Journal Club by Oleg Chalaev

Nonequilibrium mesoscopic conductance fluctuations

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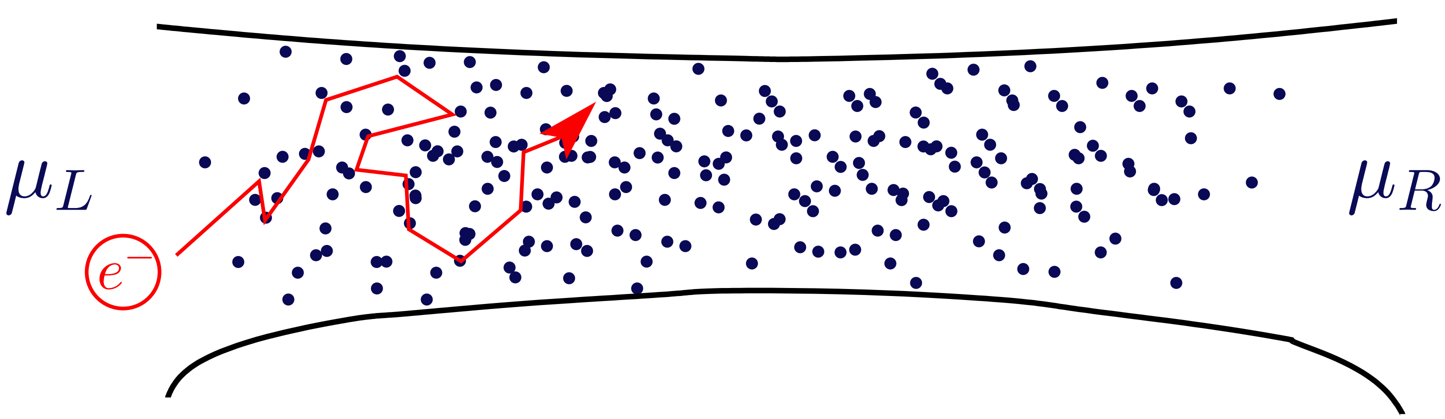
Universal conductance fluctuations in equilibrium

Altshuler'85; Lee& Stone'85:
The conductance of a metalic sample with a fixed concentration of impurities exhibits fluctuations of the order of e^2/h, when $L_\phi \gtrsim L$.

$$\langle \delta\sigma^2 \rangle = \frac{8}{15}\frac{e^4}{h^2}$$

What happens out of equilibrium?



A few words about G_{K}

- Keldysh technique is very similar to the $T=0$ equilibrium technique described in Abrikosov, Gor'kov, Dzyaloshinskii
- The major difference from $T=0$ technique: now instead of scalar Green's function we operate with $G=\begin{pmatrix} G_{\mathrm{R}} & G_{\mathrm{K}} \\ 0 & G_{\mathrm{A}} \end{pmatrix}$
- Without interaction $G_{\mathrm{K}}=(1-2f_E)\left(G_{\mathrm{R}}-G_{\mathrm{A}}\right),$
  $f_E=$ energy distribution function.

Connection with the density of states ν:

$$\frac{1}{V}\sum_p\left[G^E_{\mathrm{R}}(\vec{p})-G^E_{\mathrm{A}}(\vec{p})\right]=-2\pi i\nu_E$$

The average current $\vec{j}\sim\vec{p}\langle G_{\mathrm{K}}(\vec{p})\rangle$

The two-step energy distribution function

<u>Without interaction:</u> kinetic equation or charge conservation

$$\Longrightarrow \nabla^2 \left\langle G^K \right\rangle = 0$$

Boundary conditions:

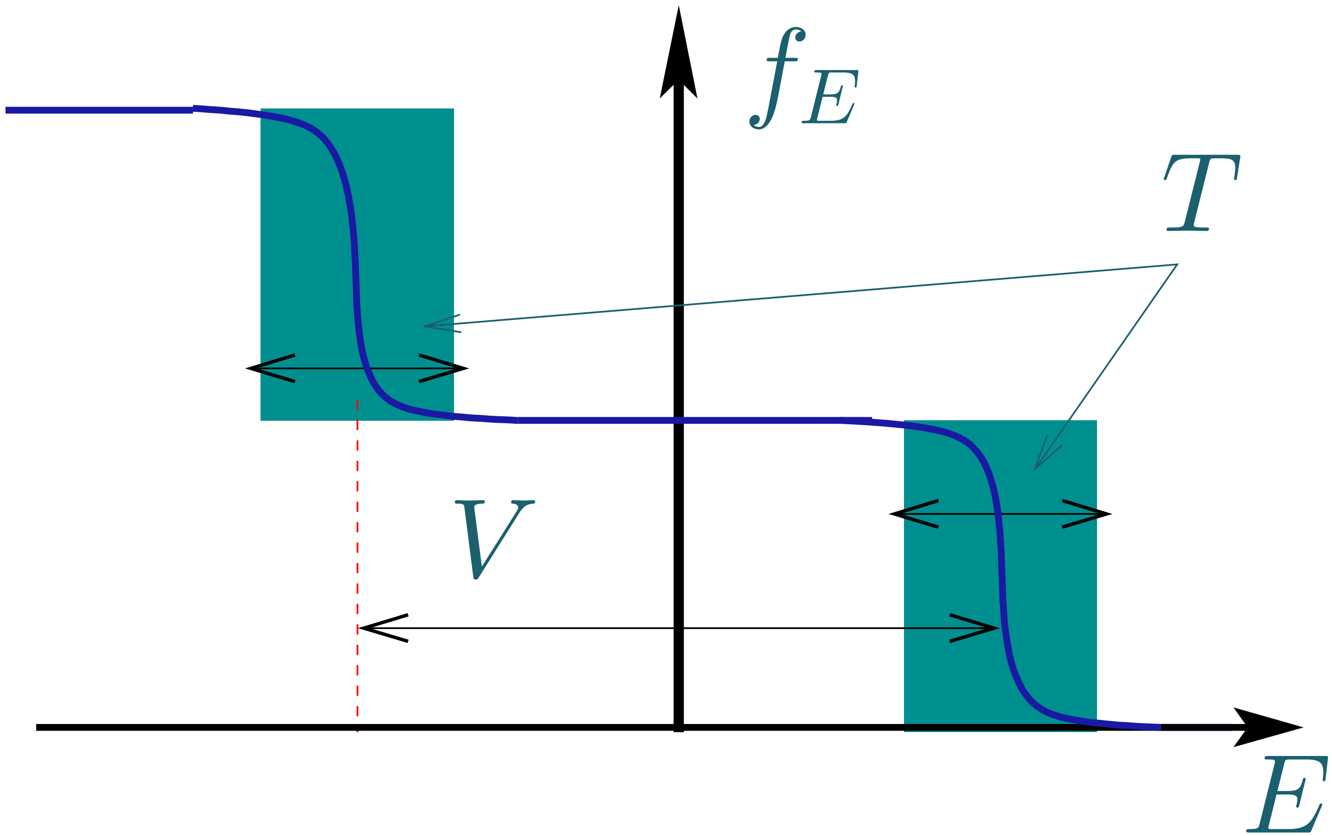
$$\left\langle G^E_{\mathrm{K}}(x) \right\rangle = -2\pi i \nu \times \begin{cases} 1 - 2f(E)\,, & x = 0 \\ 1 - 2f(E - eV)\,, & x = L \end{cases}$$

$\Longrightarrow$ we get double-step distribution function:

$$\left\langle G^E_{\mathrm{K}}(x) \right\rangle = -2\pi i \nu \left\{ 1 - 2f(E) + 2\tfrac{x}{L}\left[f(E) - f(E - eV) \right] \right\}$$

This derivation of the double-step f_E I don't understand; My understanding is more primitive, see pp. 65-66 in [1].

The two-step energy distribution function



$\Longrightarrow$ we get double-step distribution function:

$$\langle G_{\mathrm{K}}^{E}(x)\rangle = -2\pi i\nu\Big\{1 - 2f(E) + 2\tfrac{x}{L}\big[f(E) - f(E - eV)\big]\Big\}$$

This derivation of the double-step f_E I don't understand; My understanding is more primitive, see pp. 65-66 in [1].

What are the consequences of non-equilibrium

- $G_{\mathrm{R/A}}$ are not altered since they are short range objects: $G_{\mathrm{R/A}}(\vec{r},\vec{r}\,') \sim \exp\left[-\frac{|\vec{r}-\vec{r}\,'|}{l}\right], \qquad l \ll L.$

- G_{K} is changed since it depends on the energy distribution

- Cooperon/diffuson are changed since they are long-range objects, $\sim L_\omega = \sqrt{iD/\omega}$

An equation for the diffusion propagator:

$$\left\{\partial_x^2 + \frac{i\omega}{D} + \frac{ie}{D}\Big[\phi_1(x) - \phi_2(x)\Big]\right\}\Pi_\omega(x,x') = -\delta(x-x')$$

Boundary conditions: $\Pi = 0$ at $x = 0$ and $x = L$

The effect of interaction between electrons

Ideologically the authors have done (in reality – more rigorously) the following:

- got expressions with cooperons from nasty diagrams
- inserted $\tau_\phi \equiv \tau_\phi(T^*(x))$ into the denominator of the cooperon;

$$T^*(x) = eVx(1-x), \quad \tau_\phi[T^*] \sim \left(D\nu^2/T^{*2}\right)^{1/3}$$

- claiming that in this way they've taken interactions into account

Is this really all what interaction does out of equilibrium????

The result

Out of equilibrium, additional contributions appear:

$$\langle \delta g\, \delta g \rangle = \langle \delta g\, \delta g \rangle_0 + \langle \delta g\, \delta g \rangle_1 + \langle \delta g\, \delta g \rangle_2 \ ,$$

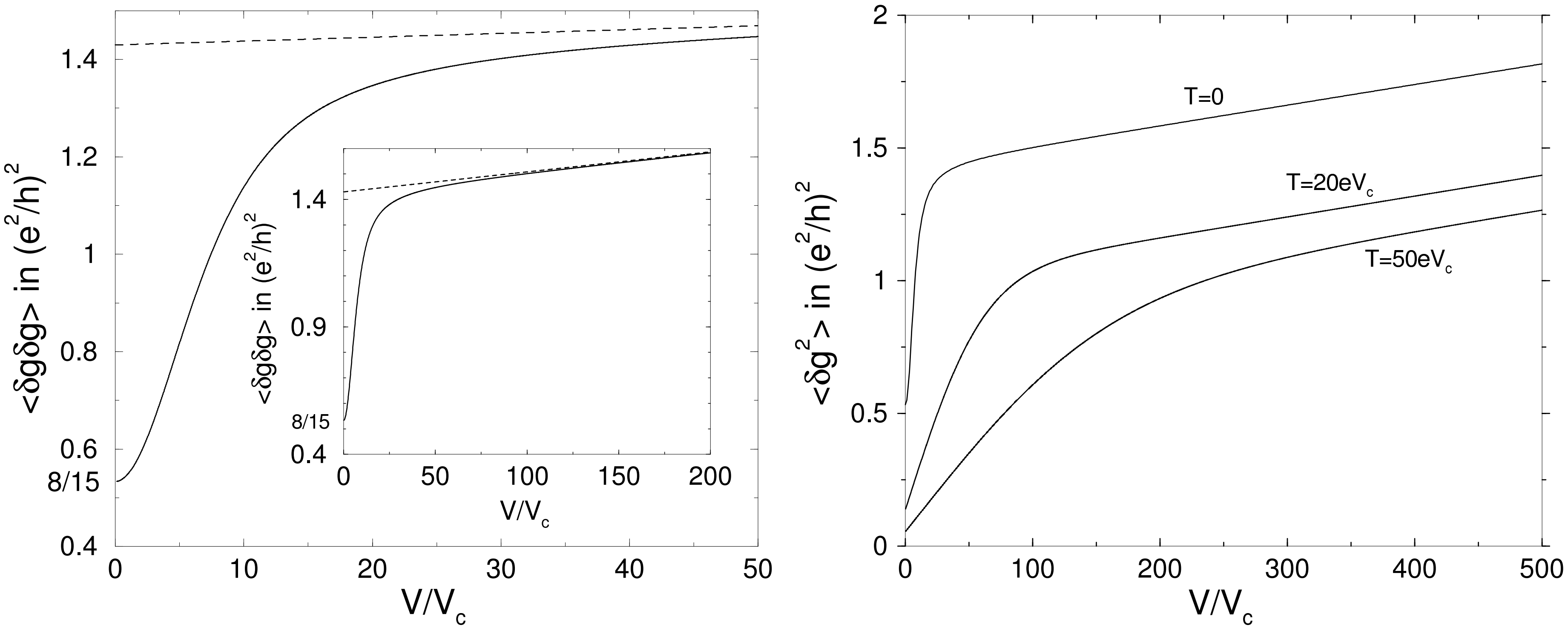
$$\Big\langle \delta g(V)\, \delta g(V) \Big\rangle_0 = 16\, \Xi_0\Big|_{\alpha=0} = \frac{8}{15}\,,$$

$$\Big\langle \delta g(V)\, \delta g(V) \Big\rangle_1 = 32 \int_0^{V/V_c} dz\, \frac{\partial}{\partial \alpha} \Xi_{z-\frac{V}{V_c}}\Big|_{\alpha=0}\,, \qquad eV_c = D/L^2$$

$$\Big\langle \delta g(V)\, \delta g(V) \Big\rangle_2 = -16 \int_0^{V/V_c} dz_1 dz_2 \frac{\partial^2}{\partial \alpha^2} \Xi_{z_1-z_2}\Big|_{\alpha=0},$$

$$\Xi_z = \int_0^1 dy_1 dy_2 \ \Big[2 \left|\Pi_z(y_1, y_2)\right|^2 + \mathrm{Re}\, \Pi_z^2(y_1, y_2) \Big]$$

The voltage and temperature dependece



Conclusions

- "nonequilibrium" terms are calculated.

- interaction is taken into account via dephasing

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References

[1] Supriyo Datta. *Electronic Transport in Mesoscopic Systems*. Cambridge uni. press, 1997.