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Nonexistence of positive solutions for a system of coupled fractional boundary value problems

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Abstract
We investigate the nonexistence of positive solutions for a system of nonlinear Riemann-Liouville fractional differential equations with coupled integral boundary conditions.

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Keywords: Riemann-Liouville fractional differential equations; integral boundary conditions; positive solutions; nonexistence

1 Introduction

Fractional differential equations describe many phenomena in various fields of engineering and scientific disciplines such as physics, biophysics, chemistry, biology (such as blood flow phenomena), economics, control theory, signal and image processing, aerodynamics, viscoelasticity, electromagnetics, and so on (see [1–6]). For some recent developments on the topic, see [7–21] and the references therein. Coupled boundary conditions appear in the study of reaction-diffusion equations and Sturm-Liouville problems, and they have applications in many fields of sciences and engineering such as thermal conduction and mathematical biology (see for example [22–28]).

In this paper, we consider the system of nonlinear fractional differential equations

$$(\mathrm{S})\qquad \begin{cases} D_{0+}^{\alpha}u(t)+\lambda f(t,u(t),v(t))=0, & t\in(0,1),\\ D_{0+}^{\beta}v(t)+\mu g(t,u(t),v(t))=0, & t\in(0,1),\end{cases}$$

with the coupled integral boundary conditions

$$(\mathrm{BC})\qquad \begin{cases} u(0)=u'(0)=\cdots=u^{(n-2)}(0)=0, & u(1)=\int_0^1 v(s)\,dH(s),\\ v(0)=v'(0)=\cdots=v^{(m-2)}(0)=0, & v(1)=\int_0^1 u(s)\,dK(s),\end{cases}$$

where $n-1<\alpha\le n$, $m-1<\beta\le m$, $n,m\in\mathbb{N}$, $n,m\ge 3$, D_{0+}^{α}, and D_{0+}^{β} denote the Riemann-Liouville derivatives of orders α and β, respectively, and the integrals from (BC) are Riemann-Stieltjes integrals.

We shall give sufficient conditions on λ, μ, f, and g such that (S)-(BC) has no positive solutions. By a positive solution of problem (S)-(BC) we mean a pair of functions $(u,v)\in$

$C([0,1]) \times C([0,1])$ satisfying (S) and (BC) with $u(t) \geq 0$, $v(t) \geq 0$ for all $t \in [0,1]$ and $(u,v) \neq (0,0)$. The existence of positive solutions for (S)-(BC) has been studied in [29] by using the Guo-Krasnosel'skii fixed point theorem. The multiplicity of positive solutions of the system (S) with $\lambda = \mu = 1, f(t,u,v) = \tilde{f}(t,v)$ and $g(t,u,v) = \tilde{g}(t,u)$ (denoted by (S_1)), with the boundary conditions (BC) was investigated in [30], where the nonlinearities f and g are nonsingular or singular functions. In [30], the authors used some theorems from the fixed point index theory and the Guo-Krasnosel'skii fixed point theorem. We also mention [31], where we studied the existence of positive solutions for (S)-(BC) ($u(t) \geq 0$, $v(t) \geq 0$ for all $t \in [0,1]$, and $u(t) > 0$, $v(t) > 0$ for all $t \in (0,1)$), where f and g are sign-changing functions. The systems (S) and (S_1) with uncoupled boundary conditions

$$(BC_1) \quad \begin{cases} u(0) = u'(0) = \cdots = u^{(n-2)}(0) = 0, & u(1) = \int_0^1 u(s)\,dH(s), \\ v(0) = v'(0) = \cdots = v^{(m-2)}(0) = 0, & v(1) = \int_0^1 v(s)\,dK(s), \end{cases}$$

were investigated in [32–35].

In Section 2, we present the necessary definitions and properties from the fractional calculus theory and some auxiliary results from [29], which investigates a nonlocal boundary value problem for fractional differential equations. In Section 3, we prove some nonexistence results for the positive solutions with respect to a cone for our problem (S)-(BC). Finally, two examples are given to illustrate our main results.

2 Auxiliary results

We present here the definitions, some lemmas from the theory of fractional calculus, and some auxiliary results from [29] that will be used to prove our main theorems.

Definition 2.1 The (left-sided) fractional integral of order $\alpha > 0$ of a function $f : (0,\infty) \to \mathbb{R}$ is given by

$$\left(I_{0+}^{\alpha} f\right)(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s)\,ds, \quad t > 0,$$

provided the right-hand side is pointwise defined on $(0,\infty)$, where $\Gamma(\alpha)$ is the Euler gamma function defined by $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t}\,dt$, $\alpha > 0$.

Definition 2.2 The Riemann-Liouville fractional derivative of order $\alpha \geq 0$ for a function $f : (0,\infty) \to \mathbb{R}$ is given by

$$\left(D_{0+}^{\alpha} f\right)(t) = \left(\frac{d}{dt}\right)^n \left(I_{0+}^{n-\alpha} f\right)(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_0^t \frac{f(s)}{(t-s)^{\alpha-n+1}}\,ds, \quad t > 0,$$

where $n = [\![\alpha]\!] + 1$, provided that the right-hand side is pointwise defined on $(0,\infty)$.

The notation $[\![\alpha]\!]$ stands for the largest integer not greater than α. If $\alpha = m \in \mathbb{N}$ then $D_{0+}^m f(t) = f^{(m)}(t)$ for $t > 0$, and if $\alpha = 0$ then $D_{0+}^0 f(t) = f(t)$ for $t > 0$.

We consider now the fractional differential system

$$\begin{cases} D_{0+}^{\alpha} u(t) + x(t) = 0, & t \in (0,1), \\ D_{0+}^{\beta} v(t) + y(t) = 0, & t \in (0,1), \end{cases} \tag{1}$$

with the coupled integral boundary conditions (BC), where $n-1<\alpha\leq n$, $m-1<\beta\leq m$, $n,m\in\mathbb{N}$, $n,m\geq 3$, and $H,K:[0,1]\to\mathbb{R}$ from (BC) are functions of bounded variation.

Lemma 2.1 ([29]) *If $H,K:[0,1]\to\mathbb{R}$ are functions of bounded variations, $\Delta=1-(\int_0^1\tau^{\alpha-1}\,dK(\tau))(\int_0^1\tau^{\beta-1}\,dH(\tau))\neq 0$ and $x,y\in C(0,1)\cap L^1(0,1)$, then the solution of problem* (1)-(BC) *is given by*

$$\begin{cases} u(t)=\int_0^1 G_1(t,s)x(s)\,ds+\int_0^1 G_2(t,s)y(s)\,ds, & t\in[0,1],\\ v(t)=\int_0^1 G_3(t,s)y(s)\,ds+\int_0^1 G_4(t,s)x(s)\,ds, & t\in[0,1],\end{cases} \tag{2}$$

where

$$\begin{cases} G_1(t,s)=g_1(t,s)+\frac{t^{\alpha-1}}{\Delta}(\int_0^1\tau^{\beta-1}\,dH(\tau))(\int_0^1 g_1(\tau,s)\,dK(\tau)),\\ G_2(t,s)=\frac{t^{\alpha-1}}{\Delta}\int_0^1 g_2(\tau,s)\,dH(\tau),\\ G_3(t,s)=g_2(t,s)+\frac{t^{\beta-1}}{\Delta}(\int_0^1\tau^{\alpha-1}\,dK(\tau))(\int_0^1 g_2(\tau,s)\,dH(\tau)),\\ G_4(t,s)=\frac{t^{\beta-1}}{\Delta}\int_0^1 g_1(\tau,s)\,dK(\tau),\end{cases} \tag{3}$$

for all $t,s\in[0,1]$ and

$$\begin{cases} g_1(t,s)=\frac{1}{\Gamma(\alpha)}\begin{cases} t^{\alpha-1}(1-s)^{\alpha-1}-(t-s)^{\alpha-1}, & 0\leq s\leq t\leq 1,\\ t^{\alpha-1}(1-s)^{\alpha-1}, & 0\leq t\leq s\leq 1,\end{cases}\\ g_2(t,s)=\frac{1}{\Gamma(\beta)}\begin{cases} t^{\beta-1}(1-s)^{\beta-1}-(t-s)^{\beta-1}, & 0\leq s\leq t\leq 1,\\ t^{\beta-1}(1-s)^{\beta-1}, & 0\leq t\leq s\leq 1.\end{cases}\end{cases} \tag{4}$$

Lemma 2.2 ([32]) *The functions g_1 and g_2 given by* (4) *have the properties*

(a) *$g_1,g_2:[0,1]\times[0,1]\to\mathbb{R}_+$ are continuous functions, and $g_1(t,s)>0$, $g_2(t,s)>0$ for all $(t,s)\in(0,1)\times(0,1)$;*

(b) *$g_1(t,s)\leq g_1(\theta_1(s),s)$, $g_2(t,s)\leq g_2(\theta_2(s),s)$, for all $(t,s)\in[0,1]\times[0,1]$;*

(c) *for any $c\in(0,1/2)$, we have*

$$\min_{t\in[c,1-c]} g_1(t,s)\geq\gamma_1 g_1(\theta_1(s),s),\qquad \min_{t\in[c,1-c]} g_2(t,s)\geq\gamma_2 g_2(\theta_2(s),s),$$

for all $s\in[0,1]$, where $\gamma_1=c^{\alpha-1}$, $\gamma_2=c^{\beta-1}$,

$$\theta_1(s)=\begin{cases}\frac{s}{1-(1-s)^{\frac{\alpha-1}{\alpha-2}}}, & s\in(0,1],\\ \frac{\alpha-2}{\alpha-1}, & s=0,\end{cases}$$

if $n-1<\alpha\leq n$, $n\geq 3$, and

$$\theta_2(s)=\begin{cases}\frac{s}{1-(1-s)^{\frac{\beta-1}{\beta-2}}}, & s\in(0,1],\\ \frac{\beta-2}{\beta-1}, & s=0,\end{cases}$$

if $m-1<\beta\leq m$, $m\geq 3$.

Lemma 2.3 ([29]) *If $H,K:[0,1]\to\mathbb{R}$ are nondecreasing functions, and $\Delta>0$, then G_i, $i=1,\ldots,4$ given by* (3) *are continuous functions on $[0,1]\times[0,1]$ and satisfy $G_i(t,s)\geq 0$*

for all $(t,s) \in [0,1] \times [0,1]$, $i = 1,\ldots,4$. *Moreover, if* $x, y \in C(0,1) \cap L^1(0,1)$ *satisfy* $x(t) \geq 0$, $y(t) \geq 0$ *for all* $t \in (0,1)$, *then the unique solution* (u,v) *of problem* (1)-(BC) (*given by* (2)) *satisfies* $u(t) \geq 0$, $v(t) \geq 0$ *for all* $t \in [0,1]$.

Lemma 2.4 ([29]) *Assume that* $H, K : [0,1] \to \mathbb{R}$ *are nondecreasing functions and* $\Delta > 0$. *Then the functions* G_i, $i = 1,\ldots,4$, *satisfy the inequalities*

(a_1) $G_1(t,s) \leq J_1(s)$, $\forall (t,s) \in [0,1] \times [0,1]$, *where*

$$J_1(s) = g_1\bigl(\theta_1(s),s\bigr) + \frac{1}{\Delta}\left(\int_0^1 \tau^{\beta-1}\,dH(\tau)\right)\left(\int_0^1 g_1(\tau,s)\,dK(\tau)\right);$$

(a_2) *for every* $c \in (0,1/2)$, *we have*

$$\min_{t\in[c,1-c]} G_1(t,s) \geq \gamma_1 J_1(s) \geq \gamma_1 G_1\bigl(t',s\bigr), \quad \forall t', s \in [0,1];$$

(b_1) $G_2(t,s) \leq J_2(s)$, $\forall (t,s) \in [0,1] \times [0,1]$, *where* $J_2(s) = \frac{1}{\Delta}\int_0^1 g_2(\tau,s)\,dH(\tau)$;

(b_2) *for every* $c \in (0,1/2)$, *we have*

$$\min_{t\in[c,1-c]} G_2(t,s) \geq \gamma_1 J_2(s) \geq \gamma_1 G_2\bigl(t',s\bigr), \quad \forall t', s \in [0,1];$$

(c_1) $G_3(t,s) \leq J_3(s)$, $\forall (t,s) \in [0,1] \times [0,1]$, *where*

$$J_3(s) = g_2\bigl(\theta_2(s),s\bigr) + \frac{1}{\Delta}\left(\int_0^1 \tau^{\alpha-1}\,dK(\tau)\right)\left(\int_0^1 g_2(\tau,s)\,dH(\tau)\right);$$

(c_2) *for every* $c \in (0,1/2)$, *we have*

$$\min_{t\in[c,1-c]} G_3(t,s) \geq \gamma_2 J_3(s) \geq \gamma_2 G_3\bigl(t',s\bigr), \quad \forall t', s \in [0,1];$$

(d_1) $G_4(t,s) \leq J_4(s)$, $\forall (t,s) \in [0,1] \times [0,1]$, *where* $J_4(s) = \frac{1}{\Delta}\int_0^1 g_1(\tau,s)\,dK(\tau)$;

(d_2) *for every* $c \in (0,1/2)$, *we have*

$$\min_{t\in[c,1-c]} G_4(t,s) \geq \gamma_2 J_4(s) \geq \gamma_2 G_4\bigl(t',s\bigr), \quad \forall t', s \in [0,1].$$

Lemma 2.5 ([29]) *Assume that* $H, K : [0,1] \to \mathbb{R}$ *are nondecreasing functions*, $\Delta > 0$, $c \in (0,1/2)$, *and* $x, y \in C(0,1) \cap L^1(0,1)$, $x(t) \geq 0$, $y(t) \geq 0$ *for all* $t \in (0,1)$. *Then the solution* $(u(t), v(t))$, $t \in [0,1]$ *of problem* (1)-(BC) *satisfies the inequalities*

$$\min_{t\in[c,1-c]} u(t) \geq \gamma_1 \max_{t'\in[0,1]} u\bigl(t'\bigr), \qquad \min_{t\in[c,1-c]} v(t) \geq \gamma_2 \max_{t'\in[0,1]} v\bigl(t'\bigr).$$

3 Main results

We present in this section intervals for λ and μ for which there exists no positive solution of problem (S)-(BC).

We present the assumptions that we shall use in the sequel.

(H1) $H, K : [0,1] \to \mathbb{R}$ are nondecreasing functions and $\Delta = 1 - (\int_0^1 \tau^{\alpha-1}\,dK(\tau))(\int_0^1 \tau^{\beta-1}\,dH(\tau)) > 0$.

(H2) The functions $f, g : [0,1] \times [0,\infty) \times [0,\infty) \to [0,\infty)$ are continuous.

For $c \in (0, 1/2)$, we introduce the following extreme limits:

$$f_0^s = \limsup_{u+v\to 0^+} \max_{t\in[0,1]} \frac{f(t,u,v)}{u+v}, \qquad g_0^s = \limsup_{u+v\to 0^+} \max_{t\in[0,1]} \frac{g(t,u,v)}{u+v},$$
$$f_0^i = \liminf_{u+v\to 0^+} \min_{t\in[c,1-c]} \frac{f(t,u,v)}{u+v}, \qquad g_0^i = \liminf_{u+v\to 0^+} \min_{t\in[c,1-c]} \frac{g(t,u,v)}{u+v},$$
$$f_\infty^s = \limsup_{u+v\to \infty} \max_{t\in[0,1]} \frac{f(t,u,v)}{u+v}, \qquad g_\infty^s = \limsup_{u+v\to \infty} \max_{t\in[0,1]} \frac{g(t,u,v)}{u+v},$$
$$f_\infty^i = \liminf_{u+v\to \infty} \min_{t\in[c,1-c]} \frac{f(t,u,v)}{u+v}, \qquad g_\infty^i = \liminf_{u+v\to \infty} \min_{t\in[c,1-c]} \frac{g(t,u,v)}{u+v}.$$

In the definitions of the extreme limits above, the variables u and v are nonnegative.

By using the functions G_i, $i = 1,\dots,4$ from Section 2 (Lemma 2.1), our problem (S)-(BC) can be written equivalently as the following nonlinear system of integral equations:

$$\begin{cases} u(t) = \lambda \int_0^1 G_1(t,s)f(s,u(s),v(s))\,ds + \mu \int_0^1 G_2(t,s)g(s,u(s),v(s))\,ds, & t \in [0,1], \\ v(t) = \mu \int_0^1 G_3(t,s)g(s,u(s),v(s))\,ds + \lambda \int_0^1 G_4(t,s)f(s,u(s),v(s))\,ds, & t \in [0,1]. \end{cases}$$

We consider the Banach space $X = C([0,1])$ with supremum norm $\|\cdot\|$, and the Banach space $Y = X \times X$ with the norm $\|(u,v)\|_Y = \|u\| + \|v\|$. We define the cone $P \subset Y$ by

$$P = \Bigl\{(u,v) \in Y; u(t) \ge 0, v(t) \ge 0, \forall t \in [0,1] \text{ and } \inf_{t\in[c,1-c]}\bigl(u(t)+v(t)\bigr) \ge \gamma \bigl\|(u,v)\bigr\|_Y\Bigr\},$$

where $\gamma = \min\{\gamma_1, \gamma_2\}$ and γ_1, γ_2 are defined in Section 2 (Lemma 2.2).

For $\lambda, \mu > 0$, we introduce the operators $T_1, T_2 : Y \to X$, and $\mathcal{T} : Y \to Y$ defined by

$$T_1(u,v)(t) = \lambda \int_0^1 G_1(t,s)f\bigl(s,u(s),v(s)\bigr)\,ds + \mu \int_0^1 G_2(t,s)g\bigl(s,u(s),v(s)\bigr)\,ds, \quad 0 \le t \le 1,$$
$$T_2(u,v)(t) = \mu \int_0^1 G_3(t,s)g\bigl(s,u(s),v(s)\bigr)\,ds + \lambda \int_0^1 G_4(t,s)f\bigl(s,u(s),v(s)\bigr)\,ds, \quad 0 \le t \le 1,$$

and $\mathcal{T}(u,v) = (T_1(u,v), T_2(u,v))$, $(u,v) \in Y$.

Lemma 3.1 ([29]) *If* (H1) *and* (H2) *hold, and* $c \in (0,1/2)$, *then* $\mathcal{T} : P \to P$ *is a completely continuous operator.*

The positive solutions of our problem (S)-(BC) coincide with the fixed points of the operator $\mathcal{T}$.

Theorem 3.1 *Assume that* (H1) *and* (H2) *hold, and* $c \in (0,1/2)$. *If* $f_0^s, f_\infty^s, g_0^s, g_\infty^s < \infty$, *then there exist positive constants* λ_0, μ_0 *such that, for every* $\lambda \in (0,\lambda_0)$ *and* $\mu \in (0,\mu_0)$, *the boundary value problem* (S)-(BC) *has no positive solution.*

Proof From the definitions of f_0^s, f_∞^s, g_0^s, g_∞^s, which are finite, we deduce that there exist $M_1, M_2 > 0$ such that

$$f(t,u,v) \le M_1(u+v), \qquad g(t,u,v) \le M_2(u+v), \quad \forall t \in [0,1],\ u,v \ge 0.$$

We define $\lambda_0 = \min\{\frac{1}{4M_1A}, \frac{1}{4M_1D}\}$, $\mu_0 = \min\{\frac{1}{4M_2B}, \frac{1}{4M_2C}\}$, where $A = \int_0^1 J_1(s)\,ds$, $B = \int_0^1 J_2(s)\,ds$, $C = \int_0^1 J_3(s)\,ds$, $D = \int_0^1 J_4(s)\,ds$. We shall show that, for every $\lambda \in (0,\lambda_0)$ and $\mu \in (0,\mu_0)$, problem (S)-(BC) has no positive solution.

Let $\lambda \in (0,\lambda_0)$ and $\mu \in (0,\mu_0)$. We suppose that (S)-(BC) has a positive solution $(u(t),v(t))$, $t \in [0,1]$. Then by using Lemma 2.4, we obtain

$$\begin{aligned} u(t) &= \bigl(T_1(u,v)\bigr)(t) = \lambda \int_0^1 G_1(t,s)f\bigl(s,u(s),v(s)\bigr)\,ds + \mu \int_0^1 G_2(t,s)g\bigl(s,u(s),v(s)\bigr)\,ds \\ &\le \lambda \int_0^1 J_1(s)f\bigl(s,u(s),v(s)\bigr)\,ds + \mu \int_0^1 J_2(s)g\bigl(s,u(s),v(s)\bigr)\,ds \\ &\le \lambda M_1 \int_0^1 J_1(s)\bigl(u(s)+v(s)\bigr)\,ds + \mu M_2 \int_0^1 J_2(s)\bigl(u(s)+v(s)\bigr)\,ds \\ &\le \lambda M_1 \int_0^1 J_1(s)\bigl(\|u\|+\|v\|\bigr)\,ds + \mu M_2 \int_0^1 J_2(s)\bigl(\|u\|+\|v\|\bigr)\,ds \\ &= (\lambda M_1 A + \mu M_2 B)\bigl\|(u,v)\bigr\|_Y, \quad \forall t \in [0,1]. \end{aligned}$$

Therefore, we conclude

$$\|u\| \le (\lambda M_1 A + \mu M_2 B)\bigl\|(u,v)\bigr\|_Y < (\lambda_0 M_1 A + \mu_0 M_2 B)\bigl\|(u,v)\bigr\|_Y \le \frac{1}{2}\bigl\|(u,v)\bigr\|_Y. \tag{5}$$

In a similar manner, we obtain

$$\begin{aligned} v(t) &= \bigl(T_2(u,v)\bigr)(t) = \mu \int_0^1 G_3(t,s)g\bigl(s,u(s),v(s)\bigr)\,ds + \lambda \int_0^1 G_4(t,s)f\bigl(s,u(s),v(s)\bigr)\,ds \\ &\le \mu \int_0^1 J_3(s)g\bigl(s,u(s),v(s)\bigr)\,ds + \lambda \int_0^1 J_4(s)f\bigl(s,u(s),v(s)\bigr)\,ds \\ &\le \mu M_2 \int_0^1 J_3(s)\bigl(u(s)+v(s)\bigr)\,ds + \lambda M_1 \int_0^1 J_4(s)\bigl(u(s)+v(s)\bigr)\,ds \\ &\le \mu M_2 \int_0^1 J_3(s)\bigl(\|u\|+\|v\|\bigr)\,ds + \lambda M_1 \int_0^1 J_4(s)\bigl(\|u\|+\|v\|\bigr)\,ds \\ &= (\mu M_2 C + \lambda M_1 D)\bigl\|(u,v)\bigr\|_Y, \quad \forall t \in [0,1]. \end{aligned}$$

Therefore, we deduce

$$\|v\| \le (\mu M_2 C + \lambda M_1 D)\bigl\|(u,v)\bigr\|_Y < (\mu_0 M_2 C + \lambda_0 M_1 D)\bigl\|(u,v)\bigr\|_Y \le \frac{1}{2}\bigl\|(u,v)\bigr\|_Y. \tag{6}$$

Hence, by (5) and (6), we conclude

$$\bigl\|(u,v)\bigr\|_Y = \|u\| + \|v\| < \frac{1}{2}\bigl\|(u,v)\bigr\|_Y + \frac{1}{2}\bigl\|(u,v)\bigr\|_Y = \bigl\|(u,v)\bigr\|_Y,$$

which is a contradiction. So the boundary value problem (S)-(BC) has no positive solution. □

Theorem 3.2 *Assume that* (H1) *and* (H2) *hold, and* $c \in (0,1/2)$. *If* $f_0^i, f_\infty^i > 0$ *and* $f(t,u,v) > 0$ *for all* $t \in [c,1-c]$, $u \ge 0$, $v \ge 0$, $u+v > 0$, *then there exists a positive constant* $\tilde{\lambda}_0$ *such that, for every* $\lambda > \tilde{\lambda}_0$ *and* $\mu > 0$, *the boundary value problem* (S)-(BC) *has no positive solution.*

Proof From the assumptions of the theorem, we deduce that there exists $m_1 > 0$ such that $f(t,u,v) \geq m_1(u+v)$ for all $t \in [c, 1-c]$ and $u, v \geq 0$. We define $\tilde{\lambda}_0 = \min\{\frac{1}{\gamma\gamma_1 m_1 \widetilde{A}}, \frac{1}{\gamma\gamma_2 m_1 \widetilde{D}}\}$, where $\widetilde{A} = \int_c^{1-c} J_1(s)\,ds$ and $\widetilde{D} = \int_c^{1-c} J_4(s)\,ds$. We shall show that, for every $\lambda > \tilde{\lambda}_0$ and $\mu > 0$, problem (S)-(BC) has no positive solution.

Let $\lambda > \tilde{\lambda}_0$ and $\mu > 0$. We suppose that (S)-(BC) has a positive solution $(u(t), v(t))$, $t \in [0,1]$.

If $\widetilde{A} \geq \widetilde{D}$, then $\tilde{\lambda}_0 = \frac{1}{\gamma\gamma_1 m_1 \widetilde{A}}$, and therefore, we obtain

$$\begin{aligned} u(c) &= \bigl(T_1(u,v)\bigr)(c) = \lambda \int_0^1 G_1(c,s) f\bigl(s,u(s),v(s)\bigr)\,ds + \mu \int_0^1 G_2(c,s) g\bigl(s,u(s),v(s)\bigr)\,ds \\ &\geq \lambda \int_0^1 G_1(c,s) f\bigl(s,u(s),v(s)\bigr)\,ds \geq \lambda \int_c^{1-c} G_1(c,s) f\bigl(s,u(s),v(s)\bigr)\,ds \\ &\geq \lambda m_1 \int_c^{1-c} G_1(c,s)\bigl(u(s)+v(s)\bigr)\,ds \geq \lambda m_1 \gamma_1 \int_c^{1-c} J_1(s)\gamma\bigl(\|u\| + \|v\|\bigr)\,ds \\ &= \lambda m_1 \gamma\gamma_1 \widetilde{A} \bigl\|(u,v)\bigr\|_Y. \end{aligned}$$

Then we conclude

$$\|u\| \geq u(c) \geq \lambda m_1 \gamma\gamma_1 \widetilde{A} \bigl\|(u,v)\bigr\|_Y > \tilde{\lambda}_0 m_1 \gamma\gamma_1 \widetilde{A} \bigl\|(u,v)\bigr\|_Y = \bigl\|(u,v)\bigr\|_Y,$$

and so $\|(u,v)\|_Y = \|u\| + \|v\| \geq \|u\| > \|(u,v)\|_Y$, which is a contradiction.

If $\widetilde{A} < \widetilde{D}$, then $\tilde{\lambda}_0 = \frac{1}{\gamma\gamma_2 m_1 \widetilde{D}}$, and therefore, we deduce

$$\begin{aligned} v(c) &= \bigl(T_2(u,v)\bigr)(c) = \mu \int_0^1 G_3(c,s) g\bigl(s,u(s),v(s)\bigr)\,ds + \lambda \int_0^1 G_4(c,s) f\bigl(s,u(s),v(s)\bigr)\,ds \\ &\geq \lambda \int_0^1 G_4(c,s) f\bigl(s,u(s),v(s)\bigr)\,ds \geq \lambda \int_c^{1-c} G_4(c,s) f\bigl(s,u(s),v(s)\bigr)\,ds \\ &\geq \lambda m_1 \int_c^{1-c} G_4(c,s)\bigl(u(s)+v(s)\bigr)\,ds \geq \lambda m_1 \gamma_2 \int_c^{1-c} J_4(s)\gamma\bigl(\|u\| + \|v\|\bigr)\,ds \\ &= \lambda m_1 \gamma\gamma_2 \widetilde{D} \bigl\|(u,v)\bigr\|_Y. \end{aligned}$$

Then we conclude

$$\|v\| \geq v(c) \geq \lambda m_1 \gamma\gamma_2 \widetilde{D} \bigl\|(u,v)\bigr\|_Y > \tilde{\lambda}_0 m_1 \gamma\gamma_2 \widetilde{D} \bigl\|(u,v)\bigr\|_Y = \bigl\|(u,v)\bigr\|_Y,$$

and so $\|(u,v)\|_Y = \|u\| + \|v\| \geq \|v\| > \|(u,v)\|_Y$, which is a contradiction.

Therefore, the boundary value problem (S)-(BC) has no positive solution. □

Theorem 3.3 *Assume that* (H1) *and* (H2) *hold, and* $c \in (0, 1/2)$. *If* $g_0^i, g_\infty^i > 0$ *and* $g(t,u,v) > 0$ *for all* $t \in [c, 1-c]$, $u \geq 0$, $v \geq 0$, $u+v > 0$, *then there exists a positive constant* $\tilde{\mu}_0$ *such that, for every* $\mu > \tilde{\mu}_0$ *and* $\lambda > 0$, *the boundary value problem* (S)-(BC) *has no positive solution*.

Proof From the assumptions of the theorem, we deduce that there exists $m_2 > 0$ such that $g(t,u,v) \geq m_2(u+v)$ for all $t \in [c, 1-c]$ and $u, v \geq 0$. We define $\tilde{\mu}_0 = \min\{\frac{1}{\gamma\gamma_1 m_2 \widetilde{B}}, \frac{1}{\gamma\gamma_2 m_2 \widetilde{C}}\}$, where $\widetilde{B} = \int_c^{1-c} J_2(s)\,ds$ and $\widetilde{C} = \int_c^{1-c} J_3(s)\,ds$. We shall show that, for every $\mu > \tilde{\mu}_0$ and $\lambda > 0$, problem (S)-(BC) has no positive solution.

Let $\mu > \tilde{\mu}_0$ and $\lambda > 0$. We suppose that (S)-(BC) has a positive solution $(u(t), v(t))$, $t \in [0,1]$.

If $\widetilde{B} \geq \widetilde{C}$, then $\tilde{\mu}_0 = \frac{1}{\gamma\gamma_1 m_2 \widetilde{B}}$, and therefore we obtain

$$\begin{aligned} u(c) &= \bigl(T_1(u,v)\bigr)(c) = \lambda \int_0^1 G_1(c,s) f\bigl(s,u(s),v(s)\bigr)\,ds + \mu \int_0^1 G_2(c,s) g\bigl(s,u(s),v(s)\bigr)\,ds \\ &\geq \mu \int_0^1 G_2(c,s) g\bigl(s,u(s),v(s)\bigr)\,ds \geq \mu \int_c^{1-c} G_2(c,s) g\bigl(s,u(s),v(s)\bigr)\,ds \\ &\geq \mu m_2 \int_c^{1-c} G_2(c,s)\bigl(u(s)+v(s)\bigr)\,ds \geq \mu m_2 \gamma_1 \int_c^{1-c} J_2(s)\gamma\bigl(\|u\|+\|v\|\bigr)\,ds \\ &= \mu m_2 \gamma\gamma_1 \widetilde{B}\bigl\|(u,v)\bigr\|_Y. \end{aligned}$$

Then we conclude

$$\|u\| \geq u(c) \geq \mu m_2 \gamma\gamma_1 \widetilde{B}\bigl\|(u,v)\bigr\|_Y > \tilde{\mu}_0 m_2 \gamma\gamma_1 \widetilde{B}\bigl\|(u,v)\bigr\|_Y = \bigl\|(u,v)\bigr\|_Y,$$

and so $\|(u,v)\|_Y = \|u\| + \|v\| \geq \|u\| > \|(u,v)\|_Y$, which is a contradiction.

If $\widetilde{B} < \widetilde{C}$, then $\tilde{\mu}_0 = \frac{1}{\gamma\gamma_2 m_2 \widetilde{C}}$, and therefore, we deduce

$$\begin{aligned} v(c) &= \bigl(T_2(u,v)\bigr)(c) = \mu \int_0^1 G_3(c,s) g\bigl(s,u(s),v(s)\bigr)\,ds + \lambda \int_0^1 G_4(c,s) f\bigl(s,u(s),v(s)\bigr)\,ds \\ &\geq \mu \int_0^1 G_3(c,s) g\bigl(s,u(s),v(s)\bigr)\,ds \geq \mu \int_c^{1-c} G_3(c,s) g\bigl(s,u(s),v(s)\bigr)\,ds \\ &\geq \mu m_2 \int_c^{1-c} G_3(c,s)\bigl(u(s)+v(s)\bigr)\,ds \geq \mu m_2 \gamma_2 \int_c^{1-c} J_3(s)\gamma\bigl(\|u\|+\|v\|\bigr)\,ds \\ &= \mu m_2 \gamma\gamma_2 \widetilde{C}\bigl\|(u,v)\bigr\|_Y. \end{aligned}$$

Then we conclude

$$\|v\| \geq v(c) \geq \mu m_2 \gamma\gamma_2 \widetilde{C}\bigl\|(u,v)\bigr\|_Y > \tilde{\mu}_0 m_2 \gamma\gamma_2 \widetilde{C}\bigl\|(u,v)\bigr\|_Y = \bigl\|(u,v)\bigr\|_Y,$$

and so $\|(u,v)\|_Y = \|u\| + \|v\| \geq \|v\| > \|(u,v)\|_Y$, which is a contradiction.

Therefore, the boundary value problem (S)-(BC) has no positive solution. □

Theorem 3.4 *Assume that* (H1) *and* (H2) *hold, and* $c \in (0,1/2)$. *If* $f_0^i, f_\infty^i, g_0^i, g_\infty^i > 0$, *and* $f(t,u,v) > 0$, $g(t,u,v) > 0$ *for all* $t \in [c, 1-c]$, $u \geq 0$, $v \geq 0$, $u + v > 0$, *then there exist positive constants* $\hat{\lambda}_0$ *and* $\hat{\mu}_0$ *such that, for every* $\lambda > \hat{\lambda}_0$ *and* $\mu > \hat{\mu}_0$, *the boundary value problem* (S)-(BC) *has no positive solution.*

Proof From the assumptions of the theorem, we deduce that there exist $m_1, m_2 > 0$ such that $f(t,u,v) \geq m_1(u+v)$, $g(t,u,v) \geq m_2(u+v)$, for all $t \in [c, 1-c]$ and $u, v \geq 0$.

We define $\hat{\lambda}_0 = \frac{1}{2\gamma\gamma_1 m_1 \widetilde{A}}$ and $\hat{\mu}_0 = \frac{1}{2\gamma\gamma_2 m_2 \widetilde{C}}$, where $\widetilde{A} = \int_c^{1-c} J_1(s)\,ds$ and $\widetilde{C} = \int_c^{1-c} J_3(s)\,ds$. Then, for every $\lambda > \hat{\lambda}_0$ and $\mu > \hat{\mu}_0$, problem (S)-(BC) has no positive solution. Indeed, let

$\lambda > \hat{\lambda}_0$ and $\mu > \hat{\mu}_0$. We suppose that (S)-(BC) has a positive solution $(u(t), v(t))$, $t \in [0,1]$. In a similar manner to that used in the proofs of Theorems 3.2 and 3.3, we obtain

$$\|u\| \geq u(c) \geq \lambda m_1 \gamma \gamma_1 \widetilde{A} \|(u,v)\|_Y, \qquad \|v\| \geq v(c) \geq \mu m_2 \gamma \gamma_2 \widetilde{C} \|(u,v)\|_Y,$$

and so

$$\begin{aligned} \|(u,v)\|_Y &= \|u\| + \|v\| \geq \lambda m_1 \gamma \gamma_1 \widetilde{A} \|(u,v)\|_Y + \mu m_2 \gamma \gamma_2 \widetilde{C} \|(u,v)\|_Y \\ &> \hat{\lambda}_0 m_1 \gamma \gamma_1 \widetilde{A} \|(u,v)\|_Y + \hat{\mu}_0 m_2 \gamma \gamma_2 \widetilde{C} \|(u,v)\|_Y \\ &= \frac{1}{2} \|(u,v)\|_Y + \frac{1}{2} \|(u,v)\|_Y = \|(u,v)\|_Y, \end{aligned}$$

which is a contradiction. Therefore, the boundary value problem (S)-(BC) has no positive solution.

We can also define $\hat{\lambda}'_0 = \frac{1}{2\gamma\gamma_2 m_1 \widetilde{D}}$ and $\hat{\mu}'_0 = \frac{1}{2\gamma\gamma_1 m_2 \widetilde{B}}$, where $\widetilde{B} = \int_c^{1-c} J_2(s)\,ds$ and $\widetilde{D} = \int_c^{1-c} J_4(s)\,ds$. Then, for every $\lambda > \hat{\lambda}'_0$ and $\mu > \hat{\mu}'_0$, problem (S)-(BC) has no positive solution. Indeed, let $\lambda > \hat{\lambda}'_0$ and $\mu > \hat{\mu}'_0$. We suppose that (S)-(BC) has a positive solution $(u(t), v(t))$, $t \in [0,1]$. In a similar manner to that used in the proofs of Theorems 3.2 and 3.3, we obtain

$$\|v\| \geq v(c) \geq \lambda m_1 \gamma \gamma_2 \widetilde{D} \|(u,v)\|_Y, \qquad \|u\| \geq u(c) \geq \mu m_2 \gamma \gamma_1 \widetilde{B} \|(u,v)\|_Y,$$

and so

$$\begin{aligned} \|(u,v)\|_Y &= \|u\| + \|v\| \geq \mu m_2 \gamma \gamma_1 \widetilde{B} \|(u,v)\|_Y + \lambda m_1 \gamma \gamma_2 \widetilde{D} \|(u,v)\|_Y \\ &> \hat{\mu}'_0 m_2 \gamma \gamma_1 \widetilde{B} \|(u,v)\|_Y + \hat{\lambda}'_0 m_1 \gamma \gamma_2 \widetilde{D} \|(u,v)\|_Y \\ &= \frac{1}{2} \|(u,v)\|_Y + \frac{1}{2} \|(u,v)\|_Y = \|(u,v)\|_Y, \end{aligned}$$

which is a contradiction. Therefore, the boundary value problem (S)-(BC) has no positive solution. □

Remark 3.1 Under the assumptions of Theorem 3.4, we have the following observations.

(a) In the case $\widetilde{A} \geq \widetilde{D}$ and $\widetilde{B} \leq \widetilde{C}$, Theorem 3.4 gives some supplementary information for the domain of λ and μ for which there is no positive solution of (S)-(BC), in comparison to Theorems 3.2 and 3.3, because $\hat{\lambda}_0 = \frac{\tilde{\lambda}_0}{2}$ and $\hat{\mu}_0 = \frac{\tilde{\mu}_0}{2}$.

(b) In the case $\widetilde{A} \leq \widetilde{D}$ and $\widetilde{B} \geq \widetilde{C}$, Theorem 3.4 gives some supplementary information for the domain of λ and μ for which there is no positive solution of (S)-(BC), in comparison to Theorems 3.2 and 3.3, because $\hat{\lambda}'_0 = \frac{\tilde{\lambda}_0}{2}$ and $\hat{\mu}'_0 = \frac{\tilde{\mu}_0}{2}$.

4 Examples

Let $\alpha = 7/3$ ($n = 3$), $\beta = 5/2$ ($m = 3$), $H(t) = t^2$,

$$K(t) = \begin{cases} 0, & t \in [0,1/3), \\ 1, & t \in [1/3,2/3), \\ 3/2, & t \in [2/3,1], \end{cases}$$

for all $t \in [0,1]$. Then $\int_0^1 v(s)\,dH(s) = 2\int_0^1 sv(s)\,ds$ and $\int_0^1 u(s)\,dK(s) = u(\frac{1}{3}) + \frac{1}{2}u(\frac{2}{3})$.

We consider the system of fractional differential equations

$$(\mathrm{S}_0)\quad \begin{cases} D_{0+}^{7/3}u(t) + \lambda f(t,u(t),v(t)) = 0, & t \in (0,1), \\ D_{0+}^{5/2}v(t) + \mu g(t,u(t),v(t)) = 0, & t \in (0,1), \end{cases}$$

with the boundary conditions

$$(\mathrm{BC}_0)\quad \begin{cases} u(0) = u'(0) = 0, & u(1) = 2\int_0^1 sv(s)\,ds, \\ v(0) = v'(0) = 0, & v(1) = u(\frac{1}{3}) + \frac{1}{2}u(\frac{2}{3}). \end{cases}$$

Then we deduce $\Delta \approx 0.70153491 > 0$, $\theta_1(s) = \frac{1}{4-6s+4s^2-s^3}$, $\theta_2(s) = \frac{1}{3-3s+s^2}$ for all $s \in [0,1]$ (see also [29]). For the functions J_i, $i = 1,\dots,4$, we obtain

$$J_1(s) = \begin{cases} \frac{1}{\Gamma(7/3)}\{\frac{s(1-s)^{4/3}}{(4-6s+4s^2-s^3)^{1/3}} + \frac{2}{21\sqrt[3]{3}\Delta}[2(1-s)^{4/3} - 2(1-3s)^{4/3} \\ \quad + (2-2s)^{4/3} - (2-3s)^{4/3}]\}, & 0 \le s < 1/3, \\ \frac{1}{\Gamma(7/3)}\{\frac{s(1-s)^{4/3}}{(4-6s+4s^2-s^3)^{1/3}} + \frac{2}{21\sqrt[3]{3}\Delta}[2(1-s)^{4/3} + (2-2s)^{4/3} \\ \quad - (2-3s)^{4/3}]\}, & 1/3 \le s < 2/3, \\ \frac{1}{\Gamma(7/3)}\{\frac{s(1-s)^{4/3}}{(4-6s+4s^2-s^3)^{1/3}} + \frac{2}{21\sqrt[3]{3}\Delta}[2(1-s)^{4/3} + (2-2s)^{4/3}]\}, & 2/3 \le s \le 1, \end{cases}$$

$$J_2(s) = \frac{16}{3\sqrt{\pi}\Delta}\left\{\frac{1}{7}(1-s)^{3/2} - \frac{1}{7}(1-s)^{7/2} - \frac{1}{5}s(1-s)^{5/2}\right\}, \quad s \in [0,1],$$

$$J_3(s) = \frac{4}{3\sqrt{\pi}}\left\{\frac{s(1-s)^{3/2}}{(3-3s+s^2)^{1/2}} + \frac{4(1+\sqrt[3]{2})}{3\sqrt[3]{3}\Delta}\left[\frac{1}{7}(1-s)^{3/2} - \frac{1}{7}(1-s)^{7/2} - \frac{1}{5}s(1-s)^{5/2}\right]\right\},$$
$$s \in [0,1],$$

$$J_4(s) = \begin{cases} \frac{1}{6\sqrt[3]{3}\Delta\Gamma(7/3)}[2(1-s)^{4/3} - 2(1-3s)^{4/3} + (2-2s)^{4/3} - (2-3s)^{4/3}], & 0 \le s < 1/3, \\ \frac{1}{6\sqrt[3]{3}\Delta\Gamma(7/3)}[2(1-s)^{4/3} + (2-2s)^{4/3} - (2-3s)^{4/3}], & 1/3 \le s < 2/3, \\ \frac{1}{6\sqrt[3]{3}\Delta\Gamma(7/3)}[2(1-s)^{4/3} + (2-2s)^{4/3}], & 2/3 \le s \le 1. \end{cases}$$

For $c = 1/4$, we deduce $\gamma_1 = 4^{-4/3} \approx 0.15749013$, $\gamma_2 = \frac{1}{8}$, $\gamma = \gamma_2$. After some computations, we conclude $A = \int_0^1 J_1(s)\,ds \approx 0.15972386$, $\widetilde{A} = \int_{1/4}^{3/4} J_1(s)\,ds \approx 0.11335535$, $B = \int_0^1 J_2(s)\,ds \approx 0.05446581$, $\widetilde{B} = \int_{1/4}^{3/4} J_2(s)\,ds \approx 0.03892266$, $C = \int_0^1 J_3(s)\,ds \approx 0.09198682$, $\widetilde{C} = \int_{1/4}^{3/4} J_3(s)\,ds \approx 0.06559293$, $D = \int_0^1 J_4(s)\,ds \approx 0.12885992$, $\widetilde{D} = \int_{1/4}^{3/4} J_4(s)\,ds \approx 0.09158825$.

Example 1 We consider the functions

$$f(t,u,v) = \frac{\sqrt[3]{t}[p_1(u+v)+1](u+v)(q_1+\sin v)}{u+v+1},$$
$$g(t,u,v) = \frac{\sqrt{1-t}[p_2(u+v)+1](u+v)(q_2+\cos u)}{u+v+1},$$

for $t \in [0,1]$, $u,v \ge 0$, where $p_1,p_2 > 0$ and $q_1,q_2 > 1$.

We obtain $f_0^s = q_1$, $g_0^s = q_2 + 1$, $f_\infty^s = p_1(q_1+1)$, $g_\infty^s = p_2(q_2+1)$, and then we can apply Theorem 3.1. So we conclude that there exist $\lambda_0,\mu_0 > 0$ such that, for every $\lambda \in (0,\lambda_0)$ and $\mu \in (0,\mu_0)$, the boundary value problem (S_0)-(BC_0) has no positive solution. By Theorem 3.1, the positive constants λ_0 and μ_0 are given by $\lambda_0 = \min\{\frac{1}{4M_1A}, \frac{1}{4M_1D}\} = \frac{1}{4M_1A}$ and

$\mu_0 = \min\{\frac{1}{4M_2B}, \frac{1}{4M_2C}\} = \frac{1}{4M_2C}$. For example, if $p_1 = 356$, $p_2 = 482$, $q_1 = 2$, $q_2 = 3$, then we obtain $M_1 = 1{,}068$, $M_2 = 1{,}928$, $\lambda_0 \approx 1.46554 \cdot 10^{-3}$, and $\mu_0 \approx 1.40964 \cdot 10^{-3}$.

Because $f_0^i = \frac{1}{\sqrt[3]{4}}q_1$ and $f_\infty^i = \frac{1}{\sqrt[3]{4}}p_1(q_1 - 1)$, we can apply Theorem 3.2. Then there exists $\tilde{\lambda}_0 > 0$ such that, for every $\lambda > \tilde{\lambda}_0$ and $\mu > 0$, problem (S_0)-(BC_0) has no positive solution. From the proof of Theorem 3.2, the positive constant $\tilde{\lambda}_0$ is given by $\tilde{\lambda}_0 = \min\{\frac{1}{\gamma\gamma_1 m_1\widetilde{A}}, \frac{1}{\gamma\gamma_2 m_1\widetilde{D}}\}$. For example, if $p_1 = 356$ and $q_1 = 2$, then we deduce $m_1 = \sqrt[3]{2}$ and $\tilde{\lambda}_0 \approx 355.67332$.

Because $g_0^i = \frac{1}{2}(q_2 + 1)$ and $g_\infty^i = \frac{1}{2}p_2(q_2 - 1)$, we can also apply Theorem 3.3. Then there exists $\tilde{\mu}_0 > 0$ such that, for every $\mu > \tilde{\mu}_0$ and $\lambda > 0$, problem (S_0)-(BC_0) has no positive solution. From the proof of Theorem 3.3, the positive constant $\tilde{\mu}_0$ is given by $\tilde{\mu}_0 = \min\{\frac{1}{\gamma\gamma_1 m_2\widetilde{B}}, \frac{1}{\gamma\gamma_2 m_2\widetilde{C}}\}$. For example, if $p_2 = 482$ and $q_2 = 3$, then we obtain $m_2 = 2$ and $\tilde{\mu}_0 \approx 487.85746$.

Example 2 We consider the functions

$$f(t,u,v) = p_1 t^{\tilde{a}}\left(u^2 + v^2\right), \qquad g(t,u,v) = p_2(1-t)^{\tilde{b}}\left(e^{u+v} - 1\right), \quad t \in [0,1],\ u,v \geq 0,$$

where $\tilde{a}, \tilde{b}, p_1, p_2 > 0$.

Because $g_0^i = 2^{-2\tilde{b}}p_2$ and $g_\infty^i = \infty$, we can apply Theorem 3.3. Then there exists $\tilde{\mu}_0$ such that, for every $\mu > \tilde{\mu}_0$ and $\lambda > 0$, problem (S_0)-(BC_0) has no positive solution. For example, if $p_2 = \tilde{b} = 1$, then we deduce $m_2 = \frac{1}{4}$ and $\tilde{\mu}_0 \approx 3{,}902.85965$.

5 Conclusions

In this paper, we give sufficient conditions on λ, μ, f, and g such that the system of nonlinear Riemann-Liouville fractional differential equations (S) with the coupled integral boundary conditions (BC) has no positive solutions. Some examples which illustrate the obtained results are also presented.

Competing interests

The authors declare that no competing interests exist.

Authors' contributions

The authors contributed equally to this paper. Both authors read and approved the final manuscript.

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