Nonextensivity and the power-law distributions for the systems with self-gravitating long-range interactions.

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We study the nonextensivity and the power-law distributions for the many-body systems with the self-gravitating long-range interactions. We assume that the entropy and the energy are both nonextensive for such many-body systems. Namely, the entropy is Tsallis entropy and the energy takes the form [1]

$$\epsilon_q^{K+P}(q) = \frac{1}{(1-q)\,\beta} \left\{ 1 - \left[1 - (1-q)\,\frac{1}{2}\,m\,v^2 \right] \left[1 + \beta\,(1-q)\,m\,\psi \right] \right\} ,$$

The nonextensive form of total energy represents the long-range inter-particle interactions and the non-local correlations within the systems. By making a natural nonextensive generalization of the conservation of energy in the q-kinetic theory, we show that the power-law distributions can be determined by the q-equilibrium in the generalized Boltzmann equation, which describe the long-range nature of the interactions and the non-local correlations within the self-gravitating system with the inhomogeneous velocity dispersion. Correspondingly, the nonextensive parameter q can be uniquely derived from the microscopic dynamical equation and thus the physical interpretation of q different from unity can be clearly presented for the system. The relation is obtained between the nonextensive parameter $q \neq 1$ and the measurable quantities of the self-gravitating system [2]: the velocity dispersion σ and the mass density ρ,

$$1 - q = -2\,\sigma\,\frac{d\,\sigma}{d\,r} \Big/ \frac{d\,\varphi}{d\,r} = -2\,\sigma\,\frac{d\,\sigma}{d\,r} \Big/ \frac{G\,M(r)}{r^2} ,$$

or

$$1 - q = -\frac{\sigma\,\nabla^2\sigma + (\nabla\sigma)^2}{2\,\pi\,G\,\rho} ,$$

Furthermore, we can derive a nonlinear differential equation for the radial density dependence of the self-gravitating system with the inhomogeneous velocity dispersion, which generalizes the form of M.P. Leubner [3] and correctly describes the density distribution for the dark matter in the above physical situation. We use this q-kinetic approach to analyze the nonextensivity of self-gravitating collisionless systems and self-gravitating gaseous dynamical systems, giving the power-law distributions the clear physical meanings. We also can prove that the so-called stellar polytropes is actually not the polytropic distribution but the *Tsallis isothermal* spheres.

[1] C. Tsallis, J. Stat. Phys. **52**, 479 (1988).
[2] J.L. Du, ApSS **312**, 47 (2007).
[3] M.P. Leubner, ApJ **632**, L1 (2005).