Nonconvexities, Retirement and the Elasticity of Labor Supply by Richard Rogerson and Johanna Wallenius Online Appendix

A. Distribution of Annual Hours from CPS

Here we present a Table similar to that of Table 1 but based on cross-section data from the CPS. In particular, Table A1 presents pooled cross-section data for annual hours worked in the previous year from the CPS for the years 2002-2004. Note that annual hours is the product of weeks worked with usual weekly hours.

Table A1

Distribution of Male Annual Hours by Age, CPS

Age	Annual Hours					
	0	$(0, 250)$	$[250, 750)$	$[750, 1250)$	$[1250, 1750)$	≥ 1750
60	.26	.02	.03	.05	.05	.59
61	.29	.02	.03	.05	.06	.55
62	.36	.02	.04	.06	.05	.47
63	.43	.02	.04	.06	.05	.39
64	.48	.03	.06	.06	.04	.34
65	.54	.03	.05	.07	.04	.27
66	.59	.03	.05	.06	.04	.22
67	.63	.04	.04	.06	.05	.19
68	.65	.03	.05	.06	.04	.16
69	.69	.03	.04	.05	.04	.15
70	.75	.03	.03	.04	.03	.12

The main point is that this table displays the same pattern as found using data from the PSID. Table A2 shows the same figures for all workers.

Table A2

Distribution of Annual Hours by Age, Male and Female, CPS

Age	Annual Hours					
	0	$(0, 250)$	$[250, 750)$	$[750, 1250)$	$[1250, 1750)$	≥ 1750
60	.34	.02	.04	.06	.06	.48
61	.37	.03	.04	.06	.06	.44
62	.43	.03	.04	.07	.06	.37
63	.50	.03	.05	.07	.05	.30
64	.54	.03	.05	.06	.04	.28
65	.61	.03	.05	.06	.05	.20
66	.65	.03	.05	.07	.05	.15
67	.68	.04	.05	.05	.04	.14
68	.71	.03	.04	.06	.03	.13
69	.74	.03	.04	.05	.03	.11
70	.79	.03	.03	.04	.02	.12

Note that including females does lead to a slightly greater share of workers who work part-time. Nonetheless, this table shows the same patterns found in Table A1 continue to hold for the overall sample. In particular, the dominant dynamic in the cross-section is the movement of workers from full-time work to no work.

B. Time-Varying Preferences and Wages

Here we consider the case with time varying preferences and/or wages. We demonstrate this in the context of an age-varying utility from leisure, given by $\alpha(t)$. The argument for time varying wages is analogous. Consistent with our desire to focus on retirement, i.e., that the period of not working in the market occurs at the end of life, we assume that the $\alpha(t)$ profile is increasing.[1] It is no longer the case that hours of work when working are constant, so we will now have an hours of work profile $h(t)$. The maximization problem is now:

$$\max_{e,h(t)} u\left(\int_0^e w(h(t)-\bar{h})dt + Y\right) + \int_0^e \alpha(t)v(1-h(t))dt + \int_e^1 \alpha(t)v(1)dt \tag{B1}$$

Assuming an interior solution the first order condition for e is:

$$u'\left(\int_0^e w(h(t)-\bar{h})dt + Y\right)w(h(e)-\bar{h}) = \alpha(e)v(1) - \alpha(e)v(1-h(e)) \tag{B2}$$

Of particular interest is the first order condition for the optimal level of hours at the time of retirement, $h(e)$. The first order condition for this value is:

$$u'\left(\int_0^e w(h(t)-\bar{h})dt + Y\right)w = \alpha(e)v'(1-h(e)) \tag{B3}$$

Dividing these two expressions gives:

$$h(e) - \bar{h} = \frac{v(1) - v(1-h(e))}{v'(1-h(e))} \tag{B4}$$

It follows that our previous calculations all go through exactly, as long as we

[1] In fact, our analysis would go through unchanged if we instead assumed that this profile were u-shaped, thereby potentially generating a period of nonwork at the beginning of life as well.

understand that the level of hours that we use in the calculation refers to the level of hours worked at the time of retirement. But with this one proviso, the calculation is entirely unchanged.[2]

We can also extend this analysis to handle the case of a single discontinuity in the α profile. If retirement occurs at the point of the discontinuity then what matters is not what the hours were just prior to retirement, but rather what the hours worked would have been at e had the individual not retired. The first order condition for optimal hours tells us that this value must satisfy

$$\alpha(e)v'(1-h(e)) = \lim_{t \to e} \alpha(t)v'(1-h(t)) \tag{B5}$$

The discontinuity in the α profile can reduce the needed nonconvexity through lowering the appropriate h to feed into the calculations. Basically, the implied level of hours is the value that equates marginal disutility of work at the margin with those periods in which the individual chose to work. However, as we know from our earlier calculations, for relatively small elasticities the effect of even moderate discontinuities on h is small, and moreover the effect of a small change in h on the required nonconvexity is also small. We conclude that the previous calculations are not much affected by allowing for time changing α or w, unless we allow for very large discontinuities.

C. Non-separable Preferences

In this appendix we derive an expression linking h, θ and $\bar{h}$ assuming the form

[2] Rogerson and Wallenius (2009) assume that productivity varies with age, but do not target working time at retirement in their calibration, so this result does not apply to their calculations.

of nonseparable preferences in the text, allowing as well for a fixed time cost of going to work, which we denote by $\bar{h}$. The optimization problem for an individual can be written as:

$$\max_{e,h,c_w,c_r} \; eu(c_w, h) + (1-e)u(c_r, 0)$$

subject to the lifetime budget constraint:

$$ec_w + (1-e)c_r = e(h - \bar{h})^{1+\theta}$$

where c_w is consumption while working, c_r is consumption while retired, h is time devoted to production, e is the fraction of life spent in employment and $\bar{h}$ is the fixed time cost associated with working. As noted in the text, the utility function is given by:

$$u(c,h) = \frac{1}{1-\eta} c^{1-\eta}[1 - \kappa(1-\eta)h^{1+\frac{1}{\phi}}]^{\eta}.$$

Letting λ denote the Lagrange multiplier on the budget equation and assuming interior solutions for all four choice variables we have the following first order conditions:

$$c_w : c_w^{-\eta}[1 - \kappa(1-\eta)h^{1+\frac{1}{\phi}}]^{\eta} = \lambda \tag{C1}$$

$$c_r : \; c_r^{-\eta} = \lambda \tag{C2}$$

$$e : \; \frac{1}{1-\eta}\{c_w^{1-\eta}[1 - \kappa(1-\eta)h^{1+\frac{1}{\phi}}]^{\eta} - c_r^{1-\eta}\} = \lambda[c_w - c_r - (h - \bar{h})^{1+\theta}] \tag{C3}$$

$$h : c_w^{1-\eta}\eta[1 - \kappa(1-\eta)h^{1+\frac{1}{\phi}}]^{\eta-1}\kappa \; (1 + \frac{1}{\phi})h^{\frac{1}{\phi}} = \lambda(1+\theta)(h - \bar{h})^{\theta} \tag{C4}$$

In characterizing the solution to this set of equations it is useful to define:

$$A = [1 - \kappa(1-\eta)h^{1+\frac{1}{\phi}}] \tag{C5}$$

Dividing equation (C1) by equation (C2) one obtains:

$$c_w = Ac_r \tag{C6}$$

Using this expression in the lifetime budget equation gives:

$$c_r = \frac{e(h-\bar{h})^{1+\theta}}{Ae+(1-e)} \tag{C7}$$

Using equation (C7) in equation (C2) yields:

$$\lambda = [\frac{e(h-\bar{h})^{1+\theta}}{Ae+(1-e)}]^{-\eta} \tag{C8}$$

Substituting equations (C6), (C7) and (C8) into (C3) and simplifying gives:

$$\frac{e}{Ae+(1-e)} = \frac{1-\eta}{\eta}\frac{1}{1-A} \tag{C9}$$

Substituting equations C67), (C7) and (C8) into equation (C4) one obtains:

$$\frac{e(h-\bar{h})}{Ae+(1-e)}(1-A) = \frac{1+\theta}{1+\frac{1}{\phi}}\frac{1-\eta}{\eta}h \tag{C10}$$

Substituting for $\frac{e}{Ae+(1-e)}$ from (C9) into (C10) yields:

$$\frac{h-\bar{h}}{h} = \frac{1+\theta}{1+\frac{1}{\phi}}$$

Similar to our previous analysis based on the separable specification, neither the exponent on consumption nor the length of working life influences the extent of nonconvexity that is required to generate retirement. Using our previous benchmark value of $h = .385$, and assuming that $\bar{h} = .1h$, Table C1 shows the required value of θ for various values of ϕ, which is equal to the IES.

Table C1

Value of θ Required for Retirement With Nonseparable Preferences, $\bar{h} = .1h$

IES=2.0	IES=1.0	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.35	.80	1.1	1.7	3.5	8.9	17.9

Relative to our earlier results, we see that this specification requires even larger nonconvexities in order to generate retirement. The reason for this is that the preference specification in equation (**??**) implies that marginal disutility of work is zero when work is zero. Relative to our earlier specification in which we had a non-zero marginal utility of leisure at zero work, this makes it even more difficult to generate retirement.

D. Derivations For Section 5

Here we provide more details for the exercise reported in the text. Solving for e^* from equation (**??**) and substituting into the objective function gives the following expression for the maximum utility U^*.:

$$U^* = \log(\frac{(1-\frac{1}{\gamma})h_f}{\alpha[1-(1-h_f)^{1-\frac{1}{\gamma}}]}) + \frac{\alpha}{1-\frac{1}{\gamma}}(1+\frac{Y}{h_f}[1-(1-h_f)^{1-\frac{1}{\gamma}}]) - 1.$$

where without loss of generality we have normalized $w_f = 1$.

Letting e_p^* denote the optimal fraction of time to spend in part-time employment when this option is included (and hence $(1-e_p^*)$ is the fraction of time spent not working), the optimal value of e_p^* is:

$$e_p^* = \frac{1-\frac{1}{\gamma}}{\alpha[1-(1-h_p)^{1-\frac{1}{\gamma}}]} - \frac{Y}{(\frac{h_p}{h_f})^{\theta}h_p}$$

Substituting into the objective function gives a maximum utility, denoted by U_p^*, equal to:

$$U_p^* = \log(\frac{(1-\frac{1}{\gamma})h_p(\frac{h_p}{h_f})^{\theta}}{\alpha[1-(1-h_p)^{1-\frac{1}{\gamma}}]}) + \frac{\alpha}{1-\frac{1}{\gamma}}(1+\frac{Y}{(\frac{h_p}{h_f})^{\theta}h_p}[1-(1-h_p)^{1-\frac{1}{\gamma}}]) - 1$$

Solving for the value of θ that equates U^* and $\hat{U}$ gives:

$$\log((\frac{h_f}{h_p})^{1+\theta}\frac{[1-(1-h_p)^{1-\frac{1}{\gamma}}]}{[1-(1-h_f)^{1-\frac{1}{\gamma}}]}) = \frac{\alpha}{1-\frac{1}{\gamma}}Y[\frac{1-(1-h_p)^{1-\frac{1}{\gamma}}}{(\frac{h_p}{h_f})^{\theta}h_p} - \frac{1-(1-h_f)^{1-\frac{1}{\gamma}}}{h_f}].$$

The unique value of θ that solves this equation is:

$$\theta = [\log(\frac{[1-(1-h_p)^{1-\frac{1}{\gamma}}]}{[1-(1-h_f)^{1-\frac{1}{\gamma}}]})/\log(h_p/h_f)] - 1.$$