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Nonlinear Control of a Spacecraft with Multiple Fuel Slosh Modes

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Abstract— **This paper studies the modeling and control problem for a spacecraft with fuel slosh dynamics. A multi-pendulum model is considered for the characterization of the most prominent sloshing modes. The control inputs are defined by the gimbal deflection angle of a non-throttable thrust engine and a pitching moment about the center of mass of the spacecraft. The control objective, as is typical in a thrust vector control design, is to control the translational velocity vector and the attitude of the spacecraft, while attenuating the sloshing modes. A nonlinear mathematical model that reflects all of these assumptions is first derived. Then, a Lyapunov-based nonlinear feedback controller is designed to achieve the control objective. Finally, a simulation example is included to demonstrate the effectiveness of the controller.**

Keywords—**Nonlinear control, fuel slosh, underactuated system.**

I. INTRODUCTION

Propellant slosh has been a problem studied in spacecraft design since the early days of large, liquid-fuel rockets. In launch vehicles or spacecraft, sloshing can be induced by propellant tank motions resulting from changes in vehicle acceleration. When the fuel tanks are only partially filled, large quantities of fuel move inside the tanks under translational and rotational accelerations and generate the slosh dynamics. The slosh dynamics interacts with the rigid body dynamics of the spacecraft. Several methods have been employed to reduce the effect of sloshing, such as introducing baffles inside the tanks or dividing a large container into a number of smaller ones, meant to limit the movement of liquid fuel to small amplitudes of high, negligible frequencies. These techniques do not completely succeed in canceling the sloshing effects. Moreover, these suppression methods involve adding to the spacecraft structural mass, thereby increasing mission cost. Hence the control system must both assure stability during the thrusting phase and achieve good attitude control while suppressing the slosh dynamics.

The existing literature on the interaction of vehicle dynamics and slosh dynamics and their control, to a large extent, treats only the case of small perturbations to the vehicle dynamics. The control approaches developed for accelerating space vehicles are mostly based on linear control design methods ([3], [21], [23]) and adaptive control methods [1]. Several related papers following a similar approach are motivated by robotic systems moving liquid filled containers ([7], [9], [10], [22], [24]-[26]). In most of these approaches, suppression of the slosh dynamics inevitably leads to excitation of the transverse vehicle motion through coupling effects; this is a major drawback which has not been adequately addressed in the published literature. In this paper, this issue is addressed by designing the control law based on the complete nonlinear translational and rotational vehicle dynamics.

The effect of liquid fuel slosh on spinning spacecraft has also been explored in the literature ([11], [12]). Different slosh motion types - surface waves, bulk fluid motion, and vortices - as well as fluid configurations during spinning are defined [11]. The design of control strategies for a launch vehicle with propellant sloshing has also been studied in several works ([2], [8], [12], [14]). In [2], an advanced linear model of the Saturn V launch vehicle is developed and a linear optimal control law is proposed to control the vehicle. The work in [8] studies the problem of robust control of a launch vehicle subject to aerodynamic, flexible, and slosh mode instabilities. It has been demonstrated that pendulum and mass-spring models can approximate complicated fluid and structural dynamics; such models have formed the basis for many studies on dynamics and control of space vehicles with fuel slosh [15].

As with any continuum model, an infinite number of sloshing modes exists but in this work we will consider finite dimensional discrete models. Two to three sloshing modes at most are sufficient to characterize the sloshing dynamics. Generally, it is more convenient to define the modes by their oscillating frequencies and their assumed damping coefficients. The damping coefficients for the sloshing masses are usually determined by experimental measurements with partially filled tanks [6].

In this paper we extend our work in [5] and [20], where we considered a spacecraft with a partially filled spherical fuel tank and included only the lowest frequency slosh mode in the dynamic model using pendulum and mass-spring analogies. Here we consider a multi-pendulum model for the characterization of the most prominent sloshing modes. The control inputs are defined by the gimbal deflection angle of a non-throttable thrust engine and a pitching moment about the center of mass of the spacecraft. The control objective, as is typical in a thrust vector control design, is to control the translational velocity vector and the attitude of the spacecraft, while attenuating the sloshing modes characterizing the internal dynamics. The main contributions in this paper are (i) the development of a full nonlinear mathematical model that reflects all of these assumptions and (ii) the design of a Lyapunov-based nonlinear feedback controller. A simulation example is included to illustrate the effectiveness of the controller.

II. Mathematical Model

This section formulates the dynamics of a spacecraft with a single propellant tank including the prominent fuel slosh modes. The spacecraft is represented as a rigid body (base body) and the sloshing fuel masses as internal bodies. The main ideas in [4] are employed to express the equations of motion in terms of the spacecraft translational velocity vector, the angular velocity, and the internal (shape) coordinates representing the slosh modes.

To summarize the formulation in [4], let $\mathbf{v} \in \mathbb{R}^3$, $\boldsymbol{\omega} \in \mathbb{R}^3$, and $\boldsymbol{\eta} \in \mathbb{R}^N$ denote the base body translational velocity vector, the base body angular velocity vector, and the vector of internal coordinates, respectively. In these coordinates, the Lagrangian has the form $L = L(\mathbf{v}, \boldsymbol{\omega}, \boldsymbol{\eta}, \dot{\boldsymbol{\eta}})$, which is $SE(3)$-invariant in the sense that it does not depend on the base body position and attitude. The generalized forces and moments on the spacecraft are assumed to consist of control inputs which can be partitioned into two parts: $\boldsymbol{\tau}_t \in \mathbb{R}^3$ (typically from thrusters) is the vector of generalized control forces that act on the base body and $\boldsymbol{\tau}_r \in \mathbb{R}^3$ (typically from symmetric rotors, reaction wheels, and thrusters) is the vector of generalized control torques that act on the base body. We also assume that the internal dissipative forces are derivable from a Rayleigh dissipation function R. Then, the equations of motion of the spacecraft with internal dynamics are shown to be given by:

$$\frac{d}{dt}\frac{\partial L}{\partial \mathbf{v}} + \hat{\boldsymbol{\omega}}\frac{\partial L}{\partial \mathbf{v}} = \boldsymbol{\tau}_t, \tag{1}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \boldsymbol{\omega}} + \hat{\boldsymbol{\omega}}\frac{\partial L}{\partial \boldsymbol{\omega}} + \hat{\mathbf{v}}\frac{\partial L}{\partial \mathbf{v}} = \boldsymbol{\tau}_r, \tag{2}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\boldsymbol{\eta}}} - \frac{\partial L}{\partial \boldsymbol{\eta}} + \frac{\partial R}{\partial \dot{\boldsymbol{\eta}}} = 0, \tag{3}$$

where $\hat{\mathbf{a}}$ denotes a 3×3 skew-symmetric matrix formed from $\mathbf{a} = [a_1,\, a_2,\, a_3]^T \in \mathbb{R}^3$:

$$\hat{\mathbf{a}} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}.$$

It must be pointed out that in the above formulation it is assumed that no control forces or torques exist that directly control the internal dynamics. The objective is to simultaneously control the rigid body dynamics and the internal dynamics using only control effectors that act on the rigid body; the control of internal dynamics must be achieved through the system coupling. In this regard, equations (1)-(3) model interesting examples of underactuated mechanical systems. The published literature on the dynamics and control of such systems includes the development of theoretical controllability and stabilizability results for a large class of systems using tools from nonlinear control theory ([16], [17]) and the development of effective nonlinear control design methodologies [18] that are applied to several practical examples, including underactuated space vehicles ([5], [20]).

We now derive a multi-pendulum model of the sloshing fuel where the oscillation frequencies of the pendula represent the prominent sloshing modes [21].

Consider a rigid spacecraft moving on a plane as shown in Figure 1, where v_x, v_z are the axial and transverse components, respectively, of the velocity of the center of the fuel tank, and θ denotes the attitude angle of the spacecraft with respect to a fixed reference. The fluid is modeled by moment of inertia I_0 assigned to a rigidly attached mass m_0 and masses $m_i,\ i = 1, \cdots, N$, attached to pendula of lengths l_i. Moments of inertia of these masses are denoted by I_i. The locations $h_0 > 0$ and $h_i > 0$ are referenced to the center of the tank. A thrust F is produced by a gimballed thrust engine as shown in Figure 1, where δ denotes the gimbal deflection angle, which is considered as one of the control inputs. A pitching moment M is also available for control purposes. The constants in the problem are the spacecraft mass m and moment of inertia I; the distance b between the body z-axis and the spacecraft center of mass location along the longitudinal axis, and the distance d from the gimbal pivot to the spacecraft center of mass. If the tank center is in front of the spacecraft center of mass then $b > 0$. The parameters $m_0, h_0, I_0, m_i, h_i, l_i, I_i,\ i = 1, \cdots, N$, depend on the shape of the fuel tank, the characteristics of the fuel and the fill ratio of the fuel tank. Although these parameters are actually time-varying, in this paper they are assumed to be constant for analysis purposes.

The sum of all the fluid masses is the same as the fuel mass m_f, i.e.,

$$m_0 + \sum_{i=1}^{N} m_i = m_f,$$

and the rigidly attached mass location h_0 satisfies

$$m_0 h_0 = \sum_{i=1}^{N} m_i (h_i - l_i).$$

Let $\hat{i}$ and $\hat{k}$ be the unit vectors along the spacecraft-fixed longitudinal and transverse axes, respectively, and denote by $(x,\, z)$ the inertial position of the center of the fuel tank. The position vector of the center of mass of the vehicle can then be expressed in the spacecraft-fixed coordinate frame as

$$\vec{r} = (x - b)\hat{i} + z\hat{k}.$$

The inertial velocity of the vehicle can be computed as

$$\dot{\vec{r}} = v_x\hat{i} + (v_z + b\dot{\theta})\hat{k},$$

where we have used the fact that $(v_x,\, v_z) = (\dot{x}+z\dot{\theta},\, \dot{z}-x\dot{\theta})$.

Similarly, the position vectors of the fuel masses $m_0,\ m_i,\ \forall i$, in the spacecraft-fixed coordinate frame are given, respectively, by

$$\vec{r}_0 = (x - h_0)\hat{i} + z\hat{k},$$

$$\vec{r}_i = (x + h_i - l_i\cos\psi_i)\hat{i} + (z + l_i\sin\psi_i)\hat{k}.$$

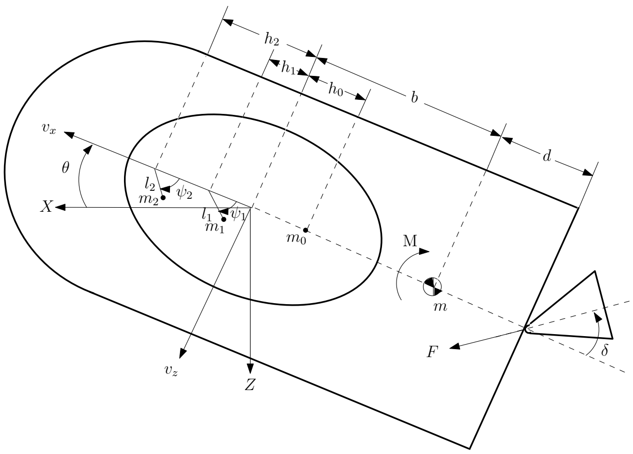

Fig. 1. A multiple slosh pendula model for a spacecraft.

Assuming h_i are constants, the inertial velocities can be computed as

$$\dot{\vec{r}}_0 = v_x\hat{\imath} + (v_z + h_0\dot{\theta})\hat{k},$$
$$\dot{\vec{r}}_i = [v_x + l_i(\dot{\theta}+\dot{\psi}_i)\sin\psi_i]\hat{\imath} + [v_z - h_i\dot{\theta} + l_i(\dot{\theta}+\dot{\psi}_i)\cos\psi_i]\hat{k}.$$

The total kinetic energy can now be expressed as

$$T = \frac{1}{2}m\dot{\vec{r}}^2 + \frac{1}{2}m_0\dot{\vec{r}}_0^2 + \frac{1}{2}(I+I_0)\dot{\theta}^2 + \frac{1}{2}\sum_{i=1}^{N}[m_i\dot{\vec{r}}_i^2 + I_i(\dot{\theta}+\dot{\psi}_i)^2].$$

Since we assume that the spacecraft is in a zero gravity environment, the potential energy is zero. Thus, the Lagrangian $(L = T - U)$ can be computed as

$$\begin{aligned} L = &\frac{1}{2}m[v_x^2 + (v_z + b\dot{\theta})^2] + \frac{1}{2}m_0[v_x^2 + (v_z + h_0\dot{\theta})^2] \\ &+ \frac{1}{2}(I + I_0)\dot{\theta}^2 + \frac{1}{2}\sum_{i=1}^{N}[m_i((v_x + l_i(\dot{\theta} + \dot{\psi}_i)\sin\psi_i)^2 \\ &+ (v_z - h_i\dot{\theta} + l_i(\dot{\theta} + \dot{\psi}_i)\cos\psi_i)^2) + I_i(\dot{\theta} + \dot{\psi}_i)^2]. \end{aligned}$$

We include dissipative effects due to fuel slosh, described by damping constants ϵ_i. A fraction of kinetic energy of sloshing fuel is dissipated during each cycle of the motion. We will include the damping via a Rayleigh dissipation function R given by

$$R = \frac{1}{2}\sum_{i=1}^{N}\epsilon_i\dot{\psi}_i^2.$$

Applying equations (1)-(3) with

$$\boldsymbol{\eta} = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}, \ \mathbf{v} = \begin{pmatrix} v_x \\ 0 \\ v_z \end{pmatrix}, \ \boldsymbol{\omega} = \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix},$$

$$\boldsymbol{\tau}_t = \begin{pmatrix} F\cos\delta \\ 0 \\ F\sin\delta \end{pmatrix}, \ \boldsymbol{\tau}_r = \begin{pmatrix} 0 \\ M + F(b+d)\sin\delta \\ 0 \end{pmatrix},$$

the equations of motion can be obtained as

$$\begin{aligned} (m + m_f)a_x &+ \sum_{i=1}^{N} m_i l_i(\ddot{\theta} + \ddot{\psi}_i)\sin\psi_i + \bar{m}\bar{b}\dot{\theta}^2 \\ &+ \sum_{i=1}^{N} m_i l_i(\dot{\theta} + \dot{\psi}_i)^2\cos\psi_i = F\cos\delta, \end{aligned} \tag{4}$$

$$\begin{aligned} (m + m_f)a_z &+ \sum_{i=1}^{N} m_i l_i(\ddot{\theta} + \ddot{\psi}_i)\cos\psi_i \\ &+ \bar{m}\bar{b}\ddot{\theta} - \sum_{i=1}^{N} m_i l_i(\dot{\theta}+\dot{\psi}_i)^2\sin\psi_i = F\sin\delta, \end{aligned} \tag{5}$$

$$\begin{aligned} \bar{I}\ddot{\theta} &- \sum_{i=1}^{N} m_i l_i h_i[(\ddot{\theta} + \ddot{\psi}_i)\cos\psi_i - (\dot{\theta} + \dot{\psi}_i)^2\sin\psi_i] \\ &+ \bar{m}\bar{b}a_z - \sum_{i=1}^{N}\epsilon_i\dot{\psi}_i = M + F(b+d)\sin\delta, \end{aligned} \tag{6}$$

$$\begin{aligned} (I_i + m_i l_i^2)(\ddot{\theta} + \ddot{\psi}_i) &- m_i l_i h_i(\ddot{\theta}\cos\psi_i + \dot{\theta}^2\sin\psi_i) \\ &+ m_i l_i(a_x\sin\psi_i + a_z\cos\psi_i) + \epsilon_i\dot{\psi}_i = 0, \ \forall i, \end{aligned} \tag{7}$$

where $(a_x, a_z) = (\dot{v}_x + \dot{\theta}v_z, \dot{v}_z - \dot{\theta}v_x)$ are the axial and transverse components of the acceleration of the center of tank, and

$$\bar{m}\bar{b} = mb - \sum_{i=1}^{N} m_i l_i,$$

$$\bar{I} = I + I_0 + mb^2 + m_0h_0^2 + \sum_{i=1}^{N} m_i h_i^2.$$

The control objective is to design feedback controllers so that the controlled spacecraft accomplishes a given planar maneuver, that is a change in the translational velocity vector and the attitude of the spacecraft, while suppressing the fuel slosh modes.

III. Feedback Control Law

Consider the model of a spacecraft with a gimballed thrust engine shown in Figure 1. If the thrust F is a positive constant, and if the gimbal deflection angle and pitching moment are zero, $\delta = M = 0$, then the spacecraft and fuel slosh dynamics have a relative equilibrium defined by

$$v_z = \bar{v}_z, \ \theta = \bar{\theta}, \ \dot{\theta} = 0, \ \psi_i = 0, \ \dot{\psi}_i = 0, \ \forall i,$$

where $\bar{v}_z$ and $\bar{\theta}$ are arbitrary constants. Without loss of generality in our subsequent analysis, we consider the relative equilibrium at the origin, i.e., $\bar{v}_z = 0, \ \bar{\theta} = 0$. Note that the relative equilibrium corresponds to the vehicle axial velocity

$$v_x(t) = \frac{F}{m + m_f}t + v_{x_0}, \ t \leq t_b,$$

where v_{x_0} is the initial axial velocity of the spacecraft and t_b is the fuel burn time.

Now assume the axial acceleration term a_x is not significantly affected by small gimbal deflections, pitch changes and fuel motion (an assumption verified in simulations). Consequently, equation (4) becomes:

$$\dot{v}_x + \dot{\theta} v_z = \frac{F}{m + m_f}. \tag{8}$$

Substituting this approximation leads to the following equations of motion for the transverse, pitch and slosh dynamics:

$$(m+m_f)(\dot{v}_z - \dot{\theta} v_x(t)) + \sum_{i=1}^{N} m_i l_i (\ddot{\theta} + \ddot{\psi}_i) \cos\psi_i + \bar{m}\bar{b}\ddot{\theta} - \sum_{i=1}^{N} m_i l_i (\dot{\theta} + \dot{\psi}_i)^2 \sin\psi_i = F \sin\delta, \tag{9}$$

$$\bar{I}\ddot{\theta} - \sum_{i=1}^{N} \Big[m_i l_i h_i [(\ddot{\theta} + \ddot{\psi}_i)\cos\psi_i - (\dot{\theta}+\dot{\psi}_i)^2 \sin\psi_i] - \epsilon_i \dot{\psi}_i \Big] + \bar{m}\bar{b}(\dot{v}_z - \dot{\theta} v_x(t)) = M + F(b+d)\sin\delta, \tag{10}$$

$$\ddot{\psi}_i = \frac{m_i l_i h_i}{I_i + m_i l_i^2}(\ddot{\theta}\cos\psi_i + \dot{\theta}^2 \sin\psi_i) - \ddot{\theta} - \frac{\epsilon_i}{I_i + m_i l_i^2}\dot{\psi}_i - \frac{m_i l_i}{I_i + m_i l_i^2}\left[\frac{F}{m+m_f}\sin\psi_i + (\dot{v}_z - \dot{\theta} v_x(t))\cos\psi_i\right], \forall i. \tag{11}$$

Here $v_x(t)$ is considered as an exogenous input. Our subsequent analysis is based on the above equations of motion for the transverse, pitch and slosh dynamics of the space vehicle.

We now design a nonlinear controller to stabilize the relative equilibrium at the origin of the equations (9)-(11). Our control design is based on a Lyapunov function approach. By defining control transformations from (δ, M) to new control inputs (u_1, u_2), the equations (9)-(11) can be written as:

$$\dot{v}_z = u_1 + \dot{\theta} v_x(t), \tag{12}$$

$$\ddot{\theta} = u_2, \tag{13}$$

$$\ddot{\psi}_i = -c_i u_1 \cos\psi_i - d_i \sin\psi_i - (1 - c_i h_i \cos\psi_i) u_2 - e_i \dot{\psi}_i + c_i h_i \dot{\theta}^2 \sin\psi_i, \forall i, \tag{14}$$

where

$$c_i = \frac{m_i l_i}{I_i + m_i l_i^2},\ d_i = \frac{F c_i}{m + m_f},\ e_i = \frac{\epsilon_i}{I_i + m_i l_i^2},\ \forall i.$$

Now, we consider the following candidate Lyapunov function for the system (12)-(14):

$$V = \frac{r_1}{2} v_z^2 + \frac{r_2}{2}\theta^2 + \frac{r_3}{2}\dot{\theta}^2 + \frac{r_4}{2}\sum_{i=1}^{N}[2 d_i (1 - \cos\psi_i) + \dot{\psi}_i^2] + \frac{r_4}{2}\sum_{i=1}^{N}[2(1 - c_i h_i \cos\psi_i)\dot{\psi}_i \dot{\theta} - c_i h_i \dot{\theta}^2 \cos\psi_i],$$

where r_1, r_2, r_3, and r_4 are positive constants. We choose these constants such that

$$\mu = r_3 - r_4 \sum_{i=1}^{N}(1 - c_i h_i \cos\psi_i + c_i^2 h_i^2 \cos^2\psi_i) > 0$$

so that the function V is positive definite.

The time derivative of V along the trajectories of (12)-(14) can be computed as

$$\begin{aligned}\dot{V} = & [r_1 v_z - r_4 \sum_{i=1}^{N} c_i (\dot{\psi}_i + \dot{\theta}(1 - c_i h_i \cos\psi_i))\cos\psi_i] u_1 \\ & + [r_1 v_x(t) v_z + r_2 \theta + \mu u_2 + r_4 \sum_{i=1}^{N} e_i \dot{\psi}_i (c_i h_i \cos\psi_i - 1) \\ & + r_4 \sum_{i=1}^{N} c_i h_i ((\dot{\theta} + \dot{\psi}_i)^2 - 0.5\dot{\theta}\dot{\psi}_i - d_i)\sin\psi_i \\ & + r_4 \sum_{i=1}^{N} c_i h_i (d_i - c_i h_i \dot{\theta}^2)\cos\psi_i \sin\psi_i]\dot{\theta} - r_4 \sum_{i=1}^{N} e_i \dot{\psi}_i^2.\end{aligned}$$

Clearly, the feedback laws

$$u_1 = -K_1 r_1 v_z + K_1 r_4 \sum_{i=1}^{N} c_i (\dot{\psi}_i + \dot{\theta}(1 - c_i h_i \cos\psi_i))\cos\psi_i], \tag{15}$$

$$\begin{aligned}u_2 = & -\frac{1}{\mu}[r_2 \theta + K_2 \dot{\theta} + r_1 v_x(t) v_z + r_4 \sum_{i=1}^{N} e_i \dot{\psi}_i (c_i h_i \cos\psi_i - 1) \\ & + r_4 \sum_{i=1}^{N} c_i h_i ((\dot{\theta} + \dot{\psi}_i)^2 - 0.5\dot{\theta}\dot{\psi}_i - d_i)\sin\psi_i \\ & + r_4 \sum_{i=1}^{N} c_i h_i (d_i - c_i h_i \dot{\theta}^2)\cos\psi_i \sin\psi_i],\end{aligned} \tag{16}$$

where K_1 and K_2 are positive constants, yield

$$\dot{V} = -K_1 [r_1 v_z - r_4 \sum_{i=1}^{N} c_i (\dot{\psi}_i + \dot{\theta}(1 - c_i h_i \cos\psi_i))\cos\psi_i]^2 - K_2 \dot{\theta}^2 - r_4 \sum_{i=1}^{N} e_i \dot{\psi}_i^2,$$

which satisfies $\dot{V} \leq 0$. Using an invariance principle for time-varying systems [13], it is easy to prove asymptotic stability of the origin of the closed loop defined by the equations (12)-(14) and the feedback control laws (15)-(16). Note that the positive control parameters $r_i,\ i = 1, \cdots, 4$; and $K_j,\ j = 1, 2$, can be chosen arbitrarily to achieve good closed loop responses.

IV. SIMULATION

The feedback control laws developed in the previous sections are implemented here for a spacecraft. The first two slosh modes are included to demonstrate the effectiveness of the control laws. The physical parameters used in the simulations are given in Table 1.

The control objective is stabilization of the spacecraft in orbital transfer, suppressing the transverse and pitching motion of the spacecraft and sloshing of fuel while the spacecraft is accelerating in the axial direction. In other words, the control objective is to stabilize the relative equilibrium corresponding to a constant axial spacecraft acceleration of

TABLE I

PHYSICAL PARAMETERS.

Parameter	Value	Parameter	Value
m	$590\,kg$	F	$2250\,N$
I	$400\,kg\cdot m^2$	I_0	$75\,kg\cdot m^2$
m_0	$480\,kg$	I_1	$10\,kg\cdot m^2$
m_1	$50\,kg$	I_2	$1\,kg\cdot m^2$
m_2	$5\,kg$	l_1	$0.2\,m$
h_0	$0.05\,m$	l_2	$0.1\,m$
h_1	$0.60\,m$	ϵ_1	$3.7\,kg\cdot m^2/s$
h_2	$0.90\,m$	ϵ_2	$0.5\,kg\cdot m^2/s$
b	$1.5\,m$	d	$1.5\,m$

$2\,m/s^2$ and $v_z = \theta = \dot{\theta} = \psi_i = \dot{\psi}_i = 0,\ i = 1, 2$. In the simulation, a fuel burn time of $600\,s$ is assumed.

In this section, we demonstrate the effectiveness of the Lyapunov-based controller (15)-(16) by applying to the complete nonlinear system (4)-(7).

Time responses shown in Figures 2-4 correspond to the initial conditions $v_{x_0} = 3000\,m/s$, $v_{z_0} = 100\,m/s$, $\theta_0 = 5^o$, $\dot{\theta}_0 = 0$, $\psi_{1_0} = 30^o$, $\psi_{2_0} = -30^o$, and $\dot{\psi}_{i_0} = 0$. As can be seen in the figures, the transverse velocity, attitude angle, and the slosh states converge to the relative equilibrium at zero while the axial velocity v_x increases and $\dot{v}_x$ tends asymptotically to $2\,m/s^2$. Note that there is a trade-off between good responses for the directly actuated degrees of freedom (the transverse and pitch dynamics) and good responses for the unactuated degree of freedoms (the slosh modes); the controller given by (15)-(16) with parameters $r_1 = 1.25\times10^{-6}$, $r_2 = 400$, $r_3 = 500$, $r_4 = 10^{-3}$, $K_1 = 6000$, $K_2 = 10^4$ represents one example of this balance.

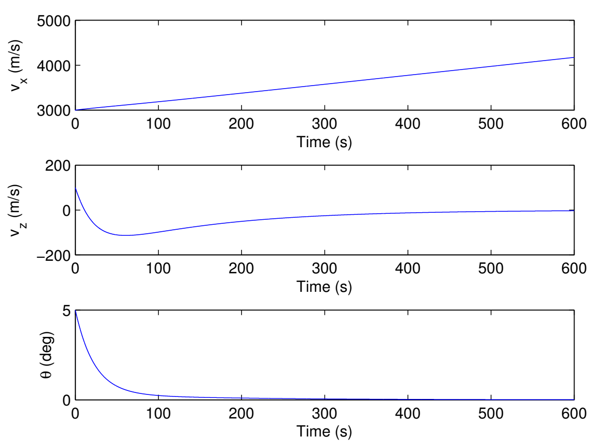

Fig. 2. Time responses of v_x, v_z and θ.

V. CONCLUSIONS

We have developed a complete nonlinear dynamical model for a spacecraft with multiple slosh modes. We have designed a Lyapunov-based nonlinear feedback control law that achieves stabilization of the pitch and transverse dynamics as well as suppression of the slosh modes, while the spacecraft accelerates in the axial direction. The effectiveness

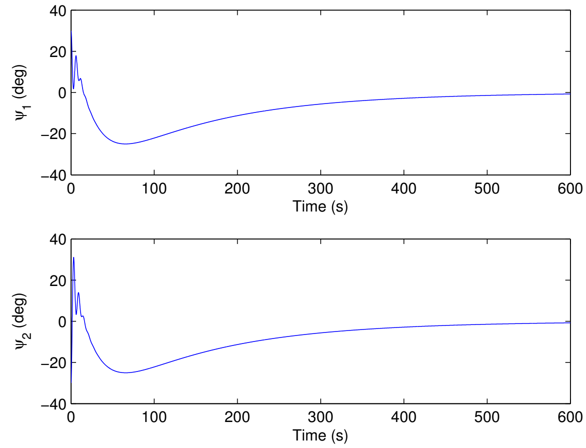

Fig. 3. Time responses of ψ_1 and ψ_2.

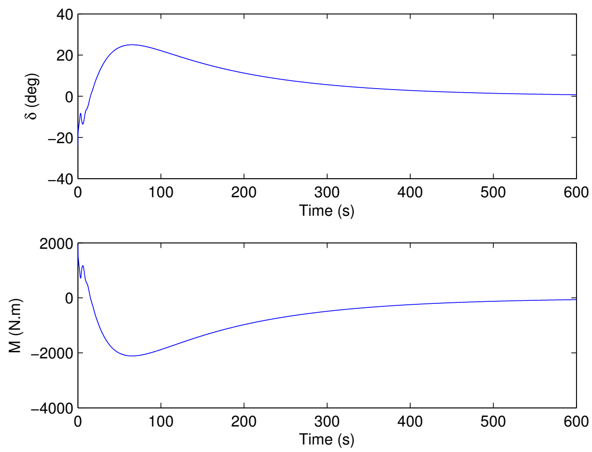

Fig. 4. Gimbal deflection angle δ and pitching moment M.

of this control feedback law has been illustrated through a simulation example. The many avenues considered for future research include problems involving multiple propellant tanks and three dimensional maneuvers. Future research also includes designing nonlinear control laws that achieve robustness, insensitivity to system and control parameters, and improved disturbance rejection.

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