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NONLINEAR BEHAVIOR OF A MAGNETIC BEARING SYSTEM

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ABSTRACT

This paper describes the results of a study into the dynamic behavior of a magnetic bearing system. The research focuses attention on the influence of nonlinearities on the forced response of a two-degree-of-freedom rotating mass suspended by magnetic bearings and subject to rotating unbalance and feedback control. Geometric coupling between the degrees of freedom leads to a pair of nonlinear ordinary differential equations which are then solved using both numerical simulation and approximate analytical techniques. The system exhibits a variety of interesting and somewhat unexpected phenomena including various amplitude driven bifurcational events, sensitivity to initial conditions and the complete loss of stability associated with the escape from the potential well in which the system can be thought to be oscillating. An approximate criterion to avoid this last possibility is developed based on concepts of limiting the response of the system. The present paper may be considered as an extension to an earlier study by the same authors which described the practical context of the work, free vibration, control aspects and derivation of the mathematical model.

NOMENCLATURE

A	coupling parameter
B	flux (T)
E	eccentricity
F	force (N)
k	dimensional proportional feedback (T/m)
V	potential energy
x, y	dimensional position (m)
X, Y	nondimensional position
α	force proportionality constant (1/m)
K	stiffness (proportional feedback)
Γ	damping (derivative feedback)
Ω	forcing frequency

C, D, G, H constants in the harmonic balance solution

INTRODUCTION

Magnetic bearings are under development for use in a number of practical applications (O'Connor, 1992) and offer compelling advantages in certain circumstances. However, they also present significant challenges to the designer of magnetic levitation systems for rotating shafts where rotor dynamic stability and robust vibration control are fundamental considerations (Nonami, 1990; Williams *et al.*, 1991; Lee and Kim, 1992).

Hebbale (1985) well illustrated several nonlinear aspects of magnetic bearings and examined the effects of coordinate coupling that arises from eddy currents during shaft rotation.

The present work examines the effects of coordinate coupling due to the geometry of the pole arrangement and the uneven flux distribution that results from nonconstant gaps when the shaft is displaced from the bearing center. These coupling forces arise even in the nonrotating case. In this work, the effects of eddy currents and other transient phenomena are neglected.

SYSTEM MODEL

Consider the active magnetic bearing shown schematically in figure 1. Each magnet pair is independently subject to linear flux control, but the forces from the actuator include coordinate coupling. Experimental measurements were used to determine the relationship between principal and normal forces. Details of the form of this coupling can be found in Knight *et al.* 1993, together with the derivation of the equations of motion. In summary, it was found that the ratio of the attractive, on-axis force between each magnet and the shaft to the normal, off-axis force was proportional to the shaft displacement in the off-axis direction. For magnet 1, for instance,

$$F_x = \alpha x F_y \quad (1)$$

The principal force is modeled using one-dimensional magnetic circuit theory. Applying independent-axis control of flux, making the flux in each magnet equal to a steady bias flux plus a control flux

$$B_i = B_b + B_{i,c} \quad (2)$$

where the control flux is of equal magnitude but opposite sign in opposing magnets, i.e.,

$$B_{1,c} = -B_{3,c} \quad (3)$$

and is proportional to shaft displacement

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$$B_{1,c} = -ky \quad (4)$$

The components of dimensionless force from all magnets acting together can be derived as

$$F_x = Kx - \frac{\Delta}{2}x(1 + K^2y^2) \quad (5a)$$

$$F_y = Ky - \frac{\Delta}{2}y(1 + K^2x^2) \quad (5b)$$

for the case of symmetric geometry and control, with no gravity loading.

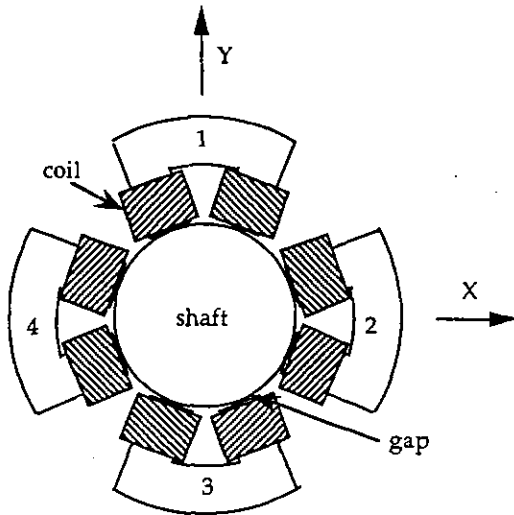


FIGURE 1. Geometry of active magnetic bearing and shaft

It is important to note that in the absence of geometric coupling of x and y forces, this system would be entirely linear. The actual forces available from a magnet arrangement will contain additional nonlinearities, but these are neglected in the present analysis. Note also that an increase in the proportional gain does not diminish the magnitude of the coupling, but increases it.

These forces can be obtained from a potential energy function:

$$V = \left(\frac{K}{2} - \frac{\Delta}{4}\right)(X^2 + Y^2) - \frac{\Delta}{4}K^2X^2Y^2 \quad (6)$$

where the terms are given in the nomenclature section. A useful analogy is to consider the free behavior of the shaft akin to the motion of a small ball rolling on this potential energy surface as shown in figure 2. Three different values of the coupling parameter are used. Due to the coupling (the last term in equation 1) these are *not* surfaces of revolution and it is precisely this apparently subtle feature that underlies much of the nonlinear behavior to be described later. If coupling and deflections are relatively strong then figure 2(c) indicates the possibility of trajectories 'escaping' from the region locally surrounding the stable equilibrium (rest position). Although this 'unstable in the large' type of behavior is less likely to occur for practical values of the system parameters (the shaft may hit the magnets) the potentially catastrophic nature of this event is given due consideration.

Derivative feedback control is also introduced and the subsequent actuator forces can be added to an unbalance forcing function which finally results in dimensionless equations of motion of the form:

$$X'' = -\frac{1}{K} \left(KX + \Gamma X' - \frac{\Delta}{2}X(1 + K^2Y^2 + 2K\Gamma Y Y' + \Gamma^2 Y'^2) \right) + E\Omega^2 \cos\Omega T \quad (7)$$

$$Y'' = -\frac{1}{K} \left(KY + \Gamma Y' - \frac{\Delta}{2}Y(1 + K^2X^2 + 2K\Gamma X X' + \Gamma^2 X'^2) \right) + E\Omega^2 \sin\Omega T \quad (8)$$

where again the terms are given in the nomenclature section. The primes indicate differentiation with respect to dimensionless time T . These equations may be integrated in time after assigning values to the system parameters K , Γ , A and E , along with appropriate initial conditions for the state variables X , Y , X' and Y' . It should be noted that the forms of the forcing function in the final terms of Equations (2) and (3) also constitute initial conditions, in the form of an assumed phase angle for the forcing function at $T = 0$. Initially, steady-state oscillations will be considered using both numerical simulation and approximate analytical techniques. This will be followed by a transient study focussing attention on initial behavior starting from the rest state. The catastrophic 'escape' scenario corresponding to figure 2(c) with forcing is described including the development of a potentially useful approximate criterion for restricting the forces to avoid this possibility (Walsh, 1990).

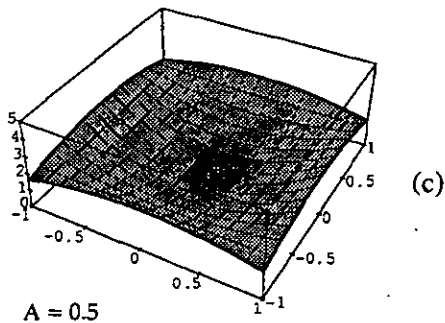
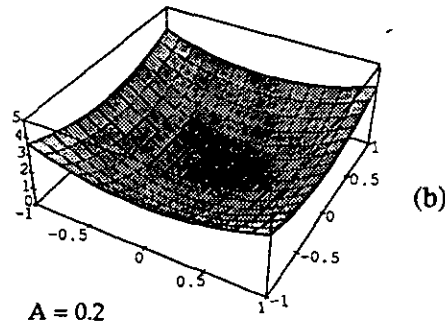
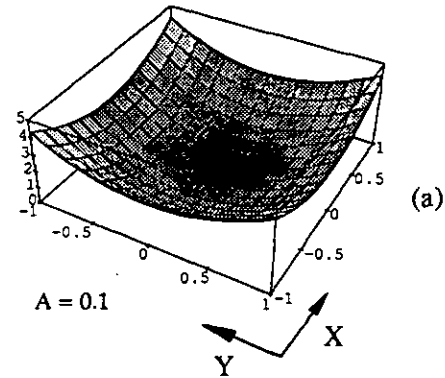


FIGURE 2. Potential energy wells for different degrees of coupling A : 0.1, 0.2, 0.5 ($K = 5.0$).

STEADY STATE RESPONSE

Two methods are used to examine the response of the system to unbalance forcing: numerical integration from arbitrary initial conditions using a fourth-order Runge-Kutta algorithm; and approximate calculation of steady state solutions by the harmonic balance method. In the former approach a sufficiently large number of transient cycles are allowed to decay before recording the magnitude of the steady-state response. These solutions may be considered exact, and a variety of sophisticated algorithms and fast computers have shifted much research interest toward numerical simulation. However, the data generated by such an approach is often cumbersome and difficult to interpret. In the latter approach the assumed solution is harmonic and hence only steady-state information can be obtained. The solutions are approximate (a series solution is truncated) but it is still useful to obtain a functional form for the solution which captures the essential response dependence on system parameters. Clearly it is useful to combine and compare these two approaches, and this is the path followed by the present paper.

Numerical simulation

The effects of four parameters are examined: K , Γ , A and E . The proportional control coefficient K determines how rapidly the flux, and thereby the force, from a magnet is reduced as the shaft approaches that magnet. A value of $K=1$ would cause the flux to be reduced to zero at contact (not counting the contribution from the derivative control coefficient Γ). Larger values of K would correspond to "stiffer" bearings. Because of the form of non-dimensionalization of Equations (2) and (3), however, an increase in K while holding Γ constant causes a decrease in the effective dissipation coefficient, by virtue of a change in the natural frequency. This must be considered when interpreting the results of parametric studies, since a straightforward increase in the dimensional quantity k (1/m) would not affect the dissipation (derivative control) coefficient. The fact that K cannot be eliminated from the equations of motion is a result of the essential nature of nonlinear systems.

The measurements of Knight *et al.* 1992, indicate that 0.15 is a reasonable value for the coordinate coupling coefficient A . These measurements were made using an actuator geometry similar in size and clearance to those in present use in magnetic bearings. In the calculations below, A is varied from 0.05 to 0.25. The values of Γ were chosen to provide dissipation of the same order as in a linear system having damping ratios between 0.1 and 0.3.

Figure 3 shows the effect of increasing the coefficient K from 1.0 in part a to 3.0 in part b to 5.0 in part c. As noted above, increasing K alone results in a smaller value of the derivative control coefficient. With this in mind, Figure 3 indicates an important feature of the system: that for some combinations of parameters, the response exhibits a split, with the motion on one axis having a much higher amplitude than that on the other axis. Associated with this split is a sudden jump in one of the amplitudes as the frequency is increased. In fact, one of the solutions of Figure 3c extends beyond an eccentricity of 1.0, which in the physical case would result in solid contact. At some frequencies near the natural frequency, however, two solutions exist that are both within the physical bounds of the system. Furthermore, the solutions are dependent on the initial conditions. For the case shown, the integration begins with both shaft position and velocity equal to zero. The numerical integration proceeds until all transients have decayed and the peak amplitudes in the two directions are sampled. The forcing function, the final terms in Equations (2) and (3), also imposes an implicit initial condition by virtue of its assumed phase. In fact, the cosine portion of the forcing function begins with a step imposition of force at time $t=0$, although all transients associated with this discontinuity have decayed before the amplitudes are sampled. Practically, loading of this sort might result from blade loss in a turbomachine. If, however, the cosine and sine parts of the force are exchanged, the solutions for X and Y are also found to have exchanged places. This dependence on phase or initial conditions is a characteristic of nonlinear systems.

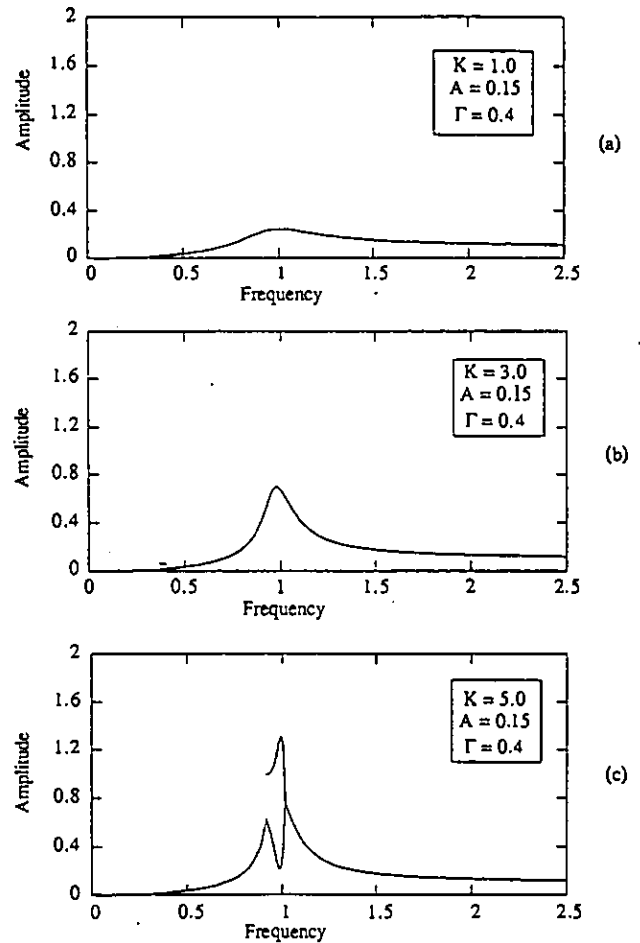


FIGURE 3. Effect of variation in dimensionless proportional control coefficient K : 1.0, 3.0, 5.0 ($A = 0.15$, $\Gamma = 0.4$).

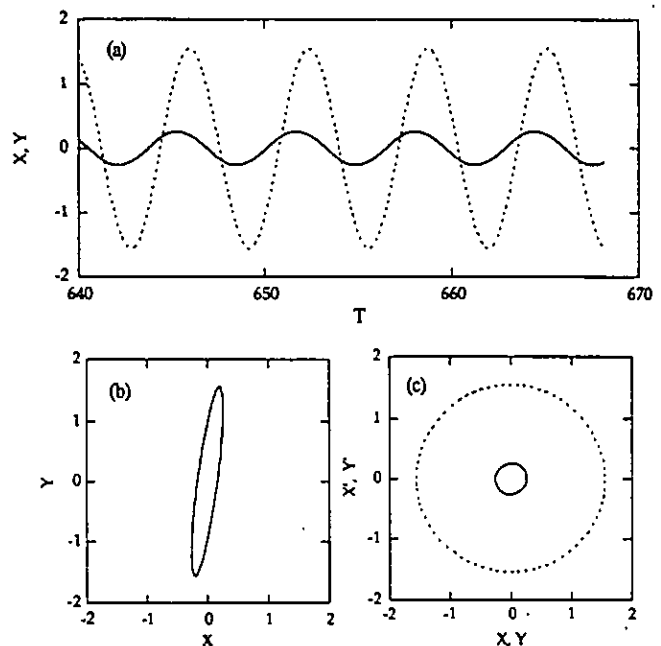


FIGURE 4. Typical time series and phase projections of the coupled oscillation ($K = 5.0$, $A = 0.15$, $\Gamma = 0.4$, $E = 0.1$, $\Omega = 0.9925$).

In Figure 4a is shown a typical time series of the X and Y displacements in the region where the symmetry of the response is broken. It is interesting to note that a linear response based on a paraboloid potential energy surface would have resulted in a phase lag of $\pi/2$ between the two responses, as well as equal amplitudes. This effect is seen more clearly in figure 4b where X and Y are plotted against each other, the phase projection of Figure 4c.

The effect of the coupling parameter can also be examined. Figure 5 illustrates the effect of increasing the value of A, while K is held constant at 0.3. The values of all parameters except A are equal to those of the case shown in Figure 3b. As A is increased beyond a threshold value, between 0.15 and 0.25, multiple solutions appear near the natural frequency. In this set of plots, the natural frequency is a constant, making this parametric variation somewhat easier to interpret than the previous one.

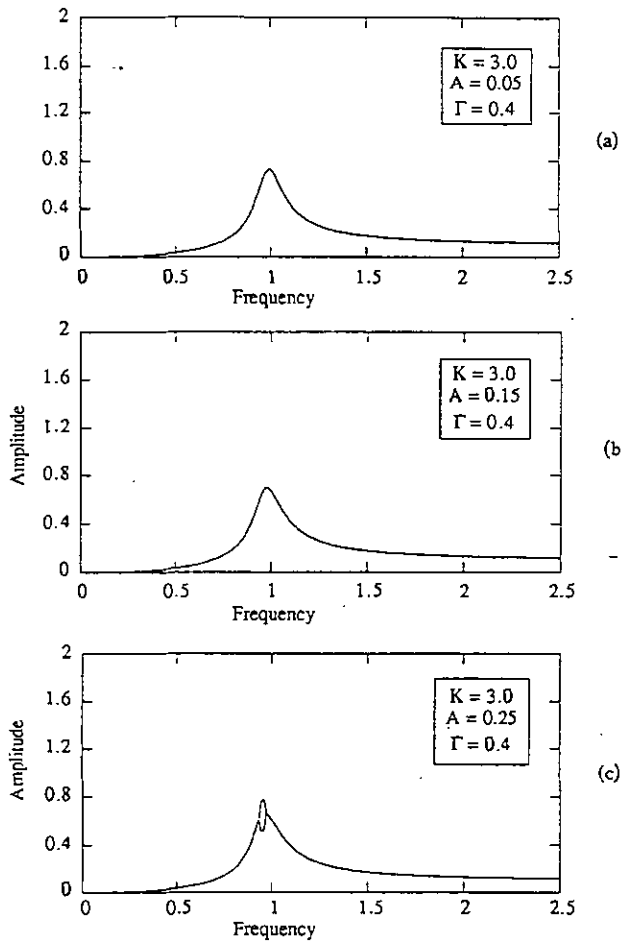


FIGURE 5. Effect of variation in the coupling parameter A: 0.05, 0.15, 0.25 ($\Gamma = 0.4$, $K = 3.0$).

Reduction of the derivative control coefficient Γ can also bring about a situation with multiple solutions, as shown in Figure 6, as can an increase of the unbalance eccentricity, not shown. The important practical implication of this type of loss of stability is that it may cause a sudden, discontinuous jump in the response.

Thus, the bifurcation seems mostly to be an amplitude-driven phenomenon, such that when a critical amplitude is exceeded, the solutions split. In all cases, the split is initial-condition-dependent. In some cases the split is followed by instability.

Solution by Harmonic Balance

The other approach to examining the steady-state response of a nonlinear system adopted in the present study is the harmonic balance method, which is approximate but analytical rather than numerical (Jordan and Smith, 1987; Virgin, 1988). It has the

advantage that both stable and unstable solutions can be located, whereas numerical integration can locate only stable solutions.

The method consists of assuming steady solutions of the form

$$X = C \cos \Omega T + D \sin \Omega T \tag{9}$$

$$Y = G \cos \Omega T + H \sin \Omega T \tag{10}$$

where C, D, G, and H are to be determined. Equations (4) and (5) are differentiated and substituted into the equations of motion. The resulting powers of trigonometric functions are expanded using trig identities, after which the harmonics higher than 1 are neglected. Because the truncation of higher harmonics is not performed until after the powers of trig functions are expanded, the solution retains its nonlinear character, although the equations have been approximated. The resulting four algebraic equations for the constants C, D, G, and H are coupled and highly nonlinear and must themselves be solved by a numerical Newton-Raphson iteration. When the constants are found, the steady amplitudes can be calculated readily.

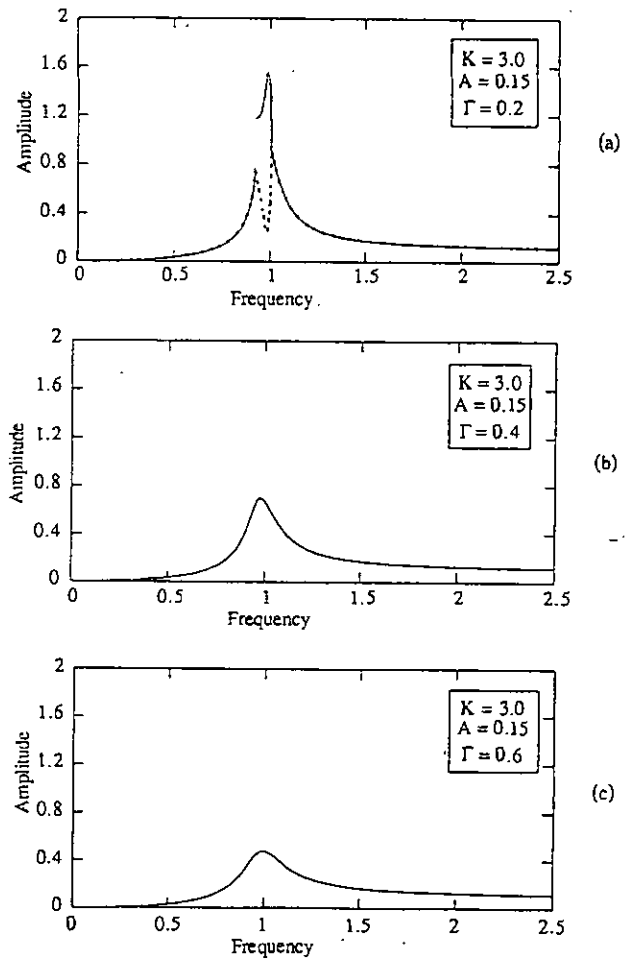


FIGURE 6. Effect of variation in derivative control coefficient Γ : 0.2, 0.4, 0.6 ($K = 3.0$, $A = 0.15$).

Figure 7 shows the amplitudes obtained by harmonic balance for the case corresponding to Figure 6a. These results indicate that in the neighborhood of the natural frequency, four solutions actually exist; two are identical. Two of the solutions are apparently unstable, but the harmonic balance method does not yield stability characteristics. Based on the results of numerical integration, however, it appears that the solutions corresponding to equal amplitudes for X and Y are unstable when they lie between the

unequal solutions. Thus the jump in one of the amplitudes stems from a change in that solution's stability. Where the equal-amplitude solutions lie below the unequal solutions, they are the stable ones. The unequal solutions are believed to exist at all frequencies, but are difficult to locate by Newton-Raphson beyond the range that is shown.

The close correspondence between the numerical and analytical results supports the validity of both methods. Neither method alone is sufficient for a complete understanding, however, because the numerical solutions are dependent on initial conditions, and the analytical solutions provide no information on stability. Stability analyses based on Floquet theory and numerical path-following techniques will form the basis of future research.

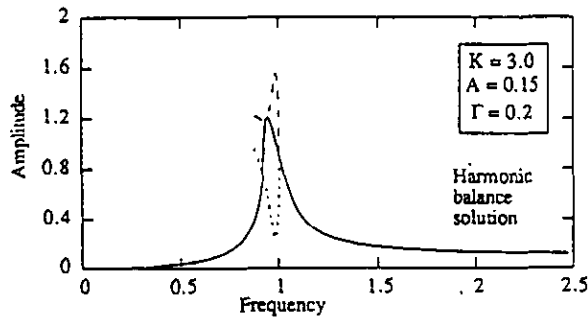


FIGURE 7. Multiple coexisting solutions obtained using the harmonic balance method ($K = 3.0$, $A = 0.15$, $\Gamma = 0.2$).

TRANSIENT RESPONSE

In a typical linear dynamic system the natural period is a constant and steady state amplitudes are independent of initial conditions, i.e. the response is unique. This is not necessarily the case for nonlinear systems and Knight et al. 1993 have shown that for the system under consideration the natural period of the coupled, unforced, undamped system is a function of amplitude and interesting precession behavior is observed. For the forced nonlinear system the initial conditions determine which steady state solution is picked up. Furthermore, in chaotic systems this sensitivity to initial conditions is extreme with adjacent trajectories diverging exponentially [Jordan and Smith, 1987] even though the system is deterministic and the response bounded.

An interesting feature of this system is found for the forced case corresponding to Figure 2c. Suppose the system is started from rest. Applying the periodic force will result in transient behavior followed by one of two general outcomes. First, the system may settle into some kind of steady-state behavior, resulting in an oscillation of the form shown in figures 3-7. However, a second and more dramatic possibility occurs when the forcing is sufficiently large to cause 'escape' from the potential energy well, i.e. enough energy is present in the system for the schematic ball to traverse the hilltop which surrounds the equilibrium point: the stiffness becomes negative. In a nonlinear system such as this the question of what constitutes 'large' forcing is not as simple as it appears. Clearly, the greater the forcing magnitude (eccentricity E) the greater the response, and proximity to the resonant frequency is also likely to cause magnified response. The essentially nonlinear phenomenon of escape is captured by mapping the combinations of forcing parameters which lead to escape. This is shown in Figure 8 for a grid of approximately 40,000 simulations. The initial conditions are zero displacement and velocity in both directions. The black areas indicate that starting from rest the solution escaped (to infinity) within four forcing cycles and the grey corresponds to escape between four and 100 cycles. The unshaded regions are those forcing parameter combinations that lead to steady state behavior. The general dependence of escape on the forcing parameters is as expected, however the boundaries between escape and no escape are fractal in nature, revealing self-similar behavior on finer and finer length scales (Walsh, 1993). A close up view of the resonance region is shown in figure 8b.

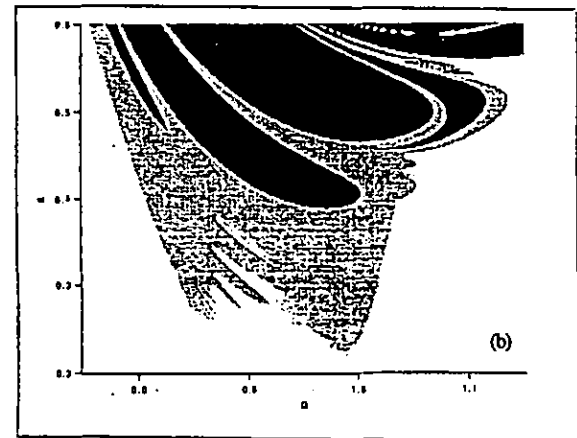
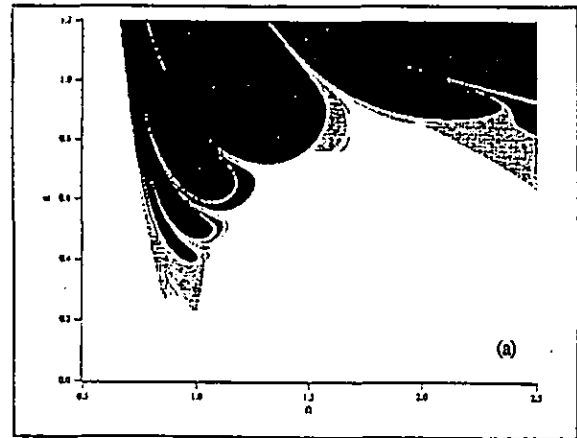


FIGURE 8. Escape eccentricity versus escape frequency ratio ($K = 3.0$, $A = 0.15$, $\Gamma = 0.2$, $E = 0.1$), the lower figure is an enlargement.

An approximate criterion

It is tempting to try to establish a criterion such that escape is avoided. This type of behavior has been encountered in other physical systems with a softening spring nonlinearity and the following approach delineates regions of the forcing parameters as safe or unsafe according to an *ad hoc* analytical criterion which can then be compared directly to the escape boundaries obtained using numerical simulation.

Returning to the approximate analytical solutions obtained using the harmonic balance method it is relatively easy to recast the equations so that given a limiting displacement the combination of forcing parameters required to achieve this response can be identified, and plotting these boundaries in the parameter space the safe (below the boundary) and unsafe (above the boundary) regions can be obtained. This is shown in figure 9a where $K = 3$, $X_{\max} = 0.4$, $A = 0.15$ and Γ is varied as indicated. Figures 9b-d show similar plots where a different parameter has been varied while the others are kept fixed. In all cases the effect of the parameters on the escape characteristics are more or less as expected. Although these results are based on steady-state oscillation, by adjusting the maximum allowable displacement it is possible to ensure that the boundaries are below the exact transient results of figure 8. As expected the proximity to resonance has the greatest influence on the likelihood of escape. Similar approaches have also been developed based on limiting the maximum velocity or total energy of the response and various safety factors have been incorporated (Virgin, 1989; Virgin et al., 1992).

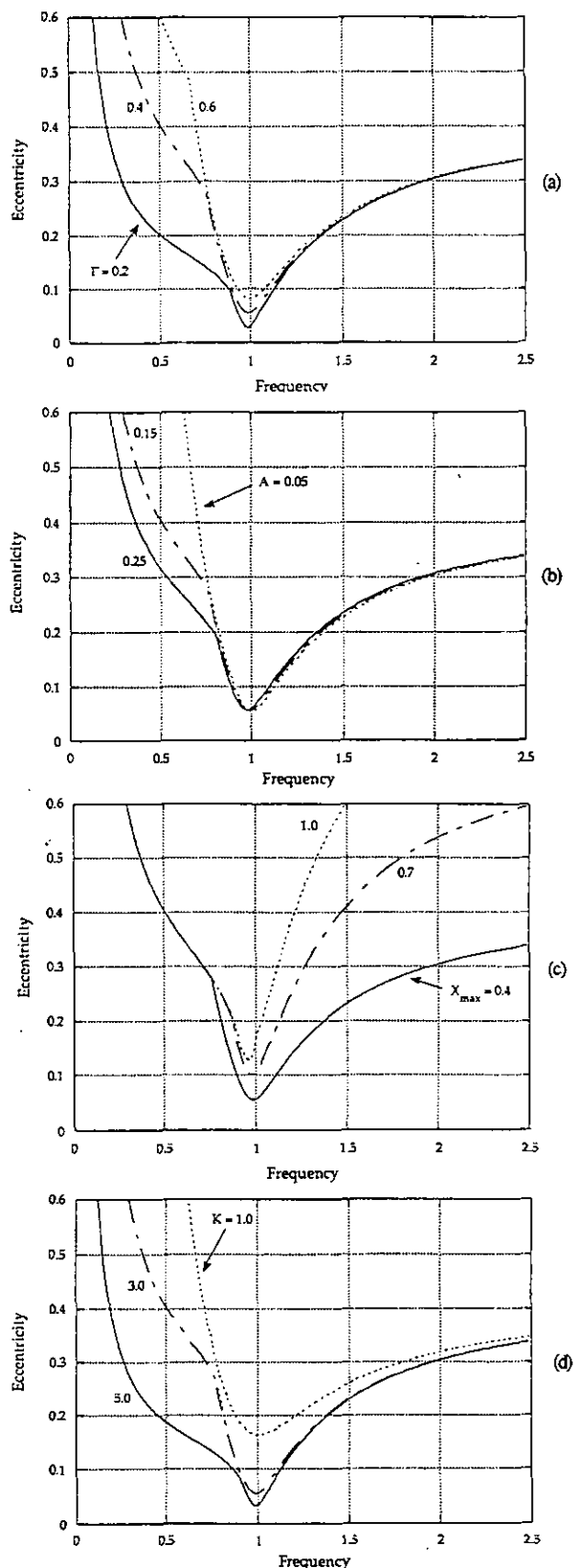


FIGURE 9. Loci of the lowest eccentricity which causes a specified displacement for a given forcing ratio. Results based on harmonic balance. Base parameters: $X_{max} = 0.4$, $A = 0.15$, $\Gamma = 0.4$, $K = 3.0$. (a) variation of Γ , (b) variation of A , (c) variation of X_{max} , (d) variation of K .

CONCLUSIONS

Equations of motion and limited parametric studies are presented for the case of a magnetic bearing subject to flux control, with geometric coordinate coupling. There are two effects of the coupling parameter on the system potential energy: a reduction of the principal stiffness, and introduction of a nonlinear normal stiffness.

The equations of motion are nonlinear and exhibit behavior that is distinctly different from that of linear systems. In forced response (rotating unbalance), the amplitudes of the system show bifurcations that are the result of changes in stability of multiple coexisting solutions. The stability seems mostly to be amplitude dependent, and the "critical" amplitude is a function of several parameters: K , G , E and A . Under certain circumstances the system may lose all stability resulting in escape. An approximate criterion to avoid this possibility is introduced.

The practical consequences of the results presented here are significant to the design of magnetic bearing actuators and controllers. The bifurcations and the associated tendency to escape, which appear to be primarily amplitude-driven phenomena, are predicted to begin within the clearance of the auxiliary bearings typical of present magnetic bearing systems. The possible interaction of coordinate coupling with the additional nonlinearity involved in accidental contact with the auxiliary bearings is beyond the scope of the present work, but should be pursued further. The discontinuities are especially likely to cause difficulty when there is a sudden change in parameters, which could result from large unbalance due to blade loss or other failure, a sudden amplitude shift due to base motion or step changes in bearing force associated with temporary control interruption. It is likely that most of these effects could be removed or compensated for if the control of axes were coupled appropriately. To the authors' knowledge, this is not being done in practice because it requires more complex control algorithms and/or additional circuitry. Because the nonlinear effects explored here all arise from the geometric coupling in the bearing actuator, the design of the actuator is of extreme importance. The independent horseshoe design used in this paper is one of two widely used geometries, the other consisting of a series of pole pairs extending inward from a continuous backing ring. The second geometry may be found to be preferable, but additional measurements are needed to determine the level of coupling to be expected there. It should not be assumed to be smaller without further study.

In the long term, successful implementation of magnetic bearings where large eccentricities may be encountered will depend on a deeper understanding of the nonlinear characteristics of the combined rotor-actuator-control system.

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