NONHYPERBOLIC REFLECTION MOVEOUT FOR AZIMUTHALLY ANISOTROPIC MEDIA

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ABSTRACT

Reflection moveout in azimuthally anisotropic media is not only azimuthally dependent but it is also nonhyperbolic. As a result, the conventional hyperbolic normal moveout (NMO) equation parameterized by the exact NMO (stacking) velocity loses accuracy with increasing offset (i.e., spreadlength). This is true even for a single-homogeneous azimuthally anisotropic layer. The most common azimuthally anisotropic models used to describe fractured media are the horizontal transverse isotropy (HTI) and the orthorhombic (ORT) symmetry.

Here, we introduce an analytic representation for the quartic coefficient of the Taylor's series expansion of the two-way traveltime for pure mode reflection (i.e., no conversion) in arbitrary anisotropic media with arbitrary strength of anisotropy. In addition, we present an analytic description of the long-spread (large-offset) nonhyperbolic reflection moveout (NHMO). In multilayered azimuthally anisotropic media, the NMO (stacking) velocity and the quartic moveout coefficient can be calculated with good accuracy using the known averaging equations for VTI media. The interval NMO velocities and the interval quartic coefficients, however, are azimuthally dependent. This allows us to extend the nonhyperbolic moveout (NHMO) equation, originally designed for VTI media, to more general horizontally stratified azimuthally anisotropic media. As a result, our formalism allows rather simple transition from VTI to azimuthally anisotropic media.

Numerical examples from reflection moveout in orthorhombic media, the focus of this paper, show that this NHMO equation accurately describes the azimuthally-dependent P-wave reflection traveltimes, even on spreadlengths twice as large as the reflector depth. This work provides analytic insight into the behavior of nonhyperbolic moveout, and it has important applications in modeling and inversion of reflection moveout in az-

imuthally anisotropic media.

INTRODUCTION

Reflection moveout in anisotropic media is generally nonhyperbolic, unless the anisotropy is elliptical. Recent studies and case histories (Lynn *et al.*, 1996; Corrigan *et al.*, 1996) have shown that wave propagation signatures, including reflection moveout and amplitude-variation-with-offset (AVO), are sensitive to the presence of azimuthal anisotropy.

Hake *et al.* (1984) derived the quartic Taylor series term A_4 of $t^2 - x^2$ reflection-moveout curves for pure modes in TI media with a vertical axis of symmetry. Tsvankin and Thomsen (1994) recasted the quartic term of Hake *et al.* (1984) in a more compact form using Thomsen's (1986) notation. They also introduced a normalization factor for the quartic term that ensures the convergence of the Taylor series traveltime expansion at infinitely large horizontal offsets for VTI media. The reflection moveout expression of Tsvankin and Thomsen (1994) will serve as a basis for our study of nonhyperbolic reflection moveout in azimuthally anisotropic media.

Transverse isotropy with a horizontal axis of symmetry (HTI) is the simplest azimuthally anisotropic model caused by vertical penny-shaped cracks embedded in an isotropic matrix such as sandstones (Crampin, 1985; Thomsen, 1988). The HTI model, however, is too simplistic model to represent realistic fractured reservoirs (e.g., fractures with different cracks shapes, multi-fracture systems, vertical cracks in anisotropic matrix, etc). The orthorhombic (ORT) model, on the other hand, is a better representative of a wide class of fractured reservoirs (e.g., orthogonal fracture systems in a purely isotropic matrix such as sandstone, or a vertical fracture system in a transverse isotropic (TI) matrix with a vertical axis of symmetry such as shale). As a result, special attention is given to orthorhombic media in this work. The ORT model is defined through nine elastic coefficients (c_{ijkl}) and it contains three orthogonal symmetry planes [e.g., (x_1,x_3), (x_2,x_3), and (x_1,x_2) in Cartesian coordinates]. Here, we assume that the symmetry planes coincide with the coordinate system principal planes. A monoclinic model, on the other hand, is the lowest order of symmetry that contains a symmetry plane. A monoclinic symmetry is defined through thirteen elastic coefficients (c_{ijkl}) and it contains one symmetry (mirror-image) plane.

The analogy between HTI and VTI media allowed Rüger (1997) and Tsvankin (1997a) to introduce Thomsen's (1986) parameters for HTI media using exactly the same expressions as for vertical transverse isotropy. Moreover, taking advantage of the analogy between the VTI symmetry and the ORT model along the symmetry planes, Tsvankin (1997b) recasted the nine elastic coefficients, which define the orthorhombic model, and introduced a convenient notation in the same fashion that Thomsen's used for VTI media. This notation is quite convenient to describe reflection moveout in orthorhombic media. Tsvankin (1997a) introduced an analytic expression for the NMO velocity for pure modes of wave propagation in an HTI layer. Recently, Grechka and Tsvankin (1998) introduced an analytic representations for the NMO velocity in or-

thorhomic media. In other publication, Grechka *et al.* (1997) introduced a generalized Dix equation for the NMO velocity in arbitrary anisotropic media.

Reflection moveout for HTI media has been studied in detail by Al-Dajani and Tsvankin (1998). In their study, Al-Dajani and Tsvankin introduced an analytic representation for the quartic coefficient of the Taylor's series expansion of the two-way traveltime $[t^2(x^2)]$ for pure modes of wave propagation and arbitrary strength of anisotropy in HTI media. They showed that the nonhyperbolic moveout (NHMO) equation originally designed by Tsvankin and Thomsen (1994) for VTI is also accurate for HTI media, provided that both the NMO velocity and the quartic coefficient of the Taylor's series expansion of the two-way traveltime $(t^2(x^2))$ honor the azimuthal dependence for HTI media. Weak anisotropy approximation for reflection moveout in HTI media also was discussed by Thomsen (1988), Sena (1991) and Li and Crampin (1993); the latter paper also treats reflection traveltimes in an orthorhombic layer.

Nonhyperbolic moveout can hamper the estimation of normal-moveout velocity using conventional hyperbolic semblance analysis (e.g., Gidlow and Fatti, 1990). In layered media, however, the magnitude of nonhyperbolic moveout may increase due to vertical velocity variations and deviations of group-velocity vectors (rays) of reflected waves from the incidence plane. Even if the exact normal-moveout velocity and the quartic coefficient for a stack of layers have been extracted from the reflection moveout, it is not clear whether the Dix-type averaging of the *interval* NMO velocities and the *interval* quartic coefficients can be sufficiently accurate, considering the fact that the VTI averaging equations are no longer strictly valid outside the symmetry planes for azimuthally anisotropic media.

In addition, Alkhalifah and Tsvankin (1995) showed that for *P*-wave data all time processing steps (e.g., time migration) are governed by just two parameters—the zero-dip NMO velocity and the "anellipticity" coefficient η. Later, Alkhalifah (1997) developed a convenient nonhyperbolic semblance analysis to estimate both parameters for VTI media. Still, the VTI formalism is two-dimensional (2-D). In this paper, we demonstrate a 3-D analogous representation in terms of zero-dip NMO velocities and "anellipticity" coefficients η(s) for *P*-wave propagation in orthorhombic media. One of the recent studies that involves velocity analysis using non-hyperbolic reflection moveout in anisotropic media of arbitrary symmetry is discussed by Tabti and Rasolofosaon (1998).

Despite these developments, some important issues pertaining to moveout analysis for azimuthally anisotropic media remained unresolved. Among them is the analytic description of long-spread (nonhyperbolic) moveout in arbitrary azimuthally anisotropic media. In this paper, we introduce a general analytic representation of the quartic coefficient of the Taylor's series expansion of the two-way traveltime $[t^2(x^2)]$ for pure modes of wave propagation and for arbitrary anisotropic media with arbitrary strength of anisotropy. This azimuthally-dependent quartic moveout term is used to generalize the results of Al-Dajani and Tsvankin (1998) for HTI media to arbitrary azimuthally anisotropic media. The quadratic and quartic moveout coefficients in multilayered media

are obtained by the same averaging equations as for vertical transverse isotropy. Special attention is given in this study toward orthorhombic media. Numerical examples are also shown to verify the accuracy of our analytic representation for P-wave reflection moveout in orthorhombic media.

ANALYTIC APPROXIMATIONS OF REFLECTION MOVEOUT

In seismic data processing, reflection moveout on common-midpoint (CMP) gathers is conventionally approximated by a hyperbolic equation:

$$t^2 = t_0^2 + \frac{x^2}{V_{\mathrm{nmo}}^2}, \tag{1}$$

where t is the reflection traveltime at the source-receiver offset x, t_0 is the two-way zero-offset traveltime, and V_{nmo} is the normal-moveout (stacking) velocity defined in the zero-spread limit.

Equation (1) is strictly valid only for a homogeneous isotropic (or elliptical anisotropic) layer. The presence of layering and/or anisotropy leads to increasing deviation of the moveout curve from the short-spread hyperbola (1). However, for vertical transverse isotropy the hyperbolic moveout equation for P-waves usually provides sufficient accuracy on conventional-length spreads close to the reflector depth (Tsvankin and Thomsen, 1994).

Nonhyperbolic moveout on longer spreads can be described by a three-term Taylor series expansion (Taner and Koehler, 1969):

$$t^2 = t_0^2 + A_2 x^2 + A_4 x^4, \tag{2}$$

where $A_2 = 1/V_{\mathrm{nmo}}^2$, and A_4 is the quartic moveout coefficient. The parameter A_4 for pure modes in horizontally layered VTI media was given by Hake et al. (1984) and represented in a more compact form by Tsvankin and Thomsen (1994). Due to the influence of the x^4 term, the quartic equation (2) becomes divergent with increasing offset and can be replaced by a more accurate nonhyperbolic moveout equation developed by Tsvankin and Thomsen (1994):

$$t^2 = t_0^2 + A_2 x^2 + \frac{A_4 x^4}{1 + A x^2}, \tag{3}$$

where $A = A_4/(1/V_{\mathrm{hor}}^2 - 1/V_{\mathrm{nmo}}^2)$ and V_{hor} is the horizontal velocity. The denominator of the nonhyperbolic term ensures the convergence of this approximation at infinitely large horizontal offsets. As a result, equation (3) provides an accurate description of P-wave traveltimes on long CMP spreads (2–3 times as large as the reflector depth), even for models with pronounced nonhyperbolic moveout.

Although equation (3) was originally designed for vertical transverse isotropy, it is generic and it can be used in *arbitrary* anisotropic media if the appropriate coefficients

A_2, A_4, and A were found. As shown by Al-Dajani and Tsvankin (1998), equation (3) is indeed accurate in the case of HTI media. Our goal is to extend this nonhyperbolic moveout approximation to single and multilayered azimuthally anisotropic media. For a CMP line parallel to a symmetry plane no generalization is necessary, since the moveout in the symmetry plane can be obtained directly from the VTI equation (3). The analogy with VTI media also holds for through-going vertical symmetry planes of multilayered models. However, for CMP lines outside the vertical symmetry planes, it is necessary to obtain the azimuthally-dependent parameters of equation (3). Below, we accomplish this task for pure mode of wave propagation in horizontally layered arbitrary media with an arbitrary strength of anisotropy.

REFLECTION MOVEOUT COEFFICIENTS FOR A SINGLE LAYER

Here, we present the exact expressions for the coefficients of the moveout equations (1)–(3) for pure mode (i.e., no conversion) of wave propagation in an azimuthally anisotropic medium. A detail derivation is provided in Apendix A.

Normal-Moveout (NMO) Velocity

The quadratic moveout coefficient A_2 (or the NMO velocity) in a single arbitrary anisotropic layer was introduced by Grechka and Tsvankin (1998) for pure mode propagation and arbitrary strength of anisotropy as an ellipse. After recasting, it is given as:

$$
\begin{aligned}
A_2(\alpha) &= \frac{1}{V_{\text{nmo}}^2(\alpha)} \\
&= A_2^{(1)} \sin^2 \alpha + A_2^{(2)} \cos^2 \alpha \\
&\quad + A_2^{(x)} \sin \alpha \cos \alpha,
\end{aligned}
\tag{4}
$$

where the superscripts (1) and (2) indicate directions along the vertical planes (x_2, x_3) and (x_1, x_3), respectively. α is the azimuth of the CMP line from one of the vertical planes [e.g., (x_1, x_3) plane]. $A_2^{(x)}$ is a cross term which absorbs the mutual influence of all principal planes.

It turned out that for any horizontal, azimuthally anisotropic medium with a horizontal symmetry plane (e.g., HTI, orthorhombic, and monoclinic), $A_2^{(x)} = 0$ and equation (4) reduces to, after recasting

$$
V_{\text{nmo}}^2(\alpha) = \frac{V_{\text{nmo},1}^2 V_{\text{nmo},2}^2}{V_{\text{nmo},1}^2 \cos^2 \alpha + V_{\text{nmo},2}^2 \sin^2 \alpha},
\tag{5}
$$

where for P–wave propagation in an orthorhombic medium, the focus of this study, the

two semi-axes of the NMO ellipse are

$$V^2_{\mathrm{nmo},1} = \frac{1}{A_2^{(1)}} = V^2_{\mathrm{P0}}(1 + 2\delta^{(1)}),$$

and

$$V^2_{\mathrm{nmo},2} = \frac{1}{A_2^{(2)}} = V^2_{\mathrm{P0}}(1 + 2\delta^{(2)}).$$

V_{P0} is the vertical P-wave velocity, while $\delta^{(1)}$ and $\delta^{(2)}$ are dimensionless anisotropic parameters defined from the stiffnesses analogously to Thomsen's coefficients in VTI media (Tsvankin, 1997b). Equivalent representation for shear wave propagation in an orthorhombic medium exists [see Grechka and Tsvankin (1998) for further discussion].

NHMO Coefficient A_4

Application of the nonhyperbolic moveout equation (3) requires knowledge of the quartic moveout coefficient A_4. Here, we introduce an exact expression for the quartic term A_4 valid for any pure mode (non-converted) of wave propagation in a homogeneous, arbitrary anisotropic layer (see Appendix A).

An Arbitrary Medium

To obtain the NHMO coefficient for any (arbitrary) model, we express the two-way traveltime of any pure reflected mode as a double Taylor's series expansion in the vicinity of the zero-offset point [in Cartesian coordinates, (x_1, x_2)]. Keeping only the quartic and lower-order terms of the two-way traveltime squared, the quartic coefficient A_4 for pure mode reflection in homogeneous arbitrary anisotropic layer is given as:

$$
\begin{aligned}
A_4(\alpha) &= A_4^{(1)} \sin^4 \alpha + A_4^{(2)} \cos^4 \alpha + A_4^{(x)} \sin^2 \alpha \cos^2 \alpha \\
&+ A_4^{(x1)} \sin \alpha \cos^3 \alpha + A_4^{(x2)} \sin^3 \alpha \cos \alpha,
\end{aligned}
\tag{6}
$$

where α is the angle between the CMP line and one of the principle vertical planes [e.g., (x_1, x_3) plane]. $A_4^{(1)}$ and $A_4^{(2)}$ are the components of quartic coefficient along the two vertical principle planes [in Cartesian coordinates, (x_2, x_3) and (x_1, x_3), respectively]. $A_4^{(x)}$, $A_4^{(x1)}$, and $A_4^{(x2)}$ are cross terms which absorb the mutual influence from all principle planes. The components of the quartic coefficient are presented in terms of the medium parameters while the azimuthal dependence is governed by the trigonometric functions.

As we should expect, the more complicated the anisotropy model (lower symmetry), the more involved the quartic coefficient, given by equation (6), would be. For example, in the case of isotropy or elliptical anisotropy, the reflection moveout is hyperbolic

$(A_4 = 0)$. For VTI symmetry, on the other hand, equation (6) reduces to the known azimuthally independent quartic coefficient A_4 given by Hake $et\ al.$ (1984) and Tsvankin and Thomsen (1994). In the case of a horizontal HTI layer with a horizontal symmetry axis parallel to x_1 of the (x_1, x_3) plane, the components $A_4^{(x)}$, $A_4^{(x1)}$, $A_4^{(x2)}$ and $A_4^{(1)}$ vanish. Hence, equation (6) reduces to the expression of Al-Dajani and Tsvankin (1998):

$$A_4 = A_4^{(2)} \cos^4 \alpha,$$

where for P-waves

$$A_4^{(2)} = \frac{-2(\epsilon^{(V)} - \delta^{(V)})(1 + 2\delta^{(V)}/f^{(V)})}{t_0^2 V_{Pvert}^4 (1 + 2\delta^{(V)})^4}.$$

Here, t_0 is the zero-offset two-way traveltime and $f^{(V)} = 1 - V_{S\perp vert}^2 / V_{Pvert}^2$. $\epsilon^{(V)}$, $\delta^{(V)}$, $V_{S\perp vert}$, and V_{Pvert} are the HTI parameters defined analogously to Thomsen's parameters for VTI media [see Al-Dajani and Tsvankin (1998) for further discussion].

On the other hand, for a horizontal monoclinic medium which is the lowest order of symmetry that contains a symmetry plane, and for the case where we have a horizontal symmetry axis (e.g., x_1), $A_4^{(x2)} = 0$. Hence, the nonhyperbolic coefficient will reduce to only four components.

In the following, we focus our discussion on the orthorhombic symmetry.

An Orthorhombic (ORT) Medium

In the case of a single homogeneous ORT layer, both $A_4^{(x1)}$ and $A_4^{(x2)}$ vanish and equation (6) reduces to:

$$A_4(\alpha) = A_4^{(1)} \sin^4 \alpha + A_4^{(2)} \cos^4 \alpha + A_4^{(x)} \sin^2 \alpha \cos^2 \alpha, \tag{7}$$

where $A_4^{(1)}$ and $A_4^{(2)}$ are the components along the two vertical symmetry planes [e.g., (x_2, x_3) and (x_1, x_3) planes, respectively]. $A_4^{(x)}$ is a cross term which absorbs the mutual influence of all symmetry planes. Equation (7) is valid for orthorhombic models with arbitrary strength of anisotropy and can be used for any pure-mode reflection.

The components of the quartic coefficient for pure wave propagation can be written in a relatively simple form as a function of the vertical slowness component ($p_3 \equiv q$) and its derivatives with respect to the horizontal components (p_1, p_2):

$$A_4^{(1)} = \frac{q^2 \left(3q_{,22}^2 + q q_{,2222}\right)}{12 t_0^2 q_{,22}^4}, \tag{8}$$

$$A_4^{(2)} = \frac{q^2 \left(3q_{,11}^2 + q q_{,1111}\right)}{12 t_0^2 q_{,11}^4}, \tag{9}$$

$$A_4^{(x)} = \frac{q^2 \left(q_{,11}q_{,22} + qq_{,1122}\right)}{2t_0^2 q_{,11}^2 q_{,22}^2}, \tag{10}$$

where t_0 is, again, the two-way zero-offset traveltime, q is the vertical component of the slowness vector, $q_{,ij} = \frac{\partial^2 q}{\partial p_i \partial p_j}$, and $q_{,ijkl} = \frac{\partial^4 q}{\partial p_i \partial p_j \partial p_k \partial p_l}$. Here, we assume no reflection point dispersal: a horizontal interface and a horizontal symmetry plane. Equations (8-10) are evaluated at normal incidence (i.e., $p_1 = p_2 = 0$).

Substituting the values of the vertical slowness component (q), and its derivatives in equations (8)–(10) in terms of the stiffnesses of the medium (c_{ijkl}), we obtain a concise representation for the quartic coefficient A_4 as a function of the medium parameters. For P–wave propagation, $q = \frac{1}{\sqrt{c_{33}}}$, and the components of the quartic coefficient are given as:

$$A_4^{(1)} = \frac{-2(\epsilon^{(1)} - \delta^{(1)})(1 + 2\delta^{(1)}/f^{(1)})}{t_0^2 V_{P0}^4(1 + 2\delta^{(1)})^4}, \tag{11}$$

$$A_4^{(2)} = \frac{-2(\epsilon^{(2)} - \delta^{(2)})(1 + 2\delta^{(2)}/f^{(2)})}{t_0^2 V_{P0}^4(1 + 2\delta^{(2)})^4}, \tag{12}$$

$$
\begin{aligned}
A_4^{(x)} = {} & \frac{2}{t_0^2 V_{P0}^4(1 + 2\delta^{(1)})^2(1 + 2\delta^{(2)})^2}\Big[\\
& \frac{(1 + 2\delta^{(1)})}{f^{(1)}}\frac{(1 + 2\delta^{(2)})}{f^{(2)}} + \\
& (1 + 2\gamma^{(2)})(1 + \frac{\delta^{(2)}}{f^{(2)}})\frac{(1 - f^{(1)})^2}{f^{(1)}} - \\
& (1 + 2\delta^{(1)})(1 + \delta^{(2)})\frac{(1 - f^{(2)})}{f^{(1)}f^{(2)}} + \\
& \left(2(1 + \delta^{(1)})(1 + \gamma^{(2)}) + \delta^{(2)}\right) . \\
& \frac{(1 - f^{(1)})}{f^{(1)}}\frac{(1 - f^{(2)})}{f^{(2)}} - \\
& \left(2(1 + \delta^{(1)})(1 + \delta^{(2)} + \gamma^{(2)}) + \delta^{(2)}(1 + 2\gamma^{(2)})\right) . \\
& \frac{(1 - f^{(1)})}{f^{(1)}f^{(2)}} - \left((1 + 2\gamma^{(2)})f^{(1)} + 2(\epsilon^{(2)} - \gamma^{(2)})\right) . \\
& \sqrt{(1 + \frac{2\delta^{(1)}}{f^{(1)}})(1 + \frac{2\delta^{(2)}}{f^{(2)}})} . \\
& \sqrt{(1 + \frac{2\delta^{(3)}(1 + 2\epsilon^{(2)})}{(1 + 2\gamma^{(2)})f^{(1)} + 2(\epsilon^{(2)} - \gamma^{(2)})})}\Big],
\end{aligned} \tag{13}
$$

where $\delta^{(3)}$, $\epsilon^{(1)}$, $\epsilon^{(2)}$, and $\gamma^{(2)}$ are dimensionless anisotropic parameters defined from the stiffnesses analogously to Thomsen's coefficients in VTI media (Tsvankin, 1997b;

see Appendix A for more information about the notation). $f^{(1)} = 1 - V_{S0_1}^2/V_{P0}^2$, and $f^{(2)} = 1 - V_{S0_2}^2/V_{P0}^2$; V_{S0_1} and V_{S0_2} are two vertical velocities of the splitted shear-wave polarized perpendicular to the two vertical symmetry planes. Analogous expressions exist for S-wave propagation which will be addressed in detail in another paper.

Due to the equivalence between VTI and ORT along the symmetry planes, equation (8) or (9) describes analytically and in compact form the quartic coefficient for pure mode reflection in VTI media. As expected, equations (11) and (12) are identical to the VTI expressions given by Tsvankin and Thomsen (1994).

Analogous to the VTI case, we can simplify equations (11)–(13) by setting the vertical shear-wave velocities (V_{S0_1} and V_{S0_2}) to zero:

$$A_4^{(1)} = \frac{-2\eta^{(1)}}{t_0^2 V_{\text{nmo},1}^4},$$

$$A_4^{(2)} = \frac{-2\eta^{(2)}}{t_0^2 V_{\text{nmo},2}^4},$$

$$A_4^{(x)} = \frac{2}{t_0^2 V_{\text{nmo},1}^2 V_{\text{nmo},2}^2}\left[1 - \sqrt{\frac{(1 + 2\eta^{(1)})(1 + 2\eta^{(2)})}{1 + 2\eta^{(3)}}}\right],$$

$$\tag{14}$$

where

$$\eta^{(1)} = \frac{\epsilon^{(1)} - \delta^{(1)}}{1 + 2\delta^{(1)}},$$

$$\eta^{(2)} = \frac{\epsilon^{(2)} - \delta^{(2)}}{1 + 2\delta^{(2)}},$$

and

$$\eta^{(3)} = \frac{\epsilon^{(1)} - \epsilon^{(2)} - \delta^{(3)}(1 + 2\epsilon^{(2)})}{(1 + 2\delta^{(3)})(1 + 2\epsilon^{(2)})}.$$

This simple representation of the quartic coefficient allows adequate development of a nonhyperbolic reflection moveout semblance analysis to achieve better imaging and, ultimately, to invert for the medium parameters. Later, we will verify the accuracy of such simplified expressions for P-wave reflection moveout in orthorhombic media.

Horizontal Velocity

To obtain the term A in the nonhyperbolic moveout equation (3), we also have to find the azimuthally dependent horizontal group velocity (V_{hor}) that controls reflection moveout at large offsets approaching infinity. Since the influence of small errors in V_{hor} for spreadlengths feasible in reflection surveys is not significant, we will ignore the difference between phase and group velocity and calculate V_{hor} as the phase velocity

evaluated at the azimuth of the CMP line. The phase velocity in the horizontal plane for any anisotropic medium with a horizontal symmetry plane is given analytically by the known VTI expression. For example, for P-wave propagation in an orthorhombic medium, the focus of our study, the horizontal velocity is given in terms of the medium parameters as follows:

$$\frac{V^2(\alpha)}{V_{P0}^2(1+2\epsilon^{(2)})} = 1 + \epsilon^{(3)}\sin^2\alpha - \frac{f^{(3)}}{2}$$
$$+ \frac{f^{(3)}}{2}\sqrt{\left(1 + \frac{2\epsilon^{(3)}\sin^2\alpha}{f^{(3)}}\right)^2 - \frac{2(\epsilon^{(3)}-\delta^{(3)})\sin^2 2\alpha}{f^{(3)}}},$$

(15)

where $\epsilon^{(3)} = \frac{(\epsilon^{(1)}-\epsilon^{(2)})}{(1+2\epsilon^{(2)})}$, and $f^{(3)} = 1 - \frac{V_{S0}^2}{V_{P0}^2(1+2\epsilon^{(2)})}$; α is the phase angle (azimuth) of the CMP line relative to a horizontal symmetry axis (e.g., x_1 in this case).

Thus, the last three sections provide the expressions for the NMO velocity, the quartic moveout coefficient, and the horizontal velocity needed to construct the non-hyperbolic moveout equation (3) for a single layer. As a result, we are now ready to verify, by numerical (synthetic) examples, the accuracy of our nonhyperbolic moveout equation. In the following section, we perform our numerical study on P-wave reflection moveout for orthorhombic media.

NUMERICAL STUDY OF P-WAVE MOVEOUT FOR A SINGLE ORT LAYER

Here, we present results of a numerical study of P–wave reflection moveout in or-thorhombic media designed to test the accuracy of the hyperbolic and nonhyperbolic moveout equations introduced above. The exact traveltimes are computed using a 3-D anisotropic ray-tracing code developed by Gajewski and Pšenčik (1987). Due to the presence of two orthogonal vertical symmetry planes in ORT media, it is sufficient to study reflection moveout in a single quadrant of azimuths (Figure 1). Here, we assume that the symmetry planes coincide with the Cartesian coordinate system.

First, let us show the influence of the nonhyperbolic portion of the reflection moveout on moveout velocity estimation. The moveout velocity on finite spreads can be obtained by least-squares fitting of a hyperbolic moveout equation to the calculated traveltimes, i.e.,

$$V_{mo}^2 = \frac{\sum_{j=1}^N x_j^2}{\sum_{j=1}^N t_j^2 - N t_0^2},$$

(16)

where x_j is the offset of the j-th trace, t_j is the corresponding two-way reflection trav-eltime, t_0 is the two-way vertical traveltime, and N is the number of traces.

As seen in Figure 2, the moveout velocity obtained from the exact traveltimes using equation (16) is generally close to the analytic NMO value [equation (5)] for

	Model 1	Model 2
$\epsilon^{(1)}$	0.05	0.3
$\delta^{(1)}$	-0.15	0.1
$\gamma^{(1)}$	0.15	0.1
$\epsilon^{(2)}$	0.2	-0.05
$\delta^{(2)}$	0.1	-0.2
$\gamma^{(2)}$	0.2	-0.1
$\delta^{(3)}$	0.0	0.1
V_{P0} (km/s)	2.0	2.5
V_{S0} (km/s)	1.5	1.3
D (km)	1.5	1.5

Table 1: Parameters of two single-layer ORT models used to generate the synthetic data. $\epsilon^{(1)}$, $\epsilon^{(2)}$, $\delta^{(1)}$, $\delta^{(2)}$, $\delta^{(3)}$, $\gamma^{(1)}$, $\gamma^{(2)}$, V_{S0}, and V_{P0} are the ORT medium parameters.

conventional-length spreads ($X/D \leq 1$) (Figure 2). Predictably, the difference between the two velocities increases on longer spreads (e.g., $X/D = 2$ in Figure 2) due to the anisotropy-induced deviations of the moveout curve from a hyperbola. The exact traveltime curves along five CMP azimuths are shown in Figure 3.

Now, let us use Model 1 and Model 2 in Table 1 to test the accuracy of the P-wave hyperbolic moveout equation (1), parameterized by the exact NMO velocity [equation (5)], and compare it to our nonhyperbolic moveout correction, parameterized by the exact NMO velocity [equation (5)], and the exact quartic coefficient [equation (7)]. Figures 3 and 4 show the time residuals as a function of spreadlength-to-depth ratio, (X/D), after applying both moveout corrections on both orthorhombic models. The inadequacy of the hyperbolic moveout equation for long spreadlength (e.g., X/D $\geq$ 1) is clearly illustrated for both models, as shown by Figures 3 and 4. In contrast, the nonhyperbolic moveout correction provides excellent accuracy for both models and for the whole range of offsets and azimuths shown in Figures 3 and 4.

MOVEOUT IN MULTILAYERED MEDIA

In multilayered anisotropic media, both the quadratic and quartic moveout coefficients reflect the combined influence of layering and anisotropy. On conventional-length spreads (spreadlength-to-depth ratio $\leq$ 1), the hyperbolic moveout equation (1) can be expected to provide an adequate description of the moveout, but the NMO velocity should be averaged over the stack of layers. In isotropic and VTI media, this averaging is performed by means of the conventional isotropic Dix (1955) equation (Hake *et al.*, 1984). Furthermore, Alkhalifah and Tsvankin (1995) showed that the Dix equation remains valid in symmetry planes of any anisotropic medium, if the interval NMO velocities are evaluated at the ray-parameter value of the zero-offset ray. A more general

Dix-type equation, which properly accounts for both azimuthal anisotropy and vertical inhomogeneity, was recently developed by Grechka *et al.* (1997) for arbitrary media. Even though the generalized NMO-velocity equation of Grechka *et al.* (1997) is exact at all azimuth directions, we have chosen, without sacrificing significant accuracy, to use the VTI averaging equation [i.e., the conventional Dix (1955) equation] for two reasons. First, we need to be consistent with our averaging for the quartic coefficient A_4. Second, the conventional Dix equation is a familiar expression and is simple. Hence, the NMO velocity is given as:

$$V_{\text{nmo}}^2 = \frac{1}{t_0} \sum_i^N V_{2i}^2 \Delta t_i \, , \tag{17}$$

where t_0 is the two-way zero-offset time to reflector N, V_{2i} is the NMO velocity for each individual layer i, and Δt_i is the two-way zero-offset time in layer i. The interval NMO velocity V_{2i} for any wave type in arbitrary anisotropic media is given by equation (4). For orthorhombic media with horizontal interfaces, the interest of this publication, the interval NMO velocity is given by equation (5). It should be mentioned that along the symmetry planes in azimuthally anisotropic media equation (17) is exact. Outside the symmetry planes, however, it is an approximation. Below, we will study the applicability of equation (17) to multilayered orthorhombic models.

To use the nonhyperbolic moveout equation (3) in multilayered media, we also need to account for the influence of layering on the quartic moveout term. The exact coefficient A_4 for pure modes in VTI media was presented by Hake *et al.* (1984):

$$
\begin{aligned}
A_4 &= \frac{(\sum_i^N V_{2i}^2 \Delta t_i)^2 - t_0 \sum_i^N V_{2i}^4 \Delta t_i}{4(\sum_i^N V_{2i}^2 \Delta t_i)^4} \\
&+ \frac{t_0 \sum_i^N A_{4i} V_{2i}^8 \Delta t_i^3}{(\sum_i^N V_{2i}^2 \Delta t_i)^4} \, ,
\end{aligned}
\tag{18}
$$

where A_{4i} is the quartic moveout coefficient for layer i.

Tsvankin and Thomsen (1994) showed that the nonhyperbolic moveout equation (3) with the quartic term given by equation (18) accurately describes P-wave reflection moveout in multilayered VTI media. Then, Al-Dajani and Tsvankin (1998) showed that the same equations which include the exact quadratic and quartic interval values for HTI media accurately describe P-wave reflection moveout in multilayered HTI media.

In stratified azimuthally anisotropic media, both phase- and group-velocity vectors deviate from the incidence plane, which violates the main assumptions behind the VTI averaging [equations (17) and (18)]. However, we can still expect both equations to provide reasonable accuracy in azimuthally anisotropic media if we use the exact expressions for the interval values V_{2i} [equation (4)] and A_{4i} [equation (7)] that honor the azimuthal dependence of the moveout coefficients. In the numerical examples below,

both the quadratic and quartic moveout coefficients in layered ORT media are calculated using the same averaging equations as for VTI media, but with the exact interval values derived for orthorhombic symmetry.

The effective horizontal velocity (V_{hor}) contained in the term A of the nonhyperbolic moveout equation (3) can be computed in several different ways, including the conventional rms averaging (Alkhalifah, 1997; Al-Dajani and Tsvankin, 1998):

$$V_{\text{hor}}^2 = \frac{1}{t_0} \sum_{i}^{N} V_{\text{hor}_i}^2 \Delta t_i \, ,$$

where V_{hor_i} is the interval horizontal velocity in layer i.

Here, we use the fourth-power averaging equation:

$$V_{\text{hor}}^4 = \frac{1}{t_0} \sum_{i}^{N} V_{\text{hor}_i}^4 \Delta t_i \, . \tag{19}$$

The interval horizontal velocity V_{hor_i} in ORT media is sufficiently approximated by equation (15) evaluated at the azimuth of the CMP line.

Despite the approximate character of our averaging calculations, especially outside the symmetry planes, it allowed us to apply concise and simple averaging equations developed for vertical transverse isotropy at the expense of partly ignoring out-of-plane phenomena in multilayered azimuthally anisotropic media. As a result, extending the VTI algorithms and their implementations to azimuthally anisotropic media can be rather simple task. The accuracy of these approximations will be studied in the next section.

NUMERICAL STUDY OF P-WAVE REFLECTION MOVEOUT IN MULTILAYERED ORT MEDIA

Here, we present results of a numerical study of P-wave reflection moveout in ORT media designed to test the accuracy of the hyperbolic and nonhyperbolic moveout equations introduced above. Again, the exact traveltimes were computed using a 3-D anisotropic ray-tracing code developed by Gajewski and Pšenčik (1987).

Consider a three-layer orthorhombic model (Model 3 in Table 2). The model geometry and the exact (ray-traced) traveltime curves for azimuths 0°, 30°, 45°, 60°, and 90° are given in Figures 5 and 6, respectively. The time residuals after applying normal-moveout (NMO) correction [equation (1)] to the exact traveltime curves are displayed in Figure 7. Clearly, the hyperbolic moveout equation based on the exact interval NMO velocities averaged by formula (17) provides a good approximation of the traveltimes on spreadlengths that do not exceed the reflector depth, as expected. Hence, the effective normal-moveout velocity calculated by rms averaging of the exact interval values [equation (17)] is sufficiently accurate for short spreadlengths. It should be mentioned, however, that the hyperbolic moveout equation breaks down if we disregard

	-	Model 3	-
	Layer 1	Layer 2	Layer 3
$\epsilon^{(1)}$	0.2	-0.05	-0.1
$\delta^{(1)}$	0.15	-0.1	-0.2
$\gamma^{(1)}$	0.1	-0.05	-0.15
$\epsilon^{(2)}$	0.25	-0.1	0.1
$\delta^{(2)}$	0.05	-0.2	-0.1
$\gamma^{(2)}$	0.05	-0.1	0.05
$\delta^{(3)}$	0.1	-0.1	0.05
V_{P0} (km/s)	2.0	3.0	4.0
V_{S0} (km/s)	1.0	1.25	1.5
Depth (km)	0.5	1.25	2.0

Table 2: Parameters of three-layer ORT model (Model 3) used to generate synthetic data in Figure 6. The symmetry planes coincide with the Cartesian coordinate system, as shown in Figure 5.

the azimuthal dependence of the interval NMO velocities described by equation (5). Moreover, application of any single value of NMO velocity would lead to misalignment of reflection events and poor stacking quality in certain ranges of azimuthal angles.

As in the homogeneous model, the error of the hyperbolic moveout equation increases with offset due to the combined influence of anisotropy and layering (Figure 7). To describe long-spread moveout in layered media, we use equation (3) with the effective values of the moveout coefficients given by equations (17), (18), and (19). Despite the approximate character of the averaging expressions, the nonhyperbolic moveout equation (3) provides excellent accuracy for multilayered media (Figure 7).

Let us add more complication to our three-layer model (Model 3 in Table 2) by rotating the symmetry planes of the second layer by 45° around the x_3 axis. The exact traveltime curves are provided in Figure 8. Not surprisingly, our conclusion remains valid, and similar to the case of Model 3, the hyperbolic moveout equation provides sufficient accuracy for short spreadlengths (Figure 9). At large offsets, however, the application of the nonhyperbolic moveout equation becomes a necessity to achieve accurate reflection moveout representation (see Figure 9).

During our discussion above, we have stated the quartic coefficient for P-wave propagation in an orthorhombic medium after setting the vertical shear-wave velocity (V_{S0}) to zero [equation (14)]. With this simplification, the quartic coefficient for a horizontal orthorhombic medium is given in terms of two NMO velocities, along the vertical symmetry planes ($V_{nmo,1}$ and $V_{nmo,2}$), and three "anellipticity" coefficients $\eta^{(1)}$, $\eta^{(2)}$, and $\eta^{(3)}$ [equation (14)]. Figure 10 shows the time residual after applying nonhyperbolic reflection moveout correction for both models of Figures 7 and 9 while using equation (14)

for the interval quartic coefficient. It should be mentioned that the vertical shear-wave velocity is also set to zero for the calculation of the horizontal velocity [equation (15)]. Notice that the difference in the time residuals using both NHMO corrections for the two models is not significant and the accuracy of the nonhyperbolic equation remains valid. Therefore, the dependence of P-wave reflection on V_{S0} can be ignored, as is the case for VTI. Moreover, instead of having nine coefficients to describe reflection moveout in orthorhombic media, we need only five parameters: $V_{\mathrm{nmo},1}$, $V_{\mathrm{nmo},1}$, $\eta^{(1)}$, $\eta^{(2)}$, and $\eta^{(3)}$ to describe P-wave data and to perform time processing and reflection moveout inversion.

DISCUSSION AND CONCLUSIONS

We have presented an analytic description for the quartic coefficient of the Taylor's series expansion of the two-way traveltime for pure mode reflection in arbitrary anisotropic media. Furthermore, we have presented an analytic description for long-spread reflection moveout in arbitrary anisotropic media. Our treatment of long-spread moveout is based on an exact expression for the azimuthally-dependent quartic moveout coefficient A_4 which has been derived for any pure mode in a homogeneous arbitrary anisotropic layer with arbitrary strength of anisotropy. The expression for A_4 has a relatively simple trigonometric form. Special attention has been given toward P-wave propagation in orthorhombic media.

For a single-layer, orthorhombic model, the hyperbolic moveout equation parameterized by the exact NMO velocity given in Grechka and Tsvankin (1998) provides a good approximation for P-wave traveltimes on short-length CMP spreads (close to the reflector depth). However, the accuracy of the hyperbolic equation rapidly decreases with offset due to the influence of anisotropy-induced nonhyperbolic moveout. To account for deviations from hyperbolic moveout on long spreads (2–3 times as large as the reflector depth), we have substituted the exact azimuthally-dependent values of the NMO velocity and the quartic moveout coefficient into the nonhyperbolic moveout equation, originally introduced for VTI media. Numerical examples show that this equation provides excellent accuracy for P-waves recorded in all azimuthal directions over an orthorhombic layer, even for models with significant velocity anisotropy and pronounced nonhyperbolic moveout.

In multilayered media, the moveout coefficients reflect the combined influence of layering, azimuthal anisotropy. Although the rays do diverge from the incidence plane on off-symmetry CMP lines in vertically inhomogeneous anisotropic media, the magnitude of these deviations usually is not sufficient to cause measurable errors with use of the Dix equation, especially for models with a similar character of the azimuthal velocity variations in all layers (e.g., media with uniform orientation of cracks). To determine the quartic moveout coefficient A_4 in stratified orthorhombic media, we use the same averaging equations as for VTI (Hake et $al.$, 1984; Tsvankin and Thomsen, 1994; Al-Dajani and Tsvankin, 1998), but with the exact interval values of V_{nmo} and A_4 in

each orthorhombic layer. Then, the NMO velocity and the quartic moveout coefficient, averaged over the stack of layers above the reflector, are used in the same nonhyperbolic moveout equation as in the single-layer model. Extensive numerical testing for stratified orthorhombic media with both uniform and depth-varying orientation of the symmetry planes demonstrates sufficient accuracy of our nonhyperbolic approximation in the description of long-spread reflection moveout.

For P–wave propagation in orthorhombic media, the dependence on the vertical shear-wave velocities can be ignored without any significant effect on the accuracy of the nonhyperbolic moveout correction. Hence, instead of having nine coefficients to describe reflection moveout in orthorhombic media, we need only five parameters to describe P-wave data and to perform time processing and reflection moveout inversion: $V_{nmo,1}$, $V_{nmo,1}$, $\eta^{(1)}$, $\eta^{(2)}$, and $\eta^{(3)}$.

The nonhyperbolic moveout equation discussed here has important applications in modeling of long-spread reflection traveltimes, in inverting for the medium parameters, and in enhancing imaging and time processing algorithms in anisotropic media.

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REFERENCES

Al-Dajani, A. and Tsvankin, I., 1998, Nonhyperbolic reflection moveout for horizontal transverse isotropy, *Geophysics, 63*, 1738–1753.

Alkhalifah, T., 1997, Velocity analysis using nonhyperbolic moveout in transversely isotropic media, *Geophysics, 62*, 1839–1854.

Alkhalifah, T., and Tsvankin, I., 1995, Velocity analysis for transversely isotropic media, *Geophysics, 60*, 1550–1566.

Corrigan, D., Withers R., Darnall J., and Skopinski T., 1996, Fracture mapping from azimuthal velocity analysis using 3D surface seismic data, *66th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts*, 1834–1837.

Crampin, S., 1985, Evidence for aligned cracks in the earth's crust, *First Break, 3*, 12–15.

Dix, C.H., 1955, Seismic velocities from surface measurements, *Geophysics, 20*, 68–86.

Gajewski, D., and Pšenčik, I., 1987, Computation of high-frequency seismic wavefields in 3-D laterally inhomogeneous anisotropic media, *Geophys. J. R. astr. Soc., 91*, 383–411.

Gidlow, P.W., and Fatti, J.L., 1990, Preserving far offset seismic data using nonhyperbolic moveout correction, *60th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts*, 1726–1729.

Grechka, V., and Tsvankin, I., 1998, 3-D description of normal moveout in anisotropic inhomogeneous media, *Geophysics, 63*, 1079–1092.

Grechka, V., Tsvankin, I., and Cohen, J.K., 1997, Generalized Dix equation and modeling of normal moveout in inhomogeneous anisotropic media, *67th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts*.

Hake, H., Helbig, K., and Mesdag, C. S., 1984, Three-term Taylor series for $t^2 - x^2$ curves over layered transversely isotropic ground, *Geophys. Prosp., 32*, 828–850.

Hale, D., Hill, N.R., and Stefani, J., 1992, Imaging salt with turning seismic waves, *Geophysics, 57*, 1453–1462.

Li, X.-Y., and Crampin, S., 1993, Approximations to shear-wave velocity and moveout equations in anisotropic media, *Geophys. Prosp., 41*, 833–858.

Lynn, H., Simon, K., Bates, C., Van Doc, R., 1996, Azimuthal anisotropy in *P*-wave 3-D (multiazimuth) data, *The Leading Edge, 15*, 923–928.

Rüger, A., 1997, *P*-wave reflection coefficients for transversely isotropic models with vertical and horizontal axis of symmetry, *Geophysics, 62*, 713–722.

Sena, A.G., 1991, Seismic traveltime equations for azimuthally anisotropic and isotropic media: Estimation of internal elastic properties, *Geophysics, 56*, 2090–2101.

Tabti, H. and Rasolofosaon, P., 1998, Velocity analysis using non-hyperbolic moveout in anisotropic media of arbitrary symmetry: Synthetic and field data studies, 8th International Workshop on Seismic Anisotropy.

Taner, M., and Koehler, F., 1969, Velocity spectra-digital computer derivation and application of velocity functions, *Geophysics, 34*, 859–881.

Thomsen, L., 1986, Weak elastic anisotropy, *Geophysics, 51*, 1954–1966.

Thomsen, L., 1988, Reflection seismology over azimuthally anisotropic media, *Geophysics*, *53*, 304–313.

Tsvankin, I., 1997a, Reflection moveout and parameter estimation for horizontal transverse isotropy, *Geophysics*, *62*, 614–629.

Tsvankin, I., 1997b, Anisotropic parameters and P-wave velocity for orthorhombic media, *Geophysics*, *62*, 1292–1309.

Tsvankin, I., and Thomsen, L., 1994, Nonhyperbolic reflection moveout in anisotropic media, *Geophysics*, *59*, 1290–1304.

APPENDIX

REFLECTION MOVEOUT COEFFICIENTS IN ANISOTROPIC MEDIA

To obtain the quadratic A_2 and the quartic A_4 coefficients of the Taylor series expansion of the squared traveltime $[t^2(x^2)]$, we find first an expression for the coefficients in terms of the one-way traveltime from the zero-offset reflection point. Since a horizontal reflector that coincides with a symmetry plane represents a mirror image, the group-velocity (ray) vector of any pure (non-converted) reflected wave represents a mirror image of the incident ray with respect to the horizontal plane (Al-Dajani and Tsvankin, 1998). Thus, there is no reflection-point dispersal on CMP gathers above a homogeneous anisotropic layer with a horizontal symmetry plane, and we can represent the two-way traveltime along the specular ray path as the sum of the traveltimes from the zero-offset reflection point to the source and receiver (Figure A-1). Following the approach suggested by Hale et $al.$ (1992) in their derivation of the normal-moveout velocity from dipping reflectors, the one-way traveltime from the reflection point to the source or receiver can be expanded in a double Taylor series in the vicinity of the zero-offset point. Here, we are interested, in particular, in deriving the quartic moveout coefficient, so we will keep the quartic and lower-order terms in the Taylor series,

$$\tau(\pm x_1, \pm x_2) = \sum_{l=0}^{4} \frac{D_{\pm x_1, \pm x_2}^l \tau}{l!}, \tag{A-1}$$

where

$$D_{\pm x_1, \pm x_2}^l \tau = \sum_{i+j=l} \frac{l!}{i!j!} (\pm x_1)^i (\pm x_2)^j \left(\frac{\partial}{\partial x_1}\right)^i \left(\frac{\partial}{\partial x_2}\right)^j \tau.$$

τ is the one way traveltime and the $(\pm x_1, \pm x_2)$ correspond to the coordinates of the source (+) and receiver (-) in the vicinity of the common-mid-point (CMP) location (O) (see Figure A-1). The derivatives are evaluated at the CMP location.

As a result, the two-way traveltime (t) is given as:

$$t = \tau_+ + \tau_-, \tag{A-2}$$

where $\tau_+ \equiv \tau(+x_1, +x_2)$ is the one-way time from the source (S) to the CMP location, while $\tau_- \equiv \tau(-x_1, -x_2)$ is the one-way traveltime from the CMP location back to the receiver (R) (Figure A-1).

Substituting equation (A-1) into equation (A-2), the two-way traveltime is given by:

$$\begin{aligned} t &= 2\tau_0 + x_1^2 \tau_{11} + 2x_1 x_2 \tau_{12} + x_2^2 \tau_{22} \\ &+ \frac{x_1^3 x_2}{3} \tau_{1112} + \frac{x_1 x_2^3}{3} \tau_{1222} + \frac{x_1^2 x_2^2}{2} \tau_{1122} \\ &+ \frac{x_1^4}{12} \tau_{1111} + \frac{x_2^4}{12} \tau_{2222}, \end{aligned} \tag{A-3}$$

where τ_0 is the one-way zero-offset time, $\tau_{ij} = \frac{\partial^2 \tau}{\partial x_i \partial x_j}$, and $\tau_{ijkl} = \frac{\partial^4 \tau}{\partial x_i \partial x_j \partial x_k \partial x_l}$. The indices i, j, k, and l take the values 1 and 2 corresponding to the Cartesian coordinates x_1 and x_2, respectively.

The coordinates x_1 and x_2 can be expressed in terms of the azimuth α and the half offset h of the CMP line, as demonstrated in Figure A-1:

$$
\begin{aligned}
x_1 &= h \cos \alpha, \\
x_2 &= h \sin \alpha.
\end{aligned}
\tag{A-4}
$$

Substituting equation (A-4) into equation (A-3) and squaring both sides of equation (A-3), we obtain (after simplification and keeping only the quartic and lower-order terms):

$$
t^2 = t_0^2 + A_2(2h)^2 + A_4(2h)^4 ,
\tag{A-5}
$$

where

$$
A_2 = \frac{t_0}{2}(\tau_{11} \cos^2 \alpha + 2\tau_{12} \sin \alpha \cos \alpha + \tau_{22} \sin^2 \alpha) ,
\tag{A-6}
$$

and

$$
\begin{aligned}
A_4 &= \cos^4 \alpha \left[\frac{\tau_{11}^2}{16} + \frac{t_0 \tau_{1111}}{96} \right] \\
&+ \sin^4 \alpha \left[\frac{\tau_{22}^2}{16} + \frac{t_0 \tau_{2222}}{96} \right] \\
&+ \sin^2 \alpha \cos^2 \alpha \left[\frac{\tau_{11}\tau_{22}}{8} + \frac{\tau_{12}^2}{4} + \frac{t_0 \tau_{1122}}{16} \right] \\
&+ \sin \alpha \cos^3 \alpha \left[\frac{\tau_{11}\tau_{12}}{4} + \frac{t_0 \tau_{1112}}{24} \right] \\
&+ \cos \alpha \sin^3 \alpha \left[\frac{\tau_{22}\tau_{12}}{4} + \frac{t_0 \tau_{2221}}{24} \right] ,
\end{aligned}
\tag{A-7}
$$

where t_0 is the two-way zero-offset time, and α is the angle between the CMP line and one of the principle vertical planes [in this case, we have chosen (x_1, x_3) plane to be our reference plane].

Furthermore, realizing that the slowness vector $\mathbf{P}$ (p_1, p_2, p_3), in Cartesian coordinates, can be written as:

$$
p_i = \frac{\partial \tau}{\partial x_i} ,
\tag{A-8}
$$

where the index i takes the values 1, 2, and 3, in Cartesian coordinates.

Therefore, substituting equation (A-8) into equation (A-6) and equation (A-7), we obtain a relatively simple and general representation of the quadratic and quartic coefficients in terms of the horizontal components of the slowness vector $(p_1$ and $p_2)$ and their spatial derivatives:

$$
A_2 = \frac{t_0}{2}(p_{1,1} \cos^2 \alpha + 2p_{1,2} \sin \alpha \cos \alpha + p_{2,2} \sin^2 \alpha) ,
\tag{A-9}
$$

and

$$
\begin{aligned}
A_4 &= \cos^4 \alpha \Big[\frac{p_{1,1}^2}{16} + \frac{t_0 p_{1,111}}{96} \Big] \\
&+ \sin^4 \alpha \Big[\frac{p_{2,2}^2}{16} + \frac{t_0 p_{2,222}}{96} \Big] \\
&+ \sin^2 \alpha \cos^2 \alpha \Big[\frac{p_{1,1} p_{2,2}}{8} + \frac{p_{1,2}^2}{4} + \frac{t_0 p_{1,122}}{16} \Big] \\
&+ \sin \alpha \cos^3 \alpha \Big[\frac{p_{1,1} p_{1,2}}{4} + \frac{t_0 p_{2,111}}{24} \Big] \\
&+ \cos \alpha \sin^3 \alpha \Big[\frac{p_{2,2} p_{1,2}}{4} + \frac{t_0 p_{1,222}}{24} \Big],
\end{aligned}
\tag{A-10}
$$

where $p_{i,j} = \frac{\partial p_i}{\partial x_j}$, $p_{i,jk} = \frac{\partial^2 p_i}{\partial x_j \partial x_k}$, and $p_{i,jkl} = \frac{\partial^3 p_i}{\partial x_j \partial x_k \partial x_l}$.

For more convenient notation, let us rewrite the coefficients in equations (A-9–A-10) as follows:

$$
\begin{aligned}
A_2(\alpha) &= A_2^{(1)} \sin^2 \alpha + A_2^{(2)} \cos^2 \alpha \\
&+ A_2^{(x)} \sin \alpha \cos \alpha,.
\end{aligned}
\tag{A-11}
$$

and

$$
\begin{aligned}
A_4(\alpha) &= A_4^{(1)} \sin^4 \alpha + A_4^{(2)} \cos^4 \alpha + A_4^{(x)} \sin^2 \alpha \cos^2 \alpha \\
&+ A_4^{(x1)} \sin \alpha \cos^3 \alpha + A_4^{(x2)} \sin^3 \alpha \cos \alpha,
\end{aligned}
\tag{A-12}
$$

where α is, again, the angle between the CMP line and one of the principle vertical planes [e.g., (x_1, x_3) plane]. The superscripts (1) and (2) are the components of coefficient along the two vertical principle planes [in Cartesian coordinates, (x_2, x_3) and (x_1, x_3), respectively]. The superscripts (x), $(x1)$, and $(x2)$ are used to represent the cross terms which absorb the mutual influence from all principle planes. The components of the coefficients are presented in terms of the medium parameters while the azimuthal dependence is governed by the trigonometric functions.

It should be mentioned that equation (A-6) and/or equation (A-11) are used in Grechka and Tsvankin (1998) to derive an expression for the normal-moveout (NMO) velocity in an azimuthally anisotropic medium. It turned out that for any horizontal, azimuthally anisotropic medium with a horizontal symmetry plane (e.g., HTI, orthorhombic, and even monoclinic), the cross term $A_2^{(x)} = 0$ in equation (A-11) which yields

$$
A_2(\alpha) = A_2^{(1)} \sin^2 \alpha + A_2^{(2)} \cos^2 \alpha.
$$

Here, our attention is focused on the quartic coefficient A_4 given by equation (A-7) and equation (A-10).

The objective now is to write the coefficients in a more compact and convenient form for wave propagation in terms of the vertical slowness component ($p_3 \equiv q$), and

its derivatives with respect to the horizontal components (p_1, p_2). In addition, we need to link this representation to the medium elastic parameters.

The vertical slowness component $q = q(p_1, p_2)$ can be found from the Christoffel equation, which can be reduced to the form $F(q, p_1, p_2) = 0$, where F is defined as:

$$F(\mathbf{p}) \equiv \det(a_{ijkl} p_j p_k - \delta_{il}) = 0, \tag{A-13}$$

where $a_{ijkl} \equiv c_{ijkl}/\rho$, the elasticity tensor normalized by the density, and δ_{il} is the symbolic Kronecker delta; p_j is the slowness vector where $p_j \equiv \partial \tau / \partial x_j$ and τ is the one-way traveltime. The indices i, j, k, l take on values from 1 to 3; summation over repeated indices is implied.

If the medium has a horizontal symmetry plane (e.g., transversely isotropic, orthorhombic, or monoclinic), F becomes a *cubic* polynomial with respect to q^2, and its roots along with the derivatives can be obtained explicitly. For example, the derivative $q_{,i}$ can be obtained by implicit differentiation as

$$q_{,i} = -\frac{F_{p_i}}{F_q},$$

where $F_{p_i} \equiv \partial F / \partial p_i$, and $F_q \equiv \partial F / \partial q$.

Similarly,

$$q_{,ij} = \frac{\partial}{\partial p_j} \left[-\frac{F_{p_i}}{F_q} \right],$$

and so on.

It should be mentioned that no assumptions have been made about the type of symmetry which the medium might pertain. The only assumption made so far is the fact that we need to have a horizontal symmetry plane in order to have analytic representation of the quartic coefficient in terms of the vertical slowness solution from the Christoffel equation. Furthermore, no assumptions have been made about the wave type or the strength of anisotropy.

Throughout the paper, however, we have focused our numerical study on P-wave propagation in media with orthorhombic (or orthotropic) symmetry that represent models for naturally fractured reservoirs with aligned vertical cracks. Such models include those containing a system of parallel vertical cracks in a horizontally-layered background medium, two different orthogonal systems of vertical cracks, or two equivalent non-orthogonal crack systems. All these models have three mutually orthogonal (one horizontal and two vertical) planes of mirror symmetry.

The orthorhombic symmetry is defined through the fourth-rank stiffness tensor c_{ijkl} as:

$$C_{\mathrm{ORT}} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix} .$$

In the case of a single homogeneous orthorhombic layer, both $A_4^{(x1)}$ and $A_4^{(x2)}$ vanish and equation (A-12) reduces to:

$$A_4(\alpha) = A_4^{(1)} \sin^4 \alpha + A_4^{(2)} \cos^4 \alpha + A_4^{(x)} \sin^2 \alpha \cos^2 \alpha . \tag{A-14}$$

Following the above procedure for orthorhombic symmetry [i.e., using the elasticity tensor c_{ijkl} for orthorhombic symmetry in equation (A-13)], the components of the quartic coefficient in equation (A-14) for pure wave propagation can be written in a relatively simple form as a function of the vertical slowness component ($p_3 \equiv q$) and its derivatives with respect to the horizontal components (p_1, p_2):

$$A_4^{(1)} = \frac{q^2 \left(3q_{,22}^2 + q q_{,2222} \right)}{12 t_0^2 q_{,22}^4} , \tag{A-15}$$

$$A_4^{(2)} = \frac{q^2 \left(3q_{,11}^2 + q q_{,1111} \right)}{12 t_0^2 q_{,11}^4} , \tag{A-16}$$

and

$$A_4^{(x)} = \frac{q^2 \left(q_{,11} q_{,22} + q q_{,1122} \right)}{2 t_0^2 q_{,11}^2 q_{,22}^2} , \tag{A-17}$$

where t_0 is, again, the two-way zero-offset traveltime, q is the vertical component of the slowness vector, $q_{,ij} = \frac{\partial^2 q}{\partial p_i \partial p_j}$, and $q_{,ijkl} = \frac{\partial^4 q}{\partial p_i \partial p_j \partial p_k \partial p_l}$. The vertical slowness and its derivatives are evaluated at normal incidence ($p_1 = 0$, $p_2 = 0$, while $q = 1/\sqrt{c_{33}}$ for P-wave propagation).

Substituting the values of the vertical slowness component (q), and its derivatives in equations (A-15-A-17) in terms of the stiffnesses of the medium (c_{ijkl}), we obtain a concise representation for the quartic coefficient A_4 as a function of the medium parameters, given in the main text as equations (11-13). After recasting the coefficient in terms of Tsvankin's (1997b) notation for orthorhombic media:

It is clear that we can represent the coefficients in equations (11-13), in the main text, in terms of the stiffnesses by simply substituting back the values of the anisotropic parameters. Tsvankin's (1997b) notation for orthorhombic symmetry is:

- V_{P0} – P-wave vertical velocity:

$$V_{P0} \equiv \sqrt{\frac{c_{33}}{\rho}} \cdot \quad .$$

- V_{S0} – the vertical velocity of S-wave polarized in the x_1-direction:

$$V_{S0} \equiv \sqrt{\frac{c_{55}}{\rho}} \cdot$$

- $\epsilon^{(2)}$ – the VTI parameter ϵ in the symmetry plane (x_1, x_3) normal to x_2-axis:

$$\epsilon^{(2)} \equiv \frac{c_{11} - c_{33}}{2\,c_{33}} \cdot$$

- $\delta^{(2)}$ – the VTI parameter δ in the (x_1, x_3) plane:

$$\delta^{(2)} \equiv \frac{(c_{13} + c_{55})^2 - (c_{33} - c_{55})^2}{2\,c_{33}\,(c_{33} - c_{55})} \cdot$$

- $\gamma^{(2)}$ – the VTI parameter γ in the plane (x_1, x_3):

$$\gamma^{(2)} \equiv \frac{c_{66} - c_{44}}{2\,c_{44}} \cdot$$

- $\epsilon^{(1)}$ – the VTI parameter ϵ in the symmetry plane (x_2, x_3) normal to x_1-axis:

$$\epsilon^{(1)} \equiv \frac{c_{22} - c_{33}}{2\,c_{33}} \cdot$$

- $\delta^{(1)}$ – the VTI parameter δ in the (x_2, x_3) plane:

$$\delta^{(1)} \equiv \frac{(c_{23} + c_{44})^2 - (c_{33} - c_{44})^2}{2\,c_{33}\,(c_{33} - c_{44})} \cdot$$

- $\gamma^{(1)}$ – the VTI parameter γ in the plane (x_2, x_3):

$$\gamma^{(1)} \equiv \frac{c_{66} - c_{55}}{2\,c_{55}} \cdot$$

- $\delta^{(3)}$ – the VTI parameter δ in the (x_1, x_2) plane (x_1 plays the role of the symmetry axis):

$$\delta^{(3)} \equiv \frac{(c_{12} + c_{66})^2 - (c_{11} - c_{66})^2}{2\,c_{11}\,(c_{11} - c_{66})} \cdot$$

It is clear that it is assumed that the orthogonal symmetry planes coincide with the principal planes of the Cartesian coordinate system.

Furthermore, by repeating the above derivation for a single homogeneous monoclinic layer, with a horizontal symmetry axis (e.g., x_1), we can conclude that $A_4^{(x2)}$ in equation (A-12) vanishes and equation (A-12) reduces to:

$$
\begin{aligned}
A_4(\alpha) \;=\; & A_4^{(1)} \sin^4 \alpha + A_4^{(2)} \cos^4 \alpha + A_4^{(x)} \sin^2 \alpha \cos^2 \alpha \\
& + A_4^{(x1)} \sin \alpha \cos^3 \alpha \,,
\end{aligned}
$$

where for a monoclinic symmetry, with a symmetry axis parallel to x_1, the fourth-rank stiffness tensor c_{ijkl} is defined as:

$$
C_{\text{Mono}} = \begin{pmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & 0 & 0 \\
c_{12} & c_{22} & c_{23} & c_{24} & 0 & 0 \\
c_{13} & c_{23} & c_{33} & c_{34} & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{55} & c_{56} \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{pmatrix} .
$$

Considering the number of stiffnesses (thirteen), however, the resulting components for the reflection moveout coefficients in terms of the medium parameters are rather complicated for monoclinic medium to be shown here, especially for the quartic coefficient.
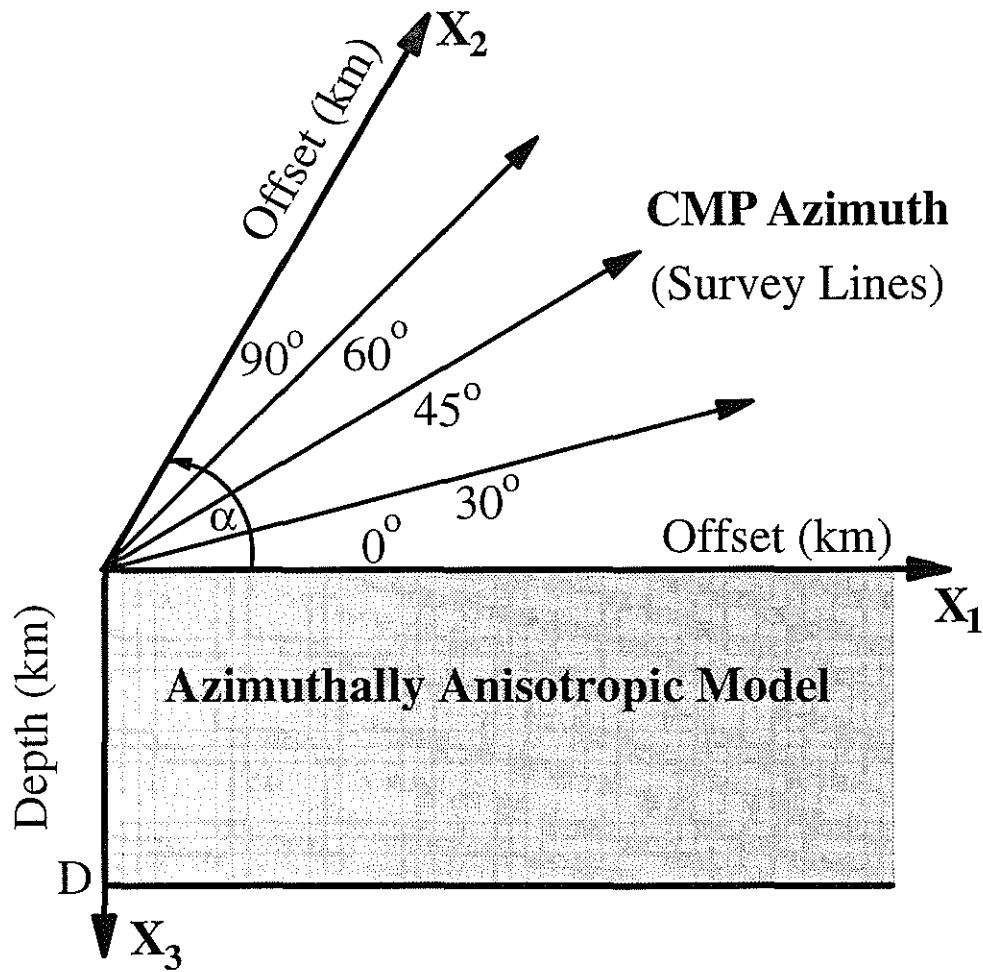
Figure 1: Orientation of the CMP azimuths (survey lines) over a horizontal ORT layer used in Figures 2-4.

Exact Vnmo vs. Vstack (X/D = 1)

Exact Vnmo vs. Vstack (X/D = 2)
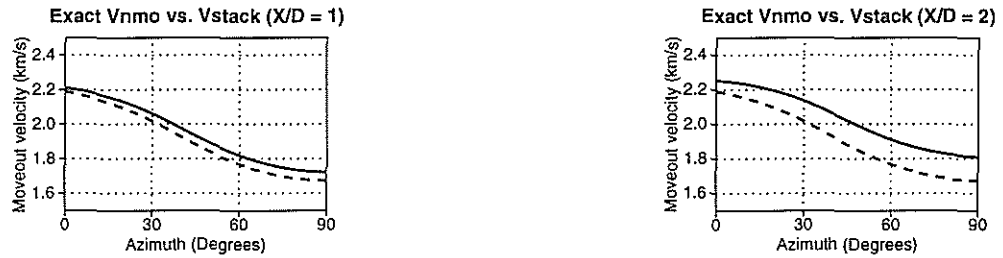
Figure 2: Comparison between the exact NMO velocity (dashed) given by equation (5) and the estimated stacking velocity (solid) from the reflection moveout [equation (16)] ignoring the fact that the reflection moveout is nonhyperbolic. The comparison was conducted at two spread-length-to-depth (X/D) ratios. The model parameters are given in Table 1 as Model 1. The geometry is given in Figure 1.
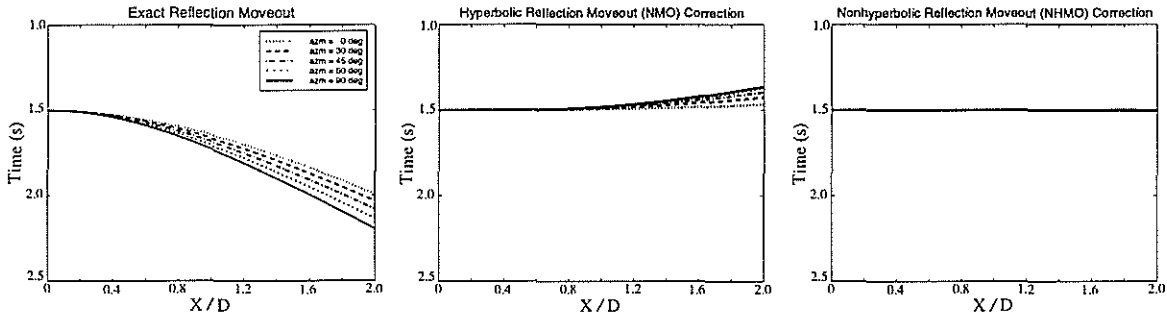
Figure 3: Exact ray-traced reflection moveout and the time residuals after applying normal-moveout (NMO) [equation (1)] and nonhyperbolic moveout (NHMO) [equation (3)] corrections, respectively. The curves correspond to CMP azimuths 0°, 30°, 45°, 60°, and 90°. The traveltime and the moveout correction are displayed as a function of offset-to-reflector-depth (X/D) ratio. The model parameters are given in Table 1 as Model 1. The geometry is given in Figure 1.
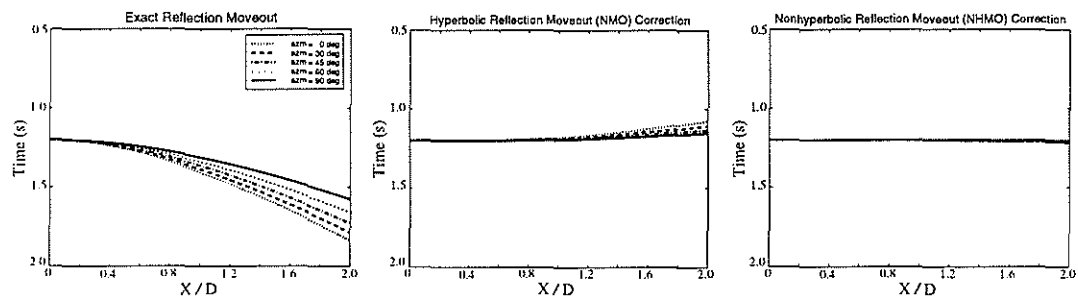
Figure 4: The same as Figure 3 but for Model 2 of Table 1. The geometry is given in Figure 1.
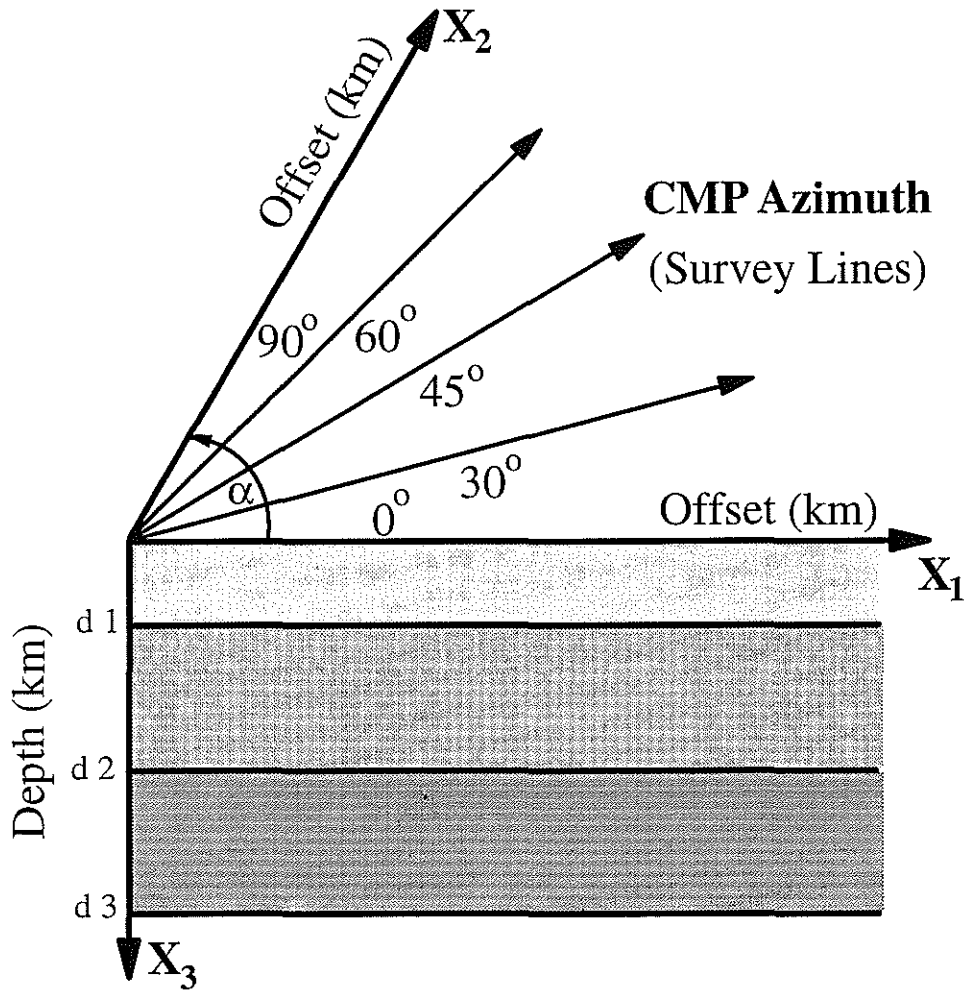
Figure 5: Orientation of the CMP azimuths (survey lines) over horizontal three-layer ORT media used to generate synthetic data for Figures 6 and 7.
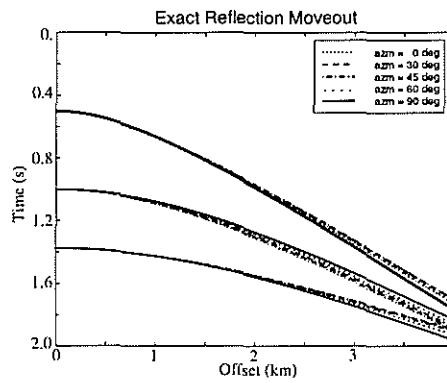
Figure 6: Ray-traced reflection moveout along CMP azimuths 0°, 30°, 45°, 60°, and 90°. The model parameters are given as Model 3 in Table 2. The geometry is given in Figure 5.
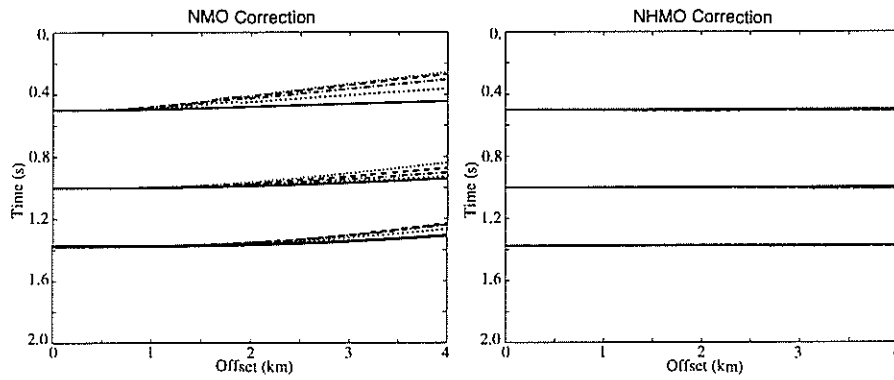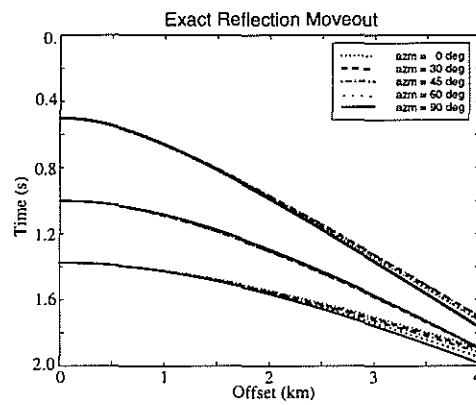
Figure 7: The time residuals after applying normal-moveout (NMO) [equation (1)] and nonhyperbolic moveout (NHMO) [equation (3)] corrections. The curves correspond to CMP azimuths 0°, 30°, 45°, 60°, and 90°. The model parameters are given as Model 3 in Table 2. The geometry is given in Figure 5.

Figure 8: Ray-traced reflection moveout along CMP azimuths 0°, 30°, 45°, 60°, and 90°. The model parameters are the same as in Model 3, Table 2. The vertical symmetry planes for the second layer, however, are rotated 45° around the x_3 axis.
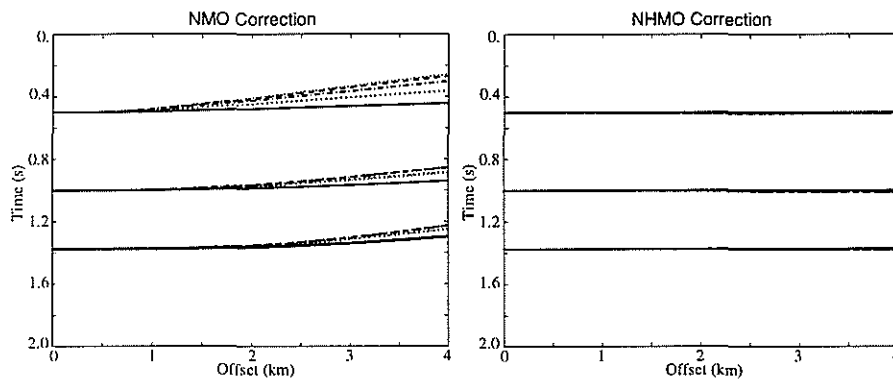
Figure 9: The same as Figure 7, except this is for model 3 with a depth-varying symmetry plane direction. The exact traveltime curves are displayed in Figure 8.
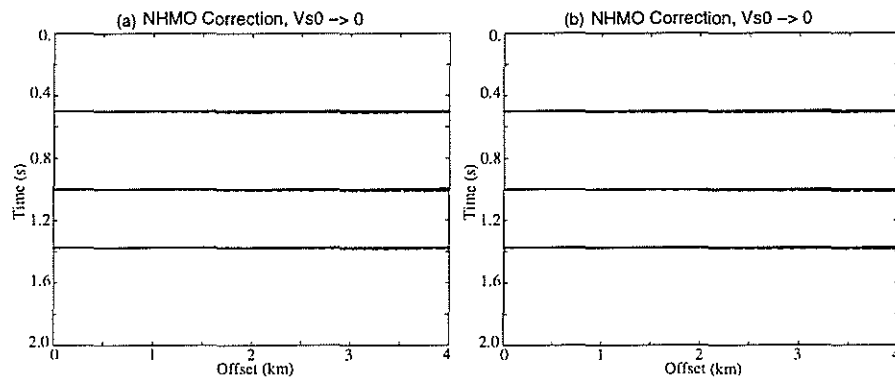
Figure 10: The time residuals after applying nonhyperbolic moveout (NHMO) correction [equation (3)]. The vertical shear velocity (V_{S0}) for the interval quartic coefficient is set to 0 [equation (14)]. (a) corresponds to the model displayed in Figure 7, while (b) corresponds to the model displayed in Figure 9.
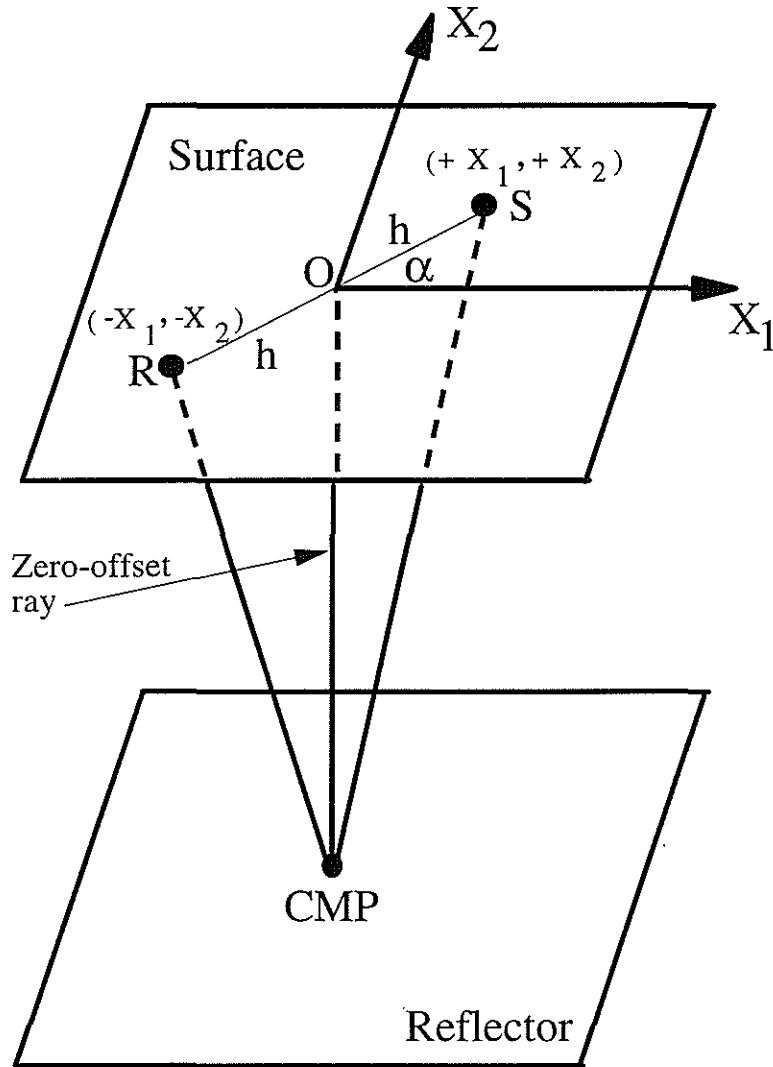
Figure A-1: For a homogeneous, azimuthally anisotropic layer with a horizontal symm etry plane, the specular reflection point for any offset coincides with the zero -offset reflection point, and there is no reflection-point dispersal on CMP gath ers. h is half the offset between the source (S) and the receiver (R), and the angle α is the azimuth of the CMP gather from the x_1 axis.