Nonlinear Continuum Mechanics and Large Inelastic Deformations

SOLID MECHANICS AND ITS APPLICATIONS
Volume 174

Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why?*, *How?*, and *How much?* The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies: vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For other titles published in this series, go to
www.springer.com/series/6557

Yuriy I. Dimitrienko

Nonlinear Continuum Mechanics and Large Inelastic Deformations

Springer

Prof. Yuriy I. Dimitrienko
Bauman Moscow State Technical University
2nd Baumanskaya St. 5
105005 Moscow
Russia
dimit@serv.bmstu.ru

ISSN 0925-0042
ISBN 978-94-007-0033-8 e-ISBN 978-94-007-0034-5
DOI 10.1007/978-94-007-0034-5
Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2010938719

Cover design: SPi Publisher Services

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Nonlinear continuum mechanics is the kernel of the general course 'Continuum Mechanics', which includes kinematics of continua, balance laws, general nonlinear theory of constitutive equations, relations at singular surfaces. Moreover, in the course of nonlinear continuum mechanics one also considers the theory of solids at finite (arbitrary) deformations. This arbitrariness of deformations makes the equations describing the behavior of continua extremely complex – nonlinear (so that sometimes the term '*strongly nonlinear*' is used), as the relationships contained in them cannot always be expressed in an explicit analytical way. If we drop the condition of the arbitrariness of continuum deformations and consider only infinitesimal deformations – usually the deformations till 1%, then the situation changes: the equations of continuum mechanics can be linearized. Hence for solving the applied problems one can exploit the wide range of analytical and numerical methods. However, many practical tasks demand the analysis not of the infinitesimal, but just the arbitrary (large) deformations of bodies, for example, such tasks include the rubber structure elements design (shock absorbers, gaskets, tires) for which the ultimate deformations can reach 100% and even higher. The various tasks of metal working under high pressure also belong to that class of problems, where large plastic deformations play a significant role, as well as the dynamical problems of barrier breakdown with a striker (aperture formation in the metal barrier while the breakdown is an example of large plastic deformations). Within this class of problems one can also find many problems of ground and rock mechanics, where there usually appears the need to consider large deformations, and modelling the processes in biological systems such as the functioning of human muscular tissue, and many others.

The theory of infinitesimal deformations of solids appeared in the XVII century in the works by Robert Hooke, who formulated one of the main assumptions of the theory: stresses are proportional to strains of bodies. Translating the assertion into mathematical language, this means that relations between stresses and displacements gradients of bodies are linear. Nowadays the theory of infinitesimal deformations is very deeply and thoroughly elaborated. On the different parts of this theory such as elasticity theory, plasticity theory, stability theory and many others there are many monographs and textbooks.

But the well-known Hooke's law does not hold for finite (or large) deformations: the basic relations between stresses and displacement gradients become 'strongly non-linear', and they cannot always be expressed analytically. The basis of finite deformations theory was laid in the XIX century by the eminent scientists A.L. Cauchy, J.L. Lagrange, L. Euler, G. Piola, A.J.C. Saint-Venant, G.R. Kirchhoff, and then developed by A.E.H. Love, G. Jaumann [28], M.A. Biot, F.D. Murnaghan [41] and other researchers. The works by M. Mooney and R.S. Rivlin written in the 1940s of the XX century contributed much to the formation of finite deformations theory as an independent part of continuum mechanics. The fundamental step was made in 1950–1960s of the XX century by the American mechanics school, first of all by B.D. Coleman [9], W. Noll and C. Truesdell [43, 54–56], who considered the nonlinear mechanics from the point of view of the formal mathematics. According to D. Hilbert, they introduced the axiomatics of nonlinear mechanics which structured the system of accumulated knowledge and made it possible to formulate the main directions of the further investigations in this theory. Together with R.S. Rivlin and A.J.M. Spencer [10, 13, 51] they elaborated the special mathematical apparatus for formulation of relationships, generalizing Hooke's law for finite deformations, namely the theory of nonlinear tensor functions. And also the tensor analysis widely used in continuum mechanics was considerably adapted to the problems of nonlinear mechanics. Equations of continuum mechanics got the invariant (i.e. independent of the choice of a reference system) form. The further development of this direction was made by A.C. Eringen, A.E. Green, W. Zerna, J.E. Adkins and others [1–7, 11, 12, 14–27, 29, 30, 32–35, 38–40, 42, 47–50, 52, 53, 57–60].

The role of Russian mechanics school in the development of contemporary nonlinear continuum mechanics principles is also quite substantial. In 1968 the first edition of the fundamental two-part textbook '*Continuum Mechanics*' by L.I. Sedov was published, which is still one of the most popular books on continuum mechanics in Russia. Outstanding results in the theory of finite elastic deformations were obtained by A.I. Lurie [36, 37], who wrote the principal monograph on the nonlinear theory of elasticity and systemized in it the problem classes of the theory of finite elastic deformations allowing for analytical solutions. Also the considerable step was made by K.F. Chernykh [8], who developed the theory of finite deformations for anisotropic media and elaborated the methods for solving the problems of nonlinear theory of shells and nonlinear theory of cracks. One can also mention the works by mechanics scientists: B.E. Pobedrya, V.I. Kondaurov, V.G. Karnauhov, A.A. Pozdeev, P.V. Trusov, Yu.I. Nyashin and many others who made considerable contributions to the theory of viscoelastic, elastoplastic and viscoplastic finite deformations.

This book is based on the lectures which the author has been giving for many years in Moscow Bauman State Technical University. The book has several fundamental traits:

1. It follows the mathematical style of course exposition, which assumes the usage of axioms, definitions, theorems and proofs.
2. It applies the tensor apparatus, mostly in the indexless form, as the latter combined with the special skills is very convenient in usage, and does not shade

the physical essence of the laws, and permits proceeding to any appropriate coordinate system.

3. It uses the divergence form of dynamic equations of deformation compatibility, that made it at last possible to write the complete system of balance laws of nonlinear mechanics in a single generalized form.
4. The theory of constitutive equations being the key part of nonlinear mechanics is for the first time exposed with the usage of all energetic couples of tensors, which were established by R. Hill [26] and K.F. Chernykh [8] and ordered by the author [12], and also with quasi-energetic couples of tensors found by the author [12].
5. To derive constitutive equations of nonlinear continuum mechanics, the author applied the theory of nonlinear tensor functions and tensor operators, elaborated by A.J.M. Spencer, R.S. Rivlin, J.L. Ericksen, V.V. Lokhin, Yu.I. Sirotin, B.E. Pobedrya, the author of this book and others.
6. The bases of theories of large elastic, viscoelastic and plastic deformations are explored from the uniform position.
7. The book uses a 'reader-friendly' style of material exposition, which can be characterized by the presence of quite detailed necessary mathematical calculations and proofs.

The axiomatic approach used in this book differs a bit from the analogous ones suggested by C. Truesdell [56] and other authors. The system of continuum mechanics axioms in the book is composed so as to minimize their total number, and give each axiom a clear physical interpretation. That is why the axioms by C. Truesdell connected with the logic relations between bodies are not included in the general list, the axioms on the bodies' mass are united into one axiom, the mass conservation law, and analogously the axioms on the existence of forces and inertial reference systems are united into one axiom, the momentum balance law. Though, the last axiom is split into the two parts: first the Sect. 3.2 considers the case of inertial reference systems, and then Sect. 4.10 deals with non-inertial ones. Unlike the axiomatics by C. Truesdell [56], in this book the axiom system includes so called principles of constitutive equations construction which play a fundamental role in the formation of a continuum mechanics equations system.

The axiomatic approach to the exploration of continuum mechanics possesses at least one merit – it permits the separation of all the values into two categories: primary and secondary. These are introduced axiomatically and consequently within the continuum mechanics there is no need to substantiate their appearance. The secondary category includes combinations of the first category's values. The axiomatic approach allows us also to distinguish from continuum mechanics statements between the definitions and corollaries of them (theorems); this is extremely useful for the initial acquaintance with the course.

To get acquainted with the specific apparatus of tensor analysis the reader is recommended to use the author's book '*Tensor Analysis and Nonlinear Tensor Functions*' [12], which uses the same main notations and definitions. All the references to the tensor analysis formulas in the text are addressed to the latter book.

This book covers the fundamental classical parts of nonlinear continuum mechanics: kinematics, balance laws, constitutive equations, relations at singular surfaces, the basics of theories of large elastic deformations, large viscoelastic deformations and large plastic deformations. Because of limits on space, important parts such as the theory of shells at large deformations, and the theory of media with phase transformations were not included in the book.

I would like to thank Professor B.E. Pobedrya (Moscow Lomonosov State University), Professor N.N. Smirnov (Moscow Lomonosov State University) and Professor V.S. Zarubin (Bauman Moscow State Technical University) for fruitful discussions and valuable advice on different problems in the book.

I am very grateful to Professor G.M.L. Gladwell of the University of Waterloo, Canada, who edited the book and improved the English text.

I also thank my wife, Dr. Irina D. Dimitrienko (Bauman Moscow State Technical University), who translated the book into English and prepared the camera-ready typescript.

I hope that the book proves to be useful for graduates and post-graduates of mathematical and natural-scientific departments of universities and for investigators, academic scientists and engineers working in solid mechanics, mechanical engineering, applied mathematics and physics. I hope that the book is of interest also for material science specialists developing advanced materials.

Russia *Yuriy Dimitrienko*

Contents