. These states were defined as eigenstates of the linear combination $\xi_1\hat{A}_1 + \xi_2\hat{A}_2 + \dots + \xi_n\hat{A}_n$, where \hat{A}_k are generators of some algebra. In the case of *two-photon algebra* these states are expressed in terms of the confluent hypergeometric function or the Bessel function, and they contain, as special cases, many other families of nonclassical states [258].

The case of the spin (angular momentum) operators was considered in [259], where the name *intelligent states* was introduced. The relations between coherent spin states, intelligent spin states, and minimum-uncertainty spin states were discussed in [260]. For recent publications see, e.g., [261]. Nowadays the 'intelligent' states are understood to be the states for which the Heisenberg uncertainty relation (11) becomes the equality, whereas the MUS are those for which the 'uncertainty product' $\Delta A \Delta B$ attains the minimal possible value (for arbitrary operators \hat{A}, \hat{B} such states may not exist [50]). The relations between squeezing and 'intelligence' were discussed, e.g., in papers [104, 258, 262]. The properties and applications of the $SU(1, 1)$ and $SU(2)$ intelligent states were considered in [263]. The intelligent states for the generators $\hat{K}_{1,2}$ of the $su(1, 1)$ algebra were named

Hermite polynomial states in [264], since they have the form $\hat{S}(z)H_m(\xi\hat{a}^\dagger)|0\rangle$, thus being finite superpositions of the *squeezed number states*. Such states were studied in [265]. The *minimum uncertainty state for sum squeezing* in the form $\hat{S}(\xi)H_{pq}(\mu\hat{a}_1^\dagger, \mu\hat{a}_2^\dagger)|00\rangle$ was found in [266]. Here H_{pq} is a special case of the family of *two-dimensional Hermite polynomials*, which are useful for many problems of quantum optics [67, 120, 267].

8. Non-Gaussian and 'coherent' states for nonoscillator systems

There exist different constructions of 'coherent states' for a particle moving in an arbitrary potential. MUS whose time evolution is as close as possible to the trajectory of a *classical* particle have been studied by Nieto and Simmons in a series of papers, beginning with [268, 269]. In [270] 'coherent' states were defined as eigenstates of operators like $\hat{A} = f(x) + i\sigma(x)d/dx$. However, such packets do not preserve their forms in the process of evolution, losing the important property of the Schrödinger nonspreading wavepackets.

At least three anharmonic potentials are of special interest in quantum mechanics. The closest to the harmonic oscillator is the 'singular oscillator' potential $x^2 + gx^{-2}$ (also known as the 'isotonic,' 'pseudoharmonic,' 'centrifugal' oscillator, or 'oscillator with centripetal barrier'). Different coherent states for this potential have been constructed in [180, 201, 269, 271–273].

MUS for the Morse potential $U_0(1 - e^{-ax})^2$ were constructed in [274]; the cases of the Pöschl–Teller and Rosen–Morse potentials, $U_0 \tan^2(ax)$ and $U_0 \tanh^2(ax)$, were considered in [269]. Algebraic coherent states for these potentials, based, in particular, on the algebras $su(1, 1)$ or $so(2, 1)$, have been proposed in [275], for recent constructions see, e.g., [272, 276]. Coherent states for the *reflectionless potentials* were constructed in [277]. The intelligent states for arbitrary potentials, with concrete applications to the Pöschl–Teller one, were considered recently in [278].

Klauder [279] has proposed a general construction of coherent states in the form

$$|z; \gamma\rangle = \sum_n \frac{z^n}{\sqrt{\rho_n}} \exp(-ie_n\gamma)|n\rangle, \quad \hat{H}|n\rangle = e_n|n\rangle, \quad (61)$$

where positive coefficients $\{\rho_n\}$ satisfy certain conditions, while the discrete energy spectrum $\{e_n\}$ may be quite arbitrary. This construction was applied to the hydrogen atom in [280]. Another special case of states (61) is the *Mittag-Leffler coherent state* [281]:

$$|z; \alpha, \beta\rangle = \mathcal{N} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{\Gamma(\alpha n + \beta)}} |n\rangle. \quad (62)$$

Penson and Solomon [282] have introduced the state $|q, z\rangle = \varepsilon(q, z\hat{a}^\dagger)|0\rangle$, where function $\varepsilon(q, z)$ is a generalization of the exponential function given by the relations:

$$d\varepsilon(q, z)/dz = \varepsilon(q, qz), \quad \varepsilon(q, z) = \sum_{n=0}^{\infty} q^{n(n-1)/2} z^n / n!.$$

The Gaussian exponential form of the coefficients ρ_n in (61) was used in [283] to construct localized wavepackets for the Coulomb problem, the planar rotor and the particle in a box. For the most recent studies see, e.g., [278, 284].

8.1. Coherent states and packets in the hydrogen atom

Introducing the nonspreading Gaussian wavepackets for the harmonic oscillator, Schrödinger wrote in the same paper [1]²

'We can definitely foresee that, in a similar way, wave groups can be constructed which would round highly quantized Kepler ellipses and are the representation by wave mechanics of the hydrogen electron. But the technical difficulties in the calculation are greater than in the especially simple case which we have treated here.'

Indeed, this problem turned out to be much more complicated than the oscillator one. One of the first attempts to construct the quasiclassical packets moving along the Kepler orbits was made by Brown [285] in 1973. A similar approach was used in [286]. Different nonspreading and squeezed Rydberg packets were considered in [287].

The symmetry explaining degeneracy of hydrogen energy levels was found by Fock [288] (see also Bargmann [289]): it is $O(4)$ for the discrete spectrum and the Lorentz group (or $O(3, 1)$) for the continuous one. The symmetry combining all the discrete levels into one irreducible representation (the dynamical group $O(4, 2)$) was found in [290] (see also [291]). Different group related coherent states connected to these dynamical symmetries were discussed in [292]. The Kustaanheimo–Stiefel transformation [293], which reduces the three-dimensional Coulomb problem to the four-dimensional constrained harmonic oscillator [294], was applied to obtain various coherent states of the hydrogen atom in [295]. For the most recent publications see, e.g., [272, 296].

8.2. Relativistic oscillator coherent states

One of the first papers on the relativistic equations with internal degrees of freedom was published by Ginzburg and Tamm [297]. The model of 'covariant relativistic oscillator' obeying the modified Dirac equation

$$(\gamma_\mu \partial/\partial x_\mu + a[\xi_\mu \xi_\mu - \partial^2/\partial \xi_\mu \partial \xi_\mu] + m_0)\psi(x, \xi) = 0, \quad (63)$$

where x is the 4-vector of the 'centre of mass', and 4-vector ξ is responsible for the 'internal' degrees of freedom of the 'extended' particle, was studied by many authors [298]. Coherent states for this model, related to the representations of the $SU(1, 1)$ group and the 'singular oscillator' coherent states of [180], have been constructed in [299]. Coherent states of relativistic particles obeying the standard Dirac or Klein–Gordon equations were discussed in [300].

Different families of coherent states for several new models of the relativistic oscillator, different from the Yukawa–Markov type (63), have been studied during the 1990s. Mir–Kasimov [301] constructed *intelligent* states (in terms of the Macdonald function $K_\nu(z)$) for the coordinate and momentum operators obeying the 'deformed relativistic uncertainty relation':

$$[\hat{x}, \hat{p}] = i\hbar \cosh[(i\hbar/2mc) d/dx].$$

² Translation given in: Schrödinger E 1978 *Collected Papers on Wave Mechanics* (New York: Chelsea) pp 41–4.

Coherent states for another model, described by the equation

$$i\partial\psi/\partial t = [\alpha_k (\hat{p}_k - im\omega\hat{x}_k\beta) + m\beta] \psi \quad (64)$$

and named 'Dirac oscillator' in [302] (although similar equations were considered earlier, e.g., in [303]), have been studied in [304]. Aldaya and Guerrero [305] introduced the coherent states based on the modified 'relativistic' commutation relations:

$$\begin{aligned} [\hat{E}, \hat{x}] &= -i\hbar \hat{p}/m, & [\hat{E}, \hat{p}] &= im\omega^2 \hbar \hat{x}, \\ [\hat{x}, \hat{p}] &= i\hbar(1 + \hat{E}/mc^2). \end{aligned}$$

These states were studied in [306].

8.3. Supersymmetric states

The concept of supersymmetry was introduced by Gol'fand and Likhtman [307]. The nonrelativistic supersymmetric quantum mechanics was proposed by Witten [308] and studied in [309]. Its super-simplified model can be described in terms of the Hamiltonian which is a sum of the free oscillator and spin parts, so that the lowering operator can be conceived as a matrix

$$\hat{H} = \hat{a}^\dagger \hat{a} - \frac{1}{2} \sigma_3, \quad \hat{A} = \begin{vmatrix} \hat{a} & 1 \\ 0 & \hat{a} \end{vmatrix}$$

(σ_3 is the Pauli matrix). Then one can try to construct various families of states applying the operators like $\exp(\alpha \hat{A}^\dagger)$ to the ground (or another) state, looking for eigenstates of \hat{A} or some functions of this operator, and so on. The first scheme was applied by Bars and Günaydin [310], who constructed group *supercoherent states*. The same (displacement operator) approach was used in [311]. The second way was chosen by Aragone and Zypman [312], who constructed the eigenstates of some 'supersymmetric' non-Hermitian operators.

Different coherent states for the Hamiltonians obtained from the harmonic oscillator Hamiltonian through various deformations of the potential (by the Darboux transformation, for example) have been studied in [313]. 'Supercoherent' and 'supersqueezed' states were studied in [314].

8.4. Binomial states

The finite combinations of the first $M + 1$ Fock states in the form [315, 316]

$$|p, M; \theta_n\rangle = \sum_{n=0}^M e^{i\theta_n} \left[\frac{M!}{n!(M-n)!} p^n (1-p)^{M-n} \right]^{1/2} |n\rangle \quad (65)$$

were named 'binomial states' in [315] (see also [317]). As a matter of fact, they were studied much earlier in the paper [199]: see equation (47). Binomial states go to the coherent states in the limit $p \rightarrow 0$ and $M \rightarrow \infty$, provided $pM = \text{const}$, so they are representatives of a wider class of intermediate, or interpolating, states. The special case of $M = 1$ (i.e., combinations of the ground and the first excited states) was named *Bernoulli states* in [315]. Other 'intermediate' states are (they are superpositions of an infinite number of Fock states) *logarithmic states* [318]

$$|\psi\rangle_{\log} = c|0\rangle + \mathcal{N} \sum_{n=1}^{\infty} \frac{z^n}{\sqrt{n}} |n\rangle \quad (66)$$

and *negative binomial states* [319]

$$|\xi, \mu; \theta_n\rangle = \sum_{n=0}^{\infty} e^{i\theta_n} \left[(1-\xi)^\mu \frac{\Gamma(\mu+n)}{\Gamma(\mu)n!} \xi^n \right]^{1/2} |n\rangle, \quad (67)$$

where $\mu > 0$, $0 \leq \xi < 1$. One can check that for $\theta_n = n\theta_0$ the state (67) coincides with the generalized phase state (45) introduced in [199, 200]. Mixed quantum states with *negative binomial distributions* of the diagonal elements of the density matrix in the Fock basis appeared in the theory of photodetectors and amplifiers in [320].

Reciprocal binomial states

$$|\Phi; M\rangle = \mathcal{N} \sum_{n=0}^M e^{in\phi} \sqrt{n!(M-n)!} |n\rangle \quad (68)$$

were introduced in [321], and schemes of their generation were discussed in [322]. *Intermediate number squeezed states* $\hat{S}(z)|\eta, M\rangle$ (where $|\eta, M\rangle$ is the binomial state) were introduced in [323] and generalized in [324]. *Multinomial states* have been introduced in [325]. Barnett [326] has introduced the modification of the *negative binomial states* of the form:

$$|\eta, -(M+1)\rangle \sim \sum_{n=M}^{\infty} \left[\frac{n!}{(n-M)!} \eta^{M+1} (1-\eta)^{n-M} \right]^{1/2} |n\rangle.$$

The binomial coherent states

$$\begin{aligned} |\lambda; M\rangle &\sim (\hat{a}^\dagger + \lambda)^M |0\rangle \sim \sum_{n=0}^M \frac{\lambda^{M-n} |n\rangle}{(M-n)! \sqrt{n!}} \\ &\sim \hat{D}(-\lambda) \hat{a}^{\dagger M} \hat{D}(\lambda) |0\rangle \end{aligned} \quad (69)$$

(here λ is real and $\hat{D}(\lambda)$ is the displacement operator) were studied in [327, 328]. State (69) is the eigenstate of operator $\hat{a}^\dagger \hat{a} + \lambda \hat{a}$ with the eigenvalue M . The superposition states $|\lambda; M\rangle + \exp(i\varphi) |-\lambda; M\rangle$ were studied in [329]. Replacing operator \hat{a}^\dagger in the right-hand side of (69) by the spin raising or lowering operators \hat{S}_\pm one arrives at the *binomial spin-coherent state* [328]. For other studies on binomial, negative binomial states and their generalizations see, e.g., [206, 330].

8.5. Kerr states and 'macroscopic superpositions'

The main suppliers of nonclassical states are the media with *nonlinear* optical characteristics. One of the simplest examples is the so-called *Kerr nonlinearity*, which can be modelled in the single-mode case by means of the Hamiltonian $\hat{H} = \omega \hat{n} + \chi \hat{n}^2$ (where $\hat{n} = \hat{a}^\dagger \hat{a}$). In 1984, Tanaš [331] demonstrated a possibility of obtaining squeezing using this kind of nonlinearity. In 1986, considering the time evolution of the initial coherent state under the action of the 'Kerr Hamiltonian', Kitagawa and Yamamoto [250] showed that the states whose initial shapes in the complex phase plane α were circles (these shapes are determined by the equation $Q(\alpha) = \text{const}$, where the *Q-function* is defined as $Q(\alpha) = \langle \alpha | \hat{\rho} | \alpha \rangle$), are transformed to some 'crescent states', with essentially reduced fluctuations of the number of photons: $\Delta n \sim \langle n \rangle^{1/6}$ (whereas in the case of the squeezed states one has $\Delta n \geq \langle n \rangle^{1/3}$).

The behaviour of the *Q-function* was also studied by Milburn [332], who (besides confirming the squeezing effect)

discovered that under certain conditions, the initial single Gaussian function is split into several well-separated Gaussian peaks. Yurke and Stoler [333] gave the analytical treatment to this problem, generalizing the nonlinear term in the Hamiltonian as \hat{n}^k (k being an integer). The initial coherent state is transformed under the action of the Kerr Hamiltonian to the Titulaer–Glauber state (32)

$$|\alpha; t\rangle = \exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \exp(-i\chi n^k t) |n\rangle, \quad (70)$$

where $\alpha_t = \alpha \exp(-i\omega t)$. Yurke and Stoler noticed that for the special value of time $t_* = \pi/2\chi$, state (70) becomes the superposition of two or four coherent states, depending on the parity of the exponent k :

$$|\alpha; t_*\rangle = \begin{cases} [e^{-i\pi/4} |\alpha_t\rangle + e^{i\pi/4} |-\alpha_t\rangle] / \sqrt{2}, & k \text{ even} \\ [|\alpha_t\rangle - |i\alpha_t\rangle + |-\alpha_t\rangle + |-i\alpha_t\rangle] / 2, & k \text{ odd.} \end{cases} \quad (71)$$

Notice that the first superposition (for k even) is different from the even or odd states (36). The even and odd states arise in the case of the two-mode nonlinear interaction $\hat{H} = \omega(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}) + \chi(\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a})^2$ considered by Mecozi and Tombesi [334]. In this case, the initial state $|0\rangle_a |\beta\rangle_b$ is transformed at the moment $t_* = \pi/4\chi$ to the superposition:

$$\begin{aligned} |\text{out}, t_*\rangle &= \frac{1}{2} e^{-i\pi/4} (|\beta_t\rangle_b - |-\beta_t\rangle_b) |0\rangle_a \\ &\quad + \frac{1}{2} |0\rangle_b (|-\beta_t\rangle_a + |i\beta_t\rangle_a). \end{aligned}$$

In the course of time, such 'macroscopic superpositions' of quantum states attracted great attention, being considered as simple models of the 'Schrödinger cat states' [335]. In particular, they were studied in detail in [336]. Other superpositions of quantum states of the electromagnetic field, which can be created in a cavity due to the interaction with a beam of two-level atoms passing one after another, were considered in [337]. Some of them, described in terms of specific solutions of the Jaynes–Cummings model [338], were named *tangent and cotangent states* in [339]. The *squeezed Kerr states* were analysed in [340].

Searching for the states giving maximal squeezing, various superpositions of states were considered. In particular, the superpositions of the Fock states $|0\rangle$ and $|1\rangle$ (*Bernoulli states*) were studied in [315, 341], similar superpositions (plus state $|2\rangle$) were studied in [342]. The superposition of two squeezed vacuum states was considered in [343]. The superpositions of the coherent states $|\alpha e^{i\varphi}\rangle$ and $|\alpha e^{-i\varphi}\rangle$ were considered in [344]. *Entangled coherent states* have been introduced by Sanders in [345]. Their generalizations and physical applications have been studied in [346]. Various superpositions of coherent, squeezed, Fock, and other states, as well as methods of their generation, were considered in [347].

Representations of nonclassical states (including quadrature squeezed and amplitude squeezed) via linear integrals over some curve \mathcal{C} (closed or open, infinite or finite) in the complex plane of parameters,

$$|g\rangle = \int_{\mathcal{C}} g(z) |z\rangle dz, \quad (72)$$

were studied in [348]. Originally, $|z\rangle$ was the coherent state, but it is clear that one may choose other families of states, as well.

8.6. Photon-added states

An interesting class of nonclassical states consists of the *photon-added states*

$$|\psi, m\rangle_{\text{add}} = \mathcal{N}_m \hat{a}^{\dagger m} |\psi\rangle, \quad (73)$$

where $|\psi\rangle$ may be an arbitrary quantum state, m is a positive integer—the number of added quanta (photons or phonons), and \mathcal{N}_m is a normalization constant (which depends on the basic state $|\psi\rangle$). Agarwal and Tara [349] introduced these states for the first time as the *photon-added coherent states* (PACS) $|\alpha, m\rangle$, identifying $|\psi\rangle$ with Glauber's coherent state $|\alpha\rangle$ (5). These states differ from the displaced number state $|n, \alpha\rangle$ (31). Taking the initial state $|\psi\rangle$ in the form of a squeezed state, one obtains *photon-added squeezed states* [350]. The *even/odd photon-added states* were studied in [351]. One can easily generalize the definition (73) to the case of mixed quantum states described in terms of the statistical operator $\hat{\rho}$: the statistical operator of the mixed photon-added state has the form (up to the normalization factor) $\hat{\rho}_m = \hat{a}^{\dagger m} \hat{\rho} \hat{a}^m$. A concrete example is the *photon-added thermal state* [352]. A distinguishing feature of all photon-added states is that the probability of detecting n quanta (photons) in these states equals exactly zero for $n < m$. These states arise in a natural way in the processes of the field–atom interaction in a cavity [349]. Replacing the creation operator \hat{a}^{\dagger} in the definition (73) by the annihilation operator \hat{a} one arrives at the *photon-subtracted state* $|\psi, m\rangle_{\text{subt}} = \mathcal{N}_m \hat{a}^m |\psi\rangle$. The methods of creating photon-added/subtracted states via conditional measurements on a beamsplitter were discussed in [353, 354].

Photon-added states are closely related to the *boson inverse operator* (40), whose properties were studied in [355]. It was shown in [356] that PACS $|\alpha, m\rangle_{\text{add}}$ is an eigenstate of the operator $\hat{a} - m\hat{a}^{\dagger-1}$ with eigenvalue α . The *photon-depleted coherent states* $|\alpha, m\rangle_{\text{depl}} = \mathcal{N}_m \hat{a}^{\dagger-m} |\alpha\rangle$ have been introduced in [356]. For the most recent studies on photon-added states see [357].

8.7. Multiphoton and ‘circular’ coherent and squeezed states

Multiphoton states, defined as eigenstates of operator \hat{a}^k , were studied in [358, 359]. The case of $k = 3$ was considered in [360]. *Four-photon states* ($k = 4$) [359, 361] can be represented as superpositions of two even/odd coherent states (36) $|\alpha\rangle_{\pm}$ and $|\alpha\rangle_{\pm}$ whose labels are rotated by 90° in the parameter complex plane. For this reason, the state $|\alpha\rangle_+ + |\alpha\rangle_+$ was called the *orthogonal-even state* in [362]. General schemes of constructing multiphoton coherent and squeezed states of arbitrary orders were given in [363].

There exist k orthogonal multiphoton states of the form

$$|\alpha; k, j\rangle = \mathcal{N} \sum_{n=0}^{\infty} \frac{\alpha^{nk+j}}{\sqrt{(nk+j)!}} |nk+j\rangle, \quad j = 0, 1, \dots, k-1, \quad (74)$$

which can also be expressed as the superpositions of k coherent states uniformly distributed along the circle:

$$|\alpha; k, j\rangle = \mathcal{N} \sum_{n=0}^{k-1} \exp(-2\pi i n j / k) |\alpha \exp(2\pi i n / k)\rangle. \quad (75)$$

Such orthogonal ‘circular’ states were studied in [364]. The difference between the Bialynicki-Birula states (33) and states (75) is in the ‘weights’ of each coherent state in the superposition. In the case of (33) these weights are different, to ensure the *Poissonian* photon statistics, whereas in the ‘circular’ case all the coefficients have the same absolute value, resulting in non-Poissonian statistics. Eigenstates of the operator $(\mu\hat{a} + \nu\hat{a}^{\dagger})^k$ were considered in [365] (with the emphasis on the case $k = 2$). Eigenstates of annihilation operators of different modes, $\hat{a}^k \hat{b}^l \dots \hat{c}^m |\eta\rangle = \eta |\eta\rangle$, have been found in [366] in the form of finite superpositions of products of coherent states. For example, in the two-mode case one has (M is an integer):

$$|\eta\rangle = \mathcal{N} \sum_{n=0}^{Mk-1} c_j \left| \alpha \exp \left[2\pi i \frac{n(M+1)}{kM} \right] \right\rangle \otimes \left| \beta \exp \left(-2\pi i \frac{n}{lM} \right) \right\rangle, \quad \eta = \alpha^k \beta^l.$$

‘Crystallized’ Schrödinger cat states have been introduced in [367]. For the most recent studies on the ‘multiphoton’ or ‘circular’ states and their generalizations see [149, 368].

8.8. ‘Intermediate’ and ‘polynomial’ states

After the papers by Pegg and Barnett [369], where the finite-dimensional ‘truncated’ Hilbert space was used to define the phase operator, various ‘truncated’ versions of nonclassical states have been studied. Properties of the state $\sum_{n=0}^M (1+n)^{-1} |n\rangle$ were discussed in [204, 370]. *Quasiphoton phase states* $\sum_{n=0}^s \exp(in\vartheta) |n\rangle_g$, where $|n\rangle_g$ is the squeezed number state, were considered in [371]. The ‘finite-dimensional’ and ‘discrete’ coherent states were considered in [372]. The *generalized geometric state* $|y; M\rangle = \sum_{n=0}^M y^{n/2} |n\rangle$ was introduced in [373], and its *even* variant in [374]. In the limit $M \rightarrow \infty$ this state goes to the *phase coherent states* (43) (called *geometric states* in [373, 374], due to the geometric progression form of the coefficients in the Fock basis). For other examples see [375].

The *Laguerre polynomial state* $L_M(-y\hat{R}^{\dagger})|0\rangle$, where $\hat{R}^{\dagger} = \hat{a}^{\dagger} \sqrt{\hat{N} + 1}$, was introduced in [376]. A more general definition $L_M(\xi \hat{J}_+) |k, 0\rangle$ (where \hat{J}_+ is one of the generators of the $su(1, 1)$ algebra, and $|k, n\rangle$ is the discrete basis state labelled by the representation index k) has been given in [377]. The properties of these states were studied in [378].

Jacobi polynomial states were introduced in [354]. Several modifications of the binomial distributions have been used to define *Pólya states* [379], *hypergeometric states* [380], and so on. For example, the states introduced in [381]

$$|N, \alpha, \beta\rangle \sim \sum_{n=0}^N \left[\frac{(\alpha+1)_n (\beta+1)_{N-n}}{n!(N-n)!} \right]^{1/2} |n\rangle$$

are related to the *Hahn polynomials*. They contain, as special or limit cases, usual binomial and negative binomial states.

9. ‘Quantum deformations’ and related states

9.1. Para-coherent states

In 1951, Wigner [196] pointed out that to obtain an equidistant spectrum of the harmonic oscillator, one could use, instead of

the canonical bosonic commutation relation (3), more weak conditions:

$$\begin{aligned} [\hat{a}_\epsilon, \hat{H}] &= \hat{a}_\epsilon, & [\hat{a}_\epsilon^\dagger, \hat{H}] &= -\hat{a}_\epsilon^\dagger, \\ \hat{H} &= (\hat{a}_\epsilon^\dagger \hat{a}_\epsilon + \hat{a}_\epsilon \hat{a}_\epsilon^\dagger) / 2. \end{aligned} \quad (76)$$

Then the spectrum of the oscillator becomes (in the dimensionless units) $E_n = n + \epsilon$, $n = 0, 1, \dots$, with an arbitrary possible lowest energy ϵ (for the usual oscillator $\epsilon = 1/2$). Wigner's observation is closely related to the problem of *parastatistics* (intermediate between the Bose and Fermi ones) [382], which was studied by many authors in the 1950s and 1960s; see [383, 384] and references therein. In 1978, Sharma *et al* [385] introduced *para-Bose coherent states* as the eigenstates of the operator \hat{a}_ϵ satisfying the relations (76)

$$\begin{aligned} |\alpha\rangle_\epsilon &\sim \sum_{n=0}^{\infty} \left\{ \Gamma\left(\left[\left[\frac{n}{2}\right]\right] + 1\right) \Gamma\left(\left[\left[\frac{n+1}{2}\right]\right] + \epsilon\right) \right\}^{-1/2} \\ &\times \left(\frac{\alpha}{\sqrt{2}}\right)^n |n\rangle_\epsilon, \end{aligned} \quad (77)$$

where $|n\rangle_\epsilon \sim \hat{a}_\epsilon^{\dagger n} |0\rangle_\epsilon$, $|0\rangle_\epsilon$ is the ground state, and $[u]$ means the greatest integer less than or equal to u . Introducing the Hermitian 'quadrature' operators in the same manner as in (19) (but with a different physical meaning), one can check that $\Delta x_\epsilon \Delta p_\epsilon = |\langle [\hat{x}_\epsilon, \hat{p}_\epsilon] \rangle|/2$ in the state (77), i.e., this state is 'intelligent' for the operators \hat{x}_ϵ and \hat{p}_ϵ [386].

Another kind of 'para-Bose operator' was considered in [387] on the basis of a nonlinear transformation of the canonical bosonic operators:

$$\hat{A} = F^*(\hat{n} + 1)\hat{a}, \quad \hat{A}^\dagger = \hat{a}^\dagger F(\hat{n} + 1), \quad \hat{n} = \hat{a}^\dagger \hat{a}. \quad (78)$$

The transformed operators satisfy the relations:

$$[\hat{A}, \hat{n}] = \hat{A}, \quad [\hat{A}^\dagger, \hat{n}] = -\hat{A}^\dagger, \quad \hat{n} = \hat{a}^\dagger \hat{a}. \quad (79)$$

Defining the 'para-coherent state' $|\lambda\rangle$ as an eigenstate of the operator \hat{A} , $\hat{A}|\lambda\rangle = \lambda|\lambda\rangle$, one obtains:

$$|\lambda\rangle = \mathcal{N} \left(|0\rangle + \sum_{n=1}^{\infty} \frac{\lambda^n |n\rangle}{\sqrt{n!} F^*(1) \cdots F^*(n)} \right). \quad (80)$$

Evidently, the choice $F(n) = \sqrt{n+2k-1}$ reduces the state (80) to the Barut-Girardello state (54). Nowadays the states (80) are known mainly under the name *nonlinear coherent states* (NLCS): see section 9.4. Various 'para-states' were considered in [388–390]. In particular, the generalized 'para-commutator relations'

$$\begin{aligned} [\hat{n}, \hat{a}^\dagger] &= \hat{a}^\dagger, & [\hat{n}, \hat{a}] &= -\hat{a}, \\ \hat{a}^\dagger \hat{a} &= \phi(\hat{n}), & \hat{a} \hat{a}^\dagger &= \phi(\hat{n} + 1) \end{aligned}$$

have been resolved in [389] by means of the nonlinear transformation to the usual bosonic operators \hat{b}, \hat{b}^\dagger

$$\hat{a} = \sqrt{\frac{\phi(\hat{n}+1)}{\hat{n}+1}} \hat{b}, \quad \hat{A} = \frac{\hat{n}+1}{\phi(\hat{n}+1)} \hat{a}, \quad [\hat{A}, \hat{a}^\dagger] = 1, \quad (81)$$

and coherent states were defined as $|\alpha\rangle \sim \exp(\alpha \hat{A}^\dagger) |0\rangle$.

9.2. *q*-coherent states

One of the most popular directions in mathematical physics of the last decades of the 20th century was related to various *deformations* of the canonical commutation relations (3) (and others). Perhaps, the first study was performed in 1951 by Iwata [391], who found eigenstates of the operator $\hat{a}_q^\dagger \hat{a}_q$, assuming that \hat{a}_q and \hat{a}_q^\dagger satisfy the relation (he used letter ρ instead of q):

$$\hat{a}_q \hat{a}_q^\dagger - q \hat{a}_q^\dagger \hat{a}_q = 1, \quad q = \text{const.} \quad (82)$$

Twenty five years later, the same relation (82) and its generalizations to the case of several dimensions were considered by Arik and Coon [392, 393], Kuryshkin [394], and by Jannussis *et al* [395]. A realization of the commutation relation (82) in terms of the usual bosonic operators \hat{a}, \hat{a}^\dagger by means of the *nonlinear* transformation was found in [395]:

$$\hat{a}_q = F(\hat{n})\hat{a}, \quad \hat{n} = \hat{a}^\dagger \hat{a}. \quad (83)$$

For *real* functions $F(n)$ equation (82) is equivalent to the recurrence relation $(n+1)[F(n)]^2 - qn[F(n-1)]^2 = 1$, whose solution is

$$F(n) = \{[n+1]_q / (n+1)\}^{1/2}, \quad (84)$$

where symbol $[n]_q$ means:

$$[n]_q = (q^n - 1) / (q - 1) \equiv 1 + q + \cdots + q^{n-1}. \quad (85)$$

The operators given by (83) obey the relations (79). Using the transformation (83) one can obtain the realizations of more general relations than (82):

$$\hat{A} \hat{A}^\dagger = 1 + \sum_{k=1}^K q_k \hat{A}^{\dagger k} \hat{A}^k.$$

The corresponding recurrence relations for $F(n)$ were given in [395].

Coherent states of the *pseudo-oscillator*, defined by the 'inverse' commutation relation $[\hat{a}, \hat{a}^\dagger] = -1$, were studied in [396]. These states satisfy relations (5) and (4), but in the right-hand side of (5) one should write $-\alpha$ instead of α .

The *q*-coherent state was introduced in [393, 395] as

$$|\alpha\rangle_q = \exp_q(-|\alpha|^2/2) \exp_q(\alpha \hat{a}_q^\dagger) |0\rangle_q, \quad (86)$$

where:

$$\exp_q(x) \equiv \sum_{n=0}^{\infty} \frac{x^n}{[n]_q!}, \quad [n]_q! \equiv [n]_q [n-1]_q \cdots [1]_q.$$

Assuming other definitions of the symbol $[n]_q$, but the same equations (83) and (84), one can construct other 'deformations' of the canonical commutation relations. For example, making the choice

$$[x]_q \equiv (q^x - q^{-x}) / (q - q^{-1}) \quad (87)$$

one arrives at the relation

$$\hat{a}_q \hat{a}_q^\dagger - q \hat{a}_q^\dagger \hat{a}_q = q^{-\hat{n}} \quad (88)$$

considered by Biedenharn in 1989 in his famous paper [397], which gave rise (together with several other publications [398])

to the ‘boom’ in the field of ‘ q -deformed’ states in quantum optics.

Squeezing properties of q -coherent states and different types of q -squeezed states were studied in [399, 400]. For example, in [399] the q -squeezed state was defined as a solution to equation $(\hat{a}_q - \alpha \hat{a}_q^\dagger) |\alpha\rangle_{q-sq} = 0$ (cf (16) and (17)). It has the following structure:

$$|\alpha\rangle_{q-sq} = \mathcal{N} \sum_{n=0}^{\infty} \alpha^n \left(\frac{[2n-1]_q!!}{[2n]_q!!} \right)^{1/2} |2n\rangle. \quad (89)$$

Two families of coherent states for the *difference analogue* of the harmonic oscillator have been studied in [401], and one of them coincided with the q -coherent states. The ‘quasicoherent’ state $\exp_q(z\hat{a}_q^\dagger) \exp_q(z^*\hat{a}_q^\dagger)|0\rangle$ was considered in [402]. Coherent states of the two-parameter quantum algebra $su_{pq}(2)$ have been introduced in [403], on the basis of the definition:

$$[x]_{pq} = (q^x - p^{-x}) / (q - p^{-1}).$$

Analogous construction, characterized by the deformations of the form

$$\begin{aligned} \hat{a}\hat{a}^\dagger &= q_1^2 \hat{a}^\dagger \hat{a} + q_2^{2\hat{n}} = q_2^2 \hat{a}^\dagger \hat{a} + q_1^{2\hat{n}}, \\ [n] &= (q_1^{2n} - q_2^{2n}) / (q_1^2 - q_2^2), \end{aligned}$$

has been studied in [404] under the name ‘*Fibonacci oscillator*.’ Even and odd q -coherent states have been studied in [405]. The q -binomial states were considered in [406]. Multimode q -coherent states were studied in [407]. Various families of ‘ q -states’, including q -analogues of the ‘standard sets’ of nonclassical states, have been studied in [186, 408]. Interrelations between ‘para’- and ‘ q -coherent’ states were elucidated in [389, 409].

9.3. Generalized k -photon and fractional photon states

The usual coherent states are generated from the vacuum by the displacement operator $\hat{U}_1 = \exp(z\hat{a}^\dagger - z^*\hat{a})$, whereas the squeezed states are generated from the vacuum by the squeeze operator $\hat{U}_2 = \exp(z\hat{a}^{\dagger 2} - z^*\hat{a}^2)$. It seems natural to suppose that one could define a much more general class of states, acting on the vacuum by the operator $\hat{U}_k = \exp(z\hat{a}^{\dagger k} - z^*\hat{a}^k)$. However, such a simple definition leads to certain troubles [410], for instance, the vacuum expectation value $\langle 0|\hat{U}_k|0\rangle$ has zero radius of convergence as a power series with respect to z , for any $k > 2$. Although this phenomenon was considered as a mathematical artefact in [411], many people preferred to modify the operators $\hat{a}^{\dagger k}$ and \hat{a}^k in the argument of the exponential in such a way that no questions on the convergence would arise.

One possibility was studied in a series of papers [412]. Instead of a simple power $\hat{a}^{\dagger k}$ in \hat{U}_k , it was proposed to use the k -photon generalized boson operator (introduced in [384])

$$\hat{A}_{(k)}^\dagger = \left(\left[\left[\frac{\hat{n}}{k} \right] \right] \frac{(\hat{n}-k)!}{\hat{n}!} \right)^{1/2} (\hat{a}^\dagger)^k \quad \hat{n} = \hat{a}^\dagger \hat{a}, \quad (90)$$

which satisfies the relations (note that $\hat{A}_{(1)} = \hat{a}$, but $\hat{A}_{(k)} \neq \hat{a}^k$ for $k \geq 2$):

$$\left[\hat{A}_{(k)}, \hat{A}_{(k)}^\dagger \right] = 1, \quad \left[\hat{n}, \hat{A}_{(k)} \right] = -k\hat{A}_{(k)}. \quad (91)$$

The related concept of coherent and squeezed *fractional photon states* was used in [413].

Bužek and Jex [414] used Hermitian superpositions of the operators $\hat{A}_{(k)}$ and $\hat{A}_{(k)}^\dagger$ in order to define the k th-order squeezing in the frameworks of the Walls–Zoller scheme (30). The multiphoton squeezing operator can be defined as $\hat{S}_{(k)} = \exp[z\hat{A}_{(k)}^\dagger - z^*\hat{A}_{(k)}]$. As a matter of fact, we have an infinite series of the products of the operators $(\hat{a}^\dagger)^l \hat{a}^{l+k}$ and their Hermitically conjugated partners in the argument of the exponential function. The generic multiphoton squeezed state can be written as $|\alpha, z\rangle_{(k)} = \hat{D}(\alpha)\hat{S}_{(k)}(z)|\psi\rangle$ (in the cited papers the initial state $|\psi\rangle$ was assumed to be the vacuum state). An alternative definition (in the simplest case of $\alpha = \psi = 0$)

$$|z; k\rangle = \exp(-|z|^2/2) \sum_{n=0}^{\infty} \frac{z^n}{n!} \hat{A}_{(k)}^{\dagger n} |0\rangle \quad (92)$$

results in the superposition of the states with $0, k, 2k, \dots$ photons of the form:

$$|z; k\rangle = \exp(-|z|^2/2) \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} |kn\rangle. \quad (93)$$

9.4. Nonlinear coherent states

The NLCS were defined in [415, 416] as the right-hand eigenstates of the product of the boson annihilation operator \hat{a} and some function $f(\hat{n})$ of the number operator \hat{n} :

$$f(\hat{n})\hat{a}|\alpha, f\rangle = \alpha|\alpha, f\rangle. \quad (94)$$

Actually, such states have been known for many years under other names. The first example is the *phase state* (43) or its generalization (45) (known nowadays as the *negative binomial state* or the $SU(1, 1)$ group coherent state), for which $f(n) = [(1+\kappa)/(1+\kappa+n)]^{1/2}$. The decomposition of the NLCS over the Fock basis has the form (80), consequently, NLCS coincide with ‘para-coherent states’ of [387]. Many nonclassical states turn out to be eigenstates of some ‘nonlinear’ generalizations of the annihilation operator. It has already been mentioned that the ‘Barut–Girardello states’ belong to the family (80). Another example is the *photon-added state* (73), which corresponds to the nonlinearity function $f(n) = 1 - m/(1+n)$ [417]. The physical meaning of NLCS was elucidated in [415, 416], where it was shown that such states may appear as stationary states of the centre-of-mass motion of a trapped ion [415], or may be related to some nonlinear processes (such as a hypothetical ‘frequency blue shift’ in high intensity photon beams [416]). Even and odd NLCS, introduced in [418], were studied in [419], and applications to the ion in the Paul trap were considered in [420]. Further generalizations, namely, NLCS on the circle, were given in [421] (also with applications to the trapped ions). Nonclassical properties of NLCS and their generalizations have been studied in [422, 423].

10. Concluding remarks

We see that the nomenclature of nonclassical states studied by theoreticians for seventy five years (most of them appeared in the last thirty years) is rather impressive. Now the main

problem is to create them in laboratory and to verify that the desired state was indeed obtained. Therefore to conclude this review, I present a few references to studies devoted to experimental aspects and applications in other areas of physics (different from quantum optics). This list is very incomplete, but the reader can find more extensive discussions in the papers cited later.

10.1. Generation and detection of nonclassical states

The first proposals on different schemes of generating ‘the most nonclassical’ n -photon (Fock) states appeared in [424]. The problem of creating Fock states and their arbitrary superpositions in a cavity (in particular, via the interaction between the field and atoms passing through the cavity), named *quantum state engineering* in [425], was studied, e.g., in [425, 426]. Different methods of producing ‘cat’ states were proposed in [427]. The use of *beam splitters* to create various types of nonclassical states was considered in [354, 428]. The problem of generating the states with ‘holes’ in the photon-number distribution was analysed in [429]. A possibility of creating nonclassical states in a cavity with *moving mirrors* (which can mimic a Kerr-like medium) was studied in [430] (for a detailed review of studies on the classical and quantum electrodynamics in cavities with moving boundaries, with the emphasis on the dynamical Casimir effect, see [431]). The results of the first experiments were described in [432] (nonclassical states of the electromagnetic field inside a cavity) and [433] (nonclassical motional states of trapped ions). For details and other proposals see, e.g., [6, 434, 435]. Various aspects of the problem of *detecting* quantum states and their ‘recognition’ or reconstruction were treated in [436–438].

Different schemes of the conditional generation of special states (in cavities, via continuous measurements, etc) were discussed, e.g., in [439].

Several specific kinds of quantum states became popular in the last decade. *Dark states* [440] are certain superpositions of the atomic eigenstates, whose typical common feature is the existence of some sharp ‘dips’ in the spectra of absorption, fluorescence, etc, due to the destructive quantum interference of transition amplitudes between different energy levels involved. These states are connected with the NLCS [415]. For reviews and references see, e.g., [441, 442]. *Greenberger–Horne–Zeilinger states* (or *GHZ-states*) [443], which are certain states of three or more correlated spin- $\frac{1}{2}$ particles, are popular in the studies related to the EPR-paradox, Bell inequalities, quantum teleportation, and so on. For methods of their generation and other references see, e.g., [444].

10.2. Applications of nonclassical states in different areas of physics

The squeezed and ‘cat’ states in *high energy physics* were considered in [445]. Applications of the squeezed and other nonclassical states to *cosmological problems* were studied in [446]. Squeezed and ‘cat’ states in *Josephson junctions* were considered in [447]. Squeezed states of *phonons* and other bosonic excitations (*polaritons*, *excitons*, etc) in condensed matter were studied in [448]. Spin-coherent states were used in [449]. Nonclassical states in *molecules* were studied in [450]. Nonclassical states of the *Bose–Einstein condensate* were considered in [435, 451].

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