

Nonexistence of Normal Quintic Abelian Surfaces in P^3

Iku NAKAMURA and Yumiko UMEZU

Hokkaido University and Toho University

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Introduction.

Normal surfaces X_d of degree d in the complex projective 3-space P^3 have simple birational structure if d is small: X_1 and X_2 are rational, and X_3 is birationally equivalent to a ruled surface (for further details, see [B-W], [H-W]), since in general $K_{X_d} \simeq \mathcal{O}_{X_d}(d-4)$. Moreover X_4 is birationally equivalent to either a ruled or a K3 surface ([Um1]).

To the contrary, various X_d may occur if $d \geq 5$. If the singularity of X_d is mild, then X_d is birationally equivalent to a surface of general type, while X_d may be birationally equivalent to a ruled surface if it has severe singularity. Moreover there are examples of X_5 which are birationally equivalent to K3 surfaces, Enriques surfaces or general elliptic surfaces ([I], [Yan], [St], [K], [Um2], [Um3], [Um4]). This leads us to the question whether there exists an X_d which is birationally an abelian or a hyperelliptic surface or not. The purpose of this note is to answer this question in the case of $d=5$. We prove:

MAIN THEOREM. *No normal quintic surface in P^3 is birationally equivalent to an abelian or a hyperelliptic surface.*

Our proof of the theorem goes as follows. First we note that if a normal quintic surface $X=X_5$ is birationally an abelian or a hyperelliptic surface, then its minimal resolution \tilde{X} is an at most 5-fold blowing-up $\mu: \tilde{X} \rightarrow \bar{X}$ of the non-singular minimal model \bar{X} . On the other hand, the pull-back of K_X to \tilde{X} minus $K_{\tilde{X}}$ is an effective divisor \tilde{D} , which reflects the property of the singularity of X fairly well. Such property of \tilde{D} and the condition of $\mu_*\tilde{D}$ as a divisor on an abelian or hyperelliptic surface finally lead us in every case to a contradiction.

CONJECTURE. *No normal hypersurface in P^3 is birationally equivalent to an abelian surface.*

Also for hyperelliptic surfaces we raise:

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PROBLEM. Are there normal hypersurfaces in P^3 which are birationally hyperelliptic surfaces?

§1. Preliminaries.

In this section we summarize some results from local theory of surface singularity, which we will use later.

Let (Y, y) be a numerically Gorenstein normal surface singularity, $\pi: \tilde{Y} \rightarrow Y$ its minimal resolution and A the exceptional set: $A = \pi^{-1}(y)$. Then there is an effective divisor D with $\text{supp } D \subseteq A$ such that $\omega_{\tilde{Y}} \cong \mathcal{O}_{\tilde{Y}}(-D)$. Let p_a or p_g stand for the arithmetic or the geometric genus of (Y, y) respectively. By definition ([W]),

$$p_a = \sup_{\substack{D' > 0 \\ \text{supp}(D') \subseteq A}} p_a(D'), \quad p_g = \dim R^1 \pi_* \mathcal{O}_{\tilde{Y}}.$$

It is known (cf. [A]) that the following conditions are equivalent:

(i) $D \neq 0$ (ii) $\text{supp } D = A$ (iii) $p_a > 0$ (iv) $p_g > 0$.

LEMMA 1.1 (Y. Koyama). $p_a \leq -D^2/8 + 1$. In particular, if $D^2 \geq -7$, then $p_a \leq 1$.

PROOF. See [Um5].

REMARK 1.2. If $D^2 = -8$ and $p_a = 2$, then $D/2$ is an integral and the unique divisor on A whose arithmetic genus is equal to 2.

In what follows (except for Corollary 1.5) we assume moreover that $p_a = 1$, i.e. our singularity (Y, y) is elliptic ([W]). Then Yau (for the minimal good resolution) and Tomari (for any resolution) defined the elliptic sequence $\{Z_1, \dots, Z_l\}$ as follows: Let E denote the minimal elliptic cycle of Laufer [L], i.e., E is the minimal effective divisor such that $\text{supp } E \subseteq A$ and $p_a(E) = 1$. For Z_1 we take the fundamental cycle. Suppose that we have defined Z_1, \dots, Z_k . If $Z_k E < 0$, we define $\{Z_1, \dots, Z_k\}$ as the elliptic sequence: $l = k$. Assume $Z_k E = 0$. Then let B_{k+1} denote the connected component containing E of the sum of the components A_i of A satisfying $Z_k A_i = 0$. We define Z_{k+1} to be the fundamental cycle of B_{k+1} . Since $\text{supp } Z_k \supseteq \text{supp } Z_{k+1}$, the elliptic sequence $\{Z_1, \dots, Z_l\}$ is defined as a finite sequence. The following results for the minimal resolution will play an important role later.

THEOREM 1.3 ([T], [Yau]). (i) $D = \sum_{i=1}^l Z_i$. (ii) $Z_l = E$. (iii) $p_g \leq l$.

From this theorem, we obtain the following

COROLLARY 1.4. $p_g \leq -D^2$.

COROLLARY 1.5. Let (Y, y) be a numerically Gorenstein normal surface singularity of geometric genus p_g , and $\pi: \tilde{Y} \rightarrow Y$ its minimal resolution. Assume that the exceptional set $\pi^{-1}(y)$ consists of a chain of curves $A_0 = E, A_1, \dots, A_m$ ($m \geq 1$) with $p_a(E) = 1, p_a(A_i) = 0$

($1 \leq i \leq m$). Then we have

- (i) $E^2 = -1$,
- (ii) $m \geq p_g - 1$ and $A_i^2 = -2$ for $1 \leq i \leq p_g - 2$.

PROOF. The fundamental cycle Z_1 coincides with $\pi^{-1}(y)$ with reduced structure, hence $p_a(Z_1) = 1$, and so (Y, y) is an elliptic singularity ([W]). Then E is the minimal elliptic cycle. Theorem 1.3 implies that for every i , Z_i contains more than $p_g - i$ components. In particular, for $i = 1$, we get $m \geq p_g - 1$; for $i = 2$, $Z_1 E = 0$ and $Z_1 A_i = 0$ ($1 \leq i \leq p_g - 2$), which proves $E^2 = -1$ and $A_i^2 = -2$ ($1 \leq i \leq p_g - 2$).

§2. Properties of divisors on the resolution.

Let X be a normal quintic surface in P^3 . Let $\pi: \tilde{X} \rightarrow X$ denote the minimal resolution of X , H a general hyperplane section of X and \tilde{H} its pull-back on \tilde{X} . Then there exists a unique effective divisor \tilde{D} on \tilde{X} such that $K_{\tilde{X}} = \tilde{H} - \tilde{D}$. This divisor \tilde{D} is supported on the exceptional sets of π which correspond to singularities with positive geometric genus. Let $\mu: \tilde{X} = X_n \xrightarrow{\mu_n} X_{n-1} \xrightarrow{\mu_{n-1}} \dots \xrightarrow{\mu_1} X_0 = \bar{X}$ be the sequence of blow-downs obtaining a non-singular minimal model \bar{X} of \tilde{X} , μ_i the induced morphism $\tilde{X} \rightarrow X_i$ ($0 \leq i \leq n$), and E_i ($1 \leq i \leq n$) the total transform on \tilde{X} of the exceptional curve of the blow-up μ_i . In what follows we fix our notations as above and assume moreover that \bar{X} is either an abelian or a hyperelliptic surface.

LEMMA 2.1. $\tilde{D}^2 = -n - 5$ and $1 \leq n \leq 5$. Moreover, if $n = 5$ and if Γ is a rational curve on \tilde{X} , then either

- (i) $\tilde{H}\Gamma = 1$ (Γ is not exceptional for π), $\tilde{D}\Gamma = 2$, $\Gamma^2 = -1$, or
- (ii) $\tilde{H}\Gamma = 0$ (Γ is exceptional for π), $\tilde{D}\Gamma = 0$, $\Gamma^2 = -2$.

PROOF. Since \bar{X} has a numerically trivial canonical bundle, $-n = K_{\tilde{X}}^2 = (\tilde{H} - \tilde{D})^2 = 5 + \tilde{D}^2$, and hence $\tilde{D}^2 = -n - 5$. Since each E_i contains at least one (-1) -curve and \tilde{X} is the minimal resolution, we have $\tilde{H}E_i > 0$, and so $5 = \tilde{H}^2 = \tilde{H}(\tilde{H} - \tilde{D}) = \tilde{H}K_{\tilde{X}} = \sum_{i=1}^n \tilde{H}E_i \geq n$. $n \geq 1$ since $\tilde{H} - \tilde{D} \neq 0$. Note that any rational curve on \tilde{X} is a component of E_i for some i since \bar{X} contains no rational curve. Assume $n = 5$. Then $\tilde{H}E_i = 1$ ($1 \leq i \leq 5$). Hence, for each i , there exists a unique component Γ_i in E_i , with multiplicity 1, such that $\tilde{H}\Gamma_i = 1$, and other components of E_i are exceptional for π and so have non-positive intersection number with \tilde{D} . Since Γ_i is a (-1) -curve, $-1 = K_{\tilde{X}}\Gamma_i = (\tilde{H} - \tilde{D})\Gamma_i$, hence $\tilde{D}\Gamma_i = 2$. By $-1 = K_{\tilde{X}}E_i = K_{\tilde{X}}\Gamma_i + K_{\tilde{X}}(E_i - \Gamma_i) = -1 + \tilde{D}(E_i - \Gamma_i)$, we see that any component Γ in $E_i - \Gamma_i$ satisfies $\tilde{D}\Gamma = 0$ and so $\Gamma^2 = -2$.

LEMMA 2.2. For each i ($1 \leq i \leq n$), the center of the blow-up μ_i lies on the singular locus of $(\mu_{i-1})_*\tilde{D}$.

PROOF. Since $-1 = K_{\tilde{X}}E_i = \tilde{H}E_i - \tilde{D}E_i$ and $\tilde{H}E_i > 0$, we have $\tilde{D}E_i \geq 2$, which implies the Lemma.

LEMMA 2.3. $\dim R^1\pi_*\mathcal{O}_{\tilde{X}}=5$.

PROOF. From the exact sequence associated with the Leray spectral sequence:

$$0 \rightarrow H^1(X, \mathcal{O}_X) \rightarrow H^1(\tilde{X}, \mathcal{O}_{\tilde{X}}) \rightarrow R^1\pi_*\mathcal{O}_{\tilde{X}} \rightarrow H^2(X, \mathcal{O}_X) \rightarrow H^2(\tilde{X}, \mathcal{O}_{\tilde{X}}) \rightarrow 0,$$

we have $\dim R^1\pi_*\mathcal{O}_{\tilde{X}}=4+q(\tilde{X})-p_g(\tilde{X})=5$.

COROLLARY 2.4. Let $\tilde{D}=\tilde{D}_1+\cdots+\tilde{D}_s$ be the decomposition of \tilde{D} into its connected components. Then

$$\sum_{i=1}^s h^0(\mathcal{O}_{\tilde{D}_i})=5.$$

PROOF. Let y_i denote the singular point on X , which is obtained by contracting \tilde{D}_i . Let $Y_i \subset X$ be a Stein neighbourhood of y_i , $\tilde{Y}_i=\pi^{-1}(Y_i)$ and $\pi_i=\pi|_{\tilde{Y}_i}$. Note that $\omega_{\tilde{Y}_i} \simeq \mathcal{O}_{\tilde{Y}_i}(-\tilde{D}_i)$. Then we have $h^0(\mathcal{O}_{\tilde{D}_i})=\dim R^1(\pi_i)_*\mathcal{O}_{\tilde{Y}_i}$, so that the Corollary follows from Lemma 2.3. In fact, consider the natural exact sequence:

$$H^1(\mathcal{O}_{\tilde{Y}_i}(-\tilde{D}_i)) \rightarrow H^1(\mathcal{O}_{\tilde{Y}_i}) \rightarrow H^1(\mathcal{O}_{\tilde{D}_i}) \rightarrow 0.$$

We see that the first term vanishes by Grauert-Riemenschneider's theorem. The second term is isomorphic to $R^1(\pi_i)_*\mathcal{O}_{\tilde{Y}_i}$, and the third is dual to $H^0(\mathcal{O}_{\tilde{D}_i})$.

LEMMA 2.5. Let D be a connected divisor on \tilde{X} with negative intersection matrix. If all irreducible components of D are rational curves, then the contraction of D is at worst rational singularity.

PROOF. Since \tilde{X} contains no rational curve, the support of D is contained in a divisor which is contracted to a non-singular point (by μ). Hence the geometric genus of the contraction of D vanishes.

LEMMA 2.6. Let C and C' be irreducible curves on an abelian surface [resp. a hyperelliptic surface] S . Then

- (i) C^2 is an even non-negative integer,
- (ii) $C^2=0$ if and only if C is a non-singular elliptic curve,
- (iii) if the desingularization of C is an elliptic curve, then C itself is smooth,
- (iv) if $CC'=0$, then $C^2=C'^2=0$ and C and C' are algebraically equivalent [resp. if $CC'=0$, then $C^2=C'^2=0$ and $C \equiv qC'$ for some positive $q \in \mathbf{Q}$],
- (v) if C and C' are elliptic curves, then they intersect transversally.

PROOF. Since S has trivial or numerically trivial canonical sheaf, $p_a(C)=C^2/2+1$. Moreover S contains no rational curves, whence (i) and (ii). For (iii)–(v), we first assume that S is an abelian surface. Then (iii) is a special case of [Ue, Theorem 10.3]. (iv) If $CC'=0$, then neither C nor C' is ample, and so $C^2=C'^2=0$. Moreover C is a fiber of the quotient morphism $S \rightarrow S/C$. Hence C and C' are algebraically equivalent. Finally, if two elliptic curves C and C' intersect, then the morphism $C' \rightarrow S/C$ is finite and

unramified, and hence (v). Suppose next that S is a hyperelliptic surface. Then there is a finite unramified covering $f: \tilde{S} \rightarrow S$ where \tilde{S} is an abelian surface. Notice that any unramified cover of an elliptic curve is a disjoint union of elliptic curves. Hence (v) is clear. (iii) Let $\tilde{C} \rightarrow C$ denote the desingularization of C . Then $\tilde{S} \times_S \tilde{C}$ is the resolution of $f^{-1}(C)$. Since $\tilde{S} \times_S \tilde{C}$ is a disjoint union of non-singular elliptic curves, we see that $f^{-1}(C)$ itself is non-singular, and hence so is C . (iv) The former part is proved in the same way as in the abelian case. Moreover, $C^2 = C'^2 = 0$ and $CC' = 0$ mean that $[C]$ and $[C']$ are not linearly independent in $NS(S) \otimes \mathbb{Q}$.

§3. Reduction to the case with only elliptic singularities.

We use the notations in §2, and assume that \bar{X} is either an abelian or a hyperelliptic surface. In this section we will prove that there exists on X no singularity with arithmetic genus greater than 1. We first notice by Lemma 2.5 that every connected component of \tilde{D} contains a non-rational curve.

(3.1) Assume that there exists in \tilde{D} an irreducible curve D_1 with $p_a(D_1) \geq 2$. Let D denote the connected component of \tilde{D} containing D_1 and D' the sum of the other components: $\tilde{D} = D + D'$. By Lemma 1.1 and 2.1, we have $-10 \leq D^2 \leq -8$ and the arithmetic genus of the singularity corresponding to D is equal to 2. Hence, by Lemma 2.6 (iii), D_1 is a non-singular curve of genus 2 and the other components of D , if exist, are all non-singular rational curves. Since $0 \geq D'^2 \geq -2$, all singular points corresponding to D' are elliptic singularities.

Case 3.1.1. $D^2 = -8$: Remark 1.2 says that $D = 2D_1$ and $D_1^2 = -2$. The exact sequence

$$0 \rightarrow \mathcal{O}_{D_1}(-D_1) \rightarrow \mathcal{O}_{2D_1} \rightarrow \mathcal{O}_{D_1} \rightarrow 0$$

shows

$$h^0(\mathcal{O}_D) = h^0(\mathcal{O}_{D_1}(-D_1)) + h^0(\mathcal{O}_{D_1}) = h^0(\omega_{D_1}) + 1 = 3.$$

Hence, by Corollary 2.4 and its proof, it follows that D' corresponds to either one singular point with geometric genus equal to 2 or two singular points both of which have geometric genus 1. Hence $D'^2 = -2$, $n = 5$ (Corollary 1.4 and Lemma 2.1). By Lemma 2.6, we have $(\mu_* D)^2 > 0$ and $(\mu_* D)(\mu_* D') > 0$ and so there is a chain of rational curves $\Gamma_1, \dots, \Gamma_k$ on \tilde{X} such that $\Gamma_i \not\subset \tilde{D}$ ($1 \leq i \leq k$) and $\Gamma_1 D_1 > 0$, $\Gamma_k D' > 0$. Both Γ_1 and Γ_k are not exceptional for π , and hence are (-1) -curves by Lemma 2.1. For $2 \leq i \leq k-1$, Γ_i is either a (-1) -curve or else an exceptional curve for π , i.e., a (-2) -curve. Therefore it turns out that $k=1$: there exists a (-1) -curve Γ such that $\tilde{D}\Gamma = (2D_1 + D')\Gamma \geq 3$, which contradicts Lemma 2.1.

Case 3.1.2. $D^2 = -9$: Let $D = mD_1 + Y$ ($Y \not\subset D_1$). Since $2 = p_a(D_1) = (D_1^2 - D_1 D)/2 + 1$,

$$D_1 D = D_1^2 - 2 \leq -3. \quad (1)$$

Therefore

$$-9 = D^2 = mD_1 D + YD \leq mD_1 D = m(D_1^2 - 2) \leq -3m, \quad (2)$$

and so

$$1 \leq m \leq 3.$$

If $m=1$, then $D_1 D = D_1^2 + D_1 Y \geq D_1^2$, which contradicts (1).

If $m=2$, then, by (1), $-3 \geq D_1 D = 2D_1^2 + D_1 Y$ and so $D_1^2 \leq -2$. Hence $D_1^2 = -2$ by (2). Hence, using (1), we have

$$D_1 Y = D_1 D - 2D_1^2 = D_1^2 - 2 - 2D_1^2 = 0.$$

This implies $Y=0$ and so $D^2 = (2D_1)^2 = -8$, a contradiction.

Assume $m=3$. Then $D_1^2 = -1$ by (2) and hence $D_1 Y = D_1 D - 3D_1^2 = 0$ by (1), i.e. $Y=0$: $D = 3D_1$. Therefore

$$h^0(\mathcal{O}_D) \leq h^0(\mathcal{O}_{D_1}(-2D_1)) + h^0(\mathcal{O}_{2D_1}) = h^0(\omega_{D_1}) + h^0(\mathcal{O}_{D_1}(-D_1)) + h^0(\mathcal{O}_{D_1}) \leq 4.$$

Hence we have $D' \neq 0$, $n=5$, and are led to a contradiction as in Case 3.1.1.

Case 3.1.3. $D^2 = -10$: In this case $D = \tilde{D}$ and $n=5$. We set $\tilde{D} = mD_1 + Y$ as before, where $\tilde{D}Y=0$ and Y consists of (-2) -curves by Lemma 2.1. Note that (1) in the previous case holds as well. Hence we have

$$-10 = \tilde{D}^2 = mD_1 \tilde{D} = m(D_1^2 - 2) \leq -3m,$$

therefore $m=1$ or 2 .

If $m=1$, then $D_1 \tilde{D} = -10$ and $D_1^2 = -8$, which is impossible because $D_1 \tilde{D} = D_1^2 + D_1 Y \geq D_1^2$.

If $m=2$, then $D_1 \tilde{D} = -5$ and $D_1^2 = -3$, hence $\tilde{D} = 2D_1 + Y$ and $D_1 Y = 1$. This implies that Y is a reduced irreducible (-2) -curve. But then we calculate

$$h^0(\mathcal{O}_{\tilde{D}}) = h^0(\mathcal{O}_{D_1}(-D_1 - Y)) + h^0(\mathcal{O}_{D_1 + Y}) = h^0(\omega_{D_1}) + 1 = 3,$$

and so get a contradiction with Corollary 2.4.

Hence we have proved with Lemma 2.6 that every non-rational component of \tilde{D} is a non-singular elliptic curve.

(3.2) Suppose that \tilde{D} has a connected component D which contains two distinct non-singular elliptic curves D_1 and D_2 . Then D corresponds to a singularity with $p_a=2$ (Lemma 1.1). We set $\tilde{D} = D + D'$.

Case 3.2.1. $D^2 = -8$: We can show a contradiction in a similar way as in Case 3.1.1, by taking a chain of reduced curves in D connecting D_1 and D_2 instead of D_1 .

Case 3.2.2. $D^2 = -9$: Set $D = m_1D_1 + m_2D_2 + Y$, $Y \not\geq D_1, D_2$.

From

$$D_i^2 = D_iD = m_iD_i^2 + m_jD_1D_2 + D_iY,$$

we have

$$(1 - m_i)D_i^2 = m_jD_1D_2 + D_iY \tag{3}$$

where $\{i, j\} = \{1, 2\}$. Since D is connected, the right hand side of (3) is positive, and so $m_i \geq 2$ ($i = 1, 2$). Also we have

$$m_1D_1^2 + m_2D_2^2 = (m_1D_1 + m_2D_2)D = D^2 - DY \geq D^2 = -9. \tag{4}$$

Consider first the case of $D_1D_2 > 0$. Then $D_1D_2 = 1$ since $p_a = 2$.

We first show that $D_i^2 \leq -2$ ($i = 1, 2$). Assume to the contrary, say $D_1^2 = -1$. Then (3) is reduced to

$$m_1 - 1 = m_2 + D_1Y, \tag{5}$$

$$(1 - m_2)D_2^2 = m_1 + D_2Y,$$

hence

$$(1 - m_2)D_2^2 = m_2 + 1 + (D_1 + D_2)Y. \tag{6}$$

From (5), we get $m_1 \geq m_2 + 1$. (6) implies $D_2^2 \leq -2$, but if $D_2^2 = -2$, then $m_2 \geq 3$, and so $m_1 \geq 4$, which contradicts (4). Therefore the unique possibility is $D_2^2 = -3$, $m_1 = 3$, $m_2 = 2$ and $D_1Y = D_2Y = 0$, i.e. $D = 3D_1 + 2D_2$ with $D_1^2 = -1$, $D_2^2 = -3$. But then we have

$$\begin{aligned} h^0(\mathcal{O}_D) &\leq h^0(\mathcal{O}_{D_1}(-2D_1 - 2D_2)) + h^0(\mathcal{O}_{D_2}(-2D_1 - D_2)) + h^0(\mathcal{O}_{D_1}(-D_1 - D_2)) \\ &\quad + h^0(\mathcal{O}_{D_1 + D_2}) \leq 4. \end{aligned}$$

Hence it follows $D' \neq 0$ and $n = 5$, and then a contradiction as in Case 3.1.1. Thus we obtain $D_1^2, D_2^2 \leq -2$.

By (4) we have $D_1^2 = D_2^2 = -2$, $m_1 = m_2 = 2$ and $DY = -1$. But then (3) implies $D_1Y = D_2Y = 0$ and hence $Y = 0$, a contradiction.

Therefore we obtain $D_1D_2 = 0$, in particular $D_1Y > 0$, $D_2Y > 0$. Then, by (3),

$$(1 - m_i)D_i^2 = D_iY \quad (i = 1, 2). \tag{7}$$

If $D_1Y = 1$, then $m_1 = 2$, $D_1^2 = -1$, and there exists a unique component Y_1 of Y such that $Y_1D_1 = 1$ and that the multiplicity of Y_1 in Y is 1. Therefore

$$\begin{aligned} -2 &= Y_1^2 - Y_1D = Y_1(-2D_1 - m_2D_2 - (Y - Y_1)) \\ &= -2 - Y_1(m_2D_2 + (Y - Y_1)) < -2. \end{aligned}$$

Hence $D_1Y \geq 2$, $D_2Y \geq 2$.

Suppose $D_i^2 = -1$. Then, since $(\mu_*D_i)^2 = 0$ (Lemma 2.6 (ii), (iii)), there exists a

unique component Y_i of Y such that $D_i Y_i = 1$ and every other component of Y is disjoint from D_i . Moreover, we see from (7) that the multiplicity of Y_i in Y is $m_i - 1$, and that $m_i \geq 3$ since $D_i Y \geq 2$. We note that $(\mu_* D_1)(\mu_* D_2) > 0$. Hence, if furthermore $D_1^2 = D_2^2 = -1$, we get $m_1 = m_2 \geq 3$ because in this situation $Y_1 = Y_2$ in the notation above. Therefore, assuming $D_1^2 \geq D_2^2$ in general, we have by (4) and (7) that there are the following possibilities:

	D_1^2	D_2^2	m_1	m_2	DY
(i)	-1	-1	3	3	-3
(ii)	-1	-1	4	4	-1
(iii)	-1	-2	3	2	-2
(iv)	-1	-2	3	3	0
(v)	-1	-3	3	2	0
(vi)	-1	-2	4	2	-1
(vii)	-1	-2	5	2	0
(viii)	-2	-2	2	2	-1

In (i) and (ii), there is a curve Y_1 of multiplicity $m_1 - 1 \geq 2$ in Y with $D_1 Y_1 = D_2 Y_1 = 1$. Note that $Y_1^2 \leq -3$, since if $Y_1^2 = -2$ then $0 = DY_1 \geq 2m_1 + (m_1 - 1)(-2) = 2$. Hence (ii) is impossible, and in (i) we have $Y_1^2 = -3$ by $DY = -3$. But then $-1 = DY_1 \geq 2m_1 + (m_1 - 1)(-3) = 0$. In (iv) and (v), all components of Y are (-2) -curves. There is a component Y_1 with multiplicity $m_1 - 1 = 2$ in Y and $D_1 Y_1 = 1$. Hence $(D - 3D_1 - 2Y_1)Y_1 = 1$, and so there is a unique component Y_2 with multiplicity 1 in Y such that $Y_1 Y_2 = 1$. This implies $D = 3D_1 + 2Y_1 + Y_2$, which is absurd. In (vi)–(viii), where $D_2^2 = -2$ and $m_2 = 2$, there is a curve Y_2 of multiplicity 2 in Y with $D_2 Y_2 = 1$. Since $DY \geq -1$, Y_2 is a (-2) -curve. Hence there is a unique curve Y_3 in $D - 2D_2 - 2Y_2$ of multiplicity 2 in it with $Y_2 Y_3 = 1$, Y_3 is a (-2) -curve if $Y_3 \leq Y$. Proceeding in this way, we find in (vi) and (vii) an infinite sequence Y_2, Y_3, \dots in Y ; in (viii) $D = 2(D_2 + Y_2 + \dots + D_1)$, contradicting $D^2 = -9$. Therefore it only remains the case (iii). Since then also $D_2^2 = -2$ and $m_2 = 2$, we can start from D_2 in the same way as above and deduce $D = 3D_1 + 2Y_1 + \dots + 2Y_k + 2D_2$, where $k \geq 1$, $Y_1^2 = -3$, $Y_2^2 = \dots = Y_k^2 = -2$ and $D_1, Y_1, \dots, Y_k, D_2$ form a chain. Hence we obtain

$$\begin{aligned} h^0(\mathcal{O}_D) &\leq h^0(\mathcal{O}_{D_1}(-2D_1 - 2Y_1 - \dots - 2Y_k - 2D_2)) \\ &\quad + h^0(\mathcal{O}_{D_1+Y_1+\dots+Y_k+D_2}(-D_1 - Y_1 - \dots - D_2)) + h^0(\mathcal{O}_{D_1+Y_1+\dots+Y_k+D_2}) \\ &\leq 4, \end{aligned}$$

and so $D' \neq 0$, $n = 5$, hence a contradiction as in Case 3.1.1.

Case 3.2.3. $D^2 = -10$: Note first that $D = \tilde{D}$ and $n = 5$. Set $\tilde{D} = m_1 D_1 + m_2 D_2 + Y$ as in Case 3.2.2. Then we have as before

$$\begin{aligned} (1 - m_i)D_i^2 &= m_j D_1 D_2 + D_i Y, \quad \{i, j\} = \{1, 2\}, \\ m_1, m_2 &\geq 2, \end{aligned} \tag{8}$$

and since Y consists of (-2) -curves by Lemma 2.1,

$$m_1 D_1^2 + m_2 D_2^2 = (m_1 D_1 + m_2 D_2) \tilde{D} = \tilde{D}^2 - \tilde{D} Y = \tilde{D}^2 = -10. \tag{9}$$

Suppose $D_1 D_2 > 0$. Then $D_1 D_2 = 1$ since $p_a = 2$, and so (8) is rewritten as

$$\begin{aligned} (1 - m_i) D_i^2 &= m_j + D_i Y, & \{i, j\} &= \{1, 2\}, \\ m_1, m_2 &\geq 2. \end{aligned} \tag{10}$$

If $D_1^2 = -1$, then $m_1 = m_2 + 1 + D_1 Y$ by (10). In particular $D_2^2 \leq -2$. If furthermore $D_2^2 \leq -3$, then $D_2^2 = -3$, $m_1 = 4$ and $m_2 = 2$ by (9), which is not compatible with (10). Hence $D_2^2 = -2$, and we see with (9) and (10) that only $(m_1, m_2) = (4, 3)$ is possible. The case of $D_i^2 \leq -2$ is easier. Then, assuming $D_1^2 \geq D_2^2$, there are two possibilities:

	D_1^2	D_2^2	m_1	m_2	$D_1 Y$	$D_2 Y$
(i)	-1	-2	4	3	0	0
(ii)	-2	-3	2	2	0	1

In (i) we get $Y = 0$: $\tilde{D} = 4D_1 + 3D_2$. But then \tilde{D} can not be obtained from \bar{X} by more than 3 blow-ups (Lemma 2.2 and Lemma 2.6 (ii), (iii)). In (ii) Y is a reduced irreducible (-2) -curve since $D_2 Y = 1$ and $m_2 = 2$. This implies

$$h^0(\mathcal{O}_{\tilde{D}}) = h^0(\mathcal{O}_{D_1 + D_2}(-D_1 - D_2 - Y)) + h^0(\mathcal{O}_{D_1 + D_2 + Y}) = h^0(\omega_{D_1 + D_2}) + 1 = 3,$$

which is impossible by Corollary 2.4.

This proves $D_1 D_2 = 0$ and so by (8)

$$(1 - m_i) D_i^2 = D_i Y \quad (i = 1, 2).$$

Then we have, as in Case 3.2.2, $m_i \geq 2$; $m_i \geq 3$ if $D_i^2 = -1$; $m_1 = m_2$ if $D_1^2 = D_2^2 = -1$. We may assume that $D_1^2 \geq D_2^2$ and that $m_1 \leq m_2$ if $D_1^2 = D_2^2$. Then, by (9), the possibilities are as follows:

	D_1^2	D_2^2	m_1	m_2
(i)	-1	-1	5	5
(ii)	-1	-2	4	3
(iii)	-1	-2	6	2
(iv)	-1	-3	4	2
(v)	-2	-2	2	3
(vi)	-2	-3	2	2

For (i), (ii) and (iv), let $Y_0 = D_1, Y_1, \dots, Y_k, Y_{k+1} = D_2$ denote the chain of curves in \tilde{D} connecting D_1 and D_2 ($k \geq 1$), and l_j the multiplicity of Y_j in \tilde{D} . Since $D_1^2 = -1$, $l_1 = m_1 - 1$. Moreover, since Y_j is a (-2) -curve for $1 \leq j \leq k$, we have $l_j - 1 \geq l_{j+1}$ for $0 \leq j \leq k$, and so $m_1 - k - 1 \geq m_2$. Hence (i) and (ii) are impossible. In (iv) we have $k = 1$, hence $\tilde{D} = 4D_1 + 3Y_1 + 2D_2$. But then there exists a (-1) -curve Γ on \tilde{X} such that $\Gamma Y_1 > 0$ and so $\tilde{D}\Gamma = 3$, which contradicts Lemma 2.1. Replace D_1 and D_2 in (iii). Then for

each of the remaining cases we notice that $D_1^2 = -2$ and $m_1 = 2$. Let $Y_0, Y_1, \dots, Y_k, Y_{k+1}$ be as above. Then we have $\tilde{D} = 2D_1 + 2Y_1 + \dots + 2Y_k + 2D_2 + Y'$, where Y' consists of (-2) -curves and is disjoint from D_1, Y_1, \dots, Y_k . Hence only the case (vi) can occur. Since $D_2\tilde{D} = -3, D_2Y' = 1$, i.e., Y' is reduced and irreducible. But then there exists a (-1) -curve Γ on \tilde{X} such that $Y'\Gamma = 1$, hence $\tilde{D}\Gamma$ is odd, contradicting Lemma 2.1.

(3.3). By what we have proved, every connected component D of \tilde{D} is the union of an elliptic curve and some exceptional rational curves for μ . Let F be a non-trivial effective divisor on \tilde{X} with $\text{supp } F \subseteq \text{supp } D$. Then $F = \mu^*\bar{F} + \sum_{i=1}^n a_i E_i$ where $\bar{F} = \mu_* F$ and $a_i \in \mathbb{Z}$ ($1 \leq i \leq n$). Therefore

$$\begin{aligned} p_a(F) &= \frac{1}{2} \left(\mu^*\bar{F} + \sum_{i=1}^n a_i E_i \right) \left(\mu^*\bar{F} + \sum_{i=1}^n a_i E_i + \sum_{i=1}^n E_i \right) + 1 \\ &= \frac{1}{2} \bar{F}^2 + 1 - \frac{1}{2} \sum_{i=1}^n a_i (a_i + 1). \end{aligned}$$

Here \bar{F} is a multiple of an elliptic curve and so $\bar{F}^2 = 0$ by Lemma 2.6. Moreover $a(a+1) \geq 0$ for any integer a . Hence $p_a(F) \leq 1$, and so the singularity corresponding to D is elliptic.

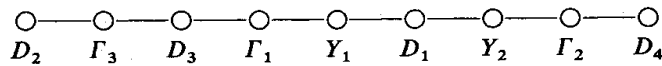
§4. The case with elliptic singularities.

We continue to use the notations in §2 and assume that \bar{X} is either an abelian or a hyperelliptic surface. To complete our proof of the Theorem, we will deduce a contradiction under the assumption that every connected component D of \tilde{D} corresponds to an elliptic singularity. Recall that then D consists of a non-singular elliptic curve and possibly some rational curves.

(4.1) Assume that there exists a connected component D of \tilde{D} which contains a rational curve. This is equivalent to that X has a singularity with $p_g \geq 2$. Set $\tilde{D} = D + D'$ and let D_1 denote the unique elliptic curve in D . Then D_1 is the minimal elliptic cycle of D . In general, we note that $\mu_* \tilde{D}$ is numerically equivalent to $\mu_* \tilde{H}$, and hence is connected and ample. In particular $D' \neq 0$.

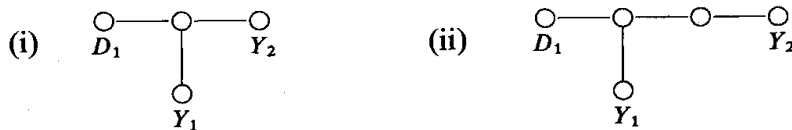
Suppose first that the rational components of D are not connected. Then $D_1^2 \leq -2$ and there are exactly two connected components of rational curves because the number of blow-ups n is bounded by 5 (Lemma 2.1). If one of them has length ≥ 2 , then the equality holds, the other has length 1, and every rational curve in D is a (-2) -curve. It follows that D can not be contracted to a numerically Gorenstein singularity (Theorem 1.3 (ii)). Hence we have that $D = 2D_1 + Y_1 + Y_2$, where Y_i is a rational curve with $D_1 Y_1 = D_1 Y_2 = 1, Y_1 Y_2 = 0$ and $D_1^2 = -2$, and that the corresponding singularity has a geometric genus equal to 2 (Theorem 1.3 (i), (iii)). Hence the sum of the geometric genera of singularities corresponding to D' is equal to 3 (Lemma 2.3). Moreover, if

$Y_1^2 \leq -3$, then $n=5$, which contradicts Lemma 2.1. Hence we have $Y_1^2 = Y_2^2 = -2$, and so there exist rational curves Γ_1 and Γ_2 such that $Y_i \Gamma_j = \delta_{ij}$. If $n=4$, then Γ_1 and Γ_2 are (-1) -curves. There is on \tilde{X} no rational curve other than Y_1, Y_2, Γ_1 and Γ_2 , and so D' consists of three disjoint elliptic curves: $D' = D_2 + D_3 + D_4$, each of which meets either Γ_1 or Γ_2 . We may assume that $D_2 \Gamma_1 > 0$ and $D_3 \Gamma_1 > 0$. But then $\mu_* D_2$ intersects $\mu_* D_3$ tangentially, contradicting Lemma 2.6 (v). If $n=5$, then Γ_1 and Γ_2 are also (-1) -curves by Lemma 2.1. There is another (-1) -curve Γ_3 which is disjoint from D, Γ_1 and Γ_2 . Moreover $D' = D_2 + D_3 + D_4$ as before. Since $\tilde{D} \Gamma_j = 2$ and $D_i \Gamma_j \leq 1$ ($1 \leq i \leq 4, 1 \leq j \leq 3$) by Lemma 2.1 and 2.6 (iii), we can assume that the dual graph of $D_1, \dots, D_4, Y_1, Y_2, \Gamma_1, \Gamma_2, \Gamma_3$ is as follows:



But then we obtain $(\mu_* D_2 + \mu_* D_3)^2 > 0$, $(\mu_* D_4)^2 = 0$ and $(\mu_* D_2 + \mu_* D_3)(\mu_* D_4) = 0$, a contradiction to Hodge Index Theorem. It follows that there is a unique rational component Y_1 in D which intersects D_1 .

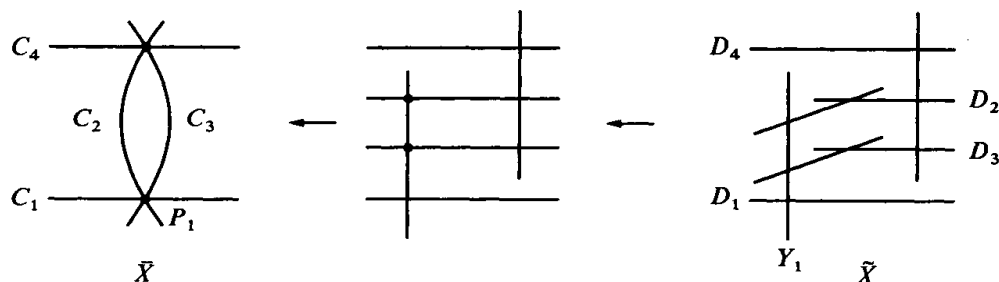
Suppose that there is a rational curve in D which intersects more than two other components in D . From Lemma 2.1, we can deduce that the dual graph of D is one of the following:



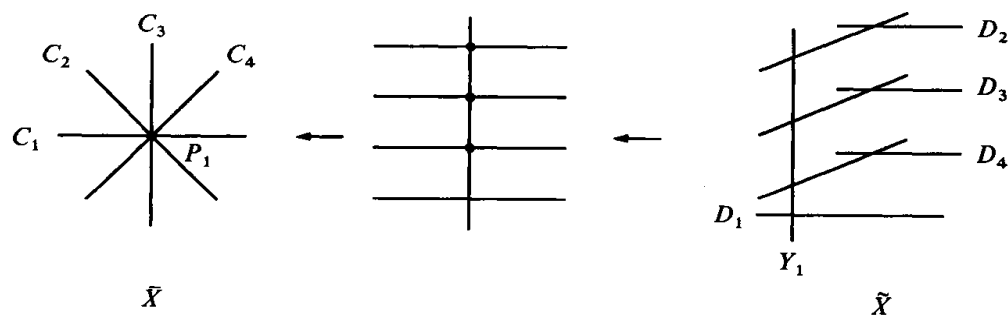
In (ii) we have $n=5$. Hence all rational components of D are (-2) -curves (Lemma 2.1), and so there is a unique rational curve Γ , which is a (-1) -curve, except the components of D . Γ intersects Y_1 or Y_2 , and also every elliptic curve D_i in D' . Therefore $\mu_* D_1$ and $\mu_* D_i$ can not intersect transversally, which contradicts Lemma 2.6 (v). In (i), if $n=4$, then there is a (-1) -curve Γ , which plays a similar role as Γ in (ii), and we are led to a contradiction. Assume $n=5$. Then every rational component in \tilde{D} is a (-2) -curve, hence again there is a unique (-1) -curve Γ_1 which intersects Y_1 or Y_2 . If $D' \Gamma_1 > 0$, we get a contradiction as in (ii). If $D' \Gamma_1 = 0$, then there is another (-1) -curve Γ_2 such that $D_1 \Gamma_2 > 1, D' \Gamma_2 > 0$. But the multiplicity of D_1 in D is greater than 1 (Theorem 1.3), and so we have $D \Gamma_2 \geq 2$, hence $\tilde{D} \Gamma_2 \geq 3$, which contradicts Lemma 2.1. Therefore we conclude that D consists of a chain D_1, Y_1, \dots, Y_m ($m \geq 1$) of an elliptic curve D_1 and rational curves Y_i .

Let $\tilde{D} = \tilde{D}_1 + \dots + \tilde{D}_s$ be the decomposition of \tilde{D} into its connected components with $\tilde{D}_1 = D$. Let p_i denote the geometric genus of the singularity corresponding to \tilde{D}_i, D_i the unique elliptic curve in \tilde{D}_i . We note that $D_i^2 = -1$ if $p_i \geq 2$ (Corollary 1.5 (i)), and that $D_i^2 \geq -3$ if $p_i = 1$, since then $\tilde{D}_i = D_i$ is to be contracted to a simple elliptic hypersurface singularity ([Sa]). Set $C_i = \mu_* D_i$, then C_i is also a non-singular elliptic curve. Recall that $C_1 + \dots + C_s$ is connected and ample. In particular $s \geq 2$ and there exists i

($2 \leq i \leq s$) such that $C_1 C_i > 0$. Since $D_1^2 = -1$, we see that $C_1 C_i = 1$. We may assume that the point $P_1 = C_1 \cdot C_i$ is the center of the first blow-up μ_1 . By our assumption, the proper transform Y_1 on \tilde{X} of $\mu_1^{-1}(P_1)$ is a component of D , and so $D_i^2 \leq -2$, and hence $p_i = 1$. Suppose that some p_j ($2 \leq j \leq s$) is greater than 1. Then we have $C_1 C_j = 0$, and so $C_i C_j > 0$ for any i such that $C_1 C_i > 0$, since $C_1 + C_i$ is ample (or by Lemma 2.6 (iv)). Hence our assumption that $p_j \geq 2$ implies $D_i^2 \leq -4$, which is impossible. Therefore we have $p_i = 1$ for $2 \leq i \leq s$, and so $p_1 + s - 1 = 5$ (Lemma 2.3). We may assume that $P_1 \in C_i$ ($1 \leq i \leq s_1$) and $P_1 \notin C_i$ ($s_1 + 1 \leq i \leq s$) for some s_1 ($2 \leq s_1 \leq s$). Since C_{s_1+1}, \dots, C_s are disjoint from C_1 , they are also disjoint from each other. Hence $C_2 \cap (\sum_{i=s_1+1}^s C_i)$ consists of greater than or equal to $s - s_1$ distinct points. This proves, with Corollary 1.5 (ii), $-3 \leq D_2^2 \leq -p_1 - (s - s_1) = s_1 - 6$. On the other hand, if $p_1 \geq 3$, then $Y_1^2 = -2$ by Corollary 1.5 (ii), and so $s_1 = 2$ (cf. Lemma 2.6 (v)), which is impossible. Hence we obtain $p_1 = 2$, $s = 4$ and $3 \leq s_1 \leq 4$. The last inequality implies $Y_1^2 \leq -3$ and hence $n \leq 4$ (Lemma 2.1). If $s_1 = 3$, then we have by Theorem 1.3 $\tilde{D}_1 = 2D_1 + Y_1$ with $Y_1^2 = -3$, $D_2^2 = D_3^2 = -3$ and $D_4^2 = -1$:



If $s_1 = 4$, then $\tilde{D}_1 = 2D_1 + Y_1$ with $Y_1^2 = -4$ and $D_2^2 = D_3^2 = D_4^2 = -2$:



In both cases we obtain $\tilde{D}^2 = -10$, contradictory to Lemma 2.1.

Thus we proved that \tilde{D} has no rational components.

(4.2) Finally let us consider the case where \tilde{D} consists of disjoint non-singular elliptic curves. Lemma 2.3 implies that \tilde{D} has five components. Set $\tilde{D} = \sum_{i=1}^5 D_i$ and $C = \mu_* D = \sum_{i=1}^5 C_i$ where $C_i = \mu_* D_i$. C_i and D_i are non-singular elliptic curves ($1 \leq i \leq 5$), D_i 's are disjoint, but C is connected. Let P_i ($1 \leq i \leq n$) denote the center of the blow-up μ_i . Then, by Lemma 2.2 and 2.6 (v), P_1, \dots, P_n are not infinitely near each other, hence

we may regard them as distinct points on \bar{X} . Set $k_i = \text{mult}_{P_i} C$. Then k_i is equal to the number of curves C_j which pass through P_i . Lemma 2.2 says $k_i \geq 2$ ($1 \leq i \leq n$), and we may assume $5 \geq k_1 \geq k_2 \geq \dots \geq k_n \geq 2$.

With these notations we have first from Lemma 2.1

$$\sum_{i=1}^n k_i = n + 5 \tag{11}$$

since $-n - 5 = \tilde{D}^2 = \sum_{j=1}^5 D_j^2 = \sum_{j=1}^5 C_j^2 - \sum_{i=1}^n k_i = -\sum_{i=1}^n k_i$. Next, let us show that $C_i C_j > 0$ for any i, j ($i \neq j$). Let s denote the maximal number of components in C , which are disjoint each other. We may assume that C_1, \dots, C_s are disjoint. Lemma 2.6 (iv) implies $C_j \equiv q_j C_1$ for $2 \leq j \leq s$, where q_j are some positive rational numbers. Hence we obtain by (11)

$$\begin{aligned} s(5-s) &\leq \left(\sum_{j=1}^s C_j \right) \left(\sum_{j=s+1}^5 C_j \right) \\ &= \sum_{i: P_i \in \bigcup_{j=1}^s C_j} (k_i - 1) \leq \sum_{i=1}^n (k_i - 1) = 5, \end{aligned}$$

and so $s = 1, 4$ or 5 . If $s = 5$, then C is not connected, which is excluded. If $s = 4$, then C_5 meets C_1, C_2, C_3 and C_4 , and hence $D_5^2 \leq -4$, which is impossible since D_5 corresponds to a simple elliptic hypersurface singularity. Therefore $s = 1$ as required.

Now we shall derive a contradiction for each n ($1 \leq n \leq 5$ by Lemma 2.1) from what we have proved.

$n = 1$: Clear since then $5 \geq k_1 = n + 5 = 6$.

$n = 2$: We have two possibilities: (i) $k_1 = 5, k_2 = 2$; (ii) $k_1 = 4, k_2 = 3$. In (i), all C_i pass through P_1 and we may assume that $P_2 \in C_1, C_2$ and $P_2 \notin C_3, C_4, C_5$. Then the intersection form of C_1, \dots, C_5 is as follows:

$$(C_i C_j) = \begin{pmatrix} 0 & 2 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix},$$

which is clearly non-degenerate. But the Picard number of \bar{X} is not greater than 4 since \bar{X} is an abelian or a hyperelliptic surface, and so we get a contradiction. In (ii), we may assume that $P_1 \notin C_1$, but then C_1 must meet every C_i ($2 \leq i \leq 5$) away from P_1 , that is at P_2 , a contradiction.

$n = 3$: There are two possibilities: (i) $k_1 = 4, k_2 = k_3 = 2$; (ii) $k_1 = k_2 = 3, k_3 = 2$. In both cases we may assume $P_1 \notin C_1$, and hence C_1 meets every C_i ($2 \leq i \leq 5$) at P_2 or P_3 , which is impossible for $k_2, k_3 < 5$ and $k_2 + k_3 < 6$.

$n = 4$: We have $k_1 = 3, k_2 = k_3 = k_4 = 2$. Assuming $P_1 \notin C_1$, we see that C_1 should meet every C_i ($2 \leq i \leq 5$) at either P_2, P_3 or P_4 , which is also impossible.

$n = 5$: In this last case, we have $k_1 = \cdots = k_5 = 2$ and so the five curves C_1, \cdots, C_5 should meet each other at only five points P_1, \cdots, P_5 with multiplicity 2, which is absurd. Thus we have completed our proof of the Main Theorem.

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Present Addresses:

IKU NAKAMURA
DEPARTMENT OF MATHEMATICS, HOKKAIDO UNIVERSITY,
SAPPORO, 060 JAPAN.

YUMIKO UMEZU
DEPARTMENT OF MATHEMATICS, SCHOOL OF MEDICINE, TOHO UNIVERSITY,
OMORI-NISHI, OTA-KU, TOKYO, 143 JAPAN.