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José M. Jiménez-Gómez**

Abstract

In this paper we use two different sets of Commonly Accepted Equity Principles to provide new characterizations of well known bankruptcy rules from an strategic viewpoint. In this sense, we extend the results obtained by Chun, 1989, and Herrero, 2003, who followed the van Damme's approach, 1986, for solving Nash type bargaining problems, but in bankruptcy problems. Specifially, by using, on the one hand, the Unanimous Concessions mechanism we justify Piniles' and the Constrained Egalitarian rules, and, on the other hand, with the Diminishing Claims procedure we retrieve the Dual of Pinile's and the Dual of Constrained Egalitarian rules.

Keywords: Bankruptcy problems, strategic approach, noncooperative justifications, Piniles' rule, Constrained Egalitarian rule.

JEL Classification: C71, D63, D71.

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1. Introduction.

How should the funds of a bank in bankruptcy be distributed among its investors? This kind of situation, where the resource are not sufficient to satisfy the agents' aggregate demand, are called bankruptcy problems. According to this, a bankruptcy rule prescribes, for each bankruptcy problem, how to share out the endowment among the agents, taking into account their claims.

In this context, Chun [3] and Herrero [8] follow the bargaining model introduced by van Damme [15], who prospects Nash equilibria of a non-cooperative game. Particularly, he defines a mechanism of successive concessions, where agents' strategies consist of the choice of a rule among a reasonable set of them. Applying this idea in bankruptcy problems, Chun [3] proposed the *Diminishing Claims* procedure to solve bankruptcy situations where the *Socially Admissible Bankruptcy* rules are determined by the agreement of all agents on a set of 'basic' requirements, P, called *'Commonly Accepted Equity Principles'.* Later, Herrero [8] modifies the *Unanimous Concessions* mechanism, provided by Marco et al. [11], for its application to bankruptcy problems.

Following this line, and more recently, García-Jurado et al [7] propose an elementary game where each agent's strategy belong to a determined closed space of possible choices. With this game, they show that any acceptable rule can be obtained as the unique allocation of the corresponding Nash equilibria depending on its associated closed interval of strategies. Finally, Herrero et al [9] provide an experimental strategic support to the *Proportional* rule, showing that this one is the choice of most of the players.

In this paper, we apply the previous methodology and the results obtained by Jiménez-Gómez and Marco-Gil [10] on two different sets of *'Commonly Accepted Equity Principles'.* First of all, we propose as basic properties the set P_1, composed by *Resource Monotonicity*, *Super-Modularity* and *Midpoint Property.* In this case we find out that in any Nash equilibrium of the game induced by the *Unanimous Concessions* procedure, the *Socially Admissible* rule is the *Dual of Piniles'* one, and by duality, in the game induced by the *Diminishing Claims procedure,* the *Piniles'* rule (Piniles [12]). Secondly, we propose the set P_2, replacing on P_1 *Super-Modularity* by *Order Preservation,* and show that the application of these procedures do not always provide desirable distributions.

The paper is organized as follows: Section 2 presents the model. Section 3 provides our main results. Section 4 summarizes our conclusions. Finally, the Appendices gathers technical proofs.

2. The model.

A **bankruptcy problem** is a pair $(E, c) \in \mathbb{R}_+ \times \mathbb{R}^n_+$, where E denotes the endowment and c is the vector of each agents' claim, c_i, for each $i \in N$, $N = \{1, ..., i, ..., n\}$, such that the agents' aggregate demand is higher than the endowment, $\sum_{i \in N} c_i > E$.

For notational convenience, $\mathcal{B}$ will denote the set of all bankruptcy problems; C the sum of the agents' claims, $C = \sum_{i \in N} c_i$; and L the total amount of losses to distribute among the agents, $L = C - E$.

In this context, a **rule** is a function, $\varphi : \mathcal{B} \rightarrow \mathbb{R}^n_+$, which associates for each $(E, c) \in \mathcal{B}$, a distribution of the endowment among the group of claimants, such that *(a)* $\sum_{i \in N} \varphi_i(E, c) = E$ and *(b)* $0 \leq \varphi_i(E, c) \leq c_i$.

Particularly, we focus on the following rules. The first one recommends equal awards to all claimants subject to no-one receiving more than her claim.

The **Constrained Equal Awards** rule, *CEA*, (Maimonides 12th Century, among others) recommends, for each $(E, c) \in \mathcal{B}$, the vector $(\min\{c_i, \mu\})_{i \in N}$, where μ is chosen so that $\sum_{i \in N} \min\{c_i, \mu\} = E$.

Next rule provides for each problem the awards that the *Constrained Equal Awards* rule recommends for $(E, c/2)$, when the endowment is less than the half-sum of the claims. Otherwise, each agent first receives her half-claim, then the *Constrained Equal Awards* rule is re-applied to the residual problem $(E - C/2, c/2)$.

Piniles' rule, *Pin*, (Piniles [12]) provides, for each $(E, c) \in \mathcal{B}$, the vector $(CEA_i(E, c/2))_{i \in N}$, if $E \leq C/2$; and $(c_i/2 + CEA_i(E - C/2, c/2))_{i \in N}$, if $E \geq C/2$.

The following rule is inspired by the *Uniform* one (Sprumont [13]), a solution to the problem of fair division when the preferences are single-peaked. It makes the minimal adjustment in the formula of the *Uniform* rule, taking the half-claims as the peaks and guaranteeing that awards are ordered in the same way as claims are.

The **Constrained Egalitarian** rule, *CE*, (Chun et al. [4]) chooses, for each $(E, c) \in \mathcal{B}$, the vector $(CEA_i(E, c/2))_{i \in N}$, if $E \leq C/2$; and $(\max\{c_i/2, \min\{c_i, \delta\}\})_{i \in N}$, if $E \geq C/2$, where δ is chosen so that $\sum_{i \in N} CE_i(E, c) = E$.

Given a rule φ, for each $(E, c) \in \mathcal{B}$ and each $i \in N$, its dual, φ^d, assigns losses in the same way as φ assigns gains (Aumann and Maschler [1]), $\varphi^d_i(E, c) = c_i - \varphi_i(L, c)$.

In this regard, we obtain that the *Constrained Equal Losses* rule, *CEL,* (Aumann and Maschler [1]), the *Dual of Piniles'* rule, *DPin*, and the *Dual of Constrained Egalitarian* rule, *DCE*, are dual of the *CEA*, the *Pin* and the *CE* rules, respectively.

On the other hand, the starting point of our results are the extended problem proposed by Jiménez-Gómez and Marco-Gil [10], called Bankruptcy Problem with Legitimate Principles. These problems are based on the fact that a society agrees in a set of basic properties or principles on which the distribution of the resource must be made in base of . That is, these are problems where all the admissible rules must satisfy the '*Commonly Accepted Equity Principles'* set. Formally,

A **Bankruptcy Problem with Legitimate Principles** is a triplet (E, c, P_t) where $(E, c) \in \mathcal{B}$ and P_t is a fixed set of principles on which a particular society has agreed.

So that, the allowed rules for this problem must satisfy the set of equity principles, P_t. That is,

An **Admissible** rule, φ, is a rule satisfying all properties in P_t.

From now on, P denotes the set of all subsets of properties of rules. Each $P_t \in P$ represents a specific society which will always apply such principles for solving its problems; $\mathcal{B}_P$ denotes the set of all *Problems with Legitimate Principles,* Φ the set of all rules and $\Phi(P_t)$ the subset of rules satisfying P_t.

Finally, we define the procedure with which we apply the previous ideas. The *Unanimous Concessions* procedure (Herrero, [8]) says that, given that agents have chosen their preferred rules, if at the initial step there is no agreement, at the second step, each agent receives her the minimal amount among all the proposed. Now, we redefine the residual bankruptcy problem, in which the endowment is the leftover resource, the claims are adjusted down by the amounts just given, and the same procedure is applied. The solution will be the limit of the procedure if it is feasible, and zero otherwise. Formally

Definition 2.1. ***Unanimous Concessions procedure****, u, (Herrero, [8]):*

Let $(E, c, P_t) \in B_P$. At the first stage, each agent chooses a rule $\varphi^i \in \Phi(P_t)$. Let $\psi = (\varphi^i)$ be the profile of rules selected. The allocation proposed by the Unanimous Concessions procedure, $u\left[\varphi, (E, c, P_t)\right]$ is obtained as follows:

[Step 1] If all agents agree on $\varphi(E, c, P_t)$, then $u\left[\varphi, (E, c, P_t)\right] = \varphi\left(E, c, P_t\right)$. Otherwise, go to next step.

[Step 2] Let us define
$s_i\left(E, c, P_t\right) = \min_{j \in N} \varphi_i^j\left(E, c, P_t\right),$
$c^2 = c - s\left(E, c, P_t\right),$ *and* $E^2 = E - \sum_{i \in N} s_i\left(E, c, P_t\right).$
Now, if all agents agree on $\varphi(E^2, c^2, P_t)$, then
$u\left[\varphi, (E, c, P_t)\right] = s\left(E, c, P_t\right) + \varphi\left(E^2, c^2, P_t\right).$ *Otherwise, go to next step.*

[Step $m+1$] Let us define
$s_i\left(E^m, c^m, P_t\right) = \min_{j \in N} \varphi_i^j\left(E^m, c^m, P_t\right),$

$c^{m+1} = c^m - s(E^m, c^m, P_t)$, *and* $E^{m+1} = E^m - \sum_{i \in N} s_i(E^m, c^m, P_t)$.

Now, if all agents agree on $\varphi(E^{m+1}, c^{m+1}, P_t)$, *then*

$u[\varphi, (E, c, P_t)] = \sum_{k=1}^{m} s(E^k, c^k, P_t) + \varphi(E^{m+1}, c^{m+1}, P_t)$. *Otherwise, go to next step.*

[Limit case] Compute $\sum_{k=1}^{\infty} s(E^k, c^k, P_t)$. *If it converges to an allocation,* x, *such that* $\sum_{i \in N} x_i \leq E$, $u[\varphi, (E, c, P_t)] = x$. *Otherwise,* $u[\varphi, (E, c, P_t)] = 0$.

From now on, let $\Gamma^u_{P_t}$ denote the game induced by the *Unanimous Concessions* procedure when agents act strategically, in which the set of players is N, the strategies for each agent are rules in $\Phi(P_t)$ and the payoffs are the sum of the amounts received by each agent in each step $m \in \mathbb{N}$. That is,

$$\Gamma^u_{P_t} = \left\{ N, \left\{ \varphi^i \in \Phi(P_t) \right\}_{i=1}^{n}, \left\{ \sum_{k=1}^{m} s_i \left(P_t^k \right) \right\}_{i=1}^{n} \right\}.$$

3. Main results.

In this section we consider two possible choices of '*Commonly Accepted Equity Principles'* by a society to apply the *Unanimous Concessions* procedure.

The first set P_1, contains *Resource Monotonicity*, *Super-Modularity* and the *Midpoint Property.*

Resource Monotonicity (Curiel et al. [5], Young [16], among others) demands that if the endowment increases, then all individuals should get at least what they received initially.

Resource Monotonicity: for each $(E, c) \in \mathcal{B}$ and each $E' \in \mathbb{R}_+$ such that $C > E' > E$, then $\varphi_i(E', c) \geq \varphi_i(E, c)$, for each $i \in N$.

A *Super-Modular* rule (Dagan et al. [6]) allocates each additional dollar in an 'order preserving' manner. In other words, when the endowment increases, agents with higher claims receive a greater part of the increment than those with lower claims.

Super-Modularity: for each $(E, c) \in \mathcal{B}$, all $E' \in \mathbb{R}_+$ and each $i, j \in N$ such that $C > E' > E$ and $c_i \geq c_j$, then $\varphi_i(E', c) - \varphi_i(E, c) \geq \varphi_j(E', c) - \varphi_j(E, c)$.

Midpoint Property (Chun, Schummer and Thomson [4]) requires that if the estate is equal to the sum of the half-claims, then all agents should receive their half-claim.

Midpoint Property: for each $(E, c) \in \mathcal{B}$ and each $i \in N$, if $E = C/2$, then $\varphi_i(E, c) = c_i/2$.

Given this equity principle set, next propositions tell us that, on the hand, if some agent announces the *DPin* rule, then the *Unanimous Concessions* procedure converge to this rule, and, on the other hand that, the *DPin* rule is a weakly dominant strategy for the agent with the highest claimant. Then, as a direct consequence of these two results, we show that in all noncooperative Nash equilibria, each agent will receive the awards recommended by the *DPin* rule

Proposition 3.1. *For each* $(E, c, P_1) \in \mathcal{B}_P$*, and each* $i \in N$*, if* $\varphi^i(E, c) \in \Phi(P_2)$ *, and for some* $j \in N$*,* $\varphi^j(E, c) = DPin(E, c)$*, then* $u[\varphi, (E, c, P_1)] = DPin(E, c)$*.*

Proof. See Appendix 2. ∎

Proposition 3.2. *In the game* $\Gamma^u_{P_1}$*, the DPin rule is a weakly dominant strategy for the agent with the highest claim.*

Proof. See Appendix 3. ∎

Theorem 3.3. *In any Nash equilibrium induced by the game* $\Gamma^u_{P_1}$*, each agent receives the amount given by the DPin rule.*

Proof. See Appendix 4. ∎

3.1. A result of non existence.

Next, we show that, in general, the Nash equilibrium induced by the application of the *Unanimous Concessions* procedure not only does not provide one of the *Admissible* rules, but also the allocation proposed by it fails one of *'Commonly Accepted Equity Principles'* on which the process is based.

With this purpose, we use the set of equity principles P_2 which contents *Resource Monotonicity*, *Midpoint Property* and *Order Preservation*.

Order Preservation (Aumann and Maschler [1]) requires respecting the ordering of the claims: if agent i's claim is at least as large as agent j's claim, he should receive and loss at least as much as agent j, does respectively.

Order Preservation: for each $(E, c) \in \mathcal{B}$, and each $i, j \in N$, such that $c_i \geq c_j$, then $\varphi_i(E, c) \geq \varphi_j(E, c)$, and $c_i - \varphi_i(E, c) \geq c_j - \varphi_j(E, c)$, that is $l_i \geq l_j$.

Note that since *Super-Modularity* implies *Order Preservation* (Thomson [14]), we obtain $P_1 \subseteq P_2$. Now, following results establish that, although for the bipersonal problems in P_2 each agent receives the awards recommended by the *DCE* rule in the Nash equilibrium induced by the *Unanimous Concessions* procedure, this conclusion cannot be generalized.

Proposition 3.4. *For each* $(E, c, P_2) \in \mathcal{B}_P$, *with* $|N| = 2$, *and each* $i \in \{1, 2\}$, *if* $\varphi^i(E, c) \in \Phi(P_2)$, *and for some* $j \in \{1, 2\}$, $\varphi^j(E, c) = DCE(E, c)$, *then* $u[\varphi, (E, c, P_2)] = DCE(E, c)$.

Proof. See Appendix 5. ■

Theorem 3.5. *For bipersonal problems, in any Nash equilibrium induced by the game* $\Gamma^u_{P_2}$, *each agent receives the amount given by the DCE rule.*

Proof. See Appendix 6. ■

Theorem 3.6. *There is a problem,* $(E, c, P_2) \in \mathcal{B}_P$, *for which if* $\varphi^i(E, c) \in \Phi(P_2)$, *and for some* $j \in N$, $\varphi^j(E, c) = DCE(E, c)$, *then* $u[\varphi, (E, c, P_2)] \neq DCE(E, c)$.

Proof. See Appendix 7. ■

Theorem 3.7. *The Nash equilibrium induced by the game* $\Gamma^u_{P_2}$ *does not fulfill Resource Monotonicity.*

Proof. See Appendix 7. ■

4. Duality.

As we have already mentioned, there are two ways of facing bankruptcy problems: gains an losses. In this line, given two properties, we say that they are dual of each other if whenever a rule satisfies one of them, its dual satisfies the other.

Two properties, $\mathcal{P}$ and $\mathcal{P}'$, are **dual** if whenever a rule, φ, satisfies $\mathcal{P}$, its dual, φ^d, satisfies $\mathcal{P}'$.

Moreover, it is worth noting here that all the proposed principles, $P_t \in \{P_1, P_2\}$, are invariant to the perspective from which the problem is thought, that is, they do not change when dividing 'what is available' or 'what is missing', so , they are *Self-Dual*. Formally,

A property, $\mathcal{P}$, is **Self-Dual** if whenever a rule satisfies the property, so does its dual.

Given this fact, and by using the dual relation among rules (Aumann and Maschler [1]), the results of the previous section can be analyzed focusing on the maximum awards that each agent can ensure, that is, the minimal losses incurred.

In this sense, the *Diminishing Claims* procedure (Chun [3]), denoted by d, says that, given that agents have chosen their preferred rules, if at the initial step there is no agreement, at the second step, we redefine the residual bankruptcy problem, in which the endowment does not change, and each agent's claim is truncated by the maximal amount among all the proposed at step 1. Then, the procedure is again applied until an agreement is reached. If this is not the case, the solution will be the limit of the procedure if is feasible, and zero otherwise. Formally,

Definition 4.1. ***Diminishing Claims procedure***, *d, (Chun, [3]):*

Let $(E, c, P_t) \in \mathcal{B}_P$. At the first stage, each agent chooses a rule $\varphi^i \in \Phi(P_t)$. Let $\psi = (\varphi^i)$ be the profile of selected rules. The division proposed by the Diminishing Claims procedure, $d[\varphi, (E, c, P_t)]$ is obtained as follows:

[Step 1] If all agents agree on $\varphi(E, c, P_t)$, then $d[\varphi, (E, c, P_t)] = \varphi(E, c, P_t)$. Otherwise, go to next step.

[Step 2] Let us define $g_i(E, c, P_t) = \max_{j \in N} \varphi_i^j(E, c, P_t)$,
$c^2 = g(E, c, P_t)$, and $E^2 = E$.
Now, if all agents agree on $\varphi(E^2, c^2, P_t)$, then
$d[\varphi, (E, c, P_t)] = \varphi(E^2, c^2, P_t)$. Otherwise, go to next step.

[Step $m+1$] Let us define $g_i(E^m, c^m, P_t) = \max_{j \in N} \varphi_i^j(E^m, c^m, P_t)$,
$c^{m+1} = g(E^m, c^m, P_t)$, and $E^{m+1} = E$.
Now, if all agents agree on $\varphi(E^{m+1}, c^{m+1}, P_t)$, then
$d[\varphi, (E, c, P_t)] = \varphi(E^{m+1}, c^{m+1}, P_t)$. Otherwise, go to next step.

[Limit case] Compute $\lim_{k \to \infty} \varphi(E^k, c^k, P_t)$. If it converges to an allocation, x, such that $\sum_{i \in N} x_i \leq E$, $d[\varphi, (E, c, P_t)] = x$. Otherwise, $d[\varphi, (E, c, P_t)] = 0$.

From now on, let $\Gamma_{P_t}^d$ denote the game induced by the *Diminishing Claims* procedure when agents act strategically, in which the set of players is N, the strategies for each agent are rules in $\Phi(P_t)$ and the payoffs are the amount recommending to each agent by the accorded rule. That is,

$$\Gamma_{P_t}^d = \left\{ N, \left\{ \varphi^i \in \Phi(P_t) \right\}_{i=1}^n, \left\{ \varphi_i \left(LB_{P_t}^m \right) \right\}_{i=1}^n \right\}.$$

It can be easily checked that the *Diminishing Claims* and the *Unanimous Concessions* procedures are dual, since the maximum amount that each agent can receive in the former mechanism can be interpreted as the minimal losses in which each agent can incur applying the latter mechanism. That is, $g_i(E^m, c^m, P_t) = c_i - s_i(L^m, c^m, P_t)$.

Therefore, by duality, we can retrieve *Piniles'* rule in P_1, and the *Constrained Egalitarian* rule for two-agents problems but not for the general case in P_2, when using the *Diminishing Claims* mechanism.

Corollary 4.2. *In any Nash equilibrium induced by the game* $\Gamma^d_{P_1}$ *each agent receives the amount given by the Pin rule.*

Corollary 4.3. *For bipersonal problems, in any Nash equilibrium induced by the game* $\Gamma^d_{P_2}$ *each agent receives the amount given by the CE rule.*

Corollary 4.4. *There is a problem* $(E, c, P_2) \in \mathcal{B}_P$*, for which if* $\varphi^i(E, c) \in \Phi(P_2)$ *and for some* $j \in N, \varphi^j(E, c) = CE(E, c)$*, then* $u[\varphi, (E, c, P_2)] \neq DCE(E, c)$.

Corollary 4.5. *The Nash equilibrium induced by the game* $\Gamma^d_{P_2}$ *does not fulfill Resource Monotonicity.*

5. Conclusions

In this paper we offer the understanding of old bankruptcy rules from a new angle. Specifically, we particularize the methodology of the *Unanimous Concessions procedure* to different sets of *'Commonly Accepted Equity Principles'* by a society.

On the one hand, we have retrieved the *DPin* rule when applying the *Unanimous Concessions* procedure with the set P_1. However, this result cannot be generalized to any equity principle set P_t, as we have shown with P_2.

By using the idea of duality and the fact all the properties proposed are *Self-Dual*, the previous results can be analyzed from the viewpoint of sharing losses, i.e., we focus on the maximum awards that each agent can ensure. In this case, we retrieve the *Pin* rule in P_1, and the *CE* rule for the two-agents problems but not for the general case in P_2, when using the *Diminishing Claims* mechanism.

Therefore, we have shown that the allocation obtained when applying the *Unanimous Concessions* and the *Diminishing Claims* procedures, may lead 'not desirable' results. Particularly, if a society agreed on choosing those rules which satisfies a determined set of equity principles, the final allocation could not satisfy the initial agreed properties. At this point, the following steps in this line should be not only the search of those properties which guarantee their fulfillment when applying both mechanisms, but also a new procedure which ensure a socially desirable distribution.

APPENDIX 1. *General Facts*

Next we present one remark, two definitions and two facts which are used in the proofs provided in the following appendices. Moreover, from now on, $m \in \mathbb{N}$ will denote the m-*th* step of the *Unanimous Concessions* procedure (see Definition 2.1), and we consider, without loss of generality, $(E, c) \in \mathcal{B}_0$, where, $\mathcal{B}_0$ denotes the set of problems in which claims are increasingly ordered, that is problems with $c_1 \leq c_2 \leq ... \leq c_n$

The remark establishes, for each $P_t \in \{P_1, P_2\}$, that the order of the agents' claims is fixed along the different steps of the procedure.

Remark 1. *(Jiménez-Gómez and Marco-Gil, 2008) For each* $(E, c, P_t) \in \mathcal{B}_P$ *with* $P_t \in \{P_1, P_2\}$, *if* $c_i^m \leq c_j^m \Rightarrow c_i^{m+1} \leq c_j^{m+1}$.

The following definitions and facts provide the *P-Safety* for P_1 and P_2.

Definition 5.1. *(Jiménez-Gómez and Marco-Gil, 2008) Given* (E, c, P_1) *in* $\mathcal{B}_P$, *the P-Safety, s, is for each* $i \in N$,

$$s_i(E, c, P_1) = \inf\{\varphi_i^*(E, c), DPin_i(E, c)\},$$

where φ^* *denotes an Admissible rule in* P_1, *such that,* $\varphi^*(E, c) \neq DPin(E, c)$.

Definition 5.2. *(Jiménez-Gómez and Marco-Gil, 2008) Given* (E, c, P_2) *in* $\mathcal{B}_p$, *the P-Safety, s, is for each* $i \in N$,

$$s_i(E, c, P_2) = \inf\{\varphi_i^*(E, c), DCE_i(E, c)\},$$

where φ^* *denotes an Admissible rule in* P_2, *such that, , such that,* $\varphi^*(E, c) \neq DCE(E, c)$.

Fact 1. *Given* (E, c, P_1) *in* $\mathcal{B}_p$ *and for each* $m \in \mathbb{N}$, $s_1(E^m, c^m, P_1) = DPin_1(E^m, c^m)$ *and* $s_n(E^m, c^m, P_1) = Pin_n(E^m, c^m)$.

Fact 2. *Given* (E, c, P_2) *in* $\mathcal{B}_p$ *and for each* $m \in \mathbb{N}$, $s_1(E^m, c^m, P_2) = DCE_1(E^m, c^m)$ *and* $s_n(E^m, c^m, P_2) = CE_n(E^m, c^m)$.

APPENDIX 2. *Proof of Proposition 3.1.*

The proof of this result is based on Remark 1, Definition 5.1, Fact 1 and three lemmas, in which φ^* denotes an *Admissible* rule in P_1, different of the *Dual Piniles'* one, $\varphi^*(E, c) \neq DPin(E, c)$.

Lemma 5.3. *(Jiménez-Gómez and Marco-Gil, 2008) For each* $(E, c) \in \mathcal{B}_0$, *if there is* $m \in \mathbb{N}$ *such that* $s_i(E^m, c^m, P_1) = DPin_i(E^m, c^m)$ *then,*

$$s_i(E^{m+h}, c^{m+h}, P_1) = 0, \text{ for each } h \in \mathbb{N}.$$

Lemma 5.4. *(Jiménez-Gómez and Marco-Gil, 2008) For each* $(E, c) \in \mathcal{B}_0$ *and each* $i \in N$, *if* $s_i(E^m, c^m, P_1) = \varphi_i^*(E^m, c^m)$ *for each* $m \in \mathbb{N}$, *then*

$$\sum_{k=1}^{\infty} s_i(E^k, c^k, P_1) \leq DPin_i(E, c).$$

Lemma 5.5. *(Jiménez-Gómez and Marco-Gil, 2008) For each* $(E, c) \in \mathcal{B}_0$ *and each* $i \in N$, *if there is* $m^* \in \mathbb{N}$, $m^* > 1$, *such that* $s_i(E^{m^*}, c^{m^*}, P_1) = DPin_i(E^{m^*}, c^{m^*})$ *and*

$$s_i(E^{m^*-1}, c^{m^*-1}, P_1) = \varphi_i^*(E^{m^*-1}, c^{m^*-1}), \text{then}$$

$$\sum_{k=1}^{m^*} s_i(E^k, c^k, P_1) = DPin_i(E, c).$$

Proof of Proposition 3.1.

[**Step** 1] If all agents agree on $\varphi(E, c, P_1) = DPin(E, c)$, then $u[\varphi, (E, c, P_1)] = DPin(E, c)$. Otherwise, go to next step.

[**Step** 2] Let $s_i(E, c, P_1) = \min_{j \in N} \varphi_i^j(E, c, P_1)$, $c^2 = c - s(E, c, P_1)$, and $E^2 = E - \sum_{i \in N} s_i(E, c, P_1)$. By Lemma 5.3, for each agent i such that $s_i(E, c, P_1) = DPin_i(E, c)$, $s_i(E^2, c^2, P_1) = 0$. If all agents agree on $\varphi(E^2, c^2, P_1)$, by Lemma 5.5, $DPin(E, c) \equiv u[\varphi, (E, c, P_1)] = s(E, c, P_1) + \varphi(E^2, c^2, P_1)$. Otherwise, go to next step.

[**Step** m+1] Let $s_i^{m+1}(E^m, c^m, P_1) = \min_{j \in N} \varphi_i^j(E^m, c^m, P_1)$, $E^{m+1} = E^m - \sum_{i \in N} s_i^m$, and $c^{m+1} = c^m - s(E^m, c^m, P_1)$. By Lemma 5.3, for each agent i such that $s_i(E^m, c^m, P_1) = DPin_i(E^m, c^m)$, $s_i(E^{m+1}, c^{m+1}, P_1) = 0$. If all agents agree on $\varphi(E^{m+1}, c^{m+1}, P_1)$, by Lemma 5.5, $DPin(E, c) \equiv u[\varphi, (E, c, P_1)] = \sum_{k=1}^{m} s(E^k, c^k, P_1) + \varphi(E^{m+1}, c^{m+1}, P_1)$. Otherwise, go to next step.

[**Limit case**] Compute $\sum_{k=1}^{\infty} s_i(E^k, c^k, P_1)$. Let us note that, by Lemmas 5.3 and 5.5 and the definition of the $DPin$ rule, for each agent $i \in N : c_i \neq c_n$, $\sum_{k=1}^{\infty} s_i(E^k, c^k, P_1) = DPin_i(E, c)$. Moreover, for the rest of agents, l, by Lemma 5.4, $\sum_{k=1}^{m} s_l(E^k, c^k, P_1) \leq DPin_l(E, c)$. Furthermore, by Fact 1 and the definition of the $DPin$ rule,

$s_l(E, c, P_1) \geq E/n \geq DPin_l(E, c)/n$; $s_l(E^2, c^2, P_1) \geq (DPin_l(E, c) - s_l(E, c, P_1))/n$; thus, $\sum_{k=0}^{m} s_l(E^k, c^k, P_1) \geq \frac{DPin_l(E,c)}{n} \left(\frac{n-1}{n}\right)^m$, i.e.,

$$\sum_{k=1}^{\infty} s_l\left(E^k, c^k, P_1\right) \geq DPin_l(E, c).$$

Therefore, $\sum_{k=1}^{\infty} s_l\left(E^k, c^k, P_1\right) = DPin_l(E, c)$. ■

APPENDIX 3. ***Proof of Proposition 3.2.***

By Remark 1, for each $m \in \mathbb{N}$, $c_1^m \leq c_2^m \leq ... \leq c_n^{m+1}$.

Moreover, let us note that for each (E, c, P_1) in $\mathcal{B}_P$ and each $\varphi \in \Phi(P_1)$,

$$DPin_n(E, c) \geq \varphi_n(E, c).$$

Finally, by Lemmas 5.3, 5.4 and 5.5, $\sum_{k=1}^{\infty} s_n(E^k, c^k, P_1) \leq DPin_n(E, c)$.

Therefore, $DPin_n(E, c, P_1) \geq u_n[\varphi, (E, c, P_1)]$, i.e., the *DPin* rule is a weakly dominant strategy for the agent with the highest claim. ■

APPENDIX 4. ***Proof of Theorem 3.3.***

Let us consider $(E, c, P_1) \in \mathcal{B}_P$. Then, each agent's outcome in any Nash equilibrium of $\Gamma_{P_1}^u$ satisfies $DPin_i(E, c) \leq u_i[\varphi, (E, c, P_1)]$, for each $i \in N$. Otherwise if for some $i \in N$, $DPin_i(E, c) > u_i[\varphi, (E, c, P_1)]$ then, by Proposition 3.1, agent i could deviate to choose $DPin$, which gives her more awards, contradicting the Nash equilibrium. Finally, if for each $i \in N$, $DPin_i(E, c) \leq u_i[\varphi, (E, c, P_1)]$, then, $u[\varphi, (E, c, P_1)] = DPin(E, c)$, since $\sum_{i \in N} u_i[\varphi, (E, c, P_1)] \leq E$.

APPENDIX 5. ***Proof of Proposition 3.4.***

The proof of this result is based on Remark 1, Definition 5.2, Fact 2 and the following three lemmas, in which φ^* denotes an *Admissible* rule in P_2, different of the *Dual Constrained Egalitarian* one $\varphi^*(E, c) \neq DCE(E, c)$.

Lemma 5.6. *(Jiménez-Gómez and Marco-Gil, 2008) For each $(E, c, P_2) \in \mathcal{B}_P$, and each $i \in \{1, 2\}$, if $s_i(E, c, P_2) = DCE_i(E, c)$, then for $m \geq 2$, $s_i(E^m, c^m, P_2) = DCE_i(E^m, c^m) = 0$.*

Lemma 5.7. *(Jiménez-Gómez and Marco-Gil, 2008) For each $(E, c, P_2) \in \mathcal{B}_P$, and each $i \in \{1, 2\}$, if $s_i(E^m, c^m, P_2) = \varphi_i^*(E^m, c^m)$ for each $m \in \mathbb{N}$, then*

$$\sum_{k=1}^{\infty} s_i(E^k, c^k, P_2) \leq DCE_i(E, c).$$

Lemma 5.8. *(Jiménez-Gómez and Marco-Gil, 2008) For each* $(E, c, P_2) \in \mathcal{B}_P$*, and each* $i \in \{1, 2\}$*, if there is* $m^* \in \mathbb{N}$*,* $m^*>1$*, such that* $s_i(E^{m^*}, c^{m^*}, P_2) = DCE_i(E^{m^*}, c^{m^*})$ *and* $s_i(E^{m^*-1}, c^{m^*-1}, P_2) = \varphi_i^*(E^{m^*-1}, c^{m^*-1})$*, then*
$\sum_{k=1}^{m^*} s_i(E^k, c^k, P_2) = DCE_i(E, c)$*.*

Proof of Proposition 3.4.

[**Step** 1] If the two agents agree on $\varphi(E, c, P_2) = DCE(E, c)$, then $u[\varphi, (E, c, P_2)] = DCE(E, c)$. Otherwise, go to next step.

[**Step** 2] Let $s_i(E, c, P_2) = \min_{j \in N} \varphi_i^j(E, c, P_2)$, $c^2 = c - s(E, c, P_2)$ and $E^2 = E - \sum_{i \in N} s_i(E, c, P_2)$. In this case, by Definition 5.2, $s_1(E, c, P_2) = DCE_1(E, c)$, and by Lemma 5.6, $s_1(E^2, c^2, P_2) = 0$. So that, if all agents agree on $\varphi(E^2, c^2, P_2)$, then, by Lemma 5.8, $u[\varphi, (E, c, P_2)] = s(E, c, P_2) + \varphi(E^2, c^2, P_2) = DCE(E, c)$. Otherwise, go to next step.

[**Step** m+1] Let $s_i^{m+1}(E^m, c^m, P_2) = \min_{j \in N} \varphi_i^j(E^m, c^m, P_2)$, $E^{m+1} = E^m - \sum_{i \in N} s_i^m$, and $c^{m+1} = c^m - s(E^m, c^m, P_2)$. By Lemma 5.6, $s_1(E^m, c^m, P_2) = 0$. So that, if all agents agree on $\varphi(E^m, c^m, P_2)$, then, by Lemma 5.8, $u[\varphi, (E, c, P_2)] = \sum_{k=1}^{m} s(E^k, c^k, P_2) + \varphi(E^{m+1}, c^{m+1}, P_2) = DCE(E, c)$. Otherwise, go to next step.

[**Limit case**] Compute $\sum_{k=1}^{\infty} s_i(E^k, c^k, P_2)$. Let us note that, by Lemmas 5.6 and 5.8 and the definition of the DCE rule, for agent 1, $\sum_{k=1}^{\infty} s_1(E^k, c^k, P_2) = DPin_1(E, c)$.

Moreover, by Lemma 5.7, $\sum_{k=1}^{m} s_2(E^k, c^k, P_2) \leq DCE_2(E, c)$. Furthermore, by Fact 2 and the definition of the DCE rule,

$$s_2(E, c, P_2) \geq \; E/2 \geq DCE_2(E, c)/2, \;\; s_2(E^2, c^2, P_2) \geq \frac{DCE_2(E, c) - s_2(E, c, P_2)}{2};$$

thus,

$$\sum_{k=0}^{m} s_2\left(E^k, c^k, P_2\right) \geq \; \frac{DCE_2(E, c)}{2}\left(\frac{1}{2}\right)^m, \text{ i.e., } \sum_{k=1}^{\infty} s_2\left(E^k, c^k, P_2\right) \geq DCE_2(E, c).$$

Therefore, $\sum_{k=1}^{\infty} s_2(E^k, c^k, P_2) = DCE_2(E, c)$. ■

APPENDIX 6. *Proof of Theorem 3.5.*

Let us consider $(E, c, P_2) \in \mathcal{B}_P$. Then, each agent's outcome, in any Nash equilibrium of $\Gamma^u_{P_2}$ satisfies $DCE_i\,(E, c) \leq u_i\,[\varphi, (E, c, P_2)]$ for each $i \in N$, with $|N| = 2$. Otherwise if for some $i \in \{1, 2\}$, $DCE_i\,(E, c) > u_i\,[\varphi, (E, c, P_2)]$ then, by Proposition 3.4, agent i could deviate to choose DCE, which gives her more awards, contradicting the Nash equilibrium. Finally, if for each $i \in \{1, 2\}$, $DCE_i\,(E, c) \leq u_i\,[\varphi, (E, c, P_2)]\,,$ then, $u\,[\varphi, (E, c, P_2)] = DCE\,(E, c)\,,$ since $\sum_{i\in N} u_i\,[\varphi, (E, c, P_2)] \leq E$.

APPENDIX 7. *Proof of Theorems 3.6 and 3.7.*

First of all, let us note the following fact.

Fact 3. *(Jiménez-Gómez and Marco-Gil, 2008) By the definition for the DCE rule, we know that it can be written as follows,*

given $(E, c) \in \mathcal{B}_0$, $i \in N$,

$$DCE_i(E, c) \equiv \begin{cases} c_i - \gamma_i & \text{if } E \leq C/2 \\ c_i - \gamma_i & \text{if } E \geq C/2 \end{cases},$$

where γ_i *is chosen such that* $\sum\limits_{i\in N} DCE_i(E, c) = E$.

Therefore,

Case a: $E \leq C/2$. *We can compute* γ_i *as:*

$$\gamma_i = \begin{cases} c_i & \forall\ i < l \\ \max\{c_i/2, \alpha_i\} & \forall\ i \geq l \end{cases},$$

where agent l *is that one such that* $\sum\limits_{j>l} \min\{c_j - c_l; c_j/2\} < E$, *and*

either $\sum\limits_{j>l-1} \min\{c_j - c_{l-1}; c_j/2\} \geq E$, *either* $l = 1$. *Otherwise,* $l = n$.

Then, $\forall i \geq l$,

$$\alpha_i = \frac{L - \sum\limits_{j=1}^{k-1} c_j - \sum\limits_{j>i} \gamma_j}{i - l + 1}.$$

Note also that we should compute α *from the highest claimant to the smallest one.*

Case b: $E \geq C/2$. *Then,* γ_i *will denote the losses incurred by agent* i *when the losses from the claim vector are equal to all agents subject to no-one obtaining less than her half-claim.*

Proof of Theorem 3.6.

Let us consider the following problem $(E,c) \in \mathcal{B} = (21,(5;19.5;20))$ and for each step $m \in \mathbb{N}$,

$$\psi\left(E^m, c^m, P_2\right) = \left(CE\left(E^m, c^m\right), \varphi_2^m\left(E^m, c^m\right), DCE\left(E^m, c^m\right)\right).$$

Thus, given the definitions of the CE rule and its dual and Fact 3, we get at step $m=1$, $(E^1,c^1) = (21,(5;19.5;20))$, $CE(E^1,c^1) = (2.5;9.25;9.25)$, and $DCE(E^1,c^1) = (1.25;9.75;10)$.

[**Step** 1] Since there is no agreement, go to next step.

[**Step** 2] $s(E,c,P_2) = (1.25;9.25;9.25)$., and $E^2 = 1.25$. So,
$\left(E^2,c^2\right) = (1.25,(3.75;10.25;10.75))$, $CE(E^2,c^2) = (0.416;0.416;0.416)$, and $DCE(E^2,c^2) = (0;0.375;0.875)$, and since there is no agreement, go to next step.

[**Step** 3] $s(E^2,c^2,P_2) = (0;0.375;0.416)$, and $E^3 = 0.459$.So,
$\left(E^3,c^3\right) = (0.459,(3.75;9.875;10.334))$, $CE(E^3,c^3) = (0.153;0.153;0.153)$, and $DCE(E^3,c^3) = (0;0;0.459)$, and since there is no agreement, go to next step.

[**Step** 4] $s(E^3,c^3,P_2) = (0;0;0.153)$, and $E^4 = 0.306$.So,
$\left(E^4,c^4\right) = (0.306,(3.75;9.875;10.181))$, $CE(E^3,c^3) = (0.102;0.102;0.102)$, and $DCE(E^3,c^3) = (0;0;0.306)$, and since there is no agreement, go to next step.

[**Limit case**] Let us note that, since $E^m \leq c_3^m - c_2^m$, and $c_i^m \geq E^m/3$, for each step $m \geq 3$, $DCE(E^m,c^m) = (0;0;E^m)$ and $CE(E^m,c^m) = (E^m/3;E^m/3;E^m/3)$. Thus, $s(E^m,c^m,P_2) = (0;0;E^m/3)$. Therefore, $u\left[\varphi,(E,c,P_2)\right] = \sum_{k=1}^{\infty} s\left(E^k,c^k,P_2\right) = (1.25;9.625;10.125)$.

At this point, we have to show that the CE and DCE are the weakly dominant strategies for agents 1 and 3 in this example, respectively.

To this respect, next we can observe that agent 3 cannot increase her payoff by changing her strategy.

Let us consider the following problem $(\bar{E},c) \in \mathcal{B} = (21,(5;19.5;20))$, and for each step $m \in \mathbb{N}$,

$$\psi\left(\bar{B}^m_{P_2}\right) = \left(CE\left(\bar{E}^m, c^m\right), \varphi^2\left(\bar{B}^m_{P_2}\right), \varphi^3\left(\bar{B}^m_{P_2}\right)\right).$$

Thus, given the definitions of the CE rule and its dual and Fact 8, we get at step $m=1$, $(\bar{E}^1,c^1) = (21,(5;19.5;20))$, $CE(\bar{E}^1,c^1) = (2.5;9.25;9.25)$, and $DCE(\bar{E}^1,c^1) = (1.25;9.75;10)$.

[**Step** 1] Since there is no agreement, go to next step.

[**Step** 2] By construction,

$$\begin{aligned}
\varphi_1\left(\bar{E}^1, c^1, P_2\right) &\geq DCE_1(\bar{E}^1, c^1),\ \varphi_1\left(\bar{E}^1, c^1, P_2\right) \leq CE_1(\bar{E}^1, c^1),\\
\varphi_2^3\left(\bar{E}^1, c^1, P_2\right) &\geq CE_2(\bar{E}^1, c^1),\ \varphi_2^2\left(\bar{E}^1, c^1, P_2\right) \geq CE_2(\bar{E}^1, c^1),\\
\varphi_3\left(\bar{E}^1, c^1, P_2\right) &\geq CE_3(\bar{E}^1, c^1),\ \varphi_3\left(\bar{E}^1, c^1, P_2\right) \leq DCE_2(\bar{E}^1, c^1).
\end{aligned}$$

Then,
$s(\bar{E}^1, c^1, P_2) = \left(\min\left\{\varphi_1^2\bar{E}^1, c^1, P_2, \varphi_1^3\left(\bar{E}^1, c^1, P_2\right)\right\}; 9.25; 9.25\right).$
Note that,
$\min\left\{\varphi_1^2\left(\bar{E}^1, c^1, P_2\right), \varphi_1^3\left(\bar{E}^1, c^1, P_2\right)\right\} = DCE_1(\bar{E}^1, c^1) + \alpha$, where $\alpha \in \mathbb{R}_+ :$ $DCE_1(\bar{E}^1, c^1) \leq DCE_1(\bar{E}^1, c^1) + \alpha \leq CE_1(\bar{E}^1, c^1)$,
and $s_{3.}(\bar{E}^1, c^1, P_2) = s_3(E^1, c^1, P_2)$.
Thus,
$\bar{E}^2 = 1.25 - \alpha \leq E^2$. So,
$\left(\bar{E}^2, c^2\right) = (1.25 - \alpha, (3.75 - \alpha; 10.25; 10.75))$,
$CE(\bar{E}^2, c^2) = (0.416 - \alpha/3; 0.416 - \alpha/3; 0.416 - \alpha/3)$, and
$DCE(\bar{E}^2, c^2) = (0; 0.375 - \alpha/2; 0.875 - \alpha/2)$, and since there is no agreement, go to next step.

[**Step** 3] By construction,

$$\begin{aligned}
\varphi_1\left(\bar{E}^2, c^2, P_2\right) &\geq DCE_1(\bar{E}^2, c^2),\ \varphi_1\left(\bar{E}^2, c^2, P_2\right) \leq CE_1(\bar{E}^2, c^2),\\
\varphi_2\left(\bar{E}^2, c^2, P_2\right) &\geq DCE_2(\bar{E}^2, c^2),\ \varphi_2\left(\bar{E}^2, c^2, P_2\right) \leq CE_2(\bar{E}^2, c^2),\\
\varphi_3\left(\bar{E}^2, c^2, P_2\right) &\geq CE_3(\bar{E}^2, c^2),\ \varphi_3\left(\bar{E}^2, c^2, P_2\right) \leq DCE_2(\bar{E}^2, c^2).
\end{aligned}$$

Then,
$s(\bar{E}^2, c^2, P_2) =$
$= \left(\min\left\{\varphi_1^2\left(\bar{B}^m_{P_2}\right), \varphi_1^3\left(\bar{B}^m_{P_2}\right)\right\}; \min\left\{\varphi_1^2\left(\bar{B}^m_{P_2}\right), \varphi_1^3\left(\bar{B}^m_{P_2}\right)\right\}; 0.416 - \alpha/3\right).$

Note that,

$\min\left\{\varphi_1^2\left(\bar{E}^2, c^2, P_2\right), \varphi_1^3\left(\bar{E}^2, c^2, P_2\right)\right\} = DCE_1(\bar{E}^2, c^2) + \alpha^2$, where $\alpha^2 \in \mathbb{R}_+$: $DCE_1(\bar{E}^2, c^2) \leq DCE_1(\bar{E}^2, c^2) + \alpha^2 \leq CE_1(\bar{E}^2, c^2)$,

$\min\left\{\varphi_2^2\left(\bar{E}^2, c^2, P_2\right), \varphi_2^3\left(\bar{E}^2, c^2, P_2\right)\right\} = DCE_2(\bar{E}^2, c^2) + \varepsilon^2$, where $\varepsilon^2 \in \mathbb{R}_+$: $DCE_2(\bar{E}^2, c^2) \leq DCE_2(\bar{E}^2, c^2) + \varepsilon^2 \leq CE_2(\bar{E}^2, c^2)$,

and $s_3(\bar{E}^2, c^2, P_2) \leq s_3(E^2, c^2, P_2)$.

Thus,

$\bar{E}^3 = (1.25 - \alpha) - \alpha^2 - \left(0.375 - \frac{\alpha}{2} - \varepsilon^2\right) - \left(0.416 - \frac{\alpha}{3}\right) = 0.459 - \frac{\alpha}{6} - \alpha^2 - \varepsilon^2 \leq E^3$.

So,

$\left(\bar{E}^3, c^3\right) = \left(0.459 - \frac{\alpha}{6} - \alpha^2 - \varepsilon^2, \left(3.75 - \alpha; 9.875 - \varepsilon^2; 10.334\right)\right)$,

$CE(\bar{E}^3, c^3) = \left(\bar{E}^3/3; \bar{E}^3/3; \bar{E}^3/3\right)$, and

$DCE(\bar{E}^3, c^3) = \left(0; 0; \bar{E}^3\right)$, and since there is no agreement, go to next step.

[**Step** 4] By construction,

$$
\begin{aligned}
\varphi_1\left(\bar{E}^3, c^3, P_2\right) &\geq DCE_1(\bar{E}^3, c^3),\ \varphi_1\left(\bar{E}^3, c^3, P_2\right) \leq CE_1(\bar{E}^3, c^3),\\
\varphi_2\left(\bar{E}^3, c^3, P_2\right) &\geq DCE_2(\bar{E}^3, c^3),\ \varphi_2\left(\bar{E}^3, c^3, P_2\right) \leq CE_2(\bar{E}^3, c^3),\\
\varphi_3\left(\bar{E}^3, c^3, P_2\right) &\geq CE_3(\bar{E}^3, c^3),\ \varphi_3\left(\bar{E}^3, c^3, P_2\right) \leq DCE_2(\bar{E}^3, c^3).
\end{aligned}
$$

Then,

$s(\bar{E}^3, c^3, P_2) = (\min\left\{\varphi_1^2\left(\bar{E}^3, c^3, P_2\right), \varphi_1^3\left(\bar{E}^3, c^3, P_2\right)\right\};$
$\min\left\{\varphi_1^2\left(\bar{E}^3, c^3, P_2\right), \varphi_1^3\left(\bar{E}^3, c^3, P_2\right)\right\}; \bar{E}^3/3)$.

Note that,

$\min\left\{\varphi_1^2\left(\bar{E}^3, c^3, P_2\right), \varphi_1^3\left(\bar{E}^3, c^3, P_2\right)\right\} = DCE_1(\bar{E}^3, c^3) + \alpha^3$, where $\alpha^3 \in \mathbb{R}_+$: $DCE_1(\bar{E}^3, c^3) \leq DCE_1(\bar{E}^3, c^3) + \alpha^3 \leq CE_1(\bar{E}^3, c^3)$,

$\min\left\{\varphi_2^2\left(\bar{E}^3, c^3, P_2\right), \varphi_2^3\left(\bar{E}^3, c^3, P_2\right)\right\} = DCE_2(\bar{E}^3, c^3) + \varepsilon^3$, where $\varepsilon^3 \in \mathbb{R}_+$: $DCE_2(\bar{E}^3, c^3) \leq DCE_2(\bar{E}^3, c^3) + \varepsilon^3 \leq CE_2(\bar{E}^3, c^3)$,

$s_3(\bar{E}^3, c^3, P_2) \leq s_3(E^3, c^3, P_2)$.

Thus,

$\bar{E}^4 = \bar{E}^3 - \alpha^3 - \varepsilon^3 - \bar{E}^3/3 \leq E^4$.

Since there is no agreement, go to next step.

[**Limit case**] Note that, since $\bar{E}^m \leq E^m$, for each step $m \geq 1$.

Thus,

$u_3\left[\varphi, \left(\bar{E}, c, P_2\right)\right] = \sum_{k=1}^{\infty} s_3\left(\bar{E}^k, c^k, P_2\right) \leq 10.125 = u_n\left[\varphi, (E, c, P_2)\right]$,

i.e., the *DCE* rule is a weakly dominant strategy for the agent 3.

Finally, by duality we can easily get that agent 2 cannot increase her payoff by changing her strategy, i.e., the *CE* rule is a weakly dominant strategy for the agent 2.

Therefore, in the *Nash equilibrium* induced by the game $\Gamma^u_{P_2}$ for this problem, $u\left[\varphi, (E, c, P_2)\right] \neq DCE(E, c)$. ■

Proof of Theorem 3.7.

Let us consider the two following *Problems with Legitimate Principles:*

$(E, c, P_2) = (22.25; (5; 19.5; 20), P_2)$, and $(E', c, P_2) = (21, (5; 19.5; 20), P_2)$.

In this case,

$$u\left[\varphi, (E, c, P_2)\right] = (2.5; 9.75; 10),$$

and

$$u\left[\varphi, \left(E', c, P_2\right)\right] = (1.25; 9.625; 10.125).$$

Obviously, these two distributions contradict *Resource Monotonicity* since the highest claimant receives less when the endowment increases. ■

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