

Lecture Notes in Mathematics

1932

Editors:

J.-M. Morel, Cachan

F. Takens, Groningen

B. Teissier, Paris



**FONDAZIONE
CIME
ROBERTO CONTI**

CENTRO INTERNAZIONALE MATEMATICO ESTIVO
INTERNATIONAL MATHEMATICAL SUMMER CENTER

C.I.M.E. means Centro Internazionale Matematico Estivo, that is, International Mathematical Summer Center. Conceived in the early fifties, it was born in 1954 and made welcome by the world mathematical community where it remains in good health and spirit. Many mathematicians from all over the world have been involved in a way or another in C.I.M.E.'s activities during the past years.

So they already know what the C.I.M.E. is all about. For the benefit of future potential users and co-operators the main purposes and the functioning of the Centre may be summarized as follows: every year, during the summer, Sessions (three or four as a rule) on different themes from pure and applied mathematics are offered by application to mathematicians from all countries. Each session is generally based on three or four main courses (24–30 hours over a period of 6-8 working days) held from specialists of international renown, plus a certain number of seminars.

A C.I.M.E. Session, therefore, is neither a Symposium, nor just a School, but maybe a blend of both. The aim is that of bringing to the attention of younger researchers the origins, later developments, and perspectives of some branch of live mathematics.

The topics of the courses are generally of international resonance and the participation of the courses cover the expertise of different countries and continents. Such combination, gave an excellent opportunity to young participants to be acquainted with the most advance research in the topics of the courses and the possibility of an interchange with the world famous specialists. The full immersion atmosphere of the courses and the daily exchange among participants are a first building brick in the edifice of international collaboration in mathematical research.

C.I.M.E. Director	C.I.M.E. Secretary
Pietro ZECCA	Elvira MASCOLO
Dipartimento di Energetica "S. Stecco"	Dipartimento di Matematica
Università di Firenze	Università di Firenze
Via S. Marta, 3	viale G.B. Morgagni 67/A
50139 Florence	50134 Florence
Italy	Italy
e-mail: zecca@unifi.it	e-mail: mascolo@math.unifi.it

For more information see CIME's homepage: <http://www.cime.unifi.it>

CIME's activity is supported by:

- Istituto Nazionale di Alta Matematica "F. Severi"
- Ministero dell'Istruzione, dell'Università e delle Ricerca
- Ministero degli Affari Esteri, Direzione Generale per la Promozione e la Cooperazione, Ufficio V
- E.U. under the Training and Mobility of Researchers Programme UNESCO ROSTE
- This course was also supported by the research project PRIN 2004 "Control, Optimization and Stability of Nonlinear Systems: Geometric and Analytic Methods"

Andrei A. Agrachev · A. Stephen Morse
Eduardo D. Sontag · Héctor J. Sussmann
Vadim I. Utkin

Nonlinear and Optimal Control Theory

Lectures given at the
C.I.M.E. Summer School
held in Cetraro, Italy
June 19–29, 2004

Editors:
Paolo Nistri
Gianna Stefani

 Springer



Andrei A. Agrachev
SISSA-ISAS
International School for Advanced Studies
via Beirut 4
34014 Trieste, Italy
agrachev@sissa.it

Eduardo D. Sontag
Héctor J. Sussmann
Department of Mathematics, Hill Center
Rutgers University
110 Frelinghuysen Rd
Piscataway, NJ 08854-8019, USA
sontag@math.rutgers.edu
sussmann@math.rutgers.edu

A. Stephen Morse
Department of Electrical Engineering
Yale University
PO Box 208267
New Haven CT 06520-8284, USA
morse@sycs.eng.yale.edu

Vadim I. Utkin
Department of Electrical Engineering
205 Dreese Laboratory
The Ohio State University
2015 Neil Avenue
Columbus, OH 43210, USA
utkin@ee.eng.ohio-state.edu

Paolo Nistri
Dipartimento di Ingegneria
dell'Informazione
Facoltà di Ingegneria
Università di Siena
via Roma 56
53100 Siena, Italia
pnistri@dii.unisi.it
<http://www.dii.unisi.it/~pnistri/>

Gianna Stefani
Dipartimento di Matematica Applicata
"G. Sansone"
Facoltà di Ingegneria
Università di Firenze
via di S. Marta 3
50139 Firenze, Italia
gianna.stefani@unifi.it
<http://poincare.dma.unifi.it/~stefani/>

ISBN: 978-3-540-77644-4
DOI: 10.1007/978-3-540-77653-6

e-ISBN: 978-3-540-77653-6

Lecture Notes in Mathematics ISSN print edition: 0075-8434
ISSN electronic edition: 1617-9692

Library of Congress Control Number: 2007943246

Mathematics Subject Classification (2000): 93B50, 93B12, 93D25, 49J15, 49J24

© 2008 Springer-Verlag Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Cover design: *design & production* GmbH, Heidelberg

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

Preface

Mathematical Control Theory is a branch of Mathematics having as one of its main aims the establishment of a sound mathematical foundation for the control techniques employed in several different fields of applications, including engineering, economy, biology and so forth. The systems arising from these applied Sciences are modeled using different types of mathematical formalism, primarily involving Ordinary Differential Equations, or Partial Differential Equations or Functional Differential Equations. These equations depend on one or more parameters that can be varied, and thus constitute the control aspect of the problem. The parameters are to be chosen so as to obtain a desired behavior for the system. From the many different problems arising in Control Theory, the C.I.M.E. school focused on some aspects of the control and optimization of nonlinear, not necessarily smooth, dynamical systems. Two points of view were presented: Geometric Control Theory and Nonlinear Control Theory. The C.I.M.E. session was arranged in five six-hours courses delivered by Professors A.A. Agrachev (SISSA-ISAS, Trieste and Steklov Mathematical Institute, Moscow), A.S. Morse (Yale University, USA), E.D. Sontag (Rutgers University, NJ, USA), H.J. Sussmann (Rutgers University, NJ, USA) and V.I. Utkin (Ohio State University Columbus, OH, USA).

We now briefly describe the presentations.

Agrachev's contribution began with the investigation of second order information in smooth optimal control problems as a means of explaining the variational and dynamical nature of powerful concepts and results such as Jacobi fields, Morse's index formula, Levi-Civita connection, Riemannian curvature. These are primarily known only within the framework of Riemannian Geometry. The theory presented is part of a beautiful project aimed at investigating the connections between Differential Geometry, Dynamical Systems and Optimal Control Theory.

The main objective of Morse's lectures was to give an overview of a variety of methods for synthesizing and analyzing logic-based switching control systems. The term "logic-based switching controller" is used to denote a controller whose subsystems include not only familiar dynamical components

(integrators, summers, gains, etc.) but logic-driven elements as well. An important category of such control systems are those consisting of a process to be controlled, a family of fixed-gain or variable-gain candidate controllers, and an “event-drive switching logic” called a supervisor whose job is to determine in real time which controller should be applied to the process. Examples of supervisory control systems include re-configurable systems, and certain types of parameter-adaptive systems.

Sontag’s contribution was devoted to the input to state stability (ISS) paradigm which provides a way of formulating questions of stability with respect to disturbances, as well as a method to conceptually unify detectability, input/output stability, minimum-phase behavior, and other systems properties. The lectures discussed the main theoretical results concerning ISS and related notions. The proofs of the results showed in particular connections to relaxations for differential inclusions, converse Lyapunov theorems, and nonsmooth analysis.

Sussmann’s presentation involved the technical background material for a version of the Pontryagin Maximum Principle with state space constraints and very weak technical hypotheses. It was based primarily on an approach that used generalized differentials and packets of needle variations. In particular, a detailed account of two theories of generalized differentials, the “generalized differential quotients” (GDQs) and the “approximate generalized differential quotients” (AGDQs), was presented. Then the resulting version of the Maximum Principle was stated.

Finally, Utkin’s contribution concerned the Sliding Mode Control concept that for many years has been recognized as one of the key approaches for the systematic design of robust controllers for complex nonlinear dynamic systems operating under uncertainty conditions. The design of feedback control in systems with sliding modes implies design of manifolds in the state space where control components undergo discontinuities, and control functions enforcing motions along the manifolds. The design methodology was illustrated by sliding mode control to achieve different objectives: eigenvalue placement, optimization, disturbance rejection, identification.

The C.I.M.E. course was attended by fifty five participants from several countries. Both graduate students and senior mathematicians intensively followed the lectures, seminars and discussions in a friendly and co-operative atmosphere.

As Editors of these Lectures Notes we would like to thank the persons and institutions that contributed to the success of the course. It is our pleasure to thank the Scientific Committee of C.I.M.E. for supporting our project: the Director, Prof. Pietro Zecca and the Secretary, Prof. Elvira Mascolo for their support during the organization. We would like also to thank Carla Dionisi for her valuable and efficient work in preparing the final manuscript for this volume.

Our special thanks go to the lecturers for their early preparation of the material to be distributed to the participants, for their excellent performance in teaching the courses and their stimulating scientific contributions.

We dedicate this volume to our teacher Prof. Roberto Conti, one of the pioneers of Mathematical Control Theory, who contributed in a decisive way to the development and to the international success of Fondazione C.I.M.E.

Siena and Firenze, May 2006

*Paolo Nistri
Gianna Stefani*

Contents

Geometry of Optimal Control Problems and Hamiltonian Systems

<i>A.A. Agrachev</i>	1
1 Lagrange Multipliers' Geometry	1
1.1 Smooth Optimal Control Problems	1
1.2 Lagrange Multipliers	4
1.3 Extremals	6
1.4 Hamiltonian System	7
1.5 Second Order Information	10
1.6 Maslov Index	14
1.7 Regular Extremals	22
2 Geometry of Jacobi Curves	25
2.1 Jacobi Curves	25
2.2 The Cross-Ratio	26
2.3 Coordinate Setting	28
2.4 Curves in the Grassmannian	29
2.5 The Curvature	30
2.6 Structural Equations	33
2.7 Canonical Connection	35
2.8 Coordinate Presentation	38
2.9 Affine Foliations	39
2.10 Symplectic Setting	41
2.11 Monotonicity	44
2.12 Comparison Theorem	49
2.13 Reduction	51
2.14 Hyperbolicity	53
References	58

Lecture Notes on Logically Switched Dynamical Systems

<i>A.S. Morse</i>	61
1 The Quintessential Switched Dynamical System Problem	62
1.1 Dwell-Time Switching	62

1.2	Switching Between Stabilizing Controllers	65
1.3	Switching Between Graphs	66
2	Switching Controls with Memoryless Logics	67
2.1	Introduction	67
2.2	The Problem	67
2.3	The Solution	67
2.4	Analysis	68
3	Collaborations	68
4	The Curse of the Continuum	69
4.1	Process Model Class	69
4.2	Controller Covering Problem	73
4.3	A Natural Approach	74
4.4	A Different Approach	75
4.5	Which Metric?	75
4.6	Construction of a Control Cover	76
5	Supervisory Control	76
5.1	The System	77
5.2	Slow Switching	86
5.3	Analysis	87
5.4	Analysis of the Dwell Time Switching Logic	102
6	Flocking	110
6.1	Leaderless Coordination	111
6.2	Symmetric Neighbor Relations	142
6.3	Measurement Delays	148
6.4	Asynchronous Flocking	155
6.5	Leader Following	158
	References	159

Input to State Stability: Basic Concepts and Results

	<i>E.D. Sontag</i>	163
1	Introduction	163
2	ISS as a Notion of Stability of Nonlinear I/O Systems	163
2.1	Desirable Properties	164
2.2	Merging Two Different Views of Stability	165
2.3	Technical Assumptions	166
2.4	Comparison Function Formalism	166
2.5	Global Asymptotic Stability	167
2.6	0-GAS Does Not Guarantee Good Behavior with Respect to Inputs	168
2.7	Gains for Linear Systems	168
2.8	Nonlinear Coordinate Changes	169
2.9	Input-to-State Stability	171
2.10	Linear Case, for Comparison	172
2.11	Feedback Redesign	173
2.12	A Feedback Redesign Theorem for Actuator Disturbances	174

3 Equivalences for ISS 176

 3.1 Nonlinear Superposition Principle 176

 3.2 Robust Stability 177

 3.3 Dissipation 178

 3.4 Using “Energy” Estimates Instead of Amplitudes 180

4 Cascade Interconnections 180

 4.1 An Example of Stabilization Using the ISS Cascade Approach .. 182

5 Integral Input-to-State Stability 183

 5.1 Other Mixed Notions 183

 5.2 Dissipation Characterization of iISS 184

 5.3 Superposition Principles for iISS 185

 5.4 Cascades Involving iISS Systems 186

 5.5 An iISS Example 188

6 Input to State Stability with Respect to Input Derivatives 190

 6.1 Cascades Involving the D^k ISS Property 190

 6.2 Dissipation Characterization of D^k ISS 191

 6.3 Superposition Principle for D^k ISS 191

 6.4 A Counter-Example Showing that D^1 ISS \neq ISS 192

7 Input-to-Output Stability 192

8 Detectability and Observability Notions 194

 8.1 Detectability 195

 8.2 Dualizing ISS to OSS and IOSS 196

 8.3 Lyapunov-Like Characterization of IOSS 196

 8.4 Superposition Principles for IOSS 197

 8.5 Norm-Estimators 197

 8.6 A Remark on Observers and Incremental IOSS 198

 8.7 Variations of IOSS 199

 8.8 Norm-Observability 200

9 The Fundamental Relationship Among ISS, IOS, and IOSS 201

10 Systems with Separate Error and Measurement Outputs 202

 10.1 Input-Measurement-to-Error Stability 202

 10.2 Review: Viscosity Subdifferentials 203

 10.3 RES-Lyapunov Functions 204

11 Output to Input Stability and Minimum-Phase 205

12 Response to Constant and Periodic Inputs 205

13 A Remark Concerning ISS and H_∞ Gains 206

14 Two Sample Applications 207

15 Additional Discussion and References 209

References 213

**Generalized Differentials, Variational Generators,
and the Maximum Principle with State Constraints**

H.J. Sussmann 221

1 Introduction 221

XII Contents

2	Preliminaries and Background	222
2.1	Review of Some Notational Conventions and Definitions	222
2.2	Generalized Jacobians, Derivate Containers, and Michel–Penot Subdifferentials	228
2.3	Finitely Additive Measures	229
3	Cellina Continuously Approximable Maps	230
3.1	Definition and Elementary Properties	231
3.2	Fixed Point Theorems for CCA Maps	234
4	GDQs and AGDQs	243
4.1	The Basic Definitions	244
4.2	Properties of GDQs and AGDQs	246
4.3	The Directional Open Mapping and Transversality Properties	255
5	Variational Generators	267
5.1	Linearization Error and Weak GDQs	267
5.2	GDQ Variational Generators	269
5.3	Examples of Variational Generators	270
6	Discontinuous Vector Fields	277
6.1	Co-Integrably Bounded Integrally Continuous Maps	277
6.2	Points of Approximate Continuity	280
7	The Maximum Principle	281
	References	285

**Sliding Mode Control: Mathematical Tools, Design
and Applications**

<i>V.I. Utkin</i>	289
1	Introduction	289
2	Examples of Dynamic Systems with Sliding Modes	289
3	VSS in Canonical Space	296
3.1	Control of Free Motion	298
3.2	Disturbance Rejection	300
3.3	Comments for VSS in Canonical Space	301
3.4	Preliminary Mathematical Remark	302
4	Sliding Modes in Arbitrary State Spaces: Problem Statements	303
5	Sliding Mode Equations: Equivalent Control Method	305
5.1	Problem Statement	305
5.2	Regularization	306
5.3	Boundary Layer Regularization	311
6	Sliding Mode Existence Conditions	313
7	Design Principles	316
7.1	Decoupling and Invariance	316
7.2	Regular Form	318
7.3	Block Control Principle	320
7.4	Enforcing Sliding Modes	322
7.5	Unit Control	325
8	The Chattering Problem	327

9	Discrete-Time Systems	330
9.1	Discrete-Time Sliding Mode Concept	331
9.2	Linear Discrete-Time Systems with Known Parameters.....	333
9.3	Linear Discrete-Time Systems with Unknown Parameters	335
10	Infinite-Dimensional Systems	336
10.1	Distributed Control of Heat Process	337
10.2	Flexible Mechanical System.....	338
11	Control of Induction Motor	340
	References	344
	List of Participants	349