Progress of Theoretical Physics, Vol. 38, No. 4, October 1967

Nonleptonic Hyperon Decay in the Quark Model

Kimio MIURA and Toshiyuki MINAMIKAWA

Department of Physics, Tokyo University of Education, Tokyo

(Received February 22, 1967)

Nonleptonic hyperon and Ω^- decays are discussed in the nonrelativistic quark model and the $SU(6)$ theory, where we consider the pole diagram for quark, the four-body weak current-current interaction $J_1^2J_3^1$ and the meson exchange interaction. We assume two models, and can explain the experiments for parity-violating and parity-conserving amplitudes of nonleptonic hyperon decays. In particular the $\Delta I=1/2$ rule is derived, though the weak current-current interaction $J_1^2J_3^1$ does not satisfy $\Delta I=1/2$. As to the decay of Ω^- two models predict the different ratio of $\Gamma(\Omega\to\Lambda+K)$ and $\Gamma(\Omega\to\Xi+\Pi)$. One is consistent with experiment, the other forbids $\Omega\to\Lambda+K$.

§ 1. Introduction

The success of the $SU(6)$[1] theory gives us some information on the structure of elementary particles. Nambu[2] proposed the naive idea that quite heavy quarks (or triplets) moving very slowly, are bound into baryon and meson by deep attractive potential. By our analysis of electromagnetic mass difference,[3] we have found that this model seems to be consistent. In this paper we consider nonleptonic hyperon decays along this line.*) We discuss this problem under the following assumptions:

(i) Baryon-octet and -decuplet belong to 56-dimensional representation of $SU(6)$.

(ii) Quarks and antiquarks are bound into baryon and meson by a deep attractive potential (or superstrong interaction) which is mediated by some scalar $SU(3)$ singlet meson.**) A small correction to the binding energy arises from the conventional meson exchange which will also play a role in the nonleptonic hyperon decays.

(iii) The coupling between quarks and meson octet is $SU(3)$ invariant.

*) Many authors have discussed the nonleptonic decays from the $SU(6)$ theoretical point of view. See, K. Kawarabayashi, Phys. Rev. Letters **14** (1965), 86 and C. Iso and M. Kato, Nuovo Cim. **37** (1965), 1734. These group theoretical considerations are not sufficient to investigate the inner structure of baryon, and they have too many parameters to make definite prediction about nonleptonic hyperon decays. Even our result based on simple Feynman diagram contains the representations of large dimensions which are not considered in their articles.

**) In the Figures of this paper we shall not write down explicitly the singlet scalar meson line which is responsible for superstrong interaction between quarks.

(iv) The weak spurion behaves like octet in $SU(3)$, such as λ_6.

(v) Four-body weak current-current interaction exists between quarks and this current is of the V-A type (c.f. Figs. 2 and 4). One of the important consequences of this paper is that the $\Delta I=1/2$ rule for nonleptonic decays of hyperons is derived from the $SU(3)$ group transformation properties of current-current interaction of the $J_1^2 J_3^1$ type, on the basis of the nonrelativistic quark model combined with the three-triplet model.[4] This interesting fact is easily proved by the theorem of the nonrelativistic version of Fierz transformation (c.f. §3). Furthermore, it is to be remarked that the $\Delta I=1/2$ spurion for quark can also be derived from the contraction of current-current interaction $\psi_2^* \langle\psi_1\psi_1^*\rangle_0\psi_3$, or from the self-energy Feynman diagram in the case of the presence of intermediate vector meson.

(vi) Some meson exchange diagrams between two quarks contribute to nonleptonic hyperon decays, i.e. by the Feynman diagram of one quark line with emission of pion and the other quark line with spurion, where meson is exchanged between two quark lines (see Fig. 3).

In §2 we assume pole diagrams (Fig. 1) for one quark line, and assume that the other two quarks do not participate in the reaction. By pole diagram we mean the diagram, in which spurion and meson vertex lie on one quark line. But we find that we cannot explain the results of experiment in terms of the results (2·4) and (2·5), evaluated from the pole diagrams. In particular we are unable to obtain the parity-conserving amplitude for Σ_+^+.*) In §3, in addition to the pole diagram, we take into account the four-body weak current-current interaction between two quarks, one of which emits a meson, by preventing the third quark from participating in the reaction (Fig. 2). We obtain good results (3·10) for the p.v. amplitude, but do not for the p.c. amplitude. The p.v. amplitude of this case satisfies the Lee-Sugawara relation[5] (3·11). Unfortunately the p.c. amplitude Σ_-^- does not vanish in this case, which contradicts the experiment (2·7). For the p.c. amplitude we must seek another diagram to explain the experiment. Therefore, in §4 (model I), we consider nonet scalar meson exchange diagrams (Fig. 3) instead of the four-body current-current interaction in §3, to explain the p.c. amplitude. Then we find that the pole diagram and the nonet scalar meson exchange diagram fairly well explain the experiments on the p.c. amplitude. In particular we obtain the vanishing p.c. amplitude of Σ_-^-, $P(\Sigma_-^-)=0$. We should mention that this nonet scalar meson exchange does not contribute to the p.v. amplitude in the nonrelativistic quark model. Using the above results, we can predict the decay ratio of Ω^-, but this seems not to explain experiments, though the experiments are not conclusive. In §5 (model II), we give another more consistent solution, by taking into account the pole diagram (Fig. 1), the above mentioned four-body weak current-current interaction (Fig. 2),

*) Hereafter parity-conserving amplitude will be abbreviated the p.c. amplitude, and parity violating amplitude the p.v. amplitude.

the four-body weak current-current interaction between two quarks accompanied with the emission of the meson by the third quark (Fig. 4) and the diagram in which spurion is attached to one quark line and the other emits the meson (Fig. 5). In this case the decay ratio of Ω^- seems to be consistent with experiment. We should remark that in model II Figs. 4 and 5 do not contribute to the p.v. amplitude in our nonrelativistic model.

Summarizing we have arrived at the conclusion that the pole diagram, the four-body weak current-current interaction (Figs. 2 and 4) and the diagram of Fig. 5 can explain the experiments for the p.v. and p.c. amplitudes. In the derivation of the p.v. amplitude we assume the negative (nonrelativistic) energy virtual quark line, while in the derivation of the p.c. amplitude we assume only the positive (nonrelativistic) energy virtual quark line. This situation resembles that of the derivation by current algebra.[6] In the derivation of the p.v. amplitude by current algebra we take into account the high energy contribution, but in the derivation of the p.c. amplitude we assume only low energy contribution. It is satisfactory that our nonrelativistic model, though crude, seems to provide a fairly satisfactory explanation for nonleptonic decays and especially for branching ratio of Ω^- decay[7] which could not be explained by current algebra.

§ 2. Pole diagram

We consider the nonleptonic hyperon decays in terms of the nonrelativistic quark model and the $SU(6)$ theory. These models, though quite crude, seem to give a consistent explanation of the low energy phenomena, in particular the static properties of baryon.[1),3)] In the first place, we take, as the simplest case, pole diagrams (Fig. 1). By the pole diagram, we mean the diagram, in which spurion and meson-emitting vertex lie on one quark line, and the other two quarks do not participate in the reaction.*)

Only the diagram of Fig. 1(a) contributes to the nonleptonic hyperon decays, because in the diagram of Fig. 1(b) a pion is not emitted in the urlambda decay. For that reason we discuss only the diagram of Fig. 1(a), and we obtain

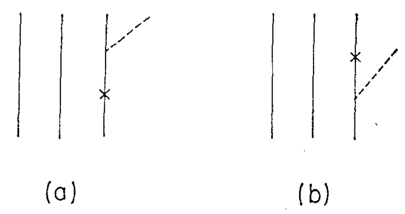

Fig. 1. These diagrams show the nonleptonic decay of one quark, together with quarks not participating in the reaction. Full (Dotted) line is quark (meson) line. Cross is the spurion. The scalar $SU(3)$ singlet meson lines which are responsible for the superstrong interaction between quarks are not written explicitly.

*) We do not consider a meson pole, but the contribution from meson pole with octet spurion or 2-body weak vertex of quark is the same as the contribution of the quark pole, (2·1) and (2·2). The 4-body weak interaction between quark and antiquark violates the $\Delta I=1/2$ rule. This term will be small from the recent argument of Iizuka on the quark model. See, J. Iizuka, Prog. Theor. Phys. **35** (1966), 117, 309. A detailed discussion of this problem will be given elsewhere.

$$\bar{\psi}^{(iA,jB,kC)}\Pi^2_A\psi_{(i3,jB,kC)} \tag{2·1}$$

for the p.v. amplitude and

$$\bar{\psi}^{(iA,jB,kC)}\Pi^2_A(\boldsymbol{k}\cdot\boldsymbol{\sigma})_i^{i'}\psi_{(i'3,jB,kC)} \tag{2·2}$$

for the p.c. amplitude, where $\psi_{(iA,jB,kC)}$ is the basis of 56-dimensional representation of $SU(6)$, i, j and k are the angular spin indices, and A, B and C are unitary spin indices. Π^A_B represents meson octet. $\boldsymbol{\sigma}$ is the spin matrix, and $\boldsymbol{k}$ is the momentum of a pion. Hereafter we shall omit all irrelevant kinematical factors in our paper. Substituting the $SU(3)\times SU(2)$ decomposition of $\psi_{(iA,jB,kC)}$,[8]

$$\psi_{(iA,jB,kC)}=\chi_{(ijk)}d_{(ABC)}+\frac{1}{3\sqrt{2}}\{\epsilon_{ij}\chi_k\epsilon_{ABD}b^D_C+\epsilon_{jk}\chi_i\epsilon_{BCD}b^D_A+\epsilon_{ki}\chi_j\epsilon_{CAD}b^D_B\}, \tag{2·3}$$

into (2·1) and (2·2), we obtain the amplitudes for the nonleptonic hyperon decays

$$\frac{1}{3}\{(\bar{b}\Pi Sb)-(\bar{b}b\Pi S)\}\bar{\chi}\chi \tag{2·4}$$

for the p.v. amplitude and

$$\frac{1}{9}\{5(\bar{b}\Pi Sb)+(\bar{b}b\Pi S)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi \tag{2·5}$$

for the p.c. amplitude, where b^A_B and $d_{(ABC)}$ represent baryon octet ($J=1/2^+$) and baryon decuplet ($J=3/2^+$) respectively, S is octet spurion. χ_i and $\chi_{(ijk)}$ are the spin function of $J=1/2$ and of $3/2$ respectively. The notation $(\bar{b}\Pi Sb)$ means trace with respect to $SU(3)$ indices, i.e. $(\bar{b}\Pi Sb)=\bar{b}^A_D\Pi^B_A S^C_B b^D_C$. Both our results (2·4) and (2·5) seem to be in conflict with experiment. (2·4) is fairly close to the experimental value, but (2·5) violently contradicts the experiment. The experimental values for the decay amplitudes[9] are given as follows:

$$\text{the p.v. amplitude} \propto -0.014\Sigma^+_+-3.3\Sigma^+_0+4.1\Sigma^-_-+3.3\Lambda^0_--4.4\Xi^-_-+3.3\Xi^0_0 \tag{2·6}$$

$$\propto\{-2(\bar{b}\Pi Sb)+5(\bar{b}b\Pi S)\}\bar{\chi}\chi \tag{2·6'}$$

and

$$\text{the p.c. amplitude} \propto 4.1\Sigma^+_++2.5\Sigma^+_0-0.04\Sigma^-_-+1.3\Lambda^0_-+0.84\Xi^-_--0.56\Xi^0_0 \tag{2·7}$$

$$\propto\{5(\bar{b}\Pi Sb)+4(\bar{b}b\Pi S)-4(\bar{b}S)(b\Pi)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi. \tag{2·7'}$$

When we get (2·7′) from (2·7), we take account of the momentum dependence which is different for various hyperon decays. When we compare our result (2·5) for the p.c. amplitude with experiment (2·7′), we find that (2·5) predicts the vanishing p.c. amplitude Σ^+_+ which considerably contradicts experiment. Therefore we cannot explain the experiments of nonleptonic hyperon decays only in terms of pole diagram.

§ 3. Four-body weak current-current interaction and parity-violating amplitude

In §2 we have found that the pole diagram is not sufficient to explain our problem. Next we assume, besides the pole diagram, the following four-body weak current-current interaction (shown in Fig. 2) between two quarks, one of which emits a meson, by making the third quark not participate in the reaction.

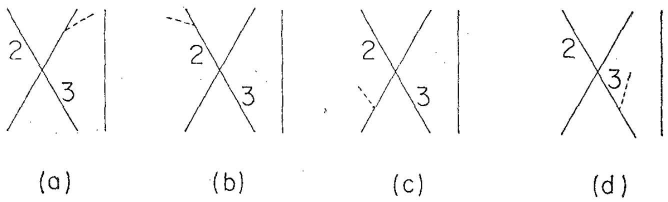

Fig. 2. Diagrams of nonleptonic decay due to the four-body weak current-current interaction between two quarks. The third quark does not participate in the reaction.

The interaction is given, in terms of the V-A current, as follows:

$$\bar{\psi}^2\gamma^\mu(1+\gamma_5)\psi_1\bar{\psi}^1\gamma_\mu(1+\gamma_5)\psi_3, \tag{3·1}$$

or in its nonrelativistic approximation, (3·1) is reduced to

$$u^{2*}u_1u^{1*}u_3 - u^{2*}\boldsymbol{\sigma}u_1u^{1*}\boldsymbol{\sigma}u_3, \tag{3·2}$$

and also $\psi(\bar{\psi})$ is reduced to $u(u^*)$. One of the important consequences of this paper is that, in our nonrelativistic quark model, (3·2) obeys the $\Delta I=1/2$ rule, in spite of its $SU(3)$ transformation property $J_1^2J_3^1$ which seems to be $(\Delta I=1/2)$-violating at first sight. In order to prove this, we must invoke nonrelativistic quark model and three-triplet model.[4] We assume that baryons are classified in $(1,n)$, where $(1,n)$ means the totally antisymmetric one-dimensional irreducible representation of $SU'(3)$ and n-dimensional irreducible representation of $SU(3)$. In the three-triplet model, baryons are totally antisymmetric state with respect to $SU'(3)$ and $SU(3)$ tensor suffices and angular spin suffices. In the three-triplet model, (3·2) should be rewritten as follows:

$$u^{2\alpha*}u_{1\alpha}u^{1\beta*}u_{3\beta} - u^{2\alpha*}\boldsymbol{\sigma}u_{1\alpha}u^{1\beta*}\boldsymbol{\sigma}u_{3\beta}, \tag{3·2′}$$

where α and β are $SU'(3)$ tensor suffices and $[u^{A\alpha*}, u_{B\beta}]_+ = \delta_{AB}\delta_{\alpha\beta}$.

Theorem. (3·2′) obeys the $\Delta I=1/2$ rule for the $(1,n)$ state, or (3·2′) effectively equals

$$2(u^{2\alpha*}u_{A\alpha}u^{A\beta*}u_{3\beta} - u^{2\alpha*}u_{3\alpha}u^{A\beta*}u_{A\beta}) \tag{3·3}$$

for the $(1,n)$ state.

Proof. Using the nonrelativistic version of the Fierz transformation identity,

$$\boldsymbol{\sigma}_i^{i'}\cdot\boldsymbol{\sigma}_j^{j'} = 2\delta_j^{i'}\delta_i^{j'} - \delta_i^{i'}\delta_j^{j'}, \tag{3·4}$$

we obtain

$$\langle(1,n)|(u^{2\alpha*}u_{1\alpha}u^{1\beta*}u_{3\beta}-u^{2\alpha*}\boldsymbol{\sigma}u_{1\alpha}u^{1\beta*}\boldsymbol{\sigma}u_{3\beta})|(1,n)\rangle$$
$$=2\langle(1,n)|(u^{2\alpha*}u_{1\alpha}u^{1\beta*}u_{3\beta}+u^{2\alpha*}u_{3\beta}u^{1\beta*}u_{1\alpha})|(1,n)\rangle,$$

which is effectively equal to

$$2\langle(1,n)|(u^{2\alpha*}u_{1\alpha}u^{1\beta*}u_{3\beta}-u^{2\alpha*}u_{3\alpha}u^{1\beta*}u_{1\beta})|(1,n)\rangle, \tag{3·5}$$

because of the antisymmetric $SU'(3)$ state of $(1,n)$. (3·5) is easily shown to be

$$2\langle(1,n)|(u^{2\alpha*}u_{A\alpha}u^{A\beta*}u_{3\beta}-u^{2\alpha*}u_{3\alpha}u^{A\beta*}u_{A\beta})|(1,n)\rangle. \qquad \text{Q.E.D.}$$

Using (3·2′) and pion-quark vertex $u^{A\alpha*}(\boldsymbol{k}\cdot\boldsymbol{\sigma})u_{B\alpha}\Pi^{B\beta}_{A\beta}$, we can establish the $\Delta I=1/2$ p.c. amplitude for the nonleptonic hyperon decay. In a similar way we can prove the theorem corresponding to (3·3) for the p.v. amplitude by replacing one of $u^*(u)$ by the nonrelativistic antiquark spinor $(v\sigma_2)^{tr}((\sigma_2 v^*)^{tr})$. Using this corresponding theorem and pion-quark vertex $((v^{B\beta}\sigma_2)^{tr}u_{A\beta}+u^{\beta B*}(\sigma_2 v_{A\beta}{}^*)^{tr})\Pi^{A\alpha}_{B\alpha}$, we can also establish the $\Delta I=1/2$ p.v. amplitude for the nonleptonic hyperon decay.

Now we shall evaluate, using $\Delta I=1/2$ obeying (3·2). We can obtain the p.v. and p.c. amplitudes from (3·2) or (3·3) as follows:

$$\bar{\psi}^{(i2,jB,kC)}\Pi^A_B\psi_{(iA,j3,kC)}-\bar{\psi}^{(i2,jB,kC)}\boldsymbol{\sigma}_i^{i'}\cdot\boldsymbol{\sigma}_j^{j'}\Pi^A_B\psi_{(i'A,j'3,kC)}, \tag{3·6}$$
$$\bar{\psi}^{(i2,jB,kC)}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_j^{j'}\Pi^A_B\psi_{(iA,j'3,kC)}-\bar{\psi}^{(i2,j'B,kC)}\boldsymbol{\sigma}_i^{i'}\cdot[(\boldsymbol{k}\cdot\boldsymbol{\sigma})\boldsymbol{\sigma}]_j^{j'}\Pi^A_B\psi_{(i'A,j'3,kC)}, \tag{3·6′}$$

corresponding to the diagram of Fig. 2(a),

$$\bar{\psi}^{(iA,jB,kC)}\Pi^2_A\psi_{(iB,j3,kC)}-\bar{\psi}^{(iA,jB,kC)}\boldsymbol{\sigma}_i^{i'}\cdot\boldsymbol{\sigma}_j^{j'}\cdot\Pi^2_A\psi_{(i'B,j'3,kC)}, \tag{3·7}$$
$$\bar{\psi}^{(iA,jB,kC)}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_i^{i'}\Pi^2_A\psi_{(i'B,j3,kC)}-\bar{\psi}^{(iA,jB,kC)}[(\boldsymbol{k}\cdot\boldsymbol{\sigma})\boldsymbol{\sigma}]_i^{i'}\boldsymbol{\sigma}_j^{j'}\Pi^2_A\psi_{(i'B,j'3,kC)}, \tag{3·7′}$$

corresponding to the diagram of Fig. 2(b), and

$$-\{\bar{\psi}^{(i2,jB,kC)}\Pi^A_B\psi_{(iA,j3,kC)}-\bar{\psi}^{(i2,jB,kC)}\boldsymbol{\sigma}_i^{i'}\cdot\boldsymbol{\sigma}_j^{j'}\Pi^A_B\psi_{(i'A,j'3,kC)}\}, \tag{3·8}$$
$$-\{\bar{\psi}^{(i2,jB,kC)}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_i^{i'}\Pi^A_B\psi_{(iA,j3,kC)}-\bar{\psi}^{(i2,jB,kC)}[\boldsymbol{\sigma}(\boldsymbol{k}\cdot\boldsymbol{\sigma})]_i^{i'}\cdot\boldsymbol{\sigma}_j^{j'}\Pi^A_B\psi_{(i'A,j'3,kC)}\}, \tag{3·8′}$$

corresponding to the diagram of Fig. 2(c), where the minus signs of (3·8) and (3·8′) are due to energy denominators. (3·6), (3·7) and (3·8) refer to the p.v. amplitude and (3·6′), (3·7′) and (3·8′) to the p.c. amplitude. We need not discuss the diagram of Fig. 2(d), because of the same reason as in the diagram of Fig. 1(b). Substituting $\psi_{(iA,jB,kC)}$ given by (2·3) into (3·6), (3·6′), (3·7), (3·7′), (3·8) and (3·8′), we get amplitudes for the nonleptonic hyperon decay as follows:

$$\frac{1}{9}[5\{(\bar{b}\Pi bS)+(\bar{b}Sb\Pi)+(\bar{b}\Pi Sb)-(\bar{b}S)(b\Pi)\}-4(\bar{b}b\Pi S)]\bar{\chi}\chi,$$

$$\frac{1}{3}\{(\bar{b}\Pi Sb)-(\bar{b}Sb\Pi)+(\bar{b}\Pi bS)+(\bar{b}S)(b\Pi)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi,$$

$$(bb\Pi S)\bar{\chi}\chi,$$

$$-\frac{1}{3}\{(\bar{b}b\Pi S)+2(\bar{b}\Pi Sb)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi,$$

$$-\frac{1}{9}[5\{(\bar{b}\Pi bS)+(\bar{b}Sb\Pi)+(\bar{b}\Pi Sb)-(\bar{b}S)(b\Pi)\}-4(\bar{b}b\Pi S)]\bar{\chi}\chi$$

and

$$-\frac{1}{3}\{(\bar{b}\Pi bS)-(\bar{b}\Pi Sb)-(\bar{b}Sb\Pi)-(\bar{b}S)(b\Pi)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi,$$

corresponding to the order of terms from (3·6) to (3·8′). Summing up the above results, we obtain

$$(\bar{b}b\Pi S)\bar{\chi}\chi \qquad (3\cdot 9)$$

for the p.v. amplitude and

$$\frac{1}{3}[2(\bar{b}S)(b\Pi)-(\bar{b}b\Pi S)]\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi \qquad (3\cdot 9')$$

for the p.c. amplitude. The first term in the square brackets of (3·9′) is preferable for the Σ nonleptonic decay. We must add (3·9) to (2·4) for the p.v. amplitude and must add (3·9′) to (2·5) for the p.c. amplitude. Thus we obtain for the p.v. amplitude

$$\text{the p.v. amplitude} \propto \left[\frac{a_1}{3}\{(\bar{b}\Pi Sb)-(\bar{b}b\Pi S)\}+b_1(\bar{b}b\Pi S)\right]\bar{\chi}\chi$$

$$=0.9\{-2(\bar{b}\Pi Sb)+5(\bar{b}b\Pi S)\}\bar{\chi}\chi$$

$$=\{-3.2\Sigma_0^+ +4.5\Sigma_-^- +3.3\Lambda_-^0 -2.3\Lambda_0^0 -4.4\Xi_-^- +3.3\Xi_0^0\}\bar{\chi}\chi \qquad (3\cdot 10)$$

with $a_1=-5.4$ and $b_1=2.7$ and obtain for the p.c. amplitude

$$\text{the p.c. amplitude} \propto \left[\frac{a_2}{9}\{5(\bar{b}\Pi Sb)+(\bar{b}b\Pi S)\}+\frac{b_2}{3}\{2(\bar{b}S)(b\Pi)-(\bar{b}b\Pi S)\}\right]\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi$$

$$=-0.52\{5(\bar{b}\Pi Sb)+4(\bar{b}b\Pi S)-6(\bar{b}S)(b\Pi)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi \qquad (3\cdot 10')$$

with $a_2=-4.7$ and $b_2=4.7$, where a_i and b_i are the adjusting parameters to be fitted to the experimental value for the p.v. and p.c. amplitudes of Λ_-^0 and Ξ_-^-. When we compare our results (3·10) and (3·10′) with experiment, we find that the p.v. amplitude is in good agreement with experiment, but that the p.c. amplitude is not. For the p.v. amplitude we have also the Lee-Sugawara relation:[5)]

$$\Lambda_-^0+2\Xi_-^-=\sqrt{3}\,\Sigma_0^+, \qquad (3\cdot 11)$$

which is an immediate consequence of odd-charge conjugation parity of V-A current-current interaction.[10)]

§4. Model I for parity-conserving amplitude

In §§2 and 3 it is shown that the pole diagram and the four-body weak current-current interaction give good results for the p.v. amplitude, but not for the p.c. amplitude. Consequently we must seek diagram which explains the experimental value of the p.c. amplitude but does not contribute to the p.v. amplitude. We find from calculation of the Feynman amplitude that scalar meson contributes only to the p.c. amplitude in nonrelativistic approximation. Hence we discuss the nonet scalar meson exchange diagram (shown in Fig. 3) between two quark lines, where a meson is emitted by one quark and spurion is attached to the other quark, and the third quark does not participate in the interaction.

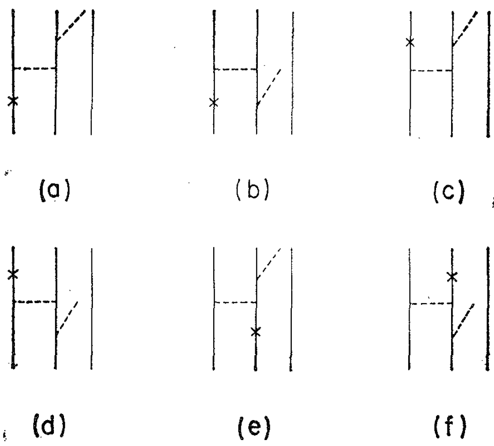

Fig. 3. Diagrams of nonleptonic decay due to nonet scalar meson exchange.

From Fig. 3, p.c. amplitudes are

$$\begin{aligned}
&\bar{\psi}^{(iA,jB,kC)}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_j^{j'}\Pi_B^2\psi_{(i3,j'A,kC)},\\
-&\bar{\psi}^{(iA,jB,kC)}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_j^{j'}\Pi_A^B\psi_{(i3,j'B,kC)},\\
-&\bar{\psi}^{(i2,jB,kC)}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_j^{j'}\Pi_B^A\psi_{(iA,j'3,kC)}
\end{aligned} \tag{4·1}$$

and

$$\bar{\psi}^{(i2,jB,kC)}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_j^{j'}\Pi_3^{B'}\psi_{(iB,j'B',kC)},$$

corresponding to the diagrams of Figs. 3(a), (b), (c) and (d) respectively, where we omit irrelevant kinematical factors as before and the minus signs of the second and third amplitudes of (4·1) result from the calculation of Feynman diagram. The fourth amplitude of (4·1) is neglected on account of the same reason as mentioned for the diagram of Fig. 1(b). The first, second, and third amplitudes of (4·1) are evaluated as follows:

$$\frac{1}{18}\{4(\bar{b}\Pi Sb)-(\bar{b}b\Pi S)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi,$$

$$\frac{1}{18}\{4(\bar{b}S)(b\Pi)+2(\bar{b}\Pi Sb)+(\bar{b}\Pi bS)+(\bar{b}Sb\Pi)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi$$

and (4·1′)

$$\frac{1}{18}\{-4(\bar{b}\Pi Sb)-2(\bar{b}S)(b\Pi)+(\bar{b}Sb\Pi)+(\bar{b}\Pi bS)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi,$$

respectively. Summing up all the terms in (4·1′), we obtain

$$\frac{1}{18}\{4(\bar{b}S)(b\Pi)-3(\bar{b}b\Pi S)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi \tag{4·2}$$

effectively, where the following identity (4·3) is used. As for observed nonleptonic hyperon decays, the following relation exists

$$(\bar{b}S)(b\Pi)-(\bar{b}\Pi Sb)-(\bar{b}b\Pi S)=(\bar{b}\Pi bS)+(\bar{b}Sb\Pi). \tag{4·3}$$

We add the meson exchange diagram (Fig. 3) to the pole diagram (Fig. 1). We find for the p.c. amplitude

$$\text{the p.c. amplitude} \propto \left[\frac{a_3}{9}\{5(\bar{b}\Pi Sb)+(\bar{b}b\Pi S)\}+\frac{b_3}{18}\{4(\bar{b}S)(b\Pi)-3(\bar{b}b\Pi S)\}\right]\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi$$

$$=-0.52\{5(\bar{b}\Pi Sb)+4(\bar{b}b\Pi S)-4(\bar{b}S)(b\Pi)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi$$

$$=4.0\Sigma_+^+ +2.8\Sigma_0^+ +1.3\Lambda_-^0-0.92\Lambda_0^0+0.84\Xi_-^- -0.59\Xi_0^0, \tag{4·4}$$

with $a_3=-4.7$, $b_3=9.4$, where a_3 and b_3 are the adjusting parameters. We find that the agreement of (4·4) with experiment (2·7) is very good. We omit diagrams such as Figs. 3(e) and (f), because these diagrams are considered as perturbation to the pole diagram (Fig. 1).

As baryon octet and baryon decuplet belong to the 56-dimensional representation of $SU(6)$, we are able to predict the decay rates of $\Omega^-\to\Xi^0+\Pi^-$, $\Omega^-\to\Xi^-+\Pi^0$ and $\Omega^-\to\Lambda+K^-$ using the above results. We suppose that the contribution to the p.v. amplitude comes from the pole diagram and the four-body weak current-current interaction, and the contribution to the p.c. amplitude comes from the pole diagram and the nonet scalar meson exchange diagram. We thus obtain the p.c. amplitude of Ω^- decay as follows:

$$P(\Omega^-\to\Xi^0+\Pi^-)=\frac{\sqrt{2}}{3}(a_3+b_3)\bar{\chi}^i\epsilon^{jk}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_k^{k'}\chi_{(ijk')},$$

$$P(\Omega^-\to\Xi^-+\Pi^0)=\frac{1}{\sqrt{2}}P(\Omega^-\to\Xi^0+\Pi^-) \tag{4·5}$$

and

$$P(\Omega^-\to\Lambda+K^-)=0.$$

In a recent experiment[11] (13 events) the decays of $\Omega\to\Xi+\Pi$ and $\Omega\to\Lambda+K$ are found in seven and six events rerpectively. Hence (4·5) is inconsistent with experiment. We suppose that the Ω^- decays as follows:

$$\Omega^-\to\Xi^{*-}(\text{virtual state})\to\begin{cases}\Xi+\Pi\\ \Lambda+K.\end{cases} \tag{4·6}$$

Then we obtain from the second process of (4·6)

$$P(\Xi^{*-}\to\Xi^0+\Pi^-)=\sqrt{2}\,P(\Xi^{*-}\to\Xi^-+\Pi^0)=\frac{\sqrt{6}}{3}P(\Xi^{*-}\to\Lambda+K^-),$$

which seems to be consistent with the above mentioned experiment.

§ 5. Model II for parity-conserving amplitude

We present the other possibility of choosing diagrams to explain the p.c. amplitude. We consider the diagrams shown in Figs. 4 and 5, besides the pole diagram in § 2 and the four-body weak current-current in § 3.

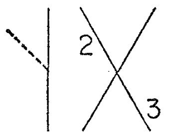

Fig. 4. Diagrams of nonleptonic decay due to the four-body weak current-current interaction of different kind from Fig. 2. A meson is emitted by the third quark.

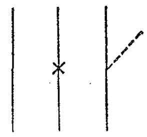

Fig. 5. Diagram of nonleptonic decay, in which spurion is attached to one quark line and the other emits a pion.

The contributions of Figs. 4 and 5 are given by

$$\bar{\psi}^{(iA,j2,kC)}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_i^{i'}\Pi_A^{A'}\psi_{(i'A',j'C,k3)}-\bar{\psi}^{(iA,j2,kC)}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_i^{i'}\boldsymbol{\sigma}_j^{j'}\cdot\boldsymbol{\sigma}_k^{k'}\Pi_A^{A'}\psi_{(i'A',j'C,k'3)}$$

and

$$\bar{\psi}^{(i2,jB,kC)}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_j^{j'}\Pi_B^{B'}\psi_{(i3,j'B',kC)},$$

and in the same manner as §§ 2, 3 and 4 we obtain

$$(\overline{b}\Pi bS)\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi \tag{5·1}$$

and

$$\frac{1}{18}\{(\overline{b}S)(b\Pi)-(\overline{b}\Pi Sb)-2(\overline{b}b\Pi S)-6(\overline{b}\Pi bS)\}\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi \tag{5·2}$$

respectively, where the relation (4·3) is used. We obtain from (2·5), (3·9′), (5·1) and (5·2),

$$\text{the p.c. amplitude} \propto\left[\frac{a}{9}\{5(\overline{b}\Pi Sb)+(\overline{b}b\Pi S)\}+\frac{b}{3}\{2(\overline{b}S)(b\Pi)-(\overline{b}b\Pi S)\}\right.$$
$$\left.+c(\overline{b}\Pi bS)+\frac{d}{18}\{(\overline{b}S)(b\Pi)-(\overline{b}\Pi Sb)-2(\overline{b}b\Pi S)-6(\overline{b}\Pi bS)\}\right]\bar{\chi}\boldsymbol{k}\cdot\boldsymbol{\sigma}\chi. \tag{5·3}$$

Clearly we reproduce the experiment (2·7′) from (5·3) with a:b:c:d = −4.0: 2.5: 2.4: 7.2. Then this model will be checked by the decay rate of Ω^-. The ratio of $\Gamma(\Omega\to\Lambda+K)$ and $\Gamma(\Omega\to\Xi+\Pi)$ is evaluated in a way similar to model I:

$$P(\Omega^-\to\Xi^0+\Pi^-)=\frac{\sqrt{2}}{3}a\bar{\chi}^i\epsilon^{jk}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_k^{k'}\chi_{(ijk')},$$

$$P(\Omega^-\to\Xi^-+\Pi^0)=\frac{1}{\sqrt{2}}P(\Omega^-\to\Xi^0+\Pi^-),$$

$$P(\Omega^-\to\Lambda+K^-)=\left(\frac{2}{\sqrt{3}}b+\frac{1}{2\sqrt{3}}d\right)\bar{\chi}^i\epsilon^{jk}(\boldsymbol{k}\cdot\boldsymbol{\sigma})_k^{k'}\chi_{(ijk')}, \tag{5·4}$$

hence

$$\frac{\Gamma(\Omega^-\to\Xi+\Pi)}{\Gamma(\Omega^-\to\Lambda+K)}\cong\frac{|\frac{1}{3}a|^2+|\frac{\sqrt{2}}{3}a|^2}{|\frac{2}{\sqrt{3}}b+\frac{1}{2\sqrt{3}}d|^2}\ \frac{k_\Pi^3}{k_K^3}\ \frac{M_\Xi}{M_\Omega}\ \frac{M_\Omega}{M_\Lambda}\cong 0.64. \tag{5·5}$$

Our result (5·5) seems to be consistent with experiment mentioned in § 4. It is to be noticed that Figs. 4 and 5 do not contribute to the p.v. decay. Thus model II is more consistent than model I. Using the experimental value of Σ_+^+ and Eqs. (5·4), we find that the life time of Ω^- is given by

$$\tau(\Omega^-)\cong 0.2\times 10^{-10}\text{sec}, \tag{5·6}$$

which seems to be inconsistent with experiment $\tau(\Omega^-)=(1.5\pm 0.5)\times 10^{-10}$ sec. This is only our weak point for model II.

§ 6. Discussion

The nonrelativistic quark model and the $SU(6)$ theory can successfully explain the various static properties of baryon, such as Cabbibo current (F-D ratio), anomalous magnetic moment, electromagnetic mass difference, and Gell-Mann-Okubo mass formula. It is quite an important task to test our model by other phenomena. We have attempted at the explanation of nonleptonic hyperon decay. In spite of this more complicated process, compared to the above mentioned static properties, we have managed to interpret the nonleptonic hyperon decay amplitude in terms of the nonrelativistic quark model and the $SU(6)$ theory. In §§ 2, 3, 4 and 5 we have shown that the main contribution to the p.v. amplitude comes from the pole diagram and the four-body weak current-current interaction, and the main contribution to the p.c. amplitude comes from the pole diagram and the meson exchange diagram in model I or from the pole diagram, the four-body weak current-current interaction, and those diagrams of Figs. 4 and 5 in model II. One of the interesting results in § 3 is the demonstration of the proof of the $\Delta I=1/2$ rule from the $J_1^2J_3^1$ current-current interaction, on the basis of nonrelativistic quark model and three-triplet model. Model II is more consistent than model I, in disregard of the usual meson exchange, and is preferable for the explanation of experiment. Not only our results seem to agree with the result of the method of current algebra, but our model can be considered as a realistic embodiment of an abstract current algebra method. Further, what is to be emphasized is that the result of model II can explain the decay ratio $\Gamma(\Omega\to\Xi+\Pi)/\Gamma(\Omega\to\Lambda+K)$ which could not be explained by current algebra. Finally it is to be remarked that we assume the applicability of the Feynman diagram method to quark.

Acknowledgements

The authors would like to express their sincere thanks to Professor Y. Miyamoto, who suggested this problem, for valuable discussions and careful reading of the manuscript. They are also grateful to Professor C. Iso and Professor K. Fujii for their valuable comments.

References

1) F. Gürsey and L.A. Radicati, Phys. Rev. Letters **13** (1964), 173.
B. Sakita, Phys. Rev. **136** (1964), 1756.
A. Pais, Phys. Rev. Letters **13** (1964), 175.
2) Y. Nambu, *Proceedings of Second Coral Gables Conference* (1965), 274.
3) Y. Miyamoto, Prog. Theor. Phys. **35** (1966), 175.
J. Arafune and Y. Iwasaki, Prog. Theor. Phys. **35** (1966), 339.
T. Minamikawa, K. Miura, Y. Miyamoto, J. Arafune and Y. Iwasaki, Prog. Theor. Phys. Suppl. Nos. 37 and 38 (1966), 56.
S. Ishida, K. Konno and H. Shimodaira, Prog. Theor. Phys. **36** (1966), 1243.
4) Y. Miyamoto, Prog. Theor. Phys. Suppl. Extra Number (1965).
S. Hori, Prog. Theor. Phys. **36** (1966), 131.
M. Y. Han and Y. Nambu, Phys. Rev. **139** (1965), B1006.
5) H. Sugawara, Prog. Theor. Phys. **31** (1964), 213.
B. W. Lee, Phys. Rev. Letters **12** (1964), 83.
6) H. Sugawara, Phys. Rev. Letters **15** (1965), 870.
M. Suzuki, Phys, Rev. Letters **15** (1965), 986.
Y. Hara, Y. Nambu and J. Schechter, Phys. Rev. Letters **16** (1966), 380.
L. S. Brown and C. M. Sommerfield, Phys. Rev. Letters **16** (1966), 751.
7) Y. Hara, Phys. Rev. **150** (1960), 1175.
8) M.A. Bèg, B.W. Lee and A. Pais, Phys. Rev. Letters **13** (1965), 514.
9) A. H. Rosenfeld et al., Table from UCRL-8030 (revised), August 31, 1966. See reference 6).
10) M. Gell-Mann, Phys. Rev. Letters **12** (1964), 155.
11) J. Allison, private communication.